

Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.1-Quadratic/1.2.1.6-
 $g+h-x^m-a+b-x+c-x^2-p-d+e-x+f-x^2-q$

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Chapter 1

Introduction

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This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [143]. This is test number [24].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. IntegrateAlgebraic under Mathematica 12.3.1 on windows 10. https://github.com/stblake/algebraic_integration. September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (143)	0.00 (0)
Mathematica	99.30 (142)	0.70 (1)
Maple	98.60 (141)	1.40 (2)
IntegrateAlgebraic	91.61 (131)	8.39 (12)
Fricas	48.25 (69)	51.75 (74)
Giac	34.97 (50)	65.03 (93)
Mupad	13.29 (19)	86.71 (124)
Maxima	10.49 (15)	89.51 (128)
Sympy	7.69 (11)	% 92.31 (132)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

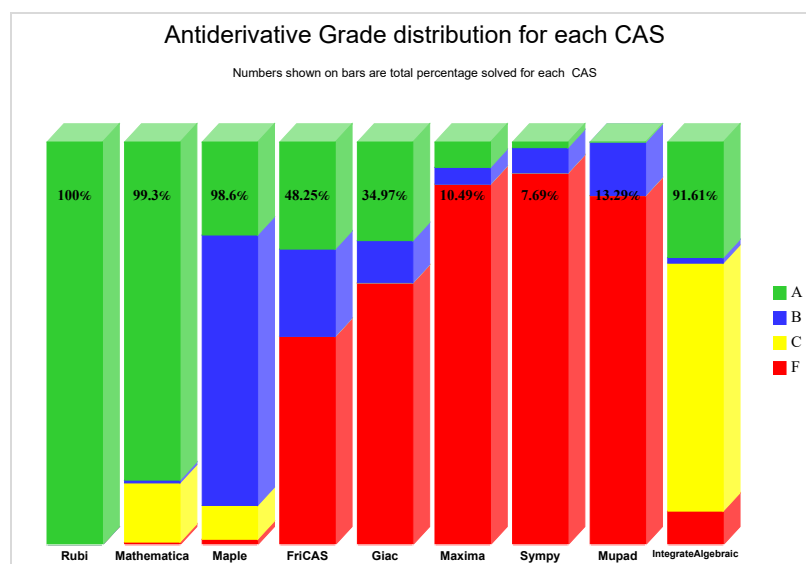
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

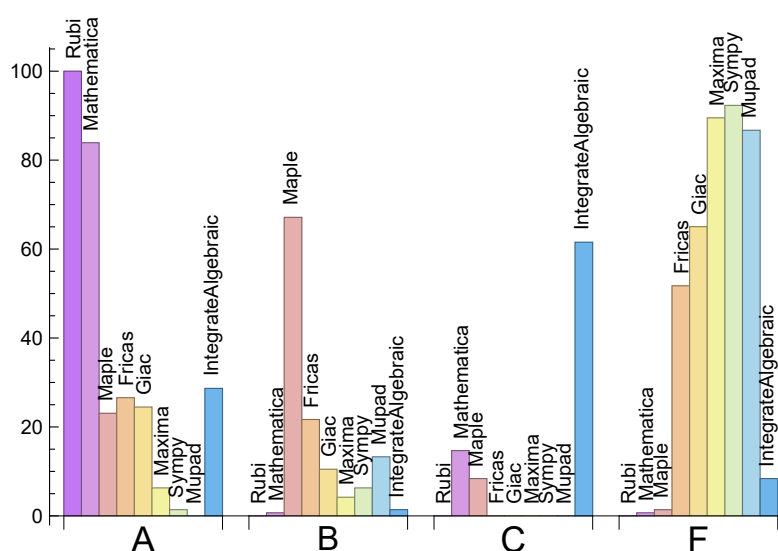
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	83.92	0.70	14.69	0.70
IntegrateAlgebraic	28.67	1.40	61.54	8.39
Fricas	26.57	21.68	0.00	51.75
Giac	24.48	10.49	0.00	65.03
Maple	23.08	67.13	8.39	1.40
Maxima	6.29	4.20	0.00	89.51
Sympy	1.40	6.29	0.00	92.31
Mupad	N/A	13.29	0.00	86.71

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure **F**.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	1	100.00 %	0.00 %	0.00 %
Maple	2	100.00 %	0.00 %	0.00 %
Fricas	74	1.35 %	98.65 %	0.00 %
IntegrateAlgebraic	12	91.67 %	8.33 %	0.00 %
Giac	93	22.58 %	18.28 %	59.14 %
Maxima	128	52.34 %	0.00 %	47.66 %
Sympy	132	87.88 %	12.12 %	0.00 %
Mupad	124	99.19 %	0.81 %	0.00 %

Table 1.4: Failure statistics for each CAS

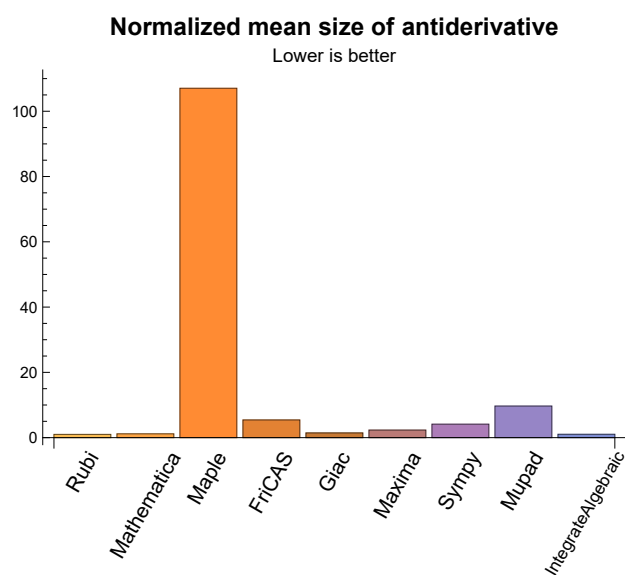
1.3 Performance

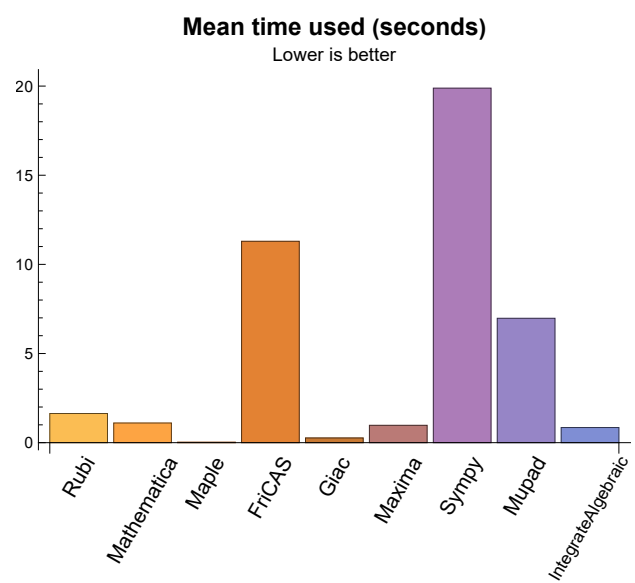
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	1.64	333.90	1.00	302.00	1.00
Mathematica	1.10	347.54	1.17	290.50	0.97
Maple	0.03	52015.57	107.04	1172.00	3.85
Maxima	0.97	388.27	2.32	220.00	1.11
Fricas	11.29	1373.01	5.42	344.00	2.21
Sympy	19.88	971.36	4.13	680.00	4.09
Giac	0.27	259.92	1.44	146.00	1.21
Mupad	6.98	7822.16	9.68	187.00	1.25
IntegrateAlgebraic	0.85	346.69	1.03	273.00	0.88

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

IntegrateAlgebraic {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {25,28,142}

IntegrateAlgebraic {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user slelievre at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

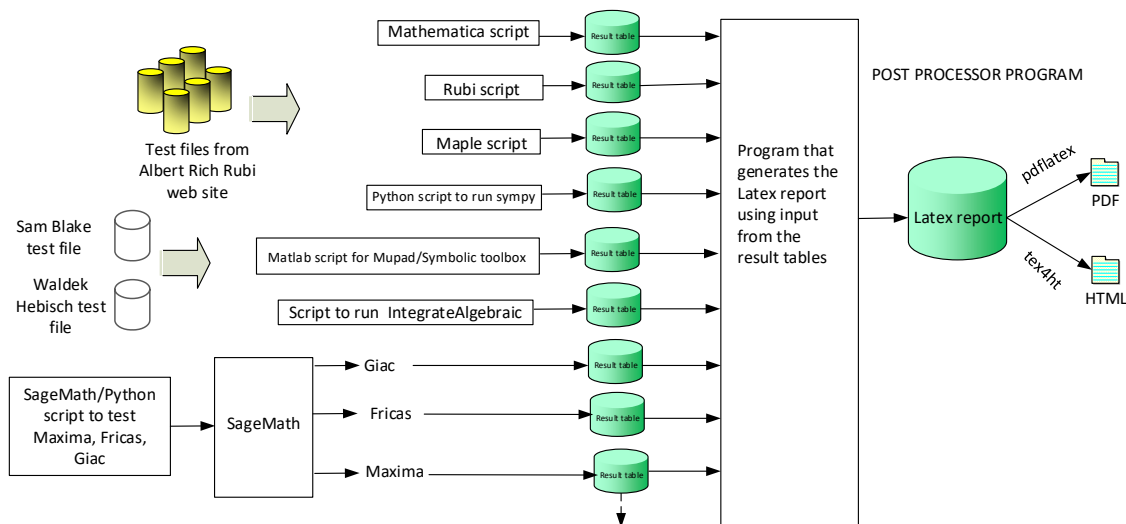
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand, the_variable)
```

Which gives $\sin(x) \sim 2/2$

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer. the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
The following field present only in Rubi and Mathematica Tables
13. integer. 1 if result was verified or 0 if not verified.
The following fields present only in Rubi Tables
14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,..}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

Chapter 2

detailed summary tables of results

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2.1 List of integrals sorted by grade for each CAS

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2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 36, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 133, 134, 135, 136, 137, 138, 139, 140, 141 }

B grade: { 35 }

C grade: { 11, 12, 31, 32, 33, 34, 37, 38, 62, 63, 75, 76, 93, 126, 127, 128, 129, 130, 131, 132, 142 }

F grade: { 143 }

2.1.3 Maple

A grade: { 1, 2, 3, 10, 22, 28, 31, 32, 33, 34, 36, 38, 39, 92, 94, 95, 96, 99, 100, 101, 126, 127, 128, 129, 130, 131, 132, 133, 134, 138, 139, 140, 141 }

B grade: { 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 29, 30, 35, 37, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 97, 98, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 135, 136, 137 }

C grade: { 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51 }

F grade: { 142, 143 }

2.1.4 Maxima

A grade: { 1, 2, 3, 10, 92, 133, 134, 138, 139 }

B grade: { 25, 26, 27, 28, 29, 30 }

C grade: { }

F grade: { 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 135, 136, 137, 140, 141, 142, 143 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 10, 13, 14, 33, 36, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 91, 92, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141 }

B grade: { 7, 11, 17, 18, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 37, 55, 66, 67, 80, 81, 82, 83, 93, 97, 98, 99, 116, 117, 135 }

C grade: { }

F grade: { 4, 5, 6, 8, 9, 12, 15, 16, 19, 20, 21, 35, 39, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 84, 85, 86, 87, 88, 89, 90, 94, 95, 96, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 118, 119, 120, 121, 122, 123, 124, 125, 142, 143 }

2.1.6 Sympy

A grade: { 33, 139 }

B grade: { 1, 2, 3, 13, 14, 17, 18, 31, 138 }

C grade: { }

F grade: { 4, 5, 6, 7, 8, 9, 10, 11, 12, 15, 16, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 140, 141, 142, 143 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 10, 13, 14, 15, 17, 18, 28, 29, 30, 36, 40, 41, 42, 43, 44, 45, 46, 47, 48, 92, 126, 127, 129, 131, 133, 134, 137, 138, 139, 140, 141 }

B grade: { 5, 11, 16, 31, 32, 33, 34, 37, 38, 51, 128, 130, 132, 135, 136 }

C grade: { }

F grade: { 6, 7, 8, 9, 12, 19, 20, 21, 22, 23, 24, 25, 26, 27, 35, 39, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 142, 143 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 13, 14, 16, 17, 18, 31, 33, 36, 133, 134, 138, 139, 140, 141 }

C grade: { }

F grade: { 6, 7, 8, 9, 10, 11, 12, 15, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 34, 35, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 135, 136, 137, 142, 143 }

2.1.9 Integrate Algebraic

A grade: { 10, 28, 29, 30, 31, 32, 33, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 91, 92, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141 }

B grade: { 24, 142 }

C grade: { 6, 7, 8, 11, 12, 19, 20, 21, 22, 23, 25, 26, 27, 34, 35, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 143 }

F grade: { 1, 2, 3, 4, 5, 9, 13, 14, 15, 16, 17, 18 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column N.S. in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, Mathematica was abbreviated to MMA and IntegrateAlgebraic to I.A.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	86	133	84	200	333	87	97	0
N.S.	1	1.00	0.91	1.41	0.89	2.13	3.54	0.93	1.03	0.00
time (sec)	N/A	0.113	0.072	0.031	0.968	0.404	1.854	0.151	3.436	0.001
Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	228	228	204	373	220	500	933	263	253	0
N.S.	1	1.00	0.89	1.64	0.96	2.19	4.09	1.15	1.11	0.00
time (sec)	N/A	0.330	0.204	0.012	0.979	0.418	7.873	0.159	0.249	0.001
Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	441	441	422	822	471	1014	1962	623	552	0
N.S.	1	1.00	0.96	1.86	1.07	2.30	4.45	1.41	1.25	0.00
time (sec)	N/A	0.621	0.459	0.008	0.989	0.438	23.623	0.164	3.792	0.001
Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	274	274	212	745	0	0	0	266	3888	0
N.S.	1	1.00	0.77	2.72	0.00	0.00	0.00	0.97	14.19	0.00
time (sec)	N/A	0.282	0.368	0.016	0.000	0.000	0.000	0.168	38.323	0.001
Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	596	596	523	9311	0	0	0	1313	23006	0
N.S.	1	1.00	0.88	15.62	0.00	0.00	0.00	2.20	38.60	0.00
time (sec)	N/A	1.772	1.842	0.055	0.000	0.000	0.000	0.205	7.534	0.001

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	331	331	322	3358	0	0	0	0	-1	523
N.S.	1	1.00	0.97	10.15	0.00	0.00	0.00	0.00	-0.00	1.58
time (sec)	N/A	0.601	0.801	0.085	0.000	0.000	0.000	0.000	0.000	0.782

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	249	249	249	714	0	6113	0	0	-1	219
N.S.	1	1.00	1.00	2.87	0.00	24.55	0.00	0.00	-0.00	0.88
time (sec)	N/A	0.204	0.240	0.024	0.000	52.196	0.000	0.000	0.000	0.427

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	381	380	440	2758	0	0	0	0	-1	658
N.S.	1	1.00	1.15	7.24	0.00	0.00	0.00	0.00	-0.00	1.73
time (sec)	N/A	0.798	0.650	0.026	0.000	0.000	0.000	0.000	0.000	1.501

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	797	796	674	6422	0	0	0	0	-1	0
N.S.	1	1.00	0.85	8.06	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	1.870	4.060	0.031	0.000	0.000	0.000	0.000	0.000	180.082

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	49	46	65	46	0	48	-1	37
N.S.	1	1.00	1.04	0.98	1.38	0.98	0.00	1.02	-0.02	0.79
time (sec)	N/A	0.034	0.008	0.018	0.973	0.629	0.000	0.246	0.000	0.225

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	78	637	0	758	0	457	-1	106
N.S.	1	1.00	0.67	5.44	0.00	6.48	0.00	3.91	-0.01	0.91
time (sec)	N/A	0.169	0.033	0.176	0.000	0.560	0.000	0.376	0.000	0.239

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	F(-1)	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	484	484	136	6871419	0	0	0	0	-1	210
N.S.	1	1.00	0.28	14197.15	0.00	0.00	0.00	0.00	-0.00	0.43
time (sec)	N/A	23.581	0.083	0.515	0.000	0.000	0.000	0.000	0.000	0.421

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	184	182	175	510	0	583	1260	191	273	0
N.S.	1	0.99	0.95	2.77	0.00	3.17	6.85	1.04	1.48	0.00
time (sec)	N/A	0.346	0.211	0.008	0.000	0.444	16.651	0.169	3.852	0.001

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	542	542	535	1672	0	1837	4663	738	893	0
N.S.	1	1.00	0.99	3.08	0.00	3.39	8.60	1.36	1.65	0.00
time (sec)	N/A	1.102	0.604	0.012	0.000	0.610	145.638	0.246	4.854	0.001

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	406	398	267	1698	0	0	0	416	-1	0
N.S.	1	0.98	0.66	4.18	0.00	0.00	0.00	1.02	-0.00	0.00
time (sec)	N/A	0.478	0.462	0.013	0.000	0.000	0.000	0.169	0.000	0.001

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	1075	1067	952	51470	0	0	0	3226	118429	0
N.S.	1	0.99	0.89	47.88	0.00	0.00	0.00	3.00	110.17	0.00
time (sec)	N/A	4.177	6.700	0.054	0.000	0.000	0.000	0.451	30.314	0.001

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	140	131	340	0	1150	709	219	395	0
N.S.	1	1.00	0.94	2.43	0.00	8.21	5.06	1.56	2.82	0.00
time (sec)	N/A	0.131	0.143	0.006	0.000	0.644	2.437	0.172	0.422	0.001

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	140	131	340	0	1130	680	207	375	0
N.S.	1	1.00	0.94	2.43	0.00	8.07	4.86	1.48	2.68	0.00
time (sec)	N/A	0.104	0.027	0.006	0.000	0.611	2.259	0.170	4.020	0.001
Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	617	615	517	16209	0	0	0	0	-1	893
N.S.	1	1.00	0.84	26.27	0.00	0.00	0.00	0.00	-0.00	1.45
time (sec)	N/A	8.998	2.086	0.051	0.000	0.000	0.000	0.000	0.000	1.207
Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	1092	1092	1627	59465	0	0	0	0	-1	2787
N.S.	1	1.00	1.49	54.46	0.00	0.00	0.00	0.00	-0.00	2.55
time (sec)	N/A	18.867	6.564	0.032	0.000	0.000	0.000	0.000	0.000	7.505
Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	416	416	393	2269	0	0	0	0	-1	278
N.S.	1	1.00	0.94	5.45	0.00	0.00	0.00	0.00	-0.00	0.67
time (sec)	N/A	2.702	4.175	0.044	0.000	0.000	0.000	0.000	0.000	0.533
Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	780	780	254	784	0	6861	0	0	-1	218
N.S.	1	1.00	0.33	1.01	0.00	8.80	0.00	0.00	-0.00	0.28
time (sec)	N/A	5.162	0.409	0.039	0.000	47.867	0.000	0.000	0.000	0.439
Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	302	302	283	1771	0	8977	0	0	-1	195
N.S.	1	1.00	0.94	5.86	0.00	29.73	0.00	0.00	-0.00	0.65
time (sec)	N/A	0.843	0.430	0.033	0.000	60.517	0.000	0.000	0.000	0.438

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	154	608	0	1515	0	0	-1	563
N.S.	1	1.00	1.52	6.02	0.00	15.00	0.00	0.00	-0.01	5.57
time (sec)	N/A	0.127	0.174	0.021	0.000	0.542	0.000	0.000	0.000	2.273
Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F	F(-1)	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	139	139	140	324	361	322	0	0	-1	149
N.S.	1	1.00	1.01	2.33	2.60	2.32	0.00	0.00	-0.01	1.07
time (sec)	N/A	0.220	0.301	0.068	1.074	0.445	0.000	0.000	0.000	0.343
Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F	F(-1)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	166	166	167	760	678	344	0	0	-1	149
N.S.	1	1.00	1.01	4.58	4.08	2.07	0.00	0.00	-0.01	0.90
time (sec)	N/A	0.221	0.378	0.027	1.112	0.442	0.000	0.000	0.000	0.394
Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F	F(-1)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	193	193	185	1560	1276	439	0	0	-1	195
N.S.	1	1.00	0.96	8.08	6.61	2.27	0.00	0.00	-0.01	1.01
time (sec)	N/A	0.266	0.596	0.020	1.263	0.442	0.000	0.000	0.000	0.535
Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	151	151	148	186	363	245	0	93	-1	109
N.S.	1	1.00	0.98	1.23	2.40	1.62	0.00	0.62	-0.01	0.72
time (sec)	N/A	0.229	0.356	0.051	1.069	0.432	0.000	0.477	0.000	0.782
Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	174	174	172	466	668	365	0	112	-1	148
N.S.	1	1.00	0.99	2.68	3.84	2.10	0.00	0.64	-0.01	0.85
time (sec)	N/A	0.255	0.570	0.019	1.099	0.446	0.000	0.499	0.000	1.019

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	46	48	0	85	0	81	49	47
N.S.	1	1.00	0.96	1.00	0.00	1.77	0.00	1.69	1.02	0.98
time (sec)	N/A	0.050	0.185	0.005	0.000	0.630	0.000	0.266	3.753	0.418
Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	165	94	0	56	0	98	-1	17
N.S.	1	1.00	9.71	5.53	0.00	3.29	0.00	5.76	-0.06	1.00
time (sec)	N/A	0.022	0.288	0.017	0.000	0.423	0.000	0.231	0.000	0.297
Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	150	123	0	132	0	163	-1	56
N.S.	1	1.00	1.74	1.43	0.00	1.53	0.00	1.90	-0.01	0.65
time (sec)	N/A	0.185	0.109	0.013	0.000	0.429	0.000	0.275	0.000	0.350
Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	108	121	0	0	0	0	-1	127
N.S.	1	1.00	0.79	0.89	0.00	0.00	0.00	0.00	-0.01	0.93
time (sec)	N/A	0.114	0.091	0.027	0.000	50.453	0.000	0.000	0.000	0.278
Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	212	212	129	103	0	175	0	117	-1	111
N.S.	1	1.00	0.61	0.49	0.00	0.83	0.00	0.55	-0.00	0.52
time (sec)	N/A	0.116	0.151	0.048	0.000	0.429	0.000	0.253	0.000	0.294
Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	117	83	0	157	0	98	-1	96
N.S.	1	1.00	0.73	0.52	0.00	0.98	0.00	0.61	-0.01	0.60
time (sec)	N/A	0.067	0.116	0.010	0.000	0.429	0.000	0.213	0.000	0.255

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	148	148	85	65	0	128	0	79	-1	87
N.S.	1	1.00	0.57	0.44	0.00	0.86	0.00	0.53	-0.01	0.59
time (sec)	N/A	0.057	0.063	0.008	0.000	0.430	0.000	0.271	0.000	0.286
Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	160	160	139	94	0	341	0	102	-1	114
N.S.	1	1.00	0.87	0.59	0.00	2.13	0.00	0.64	-0.01	0.71
time (sec)	N/A	0.119	0.143	0.009	0.000	0.445	0.000	0.247	0.000	0.305
Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	156	156	118	118	0	333	0	126	-1	111
N.S.	1	1.00	0.76	0.76	0.00	2.13	0.00	0.81	-0.01	0.71
time (sec)	N/A	0.113	0.281	0.013	0.000	0.444	0.000	0.260	0.000	0.309
Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	126	141	0	377	0	199	-1	115
N.S.	1	1.00	0.78	0.88	0.00	2.34	0.00	1.24	-0.01	0.71
time (sec)	N/A	0.116	0.116	0.015	0.000	0.440	0.000	0.275	0.000	0.409
Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	317	317	198	530	0	517	0	368	-1	263
N.S.	1	1.00	0.62	1.67	0.00	1.63	0.00	1.16	-0.00	0.83
time (sec)	N/A	0.333	0.275	0.018	0.000	0.469	0.000	0.311	0.000	1.001
Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	227	227	147	381	0	391	0	268	-1	196
N.S.	1	1.00	0.65	1.68	0.00	1.72	0.00	1.18	-0.00	0.86
time (sec)	N/A	0.127	0.131	0.013	0.000	0.452	0.000	0.283	0.000	0.700

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	316	316	302	1810	0	0	0	0	-1	327
N.S.	1	1.00	0.96	5.73	0.00	0.00	0.00	0.00	-0.00	1.03
time (sec)	N/A	0.489	0.485	0.017	0.000	0.000	0.000	0.000	0.000	0.832
Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	282	282	272	1667	0	0	0	0	-1	361
N.S.	1	1.00	0.96	5.91	0.00	0.00	0.00	0.00	-0.00	1.28
time (sec)	N/A	0.295	0.294	0.014	0.000	0.000	0.000	0.000	0.000	0.585
Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	266	266	253	1669	0	1139	0	0	-1	273
N.S.	1	1.00	0.95	6.27	0.00	4.28	0.00	0.00	-0.00	1.03
time (sec)	N/A	0.226	0.166	0.014	0.000	98.756	0.000	0.000	0.000	0.420
Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	267	267	255	1764	0	1253	0	0	-1	342
N.S.	1	1.00	0.96	6.61	0.00	4.69	0.00	0.00	-0.00	1.28
time (sec)	N/A	0.778	0.295	0.018	0.000	17.018	0.000	0.000	0.000	0.390
Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	286	286	275	1819	0	1094	0	0	-1	299
N.S.	1	1.00	0.96	6.36	0.00	3.83	0.00	0.00	-0.00	1.05
time (sec)	N/A	0.706	0.431	0.020	0.000	24.595	0.000	0.000	0.000	0.531
Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	353	353	316	1953	0	1485	0	0	-1	392
N.S.	1	1.00	0.90	5.53	0.00	4.21	0.00	0.00	-0.00	1.11
time (sec)	N/A	0.879	0.522	0.017	0.000	133.831	0.000	0.000	0.000	0.858

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	140	210	159	0	178	0	188	-1	108
N.S.	1	1.00	1.50	1.14	0.00	1.27	0.00	1.34	-0.01	0.77
time (sec)	N/A	0.501	0.527	0.019	0.000	1.278	0.000	0.203	0.000	0.416
Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	192	144	0	175	0	185	-1	99
N.S.	1	1.00	1.67	1.25	0.00	1.52	0.00	1.61	-0.01	0.86
time (sec)	N/A	0.420	0.438	0.015	0.000	1.397	0.000	0.247	0.000	0.368
Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	159	130	0	161	0	171	-1	82
N.S.	1	1.00	1.62	1.33	0.00	1.64	0.00	1.74	-0.01	0.84
time (sec)	N/A	0.198	0.187	0.009	0.000	1.036	0.000	0.226	0.000	0.335
Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	69	174	92	0	50	0	68	-1	38
N.S.	1	1.01	2.56	1.35	0.00	0.74	0.00	1.00	-0.01	0.56
time (sec)	N/A	0.062	0.168	0.011	0.000	0.875	0.000	0.185	0.000	0.249
Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	150	121	0	132	0	165	-1	62
N.S.	1	1.00	1.58	1.27	0.00	1.39	0.00	1.74	-0.01	0.65
time (sec)	N/A	0.111	0.100	0.011	0.000	0.785	0.000	0.188	0.000	0.309
Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	200	152	0	170	0	199	-1	97
N.S.	1	1.00	1.54	1.17	0.00	1.31	0.00	1.53	-0.01	0.75
time (sec)	N/A	0.417	0.437	0.019	0.000	1.055	0.000	0.201	0.000	0.333

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	151	151	225	169	0	194	0	269	-1	118
N.S.	1	1.00	1.49	1.12	0.00	1.28	0.00	1.78	-0.01	0.78
time (sec)	N/A	0.454	0.405	0.020	0.000	1.090	0.000	0.483	0.000	0.425
Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	87	147	155	88	0	85	187	92
N.S.	1	1.00	0.58	0.99	1.04	0.59	0.00	0.57	1.26	0.62
time (sec)	N/A	0.093	0.191	0.019	1.001	0.955	0.000	0.297	5.085	0.732
Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	72	96	104	73	0	70	136	77
N.S.	1	1.00	0.70	0.93	1.01	0.71	0.00	0.68	1.32	0.75
time (sec)	N/A	0.042	0.033	0.006	0.981	0.874	0.000	0.204	4.690	0.443
Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	37	163	0	53	0	63	-1	30
N.S.	1	1.00	1.32	5.82	0.00	1.89	0.00	2.25	-0.04	1.07
time (sec)	N/A	0.048	0.098	0.016	0.000	0.631	0.000	0.272	0.000	0.300
Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	114	245	0	126	0	159	-1	80
N.S.	1	1.00	1.36	2.92	0.00	1.50	0.00	1.89	-0.01	0.95
time (sec)	N/A	0.078	0.238	0.023	0.000	0.981	0.000	0.272	0.000	0.370
Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	139	139	131	306	0	186	0	232	-1	95
N.S.	1	1.00	0.94	2.20	0.00	1.34	0.00	1.67	-0.01	0.68
time (sec)	N/A	0.117	0.367	0.021	0.000	1.736	0.000	0.260	0.000	0.394

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. The column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [63] had the largest ratio of [.5556]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	1.00	25	0.160
2	A	5	4	1.00	27	0.148
3	A	5	4	1.00	27	0.148
4	A	8	8	1.00	27	0.296
5	A	9	9	1.00	27	0.333
6	A	9	6	1.00	30	0.200
7	A	5	3	1.00	30	0.100
8	A	6	4	1.00	30	0.133
9	A	7	5	1.00	30	0.167
10	A	5	4	1.00	23	0.174
11	A	5	4	1.00	23	0.174
12	A	5	4	1.00	30	0.133
13	A	6	5	0.99	28	0.179
14	A	6	5	1.00	30	0.167
15	A	9	5	0.98	30	0.167
16	A	10	6	0.99	30	0.200
17	A	5	5	1.00	34	0.147
18	A	5	5	1.00	34	0.147
19	A	9	6	1.00	32	0.188
20	A	10	7	1.00	32	0.219
21	A	5	3	1.00	32	0.094
22	A	5	3	1.00	29	0.103
23	A	5	3	1.00	29	0.103
24	A	6	6	1.00	26	0.231
25	A	5	4	1.00	30	0.133
26	A	7	6	1.00	30	0.200
27	A	7	6	1.00	30	0.200
28	A	5	3	1.00	30	0.100
29	A	6	4	1.00	30	0.133
30	A	7	5	1.00	30	0.167
31	A	2	2	1.00	26	0.077
32	A	5	5	1.00	26	0.192
33	A	2	2	1.00	24	0.083
34	A	5	5	1.00	20	0.250
35	A	6	5	1.00	36	0.139
36	A	2	2	1.00	36	0.056
37	A	2	2	1.00	32	0.062
38	A	13	9	1.00	32	0.281
39	A	5	5	1.00	38	0.132

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
40	A	6	6	1.00	35	0.171
41	A	5	5	1.00	33	0.152
42	A	5	5	1.00	32	0.156
43	A	8	8	1.00	35	0.229
44	A	8	8	1.00	35	0.229
45	A	8	8	1.00	35	0.229
46	A	6	6	1.00	38	0.158
47	A	5	5	1.00	36	0.139
48	A	5	5	1.00	35	0.143
49	A	7	6	1.00	38	0.158
50	A	7	6	1.00	38	0.158
51	A	7	6	1.00	38	0.158
52	A	9	6	1.00	27	0.222
53	A	9	6	1.00	25	0.240
54	A	8	5	1.00	24	0.208
55	A	12	9	1.00	27	0.333
56	A	18	12	1.00	27	0.444
57	A	22	13	1.00	27	0.482
58	A	10	7	1.00	27	0.259
59	A	10	7	1.00	25	0.280
60	A	9	6	1.00	24	0.250
61	A	17	11	1.00	27	0.407
62	A	21	14	1.00	27	0.518
63	A	26	15	1.00	27	0.556
64	A	10	6	1.00	27	0.222
65	A	8	5	1.00	27	0.185
66	A	5	3	1.00	25	0.120
67	A	5	3	1.00	24	0.125
68	A	10	7	1.00	27	0.259
69	A	11	8	1.00	27	0.296
70	A	15	9	1.00	27	0.333
71	A	10	7	1.00	27	0.259
72	A	6	4	1.00	27	0.148
73	A	6	4	1.00	25	0.160
74	A	6	4	1.00	24	0.167
75	A	12	9	1.00	27	0.333
76	A	14	11	1.00	27	0.407
77	A	15	9	1.00	28	0.321
78	A	9	6	1.00	28	0.214
79	A	9	6	1.00	26	0.231
80	A	8	5	1.00	25	0.200
81	A	17	9	1.00	28	0.321
82	A	16	8	1.00	28	0.286
83	A	20	10	1.00	28	0.357
84	A	17	10	1.00	28	0.357
85	A	10	7	1.00	28	0.250
86	A	10	7	1.00	26	0.269
87	A	9	6	1.00	25	0.240

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	19	11	1.00	28	0.393
89	A	18	10	1.00	28	0.357
90	A	26	13	1.00	28	0.464
91	A	9	6	1.00	24	0.250
92	A	8	6	1.00	22	0.273
93	A	10	9	1.00	17	0.529
94	A	13	7	1.00	28	0.250
95	A	10	6	1.00	28	0.214
96	A	8	5	1.00	28	0.179
97	A	5	3	1.00	26	0.115
98	A	5	3	1.00	25	0.120
99	A	9	4	1.00	28	0.143
100	A	10	5	1.00	28	0.179
101	A	13	6	1.00	28	0.214
102	A	13	9	1.00	28	0.321
103	A	9	6	1.00	28	0.214
104	A	6	4	1.00	28	0.143
105	A	6	4	1.00	26	0.154
106	A	6	4	1.00	25	0.160
107	A	12	7	1.00	28	0.250
108	A	12	7	1.00	28	0.250
109	A	9	6	1.00	30	0.200
110	A	9	6	1.00	28	0.214
111	A	8	5	1.00	27	0.185
112	A	17	9	1.00	30	0.300
113	A	23	10	1.00	30	0.333
114	A	12	6	1.00	30	0.200
115	A	8	5	1.00	30	0.167
116	A	5	3	1.00	28	0.107
117	A	5	3	1.00	27	0.111
118	A	9	4	1.00	30	0.133
119	A	12	5	1.00	30	0.167
120	A	16	7	1.00	30	0.233
121	A	10	7	1.00	30	0.233
122	A	6	4	1.00	30	0.133
123	A	6	4	1.00	28	0.143
124	A	6	4	1.00	27	0.148
125	A	12	7	1.00	30	0.233
126	A	24	14	1.00	30	0.467
127	A	20	13	1.00	30	0.433
128	A	16	12	1.00	30	0.400
129	A	6	4	1.01	28	0.143
130	A	10	8	1.00	27	0.296
131	A	17	11	1.00	30	0.367
132	A	20	12	1.00	30	0.400
133	A	8	6	1.00	34	0.176
134	A	6	5	1.00	30	0.167
135	A	3	3	1.00	34	0.088

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
136	A	5	5	1.00	34	0.147
137	A	7	7	1.00	34	0.206
138	A	1	1	1.00	17	0.059
139	A	1	1	1.00	15	0.067
140	A	2	2	1.00	23	0.087
141	A	3	3	1.00	21	0.143
142	A	2	2	1.00	40	0.050
143	A	2	2	1.00	104	0.019

Chapter 3

Listing of integrals

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3.13	$\int \frac{(A+Bx)(a+bx+cx^2)}{d+ex+fx^2} dx$	112
3.14	$\int \frac{(A+Bx)(a+bx+cx^2)^2}{d+ex+fx^2} dx$	116
3.15	$\int \frac{A+Bx}{(a+bx+cx^2)(d+ex+fx^2)} dx$	123
3.16	$\int \frac{A+Bx}{(a+bx+cx^2)^2(d+ex+fx^2)} dx$	127
3.17	$\int \frac{g+hx}{(a+bx+cx^2)(ad+bdx+cdx^2)^2} dx$	174
3.18	$\int \frac{g+hx}{(a+bx+cx^2)^2(ad+bdx+cdx^2)} dx$	178
3.19	$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2} dx$	182
3.20	$\int \frac{(A+Bx)(a+bx+cx^2)^{3/2}}{d+ex+fx^2} dx$	186
3.21	$\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+ex+fx^2}} dx$	192
3.22	$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+ex+fx^2}} dx$	197

3.23	$\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+fx^2}} dx$	203
3.24	$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+fx^2}} dx$	211
3.25	$\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x-2x^2}} dx$	215
3.26	$\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{3/2}} dx$	219
3.27	$\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{5/2}} dx$	224
3.28	$\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x+2x^2}} dx$	230
3.29	$\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{3/2}} dx$	233
3.30	$\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{5/2}} dx$	237
3.31	$\int \frac{1+x}{(4+2x+x^2)\sqrt{5+2x+x^2}} dx$	242
3.32	$\int \frac{4+x}{(4+2x+x^2)\sqrt{5+2x+x^2}} dx$	244
3.33	$\int \frac{1+2x}{(3+x+x^2)\sqrt{5+x+x^2}} dx$	247
3.34	$\int \frac{x}{(3+x+x^2)\sqrt{5+x+x^2}} dx$	250
3.35	$\int \frac{A+Bx}{\sqrt{d+ex+fx^2}(ae+bex+bf x^2)^2} dx$	253
3.36	$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^2} dx$	259
3.37	$\int \frac{3+2x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx$	262
3.38	$\int \frac{3+4x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx$	265
3.39	$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^{3/2}} dx$	269
3.40	$\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2} dx$	272
3.41	$\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2} dx$	276
3.42	$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2} dx$	279
3.43	$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x} dx$	282
3.44	$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^2} dx$	286
3.45	$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^3} dx$	290
3.46	$\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2} dx$	294
3.47	$\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2} dx$	298
3.48	$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2} dx$	302
3.49	$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x} dx$	305
3.50	$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^2} dx$	309
3.51	$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^3} dx$	313
3.52	$\int \frac{x^2\sqrt{a+cx^2}}{d+ex+fx^2} dx$	317
3.53	$\int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2} dx$	321
3.54	$\int \frac{\sqrt{a+cx^2}}{d+ex+fx^2} dx$	325
3.55	$\int \frac{\sqrt{a+cx^2}}{x(d+ex+fx^2)} dx$	329
3.56	$\int \frac{\sqrt{a+cx^2}}{x^2(d+ex+fx^2)} dx$	335

3.57	$\int \frac{\sqrt{a+cx^2}}{x^3(d+ex+fx^2)} dx$	341
3.58	$\int \frac{x^2(a+cx^2)^{3/2}}{d+ex+fx^2} dx$	347
3.59	$\int \frac{x(a+cx^2)^{3/2}}{d+ex+fx^2} dx$	352
3.60	$\int \frac{(a+cx^2)^{3/2}}{d+ex+fx^2} dx$	356
3.61	$\int \frac{(a+cx^2)^{3/2}}{x(d+ex+fx^2)} dx$	360
3.62	$\int \frac{(a+cx^2)^{3/2}}{x^2(d+ex+fx^2)} dx$	365
3.63	$\int \frac{(a+cx^2)^{3/2}}{x^3(d+ex+fx^2)} dx$	370
3.64	$\int \frac{x^3}{\sqrt{a+cx^2}(d+ex+fx^2)} dx$	376
3.65	$\int \frac{x^2}{\sqrt{a+cx^2}(d+ex+fx^2)} dx$	380
3.66	$\int \frac{x}{\sqrt{a+cx^2}(d+ex+fx^2)} dx$	384
3.67	$\int \frac{1}{\sqrt{a+cx^2}(d+ex+fx^2)} dx$	389
3.68	$\int \frac{1}{x\sqrt{a+cx^2}(d+ex+fx^2)} dx$	394
3.69	$\int \frac{1}{x^2\sqrt{a+cx^2}(d+ex+fx^2)} dx$	398
3.70	$\int \frac{1}{x^3\sqrt{a+cx^2}(d+ex+fx^2)} dx$	402
3.71	$\int \frac{x^3}{(a+cx^2)^{3/2}(d+ex+fx^2)} dx$	406
3.72	$\int \frac{x^2}{(a+cx^2)^{3/2}(d+ex+fx^2)} dx$	410
3.73	$\int \frac{x}{(a+cx^2)^{3/2}(d+ex+fx^2)} dx$	415
3.74	$\int \frac{1}{(a+cx^2)^{3/2}(d+ex+fx^2)} dx$	420
3.75	$\int \frac{1}{x(a+cx^2)^{3/2}(d+ex+fx^2)} dx$	424
3.76	$\int \frac{1}{x^2(a+cx^2)^{3/2}(d+ex+fx^2)} dx$	429
3.77	$\int \frac{x^3\sqrt{a+bx+cx^2}}{d-fx^2} dx$	434
3.78	$\int \frac{x^2\sqrt{a+bx+cx^2}}{d-fx^2} dx$	439
3.79	$\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2} dx$	443
3.80	$\int \frac{\sqrt{a+bx+cx^2}}{d-fx^2} dx$	447
3.81	$\int \frac{\sqrt{a+bx+cx^2}}{x(d-fx^2)} dx$	451
3.82	$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d-fx^2)} dx$	456
3.83	$\int \frac{\sqrt{a+bx+cx^2}}{x^3(d-fx^2)} dx$	461
3.84	$\int \frac{x^3(a+bx+cx^2)^{3/2}}{d-fx^2} dx$	467
3.85	$\int \frac{x^2(a+bx+cx^2)^{3/2}}{d-fx^2} dx$	474
3.86	$\int \frac{x(a+bx+cx^2)^{3/2}}{d-fx^2} dx$	480

3.87	$\int \frac{(a+bx+cx^2)^{3/2}}{d-fx^2} dx$	486
3.88	$\int \frac{(a+bx+cx^2)^{3/2}}{x(d-fx^2)} dx$	491
3.89	$\int \frac{(a+bx+cx^2)^{3/2}}{x^2(d-fx^2)} dx$	498
3.90	$\int \frac{(a+bx+cx^2)^{3/2}}{x^3(d-fx^2)} dx$	504
3.91	$\int \frac{(a+bx+cx^2)^{3/2}}{1-x^2} dx$	510
3.92	$\int \frac{\sqrt{-1-x+x^2}}{1-x^2} dx$	515
3.93	$\int \frac{(x+x^2)^{3/2}}{1+x^2} dx$	518
3.94	$\int \frac{x^4}{\sqrt{a+bx+cx^2}(d-fx^2)} dx$	523
3.95	$\int \frac{x^3}{\sqrt{a+bx+cx^2}(d-fx^2)} dx$	527
3.96	$\int \frac{x^2}{\sqrt{a+bx+cx^2}(d-fx^2)} dx$	531
3.97	$\int \frac{x}{\sqrt{a+bx+cx^2}(d-fx^2)} dx$	535
3.98	$\int \frac{1}{\sqrt{a+bx+cx^2}(d-fx^2)} dx$	539
3.99	$\int \frac{1}{x\sqrt{a+bx+cx^2}(d-fx^2)} dx$	543
3.100	$\int \frac{1}{x^2\sqrt{a+bx+cx^2}(d-fx^2)} dx$	548
3.101	$\int \frac{1}{x^3\sqrt{a+bx+cx^2}(d-fx^2)} dx$	552
3.102	$\int \frac{x^4}{(a+bx+cx^2)^{3/2}(d-fx^2)} dx$	556
3.103	$\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d-fx^2)} dx$	561
3.104	$\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d-fx^2)} dx$	565
3.105	$\int \frac{x}{(a+bx+cx^2)^{3/2}(d-fx^2)} dx$	569
3.106	$\int \frac{1}{(a+bx+cx^2)^{3/2}(d-fx^2)} dx$	573
3.107	$\int \frac{1}{x(a+bx+cx^2)^{3/2}(d-fx^2)} dx$	577
3.108	$\int \frac{1}{x^2(a+bx+cx^2)^{3/2}(d-fx^2)} dx$	582
3.109	$\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex+fx^2} dx$	587
3.110	$\int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2} dx$	591
3.111	$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2} dx$	595
3.112	$\int \frac{\sqrt{a+bx+cx^2}}{x(d+ex+fx^2)} dx$	599
3.113	$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex+fx^2)} dx$	604
3.114	$\int \frac{x^3}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx$	609
3.115	$\int \frac{x^2}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx$	614
3.116	$\int \frac{x}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx$	618

3.117	$\int \frac{1}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx$	626
3.118	$\int \frac{1}{x\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx$	633
3.119	$\int \frac{1}{x^2\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx$	637
3.120	$\int \frac{1}{x^3\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx$	641
3.121	$\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)} dx$	646
3.122	$\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)} dx$	651
3.123	$\int \frac{x}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)} dx$	655
3.124	$\int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)} dx$	659
3.125	$\int \frac{1}{x(a+bx+cx^2)^{3/2}(d+ex+fx^2)} dx$	664
3.126	$\int \frac{x^4}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx$	671
3.127	$\int \frac{x^3}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx$	676
3.128	$\int \frac{x^2}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx$	681
3.129	$\int \frac{x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx$	686
3.130	$\int \frac{1}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx$	689
3.131	$\int \frac{1}{x\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx$	693
3.132	$\int \frac{1}{x^2\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx$	698
3.133	$\int (2+3x)^2(30+31x-12x^2)^2\sqrt{6+17x+12x^2} dx$	703
3.134	$\int (2+3x)(30+31x-12x^2)\sqrt{6+17x+12x^2} dx$	707
3.135	$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)(30+31x-12x^2)} dx$	710
3.136	$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^2(30+31x-12x^2)^2} dx$	713
3.137	$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^3(30+31x-12x^2)^3} dx$	717
3.138	$\int (-3+2x)(-3x+x^2)^{2/3} dx$	722
3.139	$\int ((-3+x)x)^{2/3}(-3+2x) dx$	724
3.140	$\int \frac{x(9-9x+2x^2)}{\sqrt[3]{-3x+x^2}} dx$	726
3.141	$\int \frac{x(9-9x+2x^2)}{\sqrt[3]{(-3+x)x}} dx$	728
3.142	$\int \frac{g+hx}{\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}(g^2+3h^2x^2)} dx$	731
3.143	$\int \frac{g+hx}{\sqrt[3]{-\frac{c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2} \left(f\left(b^2 - \frac{-c^2g^2+bcgh+2b^2h^2}{3h^2} \right) + \frac{bfx}{c} + fx^2 \right)} dx$	734

$$3.1 \quad \int \frac{(A+Bx)(a+bx+cx^2)}{d+fx^2} dx$$

Optimal. Leaf size=94

$$-\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(-aAf + Acd + bBd)}{\sqrt{d}f^{3/2}} - \frac{\log(d + fx^2)(-aBf - Abf + Bcd)}{2f^2} + \frac{x(Ac + bB)}{f} + \frac{Bcx^2}{2f}$$

Rubi [A] time = 0.11, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1629, 635, 205, 260}

$$-\frac{\log(d + fx^2)(-aBf - Abf + Bcd)}{2f^2} - \frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(-aAf + Acd + bBd)}{\sqrt{d}f^{3/2}} + \frac{x(Ac + bB)}{f} + \frac{Bcx^2}{2f}$$

Antiderivative was successfully verified.

[In] Int[((A + B*x)*(a + b*x + c*x^2))/(d + f*x^2), x]

[Out] ((b*B + A*c)*x)/f + (B*c*x^2)/(2*f) - ((b*B*d + A*c*d - a*A*f)*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]*f^(3/2)) - ((B*c*d - A*b*f - a*B*f)*Log[d + f*x^2])/ (2*f^2)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1629

Int[(Pq_)*((d_) + (e_.)*(x_)^(m_.))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx)(a+bx+cx^2)}{d+fx^2} dx &= \int \left(\frac{bB+Ac}{f} + \frac{Bcx}{f} - \frac{bBd+Ac d - aAf + (Bcd - Abf - aBf)x}{f(d+fx^2)} \right) dx \\
&= \frac{(bB+Ac)x}{f} + \frac{Bcx^2}{2f} - \frac{\int \frac{bBd+Ac d - aAf + (Bcd - Abf - aBf)x}{d+fx^2} dx}{f} \\
&= \frac{(bB+Ac)x}{f} + \frac{Bcx^2}{2f} - \frac{(bBd+Ac d - aAf) \int \frac{1}{d+fx^2} dx}{f} - \frac{(Bcd - Abf - aBf)}{f} \\
&= \frac{(bB+Ac)x}{f} + \frac{Bcx^2}{2f} - \frac{(bBd+Ac d - aAf) \tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{\sqrt{d} f^{3/2}} - \frac{(Bcd - Abf - aBf)}{2f^2}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 86, normalized size = 0.91

$$\frac{\log(d+fx^2)(aBf+Abf-Bcd) - \frac{2\sqrt{f} \tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(-aAf+Ac d+bBd)}{\sqrt{d}} + fx(2Ac+2bB+Bcx)}{2f^2}$$

Antiderivative was successfully verified.

[In] Integrate[((A+B*x)*(a+b*x+c*x^2))/(d+f*x^2),x]

[Out] (f*x*(2*b*B+2*A*c+B*c*x) - (2*Sqrt[f]*(b*B*d+A*c*d-a*A*f)*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/Sqrt[d] + (-B*c*d)+A*b*f+a*B*f)*Log[d+f*x^2])/(2*f^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+Bx)(a+bx+cx^2)}{d+fx^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A+B*x)*(a+b*x+c*x^2))/(d+f*x^2),x]

[Out] IntegrateAlgebraic[((A+B*x)*(a+b*x+c*x^2))/(d+f*x^2),x]

fricas [A] time = 0.40, size = 200, normalized size = 2.13

$$\left[\frac{Bcdfx^2+2(Bb+Ac)dfx-(Aaf-(Bb+Ac)d)\sqrt{-df} \log\left(\frac{fx^2-2\sqrt{-df}x-d}{fx^2+d}\right) - (Bcd^2-(Ba+Ab)df) \log(fx^2+d)}{2df^2}, \frac{Bcdfx^2+2(Bb+Ac)dfx+2(Aaf-(Bb+Ac)d)\sqrt{df} \arctan\left(\frac{\sqrt{df}x}{d}\right) - (Bcd^2-(Ba+Ab)df) \log(fx^2+d)}{2df^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="fricas")

[Out] [1/2*(B*c*d*f*x^2+2*(B*b+A*c)*d*f*x-(A*a*f-(B*b+A*c)*d)*sqrt(-d*f))*log((f*x^2-2*sqrt(-d*f)*x-d)/(f*x^2+d))-(B*c*d^2-(B*a+A*b)*d*f)*log(f*x^2+d)]/(d*f^2), [1/2*(B*c*d*f*x^2+2*(B*b+A*c)*d*f*x+2*(A*a*f-(B*b+A*c)*d)*sqrt(d*f)*arctan(sqrt(d*f)*x/d)-(B*c*d^2-(B*a+A*b)*d*f)*log(f*x^2+d)]/(d*f^2)]

giac [A] time = 0.15, size = 87, normalized size = 0.93

$$\frac{(Bbd+Ac d - Aaf) \arctan\left(\frac{fx}{\sqrt{df}}\right) - (Bcd - Baf - Abf) \log(fx^2+d)}{\sqrt{df} f} + \frac{Bcfx^2+2Bbfx+2Acfx}{2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="giac")

[Out] $-(B*b*d + A*c*d - A*a*f)*\arctan(f*x/\sqrt{d*f})/(\sqrt{d*f}*f) - 1/2*(B*c*d - B*a*f - A*b*f)*\log(f*x^2 + d)/f^2 + 1/2*(B*c*f*x^2 + 2*B*b*f*x + 2*A*c*f*x)/f^2$

maple [A] time = 0.03, size = 133, normalized size = 1.41

$$\frac{Aa \arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}} - \frac{Acd \arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df} f} - \frac{Bbd \arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df} f} + \frac{Bcx^2}{2f} + \frac{Ab \ln(fx^2 + d)}{2f} + \frac{Acx}{f} + \frac{Ba \ln(fx^2 + d)}{2f} + \frac{Bbx}{f} - \frac{Bcd \ln(fx^2 + d)}{2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x+A)*(c*x^2+b*x+a)/(f*x^2+d),x)

[Out] $1/2*B*c*x^2/f + 1/f*A*c*x + 1/f*b*B*x + 1/2/f*\ln(f*x^2+d)*A*b + 1/2/f*\ln(f*x^2+d)*B*a - 1/2/f^2*\ln(f*x^2+d)*B*c*d + 1/(d*f)^{(1/2)}*\arctan(x*f/(d*f)^{(1/2)})*A*a - 1/f/(d*f)^{(1/2)}*\arctan(x*f/(d*f)^{(1/2)})*A*c*d - 1/f/(d*f)^{(1/2)}*\arctan(x*f/(d*f)^{(1/2)})*B*b*d$

maxima [A] time = 0.97, size = 84, normalized size = 0.89

$$\frac{(Aaf - (Bb + Ac)d) \arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df} f} + \frac{Bcx^2 + 2(Bb + Ac)x}{2f} - \frac{(Bcd - (Ba + Ab)f) \log(fx^2 + d)}{2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="maxima")

[Out] $(A*a*f - (B*b + A*c)*d)*\arctan(f*x/\sqrt{d*f})/(\sqrt{d*f}*f) + 1/2*(B*c*x^2 + 2*(B*b + A*c)*x)/f - 1/2*(B*c*d - (B*a + A*b)*f)*\log(f*x^2 + d)/f^2$

mupad [B] time = 3.44, size = 97, normalized size = 1.03

$$\frac{x(Ac + Bb)}{f} - \frac{\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(Acd - Aaf + Bbd)}{\sqrt{d} f^{3/2}} + \frac{Bcx^2}{2f} + \frac{\ln(fx^2 + d)(4Abdf^3 + 4Badf^3 - 4Bcd^2f^2)}{8df^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x)*(a + b*x + c*x^2))/(d + f*x^2),x)

[Out] $(x*(A*c + B*b))/f - (\operatorname{atan}((f^{(1/2)}*x)/d^{(1/2)})*(A*c*d - A*a*f + B*b*d))/(d^{(1/2)}*f^{(3/2)}) + (B*c*x^2)/(2*f) + (\log(d + f*x^2)*(4*A*b*d*f^3 + 4*B*a*d*f^3 - 4*B*c*d^2*f^2))/(8*d*f^4)$

sympy [B] time = 1.85, size = 333, normalized size = 3.54

$$\frac{Bcx^2}{2f} + x\left(\frac{Ac}{f} + \frac{Bb}{f}\right) + \left(\frac{Abf + Baf - Bcd}{2f^2} - \frac{\sqrt{-df^3}(Aaf - Acd - Bbd)}{2df^4}\right) \log\left(x + \frac{-Abdf - Bdf + Bcd + 2df^2\left(\frac{Abf + Baf - Bcd}{2f^2} - \frac{\sqrt{-df^3}(Aaf - Acd - Bbd)}{2df^4}\right)}{Aaf^2 - Acdf - Bbdf}\right) + \left(\frac{Abf + Baf - Bcd}{2f^2} + \frac{\sqrt{-df^3}(Aaf - Acd - Bbd)}{2df^4}\right) \log\left(x + \frac{-Abdf - Bdf + Bcd + 2df^2\left(\frac{Abf + Baf - Bcd}{2f^2} + \frac{\sqrt{-df^3}(Aaf - Acd - Bbd)}{2df^4}\right)}{Aaf^2 - Acdf - Bbdf}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x**2+b*x+a)/(f*x**2+d),x)

[Out] $B*c*x**2/(2*f) + x*(A*c/f + B*b/f) + ((A*b*f + B*a*f - B*c*d)/(2*f**2) - \operatorname{sqrt}(-d*f**5)*(A*a*f - A*c*d - B*b*d)/(2*d*f**4))*\log(x + (-A*b*d*f - B*a*d*f + B*c*d**2 + 2*d*f**2*((A*b*f + B*a*f - B*c*d)/(2*f**2) - \operatorname{sqrt}(-d*f**5)*(A*a*f - A*c*d - B*b*d)/(2*d*f**4)))/(A*a*f**2 - A*c*d*f - B*b*d*f)) + ((A*b*f + B*a*f - B*c*d)/(2*f**2) + \operatorname{sqrt}(-d*f**5)*(A*a*f - A*c*d - B*b*d)/(2*d*f**4))*\log(x + (-A*b*d*f - B*a*d*f + B*c*d**2 + 2*d*f**2*((A*b*f + B*a*f - B*c*d)/(2*f**2) + \operatorname{sqrt}(-d*f**5)*(A*a*f - A*c*d - B*b*d)/(2*d*f**4)))/(A*a*f**2 - A*c*d*f - B*b*d*f))$

$$3.2 \quad \int \frac{(A+Bx)(a+bx+cx^2)^2}{d+fx^2} dx$$

Optimal. Leaf size=228

$$\frac{\log(d+fx^2)(2Abf(cd-af)-B(-f(b^2d-a^2f)-2acdf+c^2d^2))}{2f^3} + \frac{x^2(2Abcf-B(-2acf+b^2(-f)+c^2d))}{2f^2}$$

Rubi [A] time = 0.33, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1012, 635, 205, 260}

$$\frac{\log(d+fx^2)(2Abf(cd-af)-B(-f(b^2d-a^2f)-2acdf+c^2d^2))}{2f^3} + \frac{x^2(2Abcf-B(-2acf+b^2(-f)+c^2d))}{2f^2} + \frac{x(-Ac(cd-2af)-B(2cd-2af)+Ab^2f)}{f^2} - \frac{\tan^{-1}\left(\frac{\sqrt{d}}{\sqrt{a}}\right)(-A(cd-af)^2-2Bd(cd-af)+Ab^2df)}{\sqrt{d}f^{3/2}} + \frac{cx^3(Ac+2bB)}{3f} + \frac{Bc^2x^4}{4f}$$

Antiderivative was successfully verified.

[In] Int[((A + B*x)*(a + b*x + c*x^2)^2)/(d + f*x^2), x]

[Out] ((A*b^2*f - A*c*(c*d - 2*a*f) - b*B*(2*c*d - 2*a*f))*x)/f^2 + ((2*A*b*c*f - B*(c^2*d - b^2*f - 2*a*c*f))*x^2)/(2*f^2) + (c*(2*b*B + A*c)*x^3)/(3*f) + (B*c^2*x^4)/(4*f) - ((A*b^2*d*f - 2*b*B*d*(c*d - a*f) - A*(c*d - a*f)^2)*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]*f^(5/2)) - ((2*A*b*f*(c*d - a*f) - B*(c^2*d^2 - 2*a*c*d*f - f*(b^2*d - a^2*f)))*Log[d + f*x^2])/(2*f^3)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1012

Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + c*x^2)^p*(d + e*x + f*x^2)^q*(g + h*x), x], x] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && IntegersQ[p, q] && (GtQ[p, 0] || GtQ[q, 0])

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx)(a+bx+cx^2)^2}{d+fx^2} dx &= \int \left(\frac{Ab^2f - Ac(cd - 2af) - bB(2cd - 2af)}{f^2} + \frac{(2Abcf - B(c^2d - b^2f - 2acf))}{f^2} \right) \\
&= \frac{(Ab^2f - Ac(cd - 2af) - bB(2cd - 2af))x}{f^2} + \frac{(2Abcf - B(c^2d - b^2f - 2acf))}{2f^2} \\
&= \frac{(Ab^2f - Ac(cd - 2af) - bB(2cd - 2af))x}{f^2} + \frac{(2Abcf - B(c^2d - b^2f - 2acf))}{2f^2} \\
&= \frac{(Ab^2f - Ac(cd - 2af) - bB(2cd - 2af))x}{f^2} + \frac{(2Abcf - B(c^2d - b^2f - 2acf))}{2f^2}
\end{aligned}$$

Mathematica [A] time = 0.20, size = 204, normalized size = 0.89

$$\frac{6 \log(d+fx^2)(B(a^2f^2-2acd f+b^2(-d)+c^2d^2)+2Abf(af-cd))+fx(4Ac(6af-3cd+cfx^2)+4bB(6af-6cd+2cfx^2)+3Bcx(4af-2cd+cfx^2)+6b^2f(2A+Bx)+12Abcfx)}{12f^3} + \frac{\tan^{-1}\left(\frac{\sqrt{d}}{\sqrt{d}}\right)(A(cd-af)^2+2bBd(cd-af)-Ab^2df)}{\sqrt{d}f^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*x)*(a + b*x + c*x^2)^2)/(d + f*x^2), x]

[Out] ((-(A*b^2*d*f) + 2*b*B*d*(c*d - a*f) + A*(c*d - a*f)^2)*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]*f^(5/2)) + (f*x*(12*A*b*c*f*x + 6*b^2*f*(2*A + B*x) + 3*B*c*x*(-2*c*d + 4*a*f + c*f*x^2) + 4*A*c*(-3*c*d + 6*a*f + c*f*x^2) + 4*b*B*(-6*c*d + 6*a*f + 2*c*f*x^2)) + 6*(2*A*b*f*(-(c*d) + a*f) + B*(c^2*d^2 - b^2*d*f - 2*a*c*d*f + a^2*f^2))*Log[d + f*x^2])/(12*f^3)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+Bx)(a+bx+cx^2)^2}{d+fx^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B*x)*(a + b*x + c*x^2)^2)/(d + f*x^2), x]

[Out] IntegrateAlgebraic[((A + B*x)*(a + b*x + c*x^2)^2)/(d + f*x^2), x]

fricas [A] time = 0.42, size = 500, normalized size = 2.19

$$\frac{[3B^2c^2f^2+4B^2bc^2f^2-4B^2b^2c^2f^2-6B^2c^2d^2f^2-4(A^2f^2+2Abf(af-cd)+2A^2c^2d^2f^2-4B^2c^2d^2f^2)+4(2Abcf+3Bcx(4af-2cd+cfx^2)+6b^2f(2A+Bx)+12Abcfx)]\sqrt{d}\arctan\left(\frac{\sqrt{d}}{\sqrt{d}}\right)+\frac{6\log(d+fx^2)(B(a^2f^2-2acd f+b^2(-d)+c^2d^2)+2Abf(af-cd))+fx(4Ac(6af-3cd+cfx^2)+4bB(6af-6cd+2cfx^2)+3Bcx(4af-2cd+cfx^2)+6b^2f(2A+Bx)+12Abcfx)}{12f^3}}{12f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+d), x, algorithm="fricas")

[Out] [1/12*(3*B*c^2*d*f^2*x^4 + 4*(2*B*b*c + A*c^2)*d*f^2*x^3 - 6*(B*c^2*d^2*f - (B*b^2 + 2*(B*a + A*b)*c)*d*f^2)*x^2 - 6*(A*a^2*f^2 + (2*B*b*c + A*c^2)*d^2 - (2*B*a*b + A*b^2 + 2*A*a*c)*d*f)*sqrt(-d*f)*log((f*x^2 - 2*sqrt(-d*f)*x - d)/(f*x^2 + d)) - 12*((2*B*b*c + A*c^2)*d^2*f - (2*B*a*b + A*b^2 + 2*A*a*c)*d*f^2)*x + 6*(B*c^2*d^3 - (B*b^2 + 2*(B*a + A*b)*c)*d^2*f + (B*a^2 + 2*A*a*b)*d*f^2)*log(f*x^2 + d))/(d*f^3), 1/12*(3*B*c^2*d*f^2*x^4 + 4*(2*B*b*c + A*c^2)*d*f^2*x^3 - 6*(B*c^2*d^2*f - (B*b^2 + 2*(B*a + A*b)*c)*d*f^2)*x^2 + 12*(A*a^2*f^2 + (2*B*b*c + A*c^2)*d^2 - (2*B*a*b + A*b^2 + 2*A*a*c)*d*f)*sqrt(d*f)*arctan(sqrt(d*f)*x/d) - 12*((2*B*b*c + A*c^2)*d^2*f - (2*B*a*b + A*b^2 + 2*A*a*c)*d*f^2)*x + 6*(B*c^2*d^3 - (B*b^2 + 2*(B*a + A*b)*c)*d^2*f + (B*a^2 + 2*A*a*b)*d*f^2)*log(f*x^2 + d))/(d*f^3)]

giac [A] time = 0.16, size = 263, normalized size = 1.15

$$\frac{(2Bbc^2 + Ac^2d^2 - 2Babdf - A^2d^2f - 2Aacdf + A^2f^2) \arctan\left(\frac{fx}{\sqrt{df}}\right) + (Bc^2d^2 - Bb^2df - 2Bacdf - 2Abcdf + B^2f^2 + 2Aabf^2) \log(fx^2 + d)}{2f^3} + \frac{3Bc^2f^4 + 8Bbcf^3d + 4A^2f^3d^2 - 6Bc^2df^2d + 6Bb^2f^3d^2 + 12Bacdf^2d + 12Abcdf^2d - 24Bc^2df^2d - 24Babdf^3d + 12A^2f^3d^2 + 24Aacdf^3d}{12f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="giac")

[Out] $(2*B*b*c*d^2 + A*c^2*d^2 - 2*B*a*b*d*f - A*b^2*d*f - 2*A*a*c*d*f + A*a^2*f^2) \arctan(f*x/\sqrt{d*f}) / (\sqrt{d*f}) * f^2 + 1/2*(B*c^2*d^2 - B*b^2*d*f - 2*B*a*c*d*f - 2*A*b*c*d*f + B*a^2*f^2 + 2*A*a*b*f^2) \log(f*x^2 + d) / f^3 + 1/12*(3*B*c^2*f^3*x^4 + 8*B*b*c*f^3*x^3 + 4*A*c^2*f^3*x^3 - 6*B*c^2*d*f^2*x^2 + 6*B*b^2*f^3*x^2 + 12*B*a*c*f^3*x^2 + 12*A*b*c*f^3*x^2 - 24*B*b*c*d*f^2*x - 12*A*c^2*d*f^2*x + 24*B*a*b*f^3*x + 12*A*b^2*f^3*x + 24*A*a*c*f^3*x) / f^4$

maple [A] time = 0.01, size = 373, normalized size = 1.64

$$\frac{Bc^2d^2 + Ac^2d^2 - 2Babdf - A^2d^2f - 2Aacdf + A^2f^2}{\sqrt{df}f^2} \arctan\left(\frac{fx}{\sqrt{df}}\right) + \frac{2Aacdf + A^2d^2f - 2Abcdf + B^2f^2 + 2Aabf^2}{\sqrt{df}f^2} \log\left(\frac{fx}{\sqrt{df}}\right) + \frac{3Bc^2f^4 + 8Bbcf^3d + 4A^2f^3d^2 - 6Bc^2df^2d + 6Bb^2f^3d^2 + 12Bacdf^2d + 12Abcdf^2d - 24Bc^2df^2d - 24Babdf^3d + 12A^2f^3d^2 + 24Aacdf^3d}{12f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+d),x)

[Out] $1/4*B*c^2*x^4/f + 1/3/f*A*x^3*c^2 + 2/3/f*B*x^3*b*c + 1/f*A*x^2*b*c + 1/f*B*x^2*a*c + 1/2/f*B*x^2*b^2 - 1/2/f^2*B*x^2*c^2*d + 2/f*A*a*c*x + 1/f*A*b^2*x - 1/f^2*A*c^2*d*x + 2/f*B*a*b*x - 2/f^2*B*b*c*d*x + 1/f*\ln(f*x^2+d)*A*a*b - 1/f^2*\ln(f*x^2+d)*A*b*c*d + 1/2/f*\ln(f*x^2+d)*B*a^2 - 1/f^2*\ln(f*x^2+d)*B*a*c*d - 1/2/f^2*\ln(f*x^2+d)*B*b^2*d + 1/2/f^3*\ln(f*x^2+d)*B*c^2*d^2 + 1/(d*f)^(1/2)*\arctan(1/(d*f)^(1/2)*f*x)*A*a^2 - 2/f/(d*f)^(1/2)*\arctan(1/(d*f)^(1/2)*f*x)*A*a*c*d - 1/f/(d*f)^(1/2)*\arctan(1/(d*f)^(1/2)*f*x)*A*b^2*d + 1/f^2/(d*f)^(1/2)*\arctan(1/(d*f)^(1/2)*f*x)*A*c^2*d^2 - 2/f/(d*f)^(1/2)*\arctan(1/(d*f)^(1/2)*f*x)*B*a*b*d + 2/f^2/(d*f)^(1/2)*\arctan(1/(d*f)^(1/2)*f*x)*B*b*c*d^2$

maxima [A] time = 0.98, size = 220, normalized size = 0.96

$$\frac{(A^2f^2 + (2Bbc + Ac^2)d^2 - (2Bab + Ab^2 + 2Aac)df) \arctan\left(\frac{fx}{\sqrt{df}}\right) + (3Bc^2f^4 + 4(2Bbc + Ac^2)f^3d - 6(Bc^2d - (Bb^2 + 2(Ba + Ab)c))f^2d - 12((2Bbc + Ac^2)d^2 - (2Bab + Ab^2 + 2Aac)df) + (Bc^2d^2 - (Bb^2 + 2(Ba + Ab)c)df + (Ba^2 + 2Aab)df^2) \log(fx^2 + d)}{2f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="maxima")

[Out] $(A*a^2*f^2 + (2*B*b*c + A*c^2)*d^2 - (2*B*a*b + A*b^2 + 2*A*a*c)*d*f) \arctan(f*x/\sqrt{d*f}) / (\sqrt{d*f}) * f^2 + 1/12*(3*B*c^2*f^3*x^4 + 4*(2*B*b*c + A*c^2)*f^3*x^3 - 6*(B*c^2*d - (B*b^2 + 2*(B*a + A*b)*c)*f)*x^2 - 12*((2*B*b*c + A*c^2)*d - (2*B*a*b + A*b^2 + 2*A*a*c)*f)*x) / f^2 + 1/2*(B*c^2*d^2 - (B*b^2 + 2*(B*a + A*b)*c)*d*f + (B*a^2 + 2*A*a*b)*f^2) \log(f*x^2 + d) / f^3$

mupad [B] time = 0.25, size = 253, normalized size = 1.11

$$\frac{x \left(\frac{A^2 + 2Bab + 2Aac}{f} - \frac{d(Ac^2 + 2Bbc)}{f^2} \right) + x^2 \left(\frac{Bb^2 + 2Acb + 2Bac}{2f} - \frac{Bc^2d}{2f^2} \right) + \frac{x^3(Ac^2 + 2Bbc)}{3f} + \frac{Bc^2d^2}{4f} + \frac{\operatorname{atan}\left(\frac{fx}{\sqrt{df}}\right) \left(A^2f^2 - 2Babdf - A^2d^2f + 2Bbcdf + A^2d^2 \right) + \ln(fx^2 + d) \left(4B^2d^2f^2 + 8Aabdf^2 - 8Bacdf^2 - 4Bb^2d^2f^2 - 8Abcd^2f^2 + 4Bc^2d^2f^2 \right)}{\sqrt{df}f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x)*(a + b*x + c*x^2)^2)/(d + f*x^2),x)

[Out] $x*((A*b^2 + 2*A*a*c + 2*B*a*b)/f - (d*(A*c^2 + 2*B*b*c))/f^2) + x^2*((B*b^2 + 2*A*b*c + 2*B*a*c)/(2*f) - (B*c^2*d)/(2*f^2)) + (x^3*(A*c^2 + 2*B*b*c))/(3*f) + (B*c^2*x^4)/(4*f) + (\operatorname{atan}((f^(1/2)*x)/d^(1/2))*(A*a^2*f^2 + A*c^2*d^2 + 2*B*b*c*d^2 - A*b^2*d*f - 2*A*a*c*d*f - 2*B*a*b*d*f))/(d^(1/2)*f^(5/2)) + (\log(d + f*x^2)*(4*B*a^2*d*f^5 - 4*B*b^2*d^2*f^4 + 4*B*c^2*d^3*f^3 + 8*A*a*b*d*f^5 - 8*A*b*c*d^2*f^4 - 8*B*a*c*d^2*f^4))/(8*d*f^6)$

sympy [B] time = 7.87, size = 933, normalized size = 4.09

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x**2+b*x+a)**2/(f*x**2+d),x)

[Out] $B*c**2*x**4/(4*f) + x**3*(A*c**2/(3*f) + 2*B*b*c/(3*f)) + x**2*(A*b*c/f + B*a*c/f + B*b**2/(2*f) - B*c**2*d/(2*f**2)) + x*(2*A*a*c/f + A*b**2/f - A*c**2*d/f**2 + 2*B*a*b/f - 2*B*b*c*d/f**2) + ((2*A*a*b*f**2 - 2*A*b*c*d*f + B*a**2*f**2 - 2*B*a*c*d*f - B*b**2*d*f + B*c**2*d**2)/(2*f**3) - \sqrt{-d*f**7}*(A*a**2*f**2 - 2*A*a*c*d*f - A*b**2*d*f + A*c**2*d**2 - 2*B*a*b*d*f + 2*B*b*c*d**2)/(2*d*f**6))*\log(x + (-2*A*a*b*d*f**2 + 2*A*b*c*d**2*f - B*a**2*d*f**2 + 2*B*a*c*d**2*f + B*b**2*d**2*f - B*c**2*d**3 + 2*d*f**3*(2*A*a*b*f**2 - 2*A*b*c*d*f + B*a**2*f**2 - 2*B*a*c*d*f - B*b**2*d*f + B*c**2*d**2)/(2*f**3) - \sqrt{-d*f**7}*(A*a**2*f**2 - 2*A*a*c*d*f - A*b**2*d*f + A*c**2*d**2 - 2*B*a*b*d*f + 2*B*b*c*d**2)/(2*d*f**6)))/(A*a**2*f**3 - 2*A*a*c*d*f**2 - A*b**2*d*f**2 + A*c**2*d**2*f - 2*B*a*b*d*f**2 + 2*B*b*c*d**2*f) + ((2*A*a*b*f**2 - 2*A*b*c*d*f + B*a**2*f**2 - 2*B*a*c*d*f - B*b**2*d*f + B*c**2*d**2)/(2*f**3) + \sqrt{-d*f**7}*(A*a**2*f**2 - 2*A*a*c*d*f - A*b**2*d*f + A*c**2*d**2 - 2*B*a*b*d*f + 2*B*b*c*d**2)/(2*d*f**6))*\log(x + (-2*A*a*b*d*f**2 + 2*A*b*c*d**2*f - B*a**2*d*f**2 + 2*B*a*c*d**2*f + B*b**2*d**2*f - B*c**2*d**3 + 2*d*f**3*(2*A*a*b*f**2 - 2*A*b*c*d*f + B*a**2*f**2 - 2*B*a*c*d*f - B*b**2*d*f + B*c**2*d**2)/(2*f**3) + \sqrt{-d*f**7}*(A*a**2*f**2 - 2*A*a*c*d*f - A*b**2*d*f + A*c**2*d**2 - 2*B*a*b*d*f + 2*B*b*c*d**2)/(2*d*f**6)))/(A*a**2*f**3 - 2*A*a*c*d*f**2 - A*b**2*d*f**2 + A*c**2*d**2*f - 2*B*a*b*d*f**2 + 2*B*b*c*d**2*f)$

$$3.3 \quad \int \frac{(A+Bx)(a+bx+cx^2)^3}{d+fx^2} dx$$

Optimal. Leaf size=441

$$\frac{\log(d+fx^2) \left(Abf(-f(b^2d-3a^2f)-6acdf+3c^2d^2) - B(cd-af) \left(-f(3b^2d-a^2f) - 2acdf + c^2d^2 \right) \right)}{2f^4} x^2$$

Rubi [A] time = 0.62, antiderivative size = 441, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1012, 635, 205, 260}

$\frac{c^2(4b^2(a^2+bx^2)+3c^2d-3c^2f^2)+3c^2f^2}{3f^2}$, $\frac{3c^2f^2(a^2+bx^2)-3c^2f^2}{3f^2}$, $\frac{3c^2f^2(a^2+bx^2)-3c^2f^2}{3f^2}$, $\frac{3c^2f^2(a^2+bx^2)-3c^2f^2}{3f^2}$, $\frac{3c^2f^2(a^2+bx^2)-3c^2f^2}{3f^2}$, $\frac{3c^2f^2(a^2+bx^2)-3c^2f^2}{3f^2}$, $\frac{3c^2f^2(a^2+bx^2)-3c^2f^2}{3f^2}$, $\frac{3c^2f^2(a^2+bx^2)-3c^2f^2}{3f^2}$, $\frac{3c^2f^2(a^2+bx^2)-3c^2f^2}{3f^2}$, $\frac{3c^2f^2(a^2+bx^2)-3c^2f^2}{3f^2}$

Antiderivative was successfully verified.

[In] Int[((A + B*x)*(a + b*x + c*x^2)^3)/(d + f*x^2), x]

[Out] -(((b^3*B*d*f + 3*A*b^2*f*(c*d - a*f) - 3*b*B*(c*d - a*f)^2 - A*c*(c^2*d^2 - 3*a*c*d*f + 3*a^2*f^2))*x)/f^3 - (((A*b*f*(3*c^2*d - b^2*f - 6*a*c*f) - B*(c^3*d^2 - 3*a*c^2*d*f + 3*a*b^2*f^2 - 3*c*f*(b^2*d - a^2*f)))*x^2)/(2*f^3) + ((b^3*B*f + 3*A*b^2*c*f - A*c^2*(c*d - 3*a*f) - 3*b*B*c*(c*d - 2*a*f))*x^3)/(3*f^2) + (c*(3*A*b*c*f - B*(c^2*d - 3*b^2*f - 3*a*c*f))*x^4)/(4*f^2) + (c^2*(3*b*B + A*c)*x^5)/(5*f) + (B*c^3*x^6)/(6*f) + ((b^3*B*d^2*f + 3*A*b^2*d*f*(c*d - a*f) - 3*b*B*d*(c*d - a*f)^2 - A*(c*d - a*f)^3)*ArcTan[(Sqrt[f]*x)/Sqrt[d]]/(Sqrt[d]*f^(7/2)) + ((A*b*f*(3*c^2*d^2 - 6*a*c*d*f - f*(b^2*d - 3*a^2*f)) - B*(c*d - a*f)*(c^2*d^2 - 2*a*c*d*f - f*(3*b^2*d - a^2*f)))*Log[d + f*x^2])/(2*f^4)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1012

Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Int[ExpandIntegrand[(a + c*x^2)^p*(d + e*x + f*x^2)^q*(g + h*x), x], x] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && IntegersQ[p, q] && (GtQ[p, 0] || GtQ[q, 0])

Rubi steps

$$\int \frac{(A + Bx)(a + bx + cx^2)^3}{d + fx^2} dx = \int \left(\frac{b^3Bdf + 3Ab^2f(cd - af) - 3bB(cd - af)^2 - Ac(c^2d^2 - 3acdf + 3a^2f^2)}{f^3} \right) dx$$

$$= \frac{(b^3Bdf + 3Ab^2f(cd - af) - 3bB(cd - af)^2 - Ac(c^2d^2 - 3acdf + 3a^2f^2))x}{f^3}$$

$$= \frac{(b^3Bdf + 3Ab^2f(cd - af) - 3bB(cd - af)^2 - Ac(c^2d^2 - 3acdf + 3a^2f^2))x}{f^3}$$

$$= \frac{(b^3Bdf + 3Ab^2f(cd - af) - 3bB(cd - af)^2 - Ac(c^2d^2 - 3acdf + 3a^2f^2))x}{f^3}$$

Mathematica [A] time = 0.46, size = 422, normalized size = 0.96

$f^3(30(4B^2d^2 + 30cd(f^2 - 3d^2)(2d^2 - 3d^2 + 3f^2)) + 30Acf(4d^2 - 2d^2 + f^2)) + (44(4B^2d^2 + 30cd(f^2 - 3d^2)(2d^2 - 3d^2 + 3f^2)) + 30(4B^2d^2 + 30cd(f^2 - 3d^2)(2d^2 - 3d^2 + 3f^2))) \times 10^9(44(4d^2 - 2d^2 + f^2) + 30(2d^2 - 2d^2 + f^2)) \times 10^9(44(4d^2 - 2d^2 + f^2) + 30(2d^2 - 2d^2 + f^2)) - 30 \log(d + f^2)(44(4d^2 - 2d^2 + f^2) + 30(2d^2 - 2d^2 + f^2)) + 30 \log(d + f^2)(44(4d^2 - 2d^2 + f^2) + 30(2d^2 - 2d^2 + f^2)) - 30 \log(d + f^2)(44(4d^2 - 2d^2 + f^2) + 30(2d^2 - 2d^2 + f^2)) - 30 \log(d + f^2)(44(4d^2 - 2d^2 + f^2) + 30(2d^2 - 2d^2 + f^2))$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x + c*x^2)^3)/(d + f*x^2), x]
[Out] ((b^3*B*d^2*f + 3*A*b^2*d*f*(c*d - a*f) - 3*b*B*d*(c*d - a*f)^2 - A*(c*d - a*f)^3)*ArcTan[(Sqrt[f]*x)/Sqrt[d]]/(Sqrt[d]*f^(7/2)) + (f*x*(10*b^3*f*(-6*B*d + 3*A*f*x + 2*B*f*x^2) + 15*b^2*f*(3*B*x*(-2*c*d + 2*a*f + c*f*x^2) + 4*A*(-3*c*d + 3*a*f + c*f*x^2)) + 3*b*(15*A*c*f*x*(-2*c*d + 4*a*f + c*f*x^2) + 4*B*(15*a^2*f^2 + 10*a*c*f*(-3*d + f*x^2) + c^2*(15*d^2 - 5*d*f*x^2 + 3*f^2*x^4))) + c*(5*B*x*(18*a^2*f^2 + 9*a*c*f*(-2*d + f*x^2) + c^2*(6*d^2 - 3*d*f*x^2 + 2*f^2*x^4)) + 4*A*(45*a^2*f^2 + 15*a*c*f*(-3*d + f*x^2) + c^2*(15*d^2 - 5*d*f*x^2 + 3*f^2*x^4)))) - 30*(A*b*f*(-3*c^2*d^2 + b^2*d*f + 6*a*c*d*f - 3*a^2*f^2) + B*(c*d - a*f)*(c^2*d^2 - 3*b^2*d*f - 2*a*c*d*f + a^2*f^2))*Log[d + f*x^2])/(60*f^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx)(a + bx + cx^2)^3}{d + fx^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((A + B*x)*(a + b*x + c*x^2)^3)/(d + f*x^2), x]
[Out] IntegrateAlgebraic[((A + B*x)*(a + b*x + c*x^2)^3)/(d + f*x^2), x]
```

fricas [A] time = 0.44, size = 1014, normalized size = 2.30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^3/(f*x^2+d), x, algorithm="fricas")
[Out] [1/60*(10*B*c^3*d*f^3*x^6 + 12*(3*B*b*c^2 + A*c^3)*d*f^3*x^5 - 15*(B*c^3*d^2*f^2 - 3*(B*b^2*c + (B*a + A*b)*c^2)*d*f^3)*x^4 - 20*((3*B*b*c^2 + A*c^3)*d^2*f^2 - (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d*f^3)*x^3 + 30*(B*c^3*d^3*f - 3*(B*b^2*c + (B*a + A*b)*c^2)*d^2*f^2 + (3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*d*f^3)*x^2 - 30*(A*a^3*f^3 - (3*B*b*c^2 + A*c^3)*d^3 + (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d^2*f - 3*(B*a^2*b + A*a*b^2 + A*a^2*c)*d*f^2)*sqrt(-d*f)*log((f*x^2 - 2*sqrt(-d*f)*x - d)/(f*x^2 + d)) + 6
```


$$0*((3*B*b*c^2 + A*c^3)*d^3*f - (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d^2*f^2 + 3*(B*a^2*b + A*a*b^2 + A*a^2*c)*d*f^3)*x - 30*(B*c^3*d^4 - 3*(B*b^2*c + (B*a + A*b)*c^2)*d^3*f + (3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*d^2*f^2 - (B*a^3 + 3*A*a^2*b)*d*f^3)*\log(f*x^2 + d))/(d*f^4), 1/60*(10*B*c^3*d*f^3*x^6 + 12*(3*B*b*c^2 + A*c^3)*d*f^3*x^5 - 15*(B*c^3*d^2*f^2 - 3*(B*b^2*c + (B*a + A*b)*c^2)*d*f^3)*x^4 - 20*((3*B*b*c^2 + A*c^3)*d^2*f^2 - (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d*f^3)*x^3 + 30*(B*c^3*d^3*f - 3*(B*b^2*c + (B*a + A*b)*c^2)*d^2*f^2 + (3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*d*f^3)*x^2 + 60*(A*a^3*f^3 - (3*B*b*c^2 + A*c^3)*d^3 + (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d^2*f - 3*(B*a^2*b + A*a*b^2 + A*a^2*c)*d*f^2)*\sqrt{d*f}*\arctan(\sqrt{d*f}*x/d) + 60*((3*B*b*c^2 + A*c^3)*d^3*f - (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d^2*f^2 + 3*(B*a^2*b + A*a*b^2 + A*a^2*c)*d*f^3)*x - 30*(B*c^3*d^4 - 3*(B*b^2*c + (B*a + A*b)*c^2)*d^3*f + (3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*d^2*f^2 - (B*a^3 + 3*A*a^2*b)*d*f^3)*\log(f*x^2 + d))/(d*f^4)]$$

giac [A] time = 0.16, size = 623, normalized size = 1.41

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)^3/(f*x^2+d),x, algorithm="giac")

[Out] $-(3*B*b*c^2*d^3 + A*c^3*d^3 - B*b^3*d^2*f - 6*B*a*b*c*d^2*f - 3*A*b^2*c*d^2*f - 3*A*a*c^2*d^2*f + 3*B*a^2*b*d*f^2 + 3*A*a*b^2*d*f^2 + 3*A*a^2*c*d*f^2 - A*a^3*f^3)*\arctan(f*x/\sqrt{d*f})/(\sqrt{d*f}*f^3) - 1/2*(B*c^3*d^3 - 3*B*b^2*c*d^2*f - 3*B*a*c^2*d^2*f - 3*A*b*c^2*d^2*f + 3*B*a*b^2*d*f^2 + A*b^3*d*f^2 + 3*B*a^2*c*d*f^2 + 6*A*a*b*c*d*f^2 - B*a^3*f^3 - 3*A*a^2*b*f^3)*\log(f*x^2 + d)/f^4 + 1/60*(10*B*c^3*f^5*x^6 + 36*B*b*c^2*f^5*x^5 + 12*A*c^3*f^5*x^5 - 15*B*c^3*d*f^4*x^4 + 45*B*b^2*c*f^5*x^4 + 45*B*a*c^2*f^5*x^4 + 45*A*b*c^2*f^5*x^4 - 60*B*b*c^2*d*f^4*x^3 - 20*A*c^3*d*f^4*x^3 + 20*B*b^3*f^5*x^3 + 120*B*a*b*c*f^5*x^3 + 60*A*b^2*c*f^5*x^3 + 60*A*a*c^2*f^5*x^3 + 30*B*c^3*d^2*f^3*x^2 - 90*B*b^2*c*d*f^4*x^2 - 90*B*a*c^2*d*f^4*x^2 - 90*A*b*c^2*d*f^4*x^2 + 90*B*a*b^2*f^5*x^2 + 30*A*b^3*f^5*x^2 + 90*B*a^2*c*f^5*x^2 + 180*A*a*b*c*f^5*x^2 + 180*B*b*c^2*d^2*f^3*x + 60*A*c^3*d^2*f^3*x - 60*B*b^3*d*f^4*x - 360*B*a*b*c*d*f^4*x - 180*A*b^2*c*d*f^4*x - 180*A*a*c^2*d*f^4*x + 180*B*a^2*b*f^5*x + 180*A*a*b^2*f^5*x + 180*A*a^2*c*f^5*x)/f^6$

maple [A] time = 0.01, size = 822, normalized size = 1.86

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x+A)*(c*x^2+b*x+a)^3/(f*x^2+d),x)

[Out] $6/f^2/(d*f)^{(1/2)}*\arctan(1/(d*f)^{(1/2)}*f*x)*B*a*b*c*d^2-1/f^2*b^3*B*d*x+3/f*A*a*b^2*x+1/6*B*c^3*x^6/f-3/2/f^2*\ln(f*x^2+d)*B*a^2*c*d-3/2/f^2*\ln(f*x^2+d)*B*a*b^2*d+3/2/f^3*\ln(f*x^2+d)*B*a*c^2*d^2+3/2/f^3*\ln(f*x^2+d)*B*b^2*c*d^2-1/f^3/(d*f)^{(1/2)}*\arctan(1/(d*f)^{(1/2)}*f*x)*A*c^3*d^3+1/f^2/(d*f)^{(1/2)}*\arctan(1/(d*f)^{(1/2)}*f*x)*b^3*B*d^2+3/f^3*B*b*c^2*d^2*x+2/f*B*x^3*a*b*c-1/f^2*B*x^3*b*c^2*d+3/f*A*x^2*a*b*c-3/2/f^2*A*x^2*b*c^2*d-3/2/f^2*B*x^2*a*c^2*d-3/2/f^2*B*x^2*b^2*c*d-3/f^2*A*a*c^2*d*x-3/f^2*A*b^2*c*d*x+3/2/f^3*\ln(f*x^2+d)*A*b*c^2*d^2+1/2/f*\ln(f*x^2+d)*B*a^3+1/(d*f)^{(1/2)}*\arctan(1/(d*f)^{(1/2)}*f*x)*A*a^3+1/3/f*B*x^3*b^3+1/2/f*A*x^2*b^3+1/5/f*A*x^5*c^3-3/f^2*\ln(f*x^2+d)*A*a*b*c*d-3/f/(d*f)^{(1/2)}*\arctan(1/(d*f)^{(1/2)}*f*x)*A*a^2*c*d-3/f/(d*f)^{(1/2)}*\arctan(1/(d*f)^{(1/2)}*f*x)*A*a*b^2*d+3/f^2/(d*f)^{(1/2)}*\arctan(1/(d*f)^{(1/2)}*f*x)*A*a*c^2*d^2+3/f^2/(d*f)^{(1/2)}*\arctan(1/(d*f)^{(1/2)}*f*x)*A*b^2*c*d^2-3/f/(d*f)^{(1/2)}*\arctan(1/(d*f)^{(1/2)}*f*x)*B*a^2*b*d-3/f^3/(d*f)^{(1/2)}*\arctan(1/(d*f)^{(1/2)}*f*x)*B*b*c^2*d^3-6/f^2*B*a*b*c*d*x+3/5/f*B*x^5*b*c^2+3/4/$

$$\frac{fAx^4bc^2+1/fAx^3ac^2+3/2/f\ln(fx^2+d)*Aa^2b-1/2/f^2\ln(fx^2+d)*Ab^3d-1/2/f^4\ln(fx^2+d)*Bc^3d^3+1/fAx^3b^2c+1/2/f^3Bx^2*c^3d^2+1/f^3A*c^3d^2*x+3/fB*a^2*b*x-1/3/f^2A*x^3*c^3*d+3/2/f*B*x^2*a^2*c+3/2/f*B*x^2*a*b^2+3/fA*a^2*c*x+3/4/fB*x^4*a*c^2+3/4/fB*x^4*b^2*c-1/4/f^2*B*x^4*c^3*d}{}$$

maxima [A] time = 0.99, size = 471, normalized size = 1.07

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)^3/(f*x^2+d),x, algorithm="maxima")

[Out] $(Aa^3f^3 - (3Bb^2c^2 + Ac^3)*d^3 + (Bb^3 + 3Aa*c^2 + 3(2B*a*b + Ab^2)*c)*d^2*f - 3(Ba^2*b + Aa*b^2 + Aa^2*c)*d*f^2)*\arctan(fx/\sqrt{df})/(\sqrt{df})f^3 + 1/60*(10Bc^3f^2*x^6 + 12(3Bb^2c^2 + Ac^3)*f^2*x^5 - 15(Bc^3*d*f - 3(Bb^2*c + (Ba + Ab)*c^2)*f^2)*x^4 - 20((3Bb^2c^2 + Ac^3)*d*f - (Bb^3 + 3Aa*c^2 + 3(2B*a*b + Ab^2)*c)*f^2)*x^3 + 30(Bc^3*d^2 - 3(Bb^2*c + (Ba + Ab)*c^2)*d*f + (3B*a*b^2 + Ab^3 + 3(Ba^2 + 2Aa*b)*c)*f^2)*x^2 + 60((3Bb^2c^2 + Ac^3)*d^2 - (Bb^3 + 3Aa*c^2 + 3(2B*a*b + Ab^2)*c)*d*f + 3(Ba^2*b + Aa*b^2 + Aa^2*c)*f^2)*x)/f^3 - 1/2*(Bc^3*d^3 - 3(Bb^2*c + (Ba + Ab)*c^2)*d^2*f + (3B*a*b^2 + Ab^3 + 3(Ba^2 + 2Aa*b)*c)*d*f^2 - (Ba^3 + 3Aa^2*b)*f^3)*\log(fx^2 + d)/f^4$

mapad [B] time = 3.79, size = 552, normalized size = 1.25

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x)*(a + b*x + c*x^2)^3)/(d + f*x^2),x)

[Out] $x^2*((Ab^3 + 3B*a*b^2 + 3B*a^2*c + 6Aa*b*c)/(2f) - (d*((3A*b*c^2 + 3B*a*c^2 + 3B*b^2*c)/f - (Bc^3*d)/f^2))/(2f)) + x*((3A*a*b^2 + 3Aa^2*c + 3B*a^2*b)/f - (d*((Bb^3 + 3Aa*c^2 + 3A*b^2*c + 6B*a*b*c)/f - (d*(Ac^3 + 3Bb*c^2))/f^2))/f) + x^3*((Bb^3 + 3Aa*c^2 + 3A*b^2*c + 6B*a*b*c)/(3f) - (d*(Ac^3 + 3Bb*c^2))/(3f^2)) + x^4*((3A*b*c^2 + 3B*a*c^2 + 3B*b^2*c)/(4f) - (Bc^3*d)/(4f^2)) + (x^5*(Ac^3 + 3Bb*c^2))/(5f) + (Bc^3*x^6)/(6f) + (\log(d + fx^2)*(4Ba^3*d*f^7 - 4Ab^3*d^2*f^6 - 4Bc^3*d^4*f^4 - 12B*a*b^2*d^2*f^6 + 12A*b*c^2*d^3*f^5 + 12B*a*c^2*d^3*f^5 - 12B*a^2*c*d^2*f^6 + 12Bb^2*c*d^3*f^5 + 12Aa^2*b*d*f^7 - 24Aa*b*c*d^2*f^6))/(8*d*f^8) + (\operatorname{atan}((f^{1/2})*x)/d^{1/2})*(Aa^3f^3 - Ac^3d^3 - 3Bb^2c^2*d^3 + Bb^3d^2*f - 3Aa*b^2*d*f^2 + 3Aa*c^2*d^2*f - 3Aa^2*c*d*f^2 - 3B*a^2*b*d*f^2 + 3A*b^2*c*d^2*f + 6B*a*b*c*d^2*f))/(d^{1/2})f^{7/2})$

sympy [B] time = 23.62, size = 1962, normalized size = 4.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x**2+b*x+a)**3/(f*x**2+d),x)

[Out] $Bc^{**3}x^{**6}/(6*f) + x^{**5}(Ac^{**3}/(5*f) + 3*B*b*c^{**2}/(5*f)) + x^{**4}(3*A*b*c^{**2}/(4*f) + 3*B*a*c^{**2}/(4*f) + 3*B*b^{**2}c/(4*f) - Bc^{**3}d/(4*f^{**2})) + x^{**3}(Aa*c^{**2}/f + Ab^{**2}c/f - Ac^{**3}d/(3*f^{**2}) + 2B*a*b*c/f + Bb^{**3}/(3*f) - Bb*c^{**2}d/f^{**2}) + x^{**2}(3Aa*b*c/f + Ab^{**3}/(2*f) - 3A*b*c^{**2}d/(2*f^{**2})) + 3B*a^{**2}c/(2*f) + 3B*a*b^{**2}/(2*f) - 3B*a*c^{**2}d/(2*f^{**2}) - 3Bb^{**2}c*d/(2*f^{**2}) + Bc^{**3}d^{**2}/(2*f^{**3})) + x*(3Aa^{**2}c/f + 3Aa*b^{**2}/f - 3Aa*c^{**2}d/f^{**2} - 3Ab^{**2}c*d/f^{**2} + Ac^{**3}d^{**2}/f^{**3} + 3B*a^{**2}b/f - 6B*$

$$\begin{aligned}
& a*b*c*d/f^{**2} - B*b^{**3}*d/f^{**2} + 3*B*b*c^{**2}*d^{**2}/f^{**3}) + ((3*A*a^{**2}*b*f^{**3} - \\
& 6*A*a*b*c*d*f^{**2} - A*b^{**3}*d*f^{**2} + 3*A*b*c^{**2}*d^{**2}*f + B*a^{**3}*f^{**3} - 3*B*a* \\
& *2*c*d*f^{**2} - 3*B*a*b^{**2}*d*f^{**2} + 3*B*a*c^{**2}*d^{**2}*f + 3*B*b^{**2}*c*d^{**2}*f - B \\
& *c^{**3}*d^{**3})/(2*f^{**4}) - \text{sqrt}(-d*f^{**9})*(A*a^{**3}*f^{**3} - 3*A*a^{**2}*c*d*f^{**2} - 3*A \\
& *a*b^{**2}*d*f^{**2} + 3*A*a*c^{**2}*d^{**2}*f + 3*A*b^{**2}*c*d^{**2}*f - A*c^{**3}*d^{**3} - 3*B* \\
& a^{**2}*b*d*f^{**2} + 6*B*a*b*c*d^{**2}*f + B*b^{**3}*d^{**2}*f - 3*B*b*c^{**2}*d^{**3})/(2*d*f* \\
& *8))*\log(x + (-3*A*a^{**2}*b*d*f^{**3} + 6*A*a*b*c*d^{**2}*f^{**2} + A*b^{**3}*d^{**2}*f^{**2} - \\
& 3*A*b*c^{**2}*d^{**3}*f - B*a^{**3}*d*f^{**3} + 3*B*a^{**2}*c*d^{**2}*f^{**2} + 3*B*a*b^{**2}*d^{**2} \\
& *f^{**2} - 3*B*a*c^{**2}*d^{**3}*f - 3*B*b^{**2}*c*d^{**3}*f + B*c^{**3}*d^{**4} + 2*d*f^{**4}*((3* \\
& A*a^{**2}*b*f^{**3} - 6*A*a*b*c*d*f^{**2} - A*b^{**3}*d*f^{**2} + 3*A*b*c^{**2}*d^{**2}*f + B*a* \\
& *3*f^{**3} - 3*B*a^{**2}*c*d*f^{**2} - 3*B*a*b^{**2}*d*f^{**2} + 3*B*a*c^{**2}*d^{**2}*f + 3*B*b \\
& **2*c*d^{**2}*f - B*c^{**3}*d^{**3})/(2*f^{**4}) - \text{sqrt}(-d*f^{**9})*(A*a^{**3}*f^{**3} - 3*A*a^{** \\
& 2*c*d*f^{**2} - 3*A*a*b^{**2}*d*f^{**2} + 3*A*a*c^{**2}*d^{**2}*f + 3*A*b^{**2}*c*d^{**2}*f - A* \\
& c^{**3}*d^{**3} - 3*B*a^{**2}*b*d*f^{**2} + 6*B*a*b*c*d^{**2}*f + B*b^{**3}*d^{**2}*f - 3*B*b*c \\
& **2*d^{**3})/(2*d*f^{**8}))/((A*a^{**3}*f^{**4} - 3*A*a^{**2}*c*d*f^{**3} - 3*A*a*b^{**2}*d*f^{**3} \\
& + 3*A*a*c^{**2}*d^{**2}*f^{**2} + 3*A*b^{**2}*c*d^{**2}*f^{**2} - A*c^{**3}*d^{**3}*f - 3*B*a^{**2}*b* \\
& d*f^{**3} + 6*B*a*b*c*d^{**2}*f^{**2} + B*b^{**3}*d^{**2}*f^{**2} - 3*B*b*c^{**2}*d^{**3}*f)) + ((3 \\
& *A*a^{**2}*b*f^{**3} - 6*A*a*b*c*d*f^{**2} - A*b^{**3}*d*f^{**2} + 3*A*b*c^{**2}*d^{**2}*f + B*a \\
& **3*f^{**3} - 3*B*a^{**2}*c*d*f^{**2} - 3*B*a*b^{**2}*d*f^{**2} + 3*B*a*c^{**2}*d^{**2}*f + 3*B* \\
& b^{**2}*c*d^{**2}*f - B*c^{**3}*d^{**3})/(2*f^{**4}) + \text{sqrt}(-d*f^{**9})*(A*a^{**3}*f^{**3} - 3*A*a* \\
& *2*c*d*f^{**2} - 3*A*a*b^{**2}*d*f^{**2} + 3*A*a*c^{**2}*d^{**2}*f + 3*A*b^{**2}*c*d^{**2}*f - A \\
& *c^{**3}*d^{**3} - 3*B*a^{**2}*b*d*f^{**2} + 6*B*a*b*c*d^{**2}*f + B*b^{**3}*d^{**2}*f - 3*B*b*c \\
& **2*d^{**3})/(2*d*f^{**8}))*\log(x + (-3*A*a^{**2}*b*d*f^{**3} + 6*A*a*b*c*d^{**2}*f^{**2} + A \\
& *b^{**3}*d^{**2}*f^{**2} - 3*A*b*c^{**2}*d^{**3}*f - B*a^{**3}*d*f^{**3} + 3*B*a^{**2}*c*d^{**2}*f^{**2} \\
& + 3*B*a*b^{**2}*d^{**2}*f^{**2} - 3*B*a*c^{**2}*d^{**3}*f - 3*B*b^{**2}*c*d^{**3}*f + B*c^{**3}*d^{** \\
& 4} + 2*d*f^{**4}*((3*A*a^{**2}*b*f^{**3} - 6*A*a*b*c*d*f^{**2} - A*b^{**3}*d*f^{**2} + 3*A*b*c \\
& **2*d^{**2}*f + B*a^{**3}*f^{**3} - 3*B*a^{**2}*c*d*f^{**2} - 3*B*a*b^{**2}*d*f^{**2} + 3*B*a*c \\
& **2*d^{**2}*f + 3*B*b^{**2}*c*d^{**2}*f - B*c^{**3}*d^{**3})/(2*f^{**4}) + \text{sqrt}(-d*f^{**9})*(A*a \\
& *3*f^{**3} - 3*A*a^{**2}*c*d*f^{**2} - 3*A*a*b^{**2}*d*f^{**2} + 3*A*a*c^{**2}*d^{**2}*f + 3*A*b \\
& **2*c*d^{**2}*f - A*c^{**3}*d^{**3} - 3*B*a^{**2}*b*d*f^{**2} + 6*B*a*b*c*d^{**2}*f + B*b^{**3} \\
& *d^{**2}*f - 3*B*b*c^{**2}*d^{**3})/(2*d*f^{**8}))/((A*a^{**3}*f^{**4} - 3*A*a^{**2}*c*d*f^{**3} - 3 \\
& *A*a*b^{**2}*d*f^{**3} + 3*A*a*c^{**2}*d^{**2}*f^{**2} + 3*A*b^{**2}*c*d^{**2}*f^{**2} - A*c^{**3}*d^{** \\
& 3}*f - 3*B*a^{**2}*b*d*f^{**3} + 6*B*a*b*c*d^{**2}*f^{**2} + B*b^{**3}*d^{**2}*f^{**2} - 3*B*b*c \\
& **2*d^{**3}*f))
\end{aligned}$$

$$3.4 \quad \int \frac{A+Bx}{(a+bx+cx^2)(d+fx^2)} dx$$

Optimal. Leaf size=274

$$\frac{\log(a+bx+cx^2)(-aBf+Abf+Bcd)}{2(f(a^2f+b^2d)-2acdf+c^2d^2)} - \frac{\log(d+fx^2)(-aBf+Abf+Bcd)}{2(f(a^2f+b^2d)-2acdf+c^2d^2)} + \frac{\sqrt{f} \tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(aAf-Acd+bBd)}{\sqrt{d}(f(a^2f+b^2d)-2acdf+c^2d^2)}$$

Rubi [A] time = 0.28, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {1023, 634, 618, 206, 628, 635, 205, 260}

$$\frac{\log(a+bx+cx^2)(-aBf+Abf+Bcd)}{2(f(a^2f+b^2d)-2acdf+c^2d^2)} - \frac{\log(d+fx^2)(-aBf+Abf+Bcd)}{2(f(a^2f+b^2d)-2acdf+c^2d^2)} + \frac{\sqrt{f} \tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(aAf-Acd+bBd)}{\sqrt{d}(f(a^2f+b^2d)-2acdf+c^2d^2)} - \frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(2Ac(cd-af)-bB(af+cd)+Ab^2f)}{\sqrt{b^2-4ac}(f(a^2f+b^2d)-2acdf+c^2d^2)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x)/((a + b*x + c*x^2)*(d + f*x^2)), x]

[Out] (Sqrt[f]*(b*B*d - A*c*d + a*A*f)*ArcTan[(Sqrt[f]*x)/Sqrt[d]]/(Sqrt[d]*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))) - ((A*b^2*f + 2*A*c*(c*d - a*f) - b*B*(c*d + a*f))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]/(Sqrt[b^2 - 4*a*c]*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))) + ((B*c*d + A*b*f - a*B*f)*Log[a + b*x + c*x^2])/(2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))) - ((B*c*d + A*b*f - a*B*f)*Log[d + f*x^2])/(2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f)))

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ

$[2*c*d - b*e, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4*a*c]$

Rule 635

$\text{Int}[(d + (e \cdot x))/(a + (c \cdot x)^2), x_Symbol] \rightarrow \text{Dist}[d, \text{Int}[1/(a + c \cdot x^2), x], x] + \text{Dist}[e, \text{Int}[x/(a + c \cdot x^2), x], x] /; \text{FreeQ}\{a, c, d, e\}, x] \&\& \text{!NiceSqrtQ}[-(a \cdot c)]$

Rule 1023

$\text{Int}[(g + (h \cdot x))/((a + (b \cdot x) + (c \cdot x)^2) \cdot ((d + (f \cdot x) \cdot (x)^2))], x_Symbol] \rightarrow \text{With}\{q = \text{Simplify}[c^2 \cdot d^2 + b^2 \cdot d \cdot f - 2 \cdot a \cdot c \cdot d \cdot f + a^2 \cdot f^2]\}, \text{Dist}[1/q, \text{Int}[\text{Simp}[g \cdot c^2 \cdot d + g \cdot b^2 \cdot f - a \cdot b \cdot h \cdot f - a \cdot g \cdot c \cdot f + c \cdot (h \cdot c \cdot d + g \cdot b \cdot f - a \cdot h \cdot f) \cdot x], x]/(a + b \cdot x + c \cdot x^2), x], x] + \text{Dist}[1/q, \text{Int}[\text{Simp}[b \cdot h \cdot d \cdot f - g \cdot c \cdot d \cdot f + a \cdot g \cdot f^2 - f \cdot (h \cdot c \cdot d + g \cdot b \cdot f - a \cdot h \cdot f) \cdot x], x]/(d + f \cdot x^2), x], x] /; \text{NeQ}[q, 0] /; \text{FreeQ}\{a, b, c, d, f, g, h\}, x] \&\& \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0]$

Rubi steps

$$\begin{aligned} \int \frac{A + Bx}{(a + bx + cx^2)(d + fx^2)} dx &= \int \frac{-abBf + A(c^2d + b^2f - acf) + c(Bcd + Abf - aBf)x}{a + bx + cx^2} dx + \int \frac{f(bBd - Acd + aAf) - f(Bcd + Abf - aBf)x}{d + fx^2} dx \\ &= \frac{(f(bBd - Acd + aAf)) \int \frac{1}{d + fx^2} dx}{c^2d^2 - 2acdf + f(b^2d + a^2f)} + \frac{(Bcd + Abf - aBf) \int \frac{b + 2cx}{a + bx + cx^2} dx}{2(c^2d^2 - 2acdf + f(b^2d + a^2f))} - \frac{f(Bcd + Abf - aBf)x}{c^2d^2 - 2acdf + f(b^2d + a^2f)} \\ &= \frac{\sqrt{f}(bBd - Acd + aAf) \tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{\sqrt{d}(c^2d^2 - 2acdf + f(b^2d + a^2f))} + \frac{(Bcd + Abf - aBf) \log(a + bx + cx^2)}{2(c^2d^2 - 2acdf + f(b^2d + a^2f))} \\ &= \frac{\sqrt{f}(bBd - Acd + aAf) \tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{\sqrt{d}(c^2d^2 - 2acdf + f(b^2d + a^2f))} - \frac{(Ab^2f + 2Ac(cd - af) - bB(cd + af))}{\sqrt{b^2 - 4ac}(c^2d^2 - 2acdf + f(b^2d + a^2f))} \end{aligned}$$

Mathematica [A] time = 0.37, size = 212, normalized size = 0.77

$$\frac{\sqrt{d}(\sqrt{4ac - b^2}(-aBf + Abf + Bcd)(\log(a + x(b + cx)) - \log(d + fx^2)) + 2 \tan^{-1}\left(\frac{b + 2cx}{\sqrt{4ac - b^2}}\right)(2Ac(cd - af) - bB(af + cd) + Ab^2f) + 2\sqrt{f}\sqrt{4ac - b^2} \tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(aAf - Acd + bBd))}{2\sqrt{d}\sqrt{4ac - b^2}(f(a^2f + b^2d) - 2acdf + c^2d^2)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x)/((a + b*x + c*x^2)*(d + f*x^2)),x]

[Out] (2*Sqrt[-b^2 + 4*a*c]*Sqrt[f]*(b*B*d - A*c*d + a*A*f)*ArcTan[(Sqrt[f]*x)/Sqrt[d]] + Sqrt[d]*(2*(A*b^2*f + 2*A*c*(c*d - a*f) - b*B*(c*d + a*f))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*c]*(B*c*d + A*b*f - a*B*f)*(-Log[d + f*x^2] + Log[a + x*(b + c*x)])))/(2*Sqrt[-b^2 + 4*a*c]*Sqrt[d]*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f)))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx}{(a + bx + cx^2)(d + fx^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x)/((a + b*x + c*x^2)*(d + f*x^2)),x]

[Out] IntegrateAlgebraic[(A + B*x)/((a + b*x + c*x^2)*(d + f*x^2)), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.17, size = 266, normalized size = 0.97

$$\frac{(Bcd - Baf + Abf) \log(cx^2 + bx + a)}{2(c^2d^2 + b^2df - 2acdf + a^2f^2)} - \frac{(Bcd - Baf + Abf) \log(fx^2 + d)}{2(c^2d^2 + b^2df - 2acdf + a^2f^2)} + \frac{(Bbdf - Acdf + Aaf^2) \arctan\left(\frac{fx}{\sqrt{df}}\right)}{(c^2d^2 + b^2df - 2acdf + a^2f^2)\sqrt{df}} - \frac{(Bbcd - 2Ac^2d + Babf - Ab^2f + 2Aacf) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(c^2d^2 + b^2df - 2acdf + a^2f^2)\sqrt{-b^2 + 4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="giac")

[Out] $\frac{1}{2}*(B*c*d - B*a*f + A*b*f)*\log(c*x^2 + b*x + a)/(c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2) - \frac{1}{2}*(B*c*d - B*a*f + A*b*f)*\log(f*x^2 + d)/(c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2) + (B*b*d*f - A*c*d*f + A*a*f^2)*\arctan(f*x/\sqrt{d*f})/((c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2)*\sqrt{d*f}) - (B*b*c*d - 2*A*c^2*d + B*a*b*f - A*b^2*f + 2*A*a*c*f)*\arctan((2*c*x + b)/\sqrt{-b^2 + 4*a*c})/((c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2)*\sqrt{-b^2 + 4*a*c})$

maple [B] time = 0.02, size = 745, normalized size = 2.72

$$\frac{2af \arctan\left(\frac{bx+2c}{\sqrt{4ac-b^2}}\right)}{(c^2d^2 + b^2df - 2acdf + a^2f^2)\sqrt{4ac-b^2}} - \frac{2af \arctan\left(\frac{fx}{\sqrt{df}}\right)}{(c^2d^2 + b^2df - 2acdf + a^2f^2)\sqrt{df}} - \frac{2af \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(c^2d^2 + b^2df - 2acdf + a^2f^2)\sqrt{4ac-b^2}} - \frac{2af \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(c^2d^2 + b^2df - 2acdf + a^2f^2)\sqrt{4ac-b^2}} - \frac{2af \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(c^2d^2 + b^2df - 2acdf + a^2f^2)\sqrt{4ac-b^2}} - \frac{2af \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(c^2d^2 + b^2df - 2acdf + a^2f^2)\sqrt{4ac-b^2}} - \frac{2af \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(c^2d^2 + b^2df - 2acdf + a^2f^2)\sqrt{4ac-b^2}} - \frac{2af \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(c^2d^2 + b^2df - 2acdf + a^2f^2)\sqrt{4ac-b^2}} - \frac{2af \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(c^2d^2 + b^2df - 2acdf + a^2f^2)\sqrt{4ac-b^2}} - \frac{2af \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(c^2d^2 + b^2df - 2acdf + a^2f^2)\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d),x)

[Out] $-\frac{1}{2}f/(a^2f^2-2a*c*d*f+b^2*d*f+c^2*d^2)*\ln(f*x^2+d)*A*b+1/2f/(a^2f^2-2a*c*d*f+b^2*d*f+c^2*d^2)*\ln(f*x^2+d)*B*a-1/2/(a^2f^2-2a*c*d*f+b^2*d*f+c^2*d^2)*\ln(f*x^2+d)*B*c*d+f^2/(a^2f^2-2a*c*d*f+b^2*d*f+c^2*d^2)/(d*f)^{(1/2)}*\arctan(1/(d*f)^{(1/2)}*f*x)*A*a-f/(a^2f^2-2a*c*d*f+b^2*d*f+c^2*d^2)/(d*f)^{(1/2)}*\arctan(1/(d*f)^{(1/2)}*f*x)*A*c*d+f/(a^2f^2-2a*c*d*f+b^2*d*f+c^2*d^2)/(d*f)^{(1/2)}*\arctan(1/(d*f)^{(1/2)}*f*x)*B*b*d+1/2/(a^2f^2-2a*c*d*f+b^2*d*f+c^2*d^2)*\ln(c*x^2+b*x+a)*A*b*f-1/2/(a^2f^2-2a*c*d*f+b^2*d*f+c^2*d^2)*\ln(c*x^2+b*x+a)*B*a*f+1/2/(a^2f^2-2a*c*d*f+b^2*d*f+c^2*d^2)*c*\ln(c*x^2+b*x+a)*B*d-2/(a^2f^2-2a*c*d*f+b^2*d*f+c^2*d^2)/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*A*a*c*f+1/(a^2f^2-2a*c*d*f+b^2*d*f+c^2*d^2)/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*A*b^2*f+2/(a^2f^2-2a*c*d*f+b^2*d*f+c^2*d^2)/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*A*c^2*d-1/(a^2f^2-2a*c*d*f+b^2*d*f+c^2*d^2)/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*B*a*b*f-1/(a^2f^2-2a*c*d*f+b^2*d*f+c^2*d^2)/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*B*b*c*d$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 38.32, size = 3888, normalized size = 14.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((A + Bx)/((d + fx^2)(a + bx + cx^2)), x)$

[Out] $(\log(B^3c^2f^2x + ((Bcd^2)/2 + (Abdf)/2 - (Bafd)/2 - (Aaf(-df)^{1/2})/2 + (Acdf(-df)^{1/2})/2 - (Bbdf(-df)^{1/2})/2) * (((Bcd^2)/2 + (Abdf)/2 - (Bafd)/2 - (Aaf(-df)^{1/2})/2 + (Acdf(-df)^{1/2})/2 - (Bbdf(-df)^{1/2})/2) * (4Aa^2c^2f^4 + 4Ac^4d^2f^2 - cf^2 * x * (3Ab^3f^2 + 4Bc^3d^2 - Babb^2f^2 + 4Ba^2c^2f^2 - 12Aabcf^2 + 12Abc^2df - 8Baac^2df - 3Bb^2cdf) - 3Ab^2c^2df^3 - 4Bb^3c^3d^2f^2 - Aab^2c^2f^4 - 8Aac^3d^2f^3 + Bb^3c^3d^2f^3 + (2cf^2 * ((Bcd^2)/2 + (Abdf)/2 - (Bafd)/2 - (Aaf(-df)^{1/2})/2 + (Acdf(-df)^{1/2})/2 - (Bbdf(-df)^{1/2})/2) * (2b^3cd^3 + 4c^4d^3x - a^2b^2f^3x + 4ab^3df^2 - 2b^3cd^2f + 4a^3cf^3x + 3b^4df^2x + 12abc^2d^2f - 14a^2b^2cdf^2 - 4ac^3d^2fx - 4a^2c^2df^2x + 3b^2c^2d^2fx - 10ab^2cdf^2x)) / (d(a^2f^2 + c^2d^2 + b^2df - 2acdf)) + 4Baacb^2df^3)) / (d(a^2f^2 + c^2d^2 + b^2df - 2acdf)) - cf^2 * x * (2B^2c^2d - 4A^2c^2f - B^2b^2f + 2B^2aacf + 2ABb^2cf) + A^2b^2c^2f^3 + B^2b^2c^2df^2 - 4ABaac^2f^3 + ABb^2c^2cf^3 - 4ABc^3df^2)) / (d(a^2f^2 + c^2d^2 + b^2df - 2acdf)) + AB^2c^2f^2 * (f * ((Bacd)/2 - (Abd)/2 + (Aa(-df)^{1/2})/2) - (Bcd^2)/2 - (Acdf(-df)^{1/2})/2 + (Bbdf(-df)^{1/2})/2)) / (c^2d^3 + a^2df^2 + b^2d^2f - 2acd^2f) - (\log(B^3c^2f^2x + ((Bcd^2)/2 + (Abdf)/2 - (Bafd)/2 + (Aaf(-df)^{1/2})/2 - (Acdf(-df)^{1/2})/2 + (Bbdf(-df)^{1/2})/2) * (((Bcd^2)/2 + (Abdf)/2 - (Bafd)/2 + (Aaf(-df)^{1/2})/2 - (Acdf(-df)^{1/2})/2 + (Bbdf(-df)^{1/2})/2) * (4Aa^2c^2f^4 + 4Ac^4d^2f^2 - cf^2 * x * (3Ab^3f^2 + 4Bc^3d^2 - Babb^2f^2 + 4Ba^2c^2f^2 - 12Aabcf^2 + 12Abc^2df - 8Baac^2df - 3Bb^2cdf) - 3Ab^2c^2df^3 - 4Bb^3c^3d^2f^2 - Aab^2c^2f^4 - 8Aac^3d^2f^3 + Bb^3c^3d^2f^3 + (2cf^2 * ((Bcd^2)/2 + (Abdf)/2 - (Bafd)/2 + (Aaf(-df)^{1/2})/2 - (Acdf(-df)^{1/2})/2 + (Bbdf(-df)^{1/2})/2) * (2b^3cd^3 + 4c^4d^3x - a^2b^2f^3x + 4ab^3df^2 - 2b^3cd^2f + 4a^3cf^3x + 3b^4df^2x + 12abc^2d^2f - 14a^2b^2cdf^2 - 4ac^3d^2fx - 4a^2c^2df^2x + 3b^2c^2d^2fx - 10ab^2cdf^2x)) / (d(a^2f^2 + c^2d^2 + b^2df - 2acdf)) + 4Baacb^2df^3)) / (d(a^2f^2 + c^2d^2 + b^2df - 2acdf)) - cf^2 * x * (2B^2c^2d - 4A^2c^2f - B^2b^2f + 2B^2aacf + 2ABb^2cf) + A^2b^2c^2f^3 + B^2b^2c^2df^2 - 4ABaac^2f^3 + ABb^2c^2cf^3 - 4ABc^3df^2)) / (d(a^2f^2 + c^2d^2 + b^2df - 2acdf)) + AB^2c^2f^2 * (f * ((Aabd)/2 - (Bacd)/2 + (Aa(-df)^{1/2})/2) + (Bcd^2)/2 - (Acdf(-df)^{1/2})/2 + (Bbdf(-df)^{1/2})/2)) / (c^2d^3 + a^2df^2 + b^2d^2f - 2acd^2f) - (\log(B^3c^2f^2x + AB^2c^2f^2 - (((Aaf(b^2 - 4aac))^{3/2} + 2Abbf(4aac - b^2) - 2BAAF(4aac - b^2) + 2Bc^2d(4aac - b^2) + 4Ac^2d(b^2 - 4aac))^{1/2} + Ab^2f(b^2 - 4aac)^{1/2} - 2BAAF(b^2 - 4aac)^{1/2} - 2Bb^2cdf(b^2 - 4aac)^{1/2}) * (cf^2 * x * (3Ab^3f^2 + 4Bc^3d^2 - Babb^2f^2 + 4Ba^2c^2f^2 - 12Aabcf^2 + 12Abc^2df - 8Baac^2df - 3Bb^2cdf) - 4Ac^4d^2f^2 - 4Aa^2c^2f^4 + 3Ab^2c^2df^3 + 4Bb^3c^3d^2f^2 + Aab^2c^2f^4 + 8Aac^3d^2f^3 - Bb^3c^3d^2f^3 - 4Baacb^2df^3 + (cf^2 * (Aaf(b^2 - 4aac))^{3/2} + 2Abbf(4aac - b^2) - 2BAAF(4aac - b^2) + 2Bc^2d(4aac - b^2) + 4Ac^2d(b^2 - 4aac))^{1/2} + Ab^2f(b^2 - 4aac)^{1/2} - 2BAAF(b^2 - 4aac)^{1/2} - 2Bb^2cdf(b^2 - 4aac)^{1/2}) * (2b^3cd^3 + 4c^4d^3x - a^2b^2f^3x + 4ab^3df^2 - 2b^3cd^2f + 4a^3cf^3x + 3b^4df^2x + 12abc^2d^2f - 14a^2b^2cdf^2 - 4ac^3d^2fx - 4a^2c^2df^2x + 3b^2c^2d^2fx - 10ab^2cdf^2x)) / (2(4aac - b^2)(a^2f^2 + c^2d^2 + b^2df - 2acd^2f))) / (4(4aac - b^2)(a^2f^2 + c^2d^2 + b^2df - 2acd^2f) - cf^2 * x * (2B^2c^2d - 4A^2c^2f - B^2b^2f + 2B^2aacf + 2ABb^2cf) + A^2b^2c^2f^3$

$$\begin{aligned} &^3 + B^2*b*c^2*d*f^2 - 4*A*B*a*c^2*f^3 + A*B*b^2*c*f^3 - 4*A*B*c^3*d*f^2)*(\\ &A*f*(b^2 - 4*a*c)^{(3/2)} + 2*A*b*f*(4*a*c - b^2) - 2*B*a*f*(4*a*c - b^2) + 2 \\ &*B*c*d*(4*a*c - b^2) + 4*A*c^2*d*(b^2 - 4*a*c)^{(1/2)} + A*b^2*f*(b^2 - 4*a*c \\ &)^{(1/2)} - 2*B*a*b*f*(b^2 - 4*a*c)^{(1/2)} - 2*B*b*c*d*(b^2 - 4*a*c)^{(1/2)))/(\\ &4*(4*a*c - b^2)*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f)))*(A*f*(b^2 - 4*a \\ &*c)^{(3/2)} + 2*A*b*f*(4*a*c - b^2) - 2*B*a*f*(4*a*c - b^2) + 2*B*c*d*(4*a*c \\ &- b^2) + 4*A*c^2*d*(b^2 - 4*a*c)^{(1/2)} + A*b^2*f*(b^2 - 4*a*c)^{(1/2)} - 2*B* \\ &a*b*f*(b^2 - 4*a*c)^{(1/2)} - 2*B*b*c*d*(b^2 - 4*a*c)^{(1/2)))/(b^2*(4*a^2*f^2 \\ &+ 4*c^2*d^2 + 4*b^2*d*f - 24*a*c*d*f) - 4*a*c*(4*a^2*f^2 + 4*c^2*d^2 - 8*a \\ &*c*d*f)) + (\log(B^3*c^2*f^2*x + A*B^2*c^2*f^2 + (((A*f*(b^2 - 4*a*c)^{(3/2)} \\ &- 2*A*b*f*(4*a*c - b^2) + 2*B*a*f*(4*a*c - b^2) - 2*B*c*d*(4*a*c - b^2) + \\ &4*A*c^2*d*(b^2 - 4*a*c)^{(1/2)} + A*b^2*f*(b^2 - 4*a*c)^{(1/2)} - 2*B*a*b*f*(b^ \\ &2 - 4*a*c)^{(1/2)} - 2*B*b*c*d*(b^2 - 4*a*c)^{(1/2)))*(4*A*a^2*c^2*f^4 + 4*A*c^ \\ &4*d^2*f^2 - c*f^2*x*(3*A*b^3*f^2 + 4*B*c^3*d^2 - B*a*b^2*f^2 + 4*B*a^2*c*f^ \\ &2 - 12*A*a*b*c*f^2 + 12*A*b*c^2*d*f - 8*B*a*c^2*d*f - 3*B*b^2*c*d*f) - 3*A* \\ &b^2*c^2*d*f^3 - 4*B*b*c^3*d^2*f^2 - A*a*b^2*c*f^4 - 8*A*a*c^3*d*f^3 + B*b^3 \\ &*c*d*f^3 + 4*B*a*b*c^2*d*f^3 + (c*f^2*(A*f*(b^2 - 4*a*c)^{(3/2)} - 2*A*b*f*(4 \\ &*a*c - b^2) + 2*B*a*f*(4*a*c - b^2) - 2*B*c*d*(4*a*c - b^2) + 4*A*c^2*d*(b^ \\ &2 - 4*a*c)^{(1/2)} + A*b^2*f*(b^2 - 4*a*c)^{(1/2)} - 2*B*a*b*f*(b^2 - 4*a*c)^{(1 \\ &/2) - 2*B*b*c*d*(b^2 - 4*a*c)^{(1/2)))*(2*b*c^3*d^3 + 4*c^4*d^3*x - a^2*b^2*f \\ &^3*x + 4*a*b^3*d*f^2 - 2*b^3*c*d^2*f + 4*a^3*c*f^3*x + 3*b^4*d*f^2*x + 12*a \\ &*b*c^2*d^2*f - 14*a^2*b*c*d*f^2 - 4*a*c^3*d^2*f*x - 4*a^2*c^2*d*f^2*x + 3*b \\ &^2*c^2*d^2*f*x - 10*a*b^2*c*d*f^2*x))/(2*(4*a*c - b^2)*(a^2*f^2 + c^2*d^2 + \\ &b^2*d*f - 2*a*c*d*f)))/(4*(4*a*c - b^2)*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2* \\ &a*c*d*f)) - c*f^2*x*(2*B^2*c^2*d - 4*A^2*c^2*f - B^2*b^2*f + 2*B^2*a*c*f + \\ &2*A*B*b*c*f) + A^2*b*c^2*f^3 + B^2*b*c^2*d*f^2 - 4*A*B*a*c^2*f^3 + A*B*b^2* \\ &c*f^3 - 4*A*B*c^3*d*f^2)*(A*f*(b^2 - 4*a*c)^{(3/2)} - 2*A*b*f*(4*a*c - b^2) + \\ &2*B*a*f*(4*a*c - b^2) - 2*B*c*d*(4*a*c - b^2) + 4*A*c^2*d*(b^2 - 4*a*c)^{(1 \\ &/2) + A*b^2*f*(b^2 - 4*a*c)^{(1/2)} - 2*B*a*b*f*(b^2 - 4*a*c)^{(1/2)} - 2*B*b*c \\ &*d*(b^2 - 4*a*c)^{(1/2)))/(4*(4*a*c - b^2)*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2* \\ &a*c*d*f)))*(A*f*(b^2 - 4*a*c)^{(3/2)} - 2*A*b*f*(4*a*c - b^2) + 2*B*a*f*(4*a* \\ &c - b^2) - 2*B*c*d*(4*a*c - b^2) + 4*A*c^2*d*(b^2 - 4*a*c)^{(1/2)} + A*b^2*f* \\ &(b^2 - 4*a*c)^{(1/2)} - 2*B*a*b*f*(b^2 - 4*a*c)^{(1/2)} - 2*B*b*c*d*(b^2 - 4*a* \\ &c)^{(1/2)))/(b^2*(4*a^2*f^2 + 4*c^2*d^2 + 4*b^2*d*f - 24*a*c*d*f) - 4*a*c*(4 \\ &*a^2*f^2 + 4*c^2*d^2 - 8*a*c*d*f)) \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+d),x)

[Out] Timed out

$$3.5 \quad \int \frac{A+Bx}{(a+bx+cx^2)^2(d+fx^2)} dx$$

Optimal. Leaf size=596

$$\frac{f^{3/2} \tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right) \left(-A(cd-af)^2 + 2bBd(cd-af) + Ab^2df\right) f \log(a+bx+cx^2) \left(B(-f(b^2d-a^2f) - 2acd\right)}{\sqrt{d} \left(f(a^2f+b^2d) - 2acdf + c^2d^2\right)^2} \frac{f \log(a+bx+cx^2) \left(B(-f(b^2d-a^2f) - 2acd\right)}{2 \left(f(a^2f+b^2d) - 2acdf + c^2d^2\right)}$$

Rubi [A] time = 1.77, antiderivative size = 596, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, number of rules / integrand size = 0.333, Rules used = {1018, 1074, 634, 618, 206, 628, 635, 205, 260}

$$\frac{f^{3/2} \tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right) \left(-A(cd-af)^2 + 2bBd(cd-af) + Ab^2df\right) f \log(a+bx+cx^2) \left(B(-f(b^2d-a^2f) - 2acd\right)}{\sqrt{d} \left(f(a^2f+b^2d) - 2acdf + c^2d^2\right)^2} \frac{f \log(a+bx+cx^2) \left(B(-f(b^2d-a^2f) - 2acd\right)}{2 \left(f(a^2f+b^2d) - 2acdf + c^2d^2\right)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x)/((a + b*x + c*x^2)^2*(d + f*x^2)), x]

[Out] (A*b*c*(c*d + a*f) - (A*b - a*B)*(2*c^2*d + b^2*f - 2*a*c*f) - c*(A*b^2*f + 2*A*c*(c*d - a*f) - b*B*(c*d + a*f))*x)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(a + b*x + c*x^2)) - (f^(3/2)*(A*b^2*d*f + 2*b*B*d*(c*d - a*f) - A*(c*d - a*f)^2)*ArcTan[(Sqrt[f]*x)/Sqrt[d]]/(Sqrt[d]*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))^2) - ((b^5*B*d*f^2 - 2*A*b^4*f^2*(c*d - a*f) - 4*A*c^2*(c*d - 3*a*f)*(c*d - a*f)^2 + b^3*B*f*(5*c^2*d^2 - 4*a*c*d*f - a^2*f^2) - 4*A*b^2*c*f*(2*c^2*d^2 - 3*a*c*d*f + 3*a^2*f^2) + 2*b*B*c*(c^3*d^3 - 7*a*c^2*d^2*f + 3*a^2*c*d*f^2 + 3*a^3*f^3))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))^2) - (f*(2*A*b*f*(c*d - a*f) + B*(c^2*d^2 - 2*a*c*d*f - f*(b^2*d - a^2*f)))*Log[a + b*x + c*x^2])/(2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))^2) + (f*(2*A*b*f*(c*d - a*f) + B*(c^2*d^2 - 2*a*c*d*f - f*(b^2*d - a^2*f)))*Log[d + f*x^2])/(2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))^2)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1018

Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f)*x))/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p + q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] && !(IntegerQ[p] && ILtQ[q, -1])

Rule 1074

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.)*(x_)^2)), x_Symbol] := With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}, Dist[1/q, Int[(A*c^2*d - a*c*C*d + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f + c*(B*c*d - b*C*d + A*b*f - a*B*f)*x)/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[(c*C*d^2 + b*B*d*f - A*c*d*f - a*C*d*f + a*A*f^2 - f*(B*c*d - b*C*d + A*b*f - a*B*f)*x)/(d + f*x^2), x], x] /; NeQ[q, 0] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\int \frac{A + Bx}{(a + bx + cx^2)^2 (d + fx^2)} dx = \frac{Abc(cd + af) - (Ab - aB)(2c^2d + b^2f - 2acf) - c(Ab^2f + 2Ac(cd - af))}{(b^2 - 4ac)(b^2df + (cd - af)^2)(a + bx + cx^2)}$$

$$= \frac{Abc(cd + af) - (Ab - aB)(2c^2d + b^2f - 2acf) - c(Ab^2f + 2Ac(cd - af))}{(b^2 - 4ac)(b^2df + (cd - af)^2)(a + bx + cx^2)}$$

$$= \frac{Abc(cd + af) - (Ab - aB)(2c^2d + b^2f - 2acf) - c(Ab^2f + 2Ac(cd - af))}{(b^2 - 4ac)(b^2df + (cd - af)^2)(a + bx + cx^2)}$$

$$= \frac{Abc(cd + af) - (Ab - aB)(2c^2d + b^2f - 2acf) - c(Ab^2f + 2Ac(cd - af))}{(b^2 - 4ac)(b^2df + (cd - af)^2)(a + bx + cx^2)}$$

$$= \frac{Abc(cd + af) - (Ab - aB)(2c^2d + b^2f - 2acf) - c(Ab^2f + 2Ac(cd - af))}{(b^2 - 4ac)(b^2df + (cd - af)^2)(a + bx + cx^2)}$$

Mathematica [A] time = 1.84, size = 523, normalized size = 0.88

$$\frac{f \log(d + f x^2) \operatorname{Ei}\left(\frac{f^2 x^2 - 2 a c d f + 2 A b (c d - a f)}{b^2 - 4 a c}\right) + f \log(a + b x + c x^2) \operatorname{Ei}\left(\frac{f^2 x^2 - 2 a c d f + 2 A b (c d - a f)}{b^2 - 4 a c}\right) - \frac{2 \sqrt{d f + (c d - a f)^2} \operatorname{ArcTan}\left(\frac{f x \sqrt{d f + (c d - a f)^2}}{d + f x^2}\right) + \frac{2 \sqrt{d f + (c d - a f)^2} \operatorname{ArcTan}\left(\frac{f x \sqrt{d f + (c d - a f)^2}}{d + f x^2}\right)}{2 \sqrt{d f + (c d - a f)^2}}}{(b^2 - 4 a c)(b^2 d f + (c d - a f)^2)(a + b x + c x^2)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x)/((a + b*x + c*x^2)^2*(d + f*x^2)),x]

[Out] $((-2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))*(A*(b^3*f + b*c*(c*d - 3*a*f) + b^2*c*f*x + 2*c^2*(c*d - a*f)*x) + B*(2*a^2*c*f - b*c^2*d*x - a*(2*c^2*d + b^2*f + b*c*f*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*f^(3/2)*(-A*b^2*d*f + A*(c*d - a*f)^2 + 2*b*B*d*(-(c*d) + a*f))*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/Sqrt[d] - (2*(b^5*B*d*f^2 - 4*A*c^2*(c*d - 3*a*f)*(c*d - a*f)^2 + 2*A*b^4*f^2*(-(c*d) + a*f) - b^3*B*f*(-5*c^2*d^2 + 4*a*c*d*f + a^2*f^2) - 4*A*b^2*c*f*(2*c^2*d^2 - 3*a*c*d*f + 3*a^2*f^2) + 2*b*B*c*(c^3*d^3 - 7*a*c^2*d^2*f + 3*a^2*c*d*f^2 + 3*a^3*f^3))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + f*(2*A*b*f*(c*d - a*f) + B*(c^2*d^2 - 2*a*c*d*f + f*(-(b^2*d) + a^2*f)))*Log[d + f*x^2] + f*(2*A*b*f*(-(c*d) + a*f) + B*(-(c^2*d^2) + 2*a*c*d*f + f*(b^2*d - a^2*f)))*Log[a + x*(b + c*x)]/(2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))^2)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx}{(a + bx + cx^2)^2 (d + fx^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x)/((a + b*x + c*x^2)^2*(d + f*x^2)),x]

[Out] IntegrateAlgebraic[(A + B*x)/((a + b*x + c*x^2)^2*(d + f*x^2)), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [B] time = 0.21, size = 1313, normalized size = 2.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="giac")

[Out]
$$-1/2*(B*c^2*d^2*f - B*b^2*d*f^2 - 2*B*a*c*d*f^2 + 2*A*b*c*d*f^2 + B*a^2*f^3 - 2*A*a*b*f^3)*\log(c*x^2 + b*x + a)/(c^4*d^4 + 2*b^2*c^2*d^3*f - 4*a*c^3*d^3*f + b^4*d^2*f^2 - 4*a*b^2*c*d^2*f^2 + 6*a^2*c^2*d^2*f^2 + 2*a^2*b^2*d*f^3 - 4*a^3*c*d*f^3 + a^4*f^4) + 1/2*(B*c^2*d^2*f - B*b^2*d*f^2 - 2*B*a*c*d*f^2 + 2*A*b*c*d*f^2 + B*a^2*f^3 - 2*A*a*b*f^3)*\log(f*x^2 + d)/(c^4*d^4 + 2*b^2*c^2*d^3*f - 4*a*c^3*d^3*f + b^4*d^2*f^2 - 4*a*b^2*c*d^2*f^2 + 6*a^2*c^2*d^2*f^2 + 2*a^2*b^2*d*f^3 - 4*a^3*c*d*f^3 + a^4*f^4) - (2*B*b*c*d^2*f^2 - A*c^2*d^2*f^2 - 2*B*a*b*d*f^3 + A*b^2*d*f^3 + 2*A*a*c*d*f^3 - A*a^2*f^4)*\arctan(f*x/\sqrt{d*f})/((c^4*d^4 + 2*b^2*c^2*d^3*f - 4*a*c^3*d^3*f + b^4*d^2*f^2 - 4*a*b^2*c*d^2*f^2 + 6*a^2*c^2*d^2*f^2 + 2*a^2*b^2*d*f^3 - 4*a^3*c*d*f^3 + a^4*f^4)*\sqrt{d*f}) + (2*B*b*c^4*d^3 - 4*A*c^5*d^3 + 5*B*b^3*c^2*d^2*f - 14*B*a*b*c^3*d^2*f - 8*A*b^2*c^3*d^2*f + 20*A*a*c^4*d^2*f + B*b^5*d*f^2 - 4*B*a*b^3*c*d*f^2 - 2*A*b^4*c*d*f^2 + 6*B*a^2*b*c^2*d*f^2 + 12*A*a*b^2*c^2*d*f^2 - 28*A*a^2*c^3*d*f^2 - B*a^2*b^3*f^3 + 2*A*a*b^4*f^3 + 6*B*a^3*b*c*f^3 - 12*A*a^2*b^2*c*f^3 + 12*A*a^3*c^2*f^3)*\arctan((2*c*x + b)/\sqrt{-b^2 + 4*a*c})/((b^2*c^4*d^4 - 4*a*c^5*d^4 + 2*b^4*c^2*d^3*f - 12*a*b^2*c^3*d^3*f + 16*a^2*c^4*d^3*f + b^6*d^2*f^2 - 8*a*b^4*c*d^2*f^2 + 22*a^2*b^2*c^2*d^2*f^2 - 24*a^3*c^3*d^2*f^2 + 2*a^2*b^4*d*f^3 - 12*a^3*b^2*c*d*f^3 + 16*a^4*c^2*d*f^3 + a^4*b^2*f^4 - 4*a^5*c*f^4)*\sqrt{-b^2 + 4*a*c}) + (2*B*a*c^4*d^3 - A*b*c^4*d^3 + 3*B*a*b^2*c^2*d^2*f - 2*A*b^3*c^2*d^2*f - 6*B*a^2*c^3*d^2*f + 5*A*a*b*c^3*d^2*f + B*a*b^4*d*f^2 - A*b^5*d*f^2 - 4*B*a^2*b^2*c*d*f^2 + 5*A*a*b^3*c*d*f^2 + 6*B*a^3*c^2*d*f^2 - 7*A*a^2*b*c^2*d*f^2 + B*a^3*b^2*f^3 - A*a^2*b^3*f^3 - 2*B*a^4*c*f^3 + 3*A*a^3*b*c*f^3 + (B*b*c^4*d^3 - 2*A*c^5*d^3 + B*b^3*c^2*d^2*f - B*a*b*c^3*d^2*f - 3*A*b^2*c^3*d^2*f + 6*A*a*c^4*d^2*f + B*a*b^3*c*d*f^2 - A*b^4*c*d*f^2 - B*a^2*b*c^2*d*f^2 + 4*A*a*b^2*c^2*d*f^2 - 6*A*a^2*c^3*d*f^2 + B*a^3*b*c*f^3 - A*a^2*b^2*c*f^3 + 2*A*a^3*c^2*f^3)*x)/((c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2)^2*(c*x^2 + b*x + a)*(b^2 - 4*a*c))$$

maple [B] time = 0.06, size = 9311, normalized size = 15.62

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+d),x)

[Out] result too large to display

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 7.53, size = 23006, normalized size = 38.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((A + Bx)/((d + fx^2)(a + bx + cx^2)^2), x)$

[Out]
$$\frac{(A^2b^3c^2f + A^2b^2c^2d - 2A^2b^2c^2d - B^2a^2c^2f - B^2a^2b^2f + 2A^2b^2c^2f - 3A^2a^2b^2c^2f)}{(4a^2c - b^2)(a^2f^2 + c^2d^2 + b^2d^2f - 2a^2c^2d^2f)} - \frac{(x(2A^2a^2c^2f - 2A^2c^3d + B^2b^2c^2d - A^2b^2c^2f + B^2a^2b^2c^2f))}{(4a^2c - b^2)(a^2f^2 + c^2d^2 + b^2d^2f - 2a^2c^2d^2f)} + \frac{\text{symsum}(\log((x(4A^3b^3c^4f^6 + 16B^3a^3c^4f^6 - 3B^3a^2b^2c^3f^6 + B^3b^2c^5d^2f^4 - 16A^3a^2b^2c^5f^6 + 20A^2B^2a^2c^5f^6 - 3A^2B^2b^4c^3f^6 + 4A^2B^2c^7d^2f^4 - 16B^3a^2c^5d^2f^5 + 6B^3a^2b^2c^4d^2f^5 - 24A^2B^2a^2c^6d^2f^5 + 6A^2B^2a^2b^3c^3f^6 - 28A^2B^2a^2b^2c^4d^2f^6 + 8A^2B^2a^2b^2c^4d^2f^6 - 4A^2B^2b^2c^6d^2f^4 - 6A^2B^2b^3c^4d^2f^5 + 8A^2B^2b^2c^5d^2f^5 + 16A^2B^2a^2b^2c^5d^2f^5))}{(16a^2c^6d^4 + a^4b^4d^4 + b^4c^4d^4 + 16a^6c^2d^2f^4 + b^8d^2f^2 - 8a^2b^2c^5d^4 - 8a^5b^2c^2d^3f + 2a^2b^6d^2f^3 - 64a^3c^5d^3f - 64a^5c^3d^2f^3 + 2b^6c^2d^3f + 96a^4c^4d^2f^2 + 54a^2b^4c^2d^2f^2 - 112a^3b^2c^3d^2f^2 - 20a^2b^4c^3d^3f - 12a^2b^6c^2d^2f^2 - 20a^3b^4c^2d^3f + 64a^4b^2c^2d^2f^3) - \text{root}(2560a^3b^2c^9d^8f^2z^4 - 1152a^2b^4c^8d^8f^2z^4 + 384a^5b^8c^3d^3f^6z^4 + 384a^2b^8c^5d^7f^2z^4 + 288a^3b^10c^3d^4f^5z^4 + 288a^2b^10c^3d^6f^3z^4 + 224a^7b^6c^2d^2f^7z^4 - 192a^10b^2c^2d^2f^8z^4 + 224a^2b^6c^7d^8f^2z^4 + 80a^2b^12c^2d^5f^4z^4 + 48a^9b^4c^2d^2f^8z^4 - 33920a^6b^2c^6d^5f^4z^4 + 27936a^5b^4c^5d^5f^4z^4 + 26112a^7b^2c^5d^4f^5z^4 + 26112a^5b^2c^7d^6f^3z^4 - 20352a^6b^4c^4d^4f^5z^4 - 20352a^4b^4c^6d^6f^3z^4 - 13080a^4b^6c^4d^5f^4z^4 - 11520a^8b^2c^4d^3f^6z^4 - 11520a^4b^2c^8d^7f^2z^4 + 8736a^5b^6c^3d^4f^5z^4 + 8736a^3b^6c^5d^6f^3z^4 + 7488a^7b^4c^3d^3f^6z^4 + 7488a^3b^4c^7d^7f^2z^4 + 3840a^3b^8c^3d^5f^4z^4 + 2560a^9b^2c^3d^2f^7z^4 - 2416a^6b^6c^2d^3f^6z^4 - 2416a^2b^6c^6d^7f^2z^4 - 2160a^4b^8c^2d^4f^5z^4 - 2160a^2b^8c^4d^6f^3z^4 - 1152a^8b^4c^2d^2f^7z^4 - 720a^2b^10c^2d^5f^4z^4 - 16b^8c^6d^8f^2z^4 - 2048a^4c^10d^8f^2z^4 + 256a^11c^3d^2f^8z^4 - 4a^8b^6d^2f^8z^4 + 48a^2b^4c^9d^9z^4 - 24b^10c^4d^7f^2z^4 - 16b^12c^2d^6f^3z^4 + 17920a^7c^7d^5f^4z^4 - 14336a^8c^6d^4f^5z^4 - 14336a^6c^8d^6f^3z^4 + 7168a^9c^5d^3f^6z^4 + 7168a^5c^9d^7f^2z^4 - 2048a^10c^4d^2f^7z^4 - 24a^4b^10d^3f^6z^4 - 16a^6b^8d^2f^7z^4 - 16a^2b^12d^4f^5z^4 - 192a^2b^2c^10d^9z^4 - 4b^14d^5f^4z^4 - 4b^6c^8d^9z^4 + 256a^3c^11d^9z^4 + 912A^2B^2a^6b^2c^3d^2f^6z^2 + 192A^2B^2a^4b^5c^2d^2f^6z^2 + 920A^2B^2a^4b^3c^3d^2f^5z^2 - 480A^2B^2a^2b^5c^3d^3f^4z^2 - 336A^2B^2a^2b^3c^5d^4f^3z^2 - 272A^2B^2a^3b^3c^4d^3f^4z^2 + 240A^2B^2a^3b^5c^2d^2f^5z^2 + 192A^2B^2a^2b^2c^8d^6f^2z^2 - 2496A^2B^2a^5b^2c^4d^2f^5z^2 + 1872A^2B^2a^4b^2c^5d^3f^4z^2 - 744A^2B^2a^5b^3c^2d^2f^6z^2 - 720A^2B^2a^2b^2c^7d^5f^2z^2 + 504A^2B^2a^2b^3c^6d^5f^2z^2 + 256A^2B^2a^3b^2c^6d^4f^3z^2 + 168A^2B^2a^2b^7c^2d^3f^4z^2 - 144A^2B^2a^2b^7c^2d^2f^5z^2 + 144A^2B^2a^2b^5c^4d^4f^3z^2 - 56B^2a^2b^2c^7d^6f^2z^2 - 36B^2a^5b^4c^2d^2f^6z^2 - 16B^2a^2b^8c^2d^3f^4z^2 - 164A^2a^3b^6c^2d^2f^6z^2 - 16A^2a^2b^8c^2d^2f^5z^2 - 96A^2B^2b^5c^5d^5f^2z^2 - 24A^2B^2b^7c^3d^4f^3z^2 - 580B^2a^4b^2c^4d^3f^4z^2 + 536B^2a^3b^4c^3d^3f^4z^2 - 348B^2a^4b^4c^2d^2f^5z^2 + 316B^2a^2b^2c^6d^5f^2z^2 + 200B^2a^5b^2c^3d^2f^5z^2 - 120B^2a^2b^4c^4d^4f^3z^2 - 66B^2a^2b^6c^2d^3f^4z^2 - 16B^2a^3b^2c^5d^4f^3z^2 + 1952A^2a^4b^2c^4d^2f^5z^2 - 1792A^2a^3b^2c^5d^3f^4z^2 - 1272A^2a^3b^4c^3d^2f^5z^2 + 976A^2a^2b^2c^6d^4f^3z^2 + 960A^2a^2b^4c^4d^3f^4z^2 + 282A^2a^2b^6c^2d^2f^5z^2 - 72A^2B^2b^3c^7d^6f^2z^2 - 16A^2B^2b^9c^2d^3f^4z^2 - 16A^2B^2a^3b^7d^2f^6z^2 + 16A^2B^2a^2b^9d^2f^5z^2 - 180B^2a^2b^4c^5d^5f^2z^2 + 132B^2a^6b^2c^2d^2f^6z^2 + 108B^2a^3b^6c^2d^2f^5z^2 + 20B^2a^2b^6c^3d^4f^3z^2 - 736A^2a^5b^2c^3d^2f^6z^2$$

$$\begin{aligned}
& + 624*A^2*a^4*b^4*c^2*d*f^6*z^2 - 416*A^2*a*b^2*c^7*d^5*f^2*z^2 - 276*A^2* \\
& a*b^4*c^5*d^4*f^3*z^2 - 196*A^2*a*b^6*c^3*d^3*f^4*z^2 + 31*B^2*b^6*c^4*d^5* \\
& f^2*z^2 + 2*B^2*b^8*c^2*d^4*f^3*z^2 - 768*B^2*a^5*c^5*d^3*f^4*z^2 + 512*B^2 \\
& *a^6*c^4*d^2*f^5*z^2 + 512*B^2*a^4*c^6*d^4*f^3*z^2 - 128*B^2*a^3*c^7*d^5*f^ \\
& 2*z^2 + 80*A^2*b^4*c^6*d^5*f^2*z^2 + 31*A^2*b^6*c^4*d^4*f^3*z^2 + 14*A^2*b^ \\
& 8*c^2*d^3*f^4*z^2 - 1152*A^2*a^3*c^7*d^4*f^3*z^2 + 1008*A^2*a^4*c^6*d^3*f^4 \\
& *z^2 + 624*A^2*a^2*c^8*d^5*f^2*z^2 - 288*A^2*a^5*c^5*d^2*f^5*z^2 - 10*B^2*a \\
& ^2*b^8*d^2*f^5*z^2 - 48*A^2*a^6*b^2*c^2*f^7*z^2 - 16*A*B*b*c^9*d^7*z^2 + 20 \\
& *B^2*b^4*c^6*d^6*f*z^2 - 128*B^2*a^7*c^3*d*f^6*z^2 + 64*A^2*b^2*c^8*d^6*f*z \\
& ^2 - 112*A^2*a^6*c^4*d*f^6*z^2 + 3*B^2*a^4*b^6*d*f^6*z^2 + 14*A^2*a^2*b^8*d \\
& *f^6*z^2 + 12*A^2*a^5*b^4*c*f^7*z^2 - 160*A^2*a*c^9*d^6*f*z^2 + 3*B^2*b^10* \\
& d^3*f^4*z^2 - A^2*b^10*d^2*f^5*z^2 + 64*A^2*a^7*c^3*f^7*z^2 + 4*B^2*b^2*c^8 \\
& *d^7*z^2 - A^2*a^4*b^6*f^7*z^2 + 16*A^2*c^10*d^7*z^2 - 160*A*B^2*a*b*c^6*d^ \\
& 4*f^2*z + 112*A*B^2*a^4*b*c^3*d*f^5*z - 24*A*B^2*a^2*b^5*c*d*f^5*z + 480*A^ \\
& 2*B*a^2*b^2*c^4*d^2*f^4*z - 176*A*B^2*a^2*b^3*c^3*d^2*f^4*z - 10*A^2*B*a*b^ \\
& 6*c*d*f^5*z + 384*A*B^2*a^2*b*c^5*d^3*f^3*z - 352*A*B^2*a^3*b*c^4*d^2*f^4*z \\
& - 288*A^2*B*a*b^2*c^5*d^3*f^3*z - 160*A^2*B*a^3*b^2*c^3*d*f^5*z - 148*A^2* \\
& B*a*b^4*c^3*d^2*f^4*z + 112*A*B^2*a*b^3*c^4*d^3*f^3*z + 72*A^2*B*a^2*b^4*c^ \\
& 2*d*f^5*z + 72*A*B^2*a*b^5*c^2*d^2*f^4*z + 48*A*B^2*a^3*b^3*c^2*d*f^5*z + 4 \\
& 8*B^3*a*b^2*c^5*d^4*f^2*z - 36*B^3*a^4*b^2*c^2*d*f^5*z - 4*B^3*a*b^4*c^3*d^ \\
& 3*f^3*z - 480*A^3*a^2*b*c^5*d^2*f^4*z - 160*A^3*a^2*b^3*c^3*d*f^5*z + 128*A \\
& ^3*a*b^3*c^4*d^2*f^4*z + 112*A^2*B*b^4*c^4*d^3*f^3*z - 64*A*B^2*b^5*c^3*d^3 \\
& *f^3*z + 16*A^2*B*b^2*c^6*d^4*f^2*z + 16*A*B^2*b^3*c^5*d^4*f^2*z - A^2*B*b^ \\
& 6*c^2*d^2*f^4*z + 448*A^2*B*a^3*c^5*d^2*f^4*z - 352*A^2*B*a^2*c^6*d^3*f^3*z \\
& - 48*A^2*B*a^4*b^2*c^2*f^6*z + 12*B^3*a^3*b^4*c*d*f^5*z - 10*B^3*a*b^6*c*d \\
& ^2*f^4*z + 416*A^3*a^3*b*c^4*d*f^5*z + 224*A^3*a*b*c^6*d^3*f^3*z + 24*A^3*a \\
& *b^5*c^2*d*f^5*z - 2*A*B^2*b^7*c*d^2*f^4*z - 272*A^2*B*a^4*c^4*d*f^5*z + 12 \\
& 8*A^2*B*a*c^7*d^4*f^2*z + 12*A^2*B*a^3*b^4*c*f^6*z - 120*B^3*a^2*b^2*c^4*d^ \\
& 3*f^3*z + 112*B^3*a^3*b^2*c^3*d^2*f^4*z + 16*A*B^2*b*c^7*d^5*f*z + 2*A*B^2* \\
& a*b^7*d*f^5*z - 2*A^3*b^7*c*d*f^5*z - 16*A^2*B*c^8*d^5*f*z + 11*B^3*b^6*c^2 \\
& *d^3*f^3*z - 8*B^3*b^4*c^4*d^4*f^2*z - 64*A^3*b^3*c^5*d^3*f^3*z + 96*A^3*a^ \\
& 3*b^3*c^2*f^6*z - 4*B^3*b^2*c^6*d^5*f*z - 32*A^3*b*c^7*d^4*f^2*z - B^3*a^2* \\
& b^6*d*f^5*z - 128*A^3*a^4*b*c^3*f^6*z - 24*A^3*a^2*b^5*c*f^6*z + 64*A^2*B*a \\
& ^5*c^3*f^6*z - A^2*B*a^2*b^6*f^6*z + A^2*B*b^8*d*f^5*z + 2*A^3*a*b^7*f^6*z \\
& + B^3*b^8*d^2*f^4*z + 32*A^3*B*a*b*c^4*d*f^4 - 18*A^2*B^2*a*b^2*c^3*d*f^4 + \\
& 32*A*B^3*a*b*c^4*d^2*f^3 - 28*A*B^3*a^2*b*c^3*d*f^4 + 6*A*B^3*a*b^3*c^2*d* \\
& f^4 - 10*A^3*B*b^3*c^3*d*f^4 - 4*A^3*B*b*c^5*d^2*f^3 - 4*A*B^3*b*c^5*d^3*f^ \\
& 2 - 28*A^3*B*a^2*b*c^3*f^5 + 6*A^3*B*a*b^3*c^2*f^5 + 9*A^2*B^2*b^2*c^4*d^2* \\
& f^3 - 3*A^2*B^2*a^2*b^2*c^2*f^5 - 10*B^4*a*b^2*c^3*d^2*f^3 - 3*B^4*a^2*b^2* \\
& c^2*d*f^4 - 10*A*B^3*b^3*c^3*d^2*f^3 + 3*A^2*B^2*b^4*c^2*d*f^4 + 36*A^2*B^2 \\
& *a^2*c^4*d*f^4 - 24*A^2*B^2*a*c^5*d^2*f^3 + 4*A^2*B^2*c^6*d^3*f^2 + 16*A^2* \\
& B^2*a^3*c^3*f^5 + 16*B^4*a^3*c^3*d*f^4 + 8*A^4*b^2*c^4*d*f^4 - 8*A^4*a*b^2* \\
& c^3*f^5 - 24*A^4*a*c^5*d*f^4 + 3*B^4*b^4*c^2*d^2*f^3 + 4*A^4*c^6*d^2*f^3 + \\
& 36*A^4*a^2*c^4*f^5 + B^4*b^2*c^4*d^3*f^2, z, k)*(root(2560*a^3*b^2*c^9*d^8* \\
& f*z^4 - 1152*a^2*b^4*c^8*d^8*f*z^4 + 384*a^5*b^8*c*d^3*f^6*z^4 + 384*a*b^8* \\
& c^5*d^7*f^2*z^4 + 288*a^3*b^10*c*d^4*f^5*z^4 + 288*a*b^10*c^3*d^6*f^3*z^4 + \\
& 224*a^7*b^6*c*d^2*f^7*z^4 - 192*a^10*b^2*c^2*d*f^8*z^4 + 224*a*b^6*c^7*d^8 \\
& *f*z^4 + 80*a*b^12*c*d^5*f^4*z^4 + 48*a^9*b^4*c*d*f^8*z^4 - 33920*a^6*b^2*c \\
& ^6*d^5*f^4*z^4 + 27936*a^5*b^4*c^5*d^5*f^4*z^4 + 26112*a^7*b^2*c^5*d^4*f^5* \\
& z^4 + 26112*a^5*b^2*c^7*d^6*f^3*z^4 - 20352*a^6*b^4*c^4*d^4*f^5*z^4 - 20352 \\
& *a^4*b^4*c^6*d^6*f^3*z^4 - 13080*a^4*b^6*c^4*d^5*f^4*z^4 - 11520*a^8*b^2*c^ \\
& 4*d^3*f^6*z^4 - 11520*a^4*b^2*c^8*d^7*f^2*z^4 + 8736*a^5*b^6*c^3*d^4*f^5*z^ \\
& 4 + 8736*a^3*b^6*c^5*d^6*f^3*z^4 + 7488*a^7*b^4*c^3*d^3*f^6*z^4 + 7488*a^3* \\
& b^4*c^7*d^7*f^2*z^4 + 3840*a^3*b^8*c^3*d^5*f^4*z^4 + 2560*a^9*b^2*c^3*d^2*f \\
& ^7*z^4 - 2416*a^6*b^6*c^2*d^3*f^6*z^4 - 2416*a^2*b^6*c^6*d^7*f^2*z^4 - 2160 \\
& *a^4*b^8*c^2*d^4*f^5*z^4 - 2160*a^2*b^8*c^4*d^6*f^3*z^4 - 1152*a^8*b^4*c^2* \\
& d^2*f^7*z^4 - 720*a^2*b^10*c^2*d^5*f^4*z^4 - 16*b^8*c^6*d^8*f*z^4 - 2048*a^ \\
& 4*c^10*d^8*f*z^4 + 256*a^11*c^3*d*f^8*z^4 - 4*a^8*b^6*d*f^8*z^4 + 48*a*b^4* \\
& c^9*d^9*z^4 - 24*b^10*c^4*d^7*f^2*z^4 - 16*b^12*c^2*d^6*f^3*z^4 + 17920*a^7
\end{aligned}$$

$$\begin{aligned}
& *c^7*d^5*f^4*z^4 - 14336*a^8*c^6*d^4*f^5*z^4 - 14336*a^6*c^8*d^6*f^3*z^4 + \\
& 7168*a^9*c^5*d^3*f^6*z^4 + 7168*a^5*c^9*d^7*f^2*z^4 - 2048*a^10*c^4*d^2*f^7 \\
& *z^4 - 24*a^4*b^10*d^3*f^6*z^4 - 16*a^6*b^8*d^2*f^7*z^4 - 16*a^2*b^12*d^4*f \\
& ^5*z^4 - 192*a^2*b^2*c^10*d^9*z^4 - 4*b^14*d^5*f^4*z^4 - 4*b^6*c^8*d^9*z^4 \\
& + 256*a^3*c^11*d^9*z^4 + 912*A*B*a^6*b*c^3*d*f^6*z^2 + 192*A*B*a^4*b^5*c*d* \\
& f^6*z^2 + 920*A*B*a^4*b^3*c^3*d^2*f^5*z^2 - 480*A*B*a^2*b^5*c^3*d^3*f^4*z^2 \\
& - 336*A*B*a^2*b^3*c^5*d^4*f^3*z^2 - 272*A*B*a^3*b^3*c^4*d^3*f^4*z^2 + 240* \\
& A*B*a^3*b^5*c^2*d^2*f^5*z^2 + 192*A*B*a*b*c^8*d^6*f*z^2 - 2496*A*B*a^5*b*c^ \\
& 4*d^2*f^5*z^2 + 1872*A*B*a^4*b*c^5*d^3*f^4*z^2 - 744*A*B*a^5*b^3*c^2*d*f^6* \\
& z^2 - 720*A*B*a^2*b*c^7*d^5*f^2*z^2 + 504*A*B*a*b^3*c^6*d^5*f^2*z^2 + 256*A \\
& *B*a^3*b*c^6*d^4*f^3*z^2 + 168*A*B*a*b^7*c^2*d^3*f^4*z^2 - 144*A*B*a^2*b^7* \\
& c*d^2*f^5*z^2 + 144*A*B*a*b^5*c^4*d^4*f^3*z^2 - 56*B^2*a*b^2*c^7*d^6*f*z^2 \\
& - 36*B^2*a^5*b^4*c*d*f^6*z^2 - 16*B^2*a*b^8*c*d^3*f^4*z^2 - 164*A^2*a^3*b^6 \\
& *c*d*f^6*z^2 - 16*A^2*a*b^8*c*d^2*f^5*z^2 - 96*A*B*b^5*c^5*d^5*f^2*z^2 - 24 \\
& *A*B*b^7*c^3*d^4*f^3*z^2 - 580*B^2*a^4*b^2*c^4*d^3*f^4*z^2 + 536*B^2*a^3*b^ \\
& 4*c^3*d^3*f^4*z^2 - 348*B^2*a^4*b^4*c^2*d^2*f^5*z^2 + 316*B^2*a^2*b^2*c^6*d \\
& ^5*f^2*z^2 + 200*B^2*a^5*b^2*c^3*d^2*f^5*z^2 - 120*B^2*a^2*b^4*c^4*d^4*f^3* \\
& z^2 - 66*B^2*a^2*b^6*c^2*d^3*f^4*z^2 - 16*B^2*a^3*b^2*c^5*d^4*f^3*z^2 + 195 \\
& 2*A^2*a^4*b^2*c^4*d^2*f^5*z^2 - 1792*A^2*a^3*b^2*c^5*d^3*f^4*z^2 - 1272*A^2 \\
& *a^3*b^4*c^3*d^2*f^5*z^2 + 976*A^2*a^2*b^2*c^6*d^4*f^3*z^2 + 960*A^2*a^2*b^ \\
& 4*c^4*d^3*f^4*z^2 + 282*A^2*a^2*b^6*c^2*d^2*f^5*z^2 - 72*A*B*b^3*c^7*d^6*f* \\
& z^2 - 16*A*B*b^9*c*d^3*f^4*z^2 - 16*A*B*a^3*b^7*d*f^6*z^2 + 16*A*B*a*b^9*d^ \\
& 2*f^5*z^2 - 180*B^2*a*b^4*c^5*d^5*f^2*z^2 + 132*B^2*a^6*b^2*c^2*d*f^6*z^2 + \\
& 108*B^2*a^3*b^6*c*d^2*f^5*z^2 + 20*B^2*a*b^6*c^3*d^4*f^3*z^2 - 736*A^2*a^5 \\
& *b^2*c^3*d*f^6*z^2 + 624*A^2*a^4*b^4*c^2*d*f^6*z^2 - 416*A^2*a*b^2*c^7*d^5* \\
& f^2*z^2 - 276*A^2*a*b^4*c^5*d^4*f^3*z^2 - 196*A^2*a*b^6*c^3*d^3*f^4*z^2 + 3 \\
& 1*B^2*b^6*c^4*d^5*f^2*z^2 + 2*B^2*b^8*c^2*d^4*f^3*z^2 - 768*B^2*a^5*c^5*d^3 \\
& *f^4*z^2 + 512*B^2*a^6*c^4*d^2*f^5*z^2 + 512*B^2*a^4*c^6*d^4*f^3*z^2 - 128* \\
& B^2*a^3*c^7*d^5*f^2*z^2 + 80*A^2*b^4*c^6*d^5*f^2*z^2 + 31*A^2*b^6*c^4*d^4*f \\
& ^3*z^2 + 14*A^2*b^8*c^2*d^3*f^4*z^2 - 1152*A^2*a^3*c^7*d^4*f^3*z^2 + 1008*A \\
& ^2*a^4*c^6*d^3*f^4*z^2 + 624*A^2*a^2*c^8*d^5*f^2*z^2 - 288*A^2*a^5*c^5*d^2* \\
& f^5*z^2 - 10*B^2*a^2*b^8*d^2*f^5*z^2 - 48*A^2*a^6*b^2*c^2*f^7*z^2 - 16*A*B* \\
& b*c^9*d^7*z^2 + 20*B^2*b^4*c^6*d^6*f*z^2 - 128*B^2*a^7*c^3*d*f^6*z^2 + 64*A \\
& ^2*b^2*c^8*d^6*f*z^2 - 112*A^2*a^6*c^4*d*f^6*z^2 + 3*B^2*a^4*b^6*d*f^6*z^2 \\
& + 14*A^2*a^2*b^8*d*f^6*z^2 + 12*A^2*a^5*b^4*c*f^7*z^2 - 160*A^2*a*c^9*d^6*f \\
& *z^2 + 3*B^2*b^10*d^3*f^4*z^2 - A^2*b^10*d^2*f^5*z^2 + 64*A^2*a^7*c^3*f^7*z \\
& ^2 + 4*B^2*b^2*c^8*d^7*z^2 - A^2*a^4*b^6*f^7*z^2 + 16*A^2*c^10*d^7*z^2 - 16 \\
& 0*A*B^2*a*b*c^6*d^4*f^2*z + 112*A*B^2*a^4*b*c^3*d*f^5*z - 24*A*B^2*a^2*b^5* \\
& c*d*f^5*z + 480*A^2*B*a^2*b^2*c^4*d^2*f^4*z - 176*A*B^2*a^2*b^3*c^3*d^2*f^4 \\
& *z - 10*A^2*B*a*b^6*c*d*f^5*z + 384*A*B^2*a^2*b*c^5*d^3*f^3*z - 352*A*B^2*a \\
& ^3*b*c^4*d^2*f^4*z - 288*A^2*B*a*b^2*c^5*d^3*f^3*z - 160*A^2*B*a^3*b^2*c^3* \\
& d*f^5*z - 148*A^2*B*a*b^4*c^3*d^2*f^4*z + 112*A*B^2*a*b^3*c^4*d^3*f^3*z + 7 \\
& 2*A^2*B*a^2*b^4*c^2*d*f^5*z + 72*A*B^2*a*b^5*c^2*d^2*f^4*z + 48*A*B^2*a^3*b \\
& ^3*c^2*d*f^5*z + 48*B^3*a*b^2*c^5*d^4*f^2*z - 36*B^3*a^4*b^2*c^2*d*f^5*z - \\
& 4*B^3*a*b^4*c^3*d^3*f^3*z - 480*A^3*a^2*b*c^5*d^2*f^4*z - 160*A^3*a^2*b^3*c \\
& ^3*d*f^5*z + 128*A^3*a*b^3*c^4*d^2*f^4*z + 112*A^2*B*b^4*c^4*d^3*f^3*z - 64 \\
& *A*B^2*b^5*c^3*d^3*f^3*z + 16*A^2*B*b^2*c^6*d^4*f^2*z + 16*A*B^2*b^3*c^5*d^ \\
& 4*f^2*z - A^2*B*b^6*c^2*d^2*f^4*z + 448*A^2*B*a^3*c^5*d^2*f^4*z - 352*A^2*B \\
& *a^2*c^6*d^3*f^3*z - 48*A^2*B*a^4*b^2*c^2*f^6*z + 12*B^3*a^3*b^4*c*d*f^5*z \\
& - 10*B^3*a*b^6*c*d^2*f^4*z + 416*A^3*a^3*b*c^4*d*f^5*z + 224*A^3*a*b*c^6*d^ \\
& 3*f^3*z + 24*A^3*a*b^5*c^2*d*f^5*z - 2*A*B^2*b^7*c*d^2*f^4*z - 272*A^2*B*a^ \\
& 4*c^4*d*f^5*z + 128*A^2*B*a*c^7*d^4*f^2*z + 12*A^2*B*a^3*b^4*c*f^6*z - 120* \\
& B^3*a^2*b^2*c^4*d^3*f^3*z + 112*B^3*a^3*b^2*c^3*d^2*f^4*z + 16*A*B^2*b*c^7* \\
& d^5*f*z + 2*A*B^2*a*b^7*d*f^5*z - 2*A^3*b^7*c*d*f^5*z - 16*A^2*B*c^8*d^5*f* \\
& z + 11*B^3*b^6*c^2*d^3*f^3*z - 8*B^3*b^4*c^4*d^4*f^2*z - 64*A^3*b^3*c^5*d^3 \\
& *f^3*z + 96*A^3*a^3*b^3*c^2*f^6*z - 4*B^3*b^2*c^6*d^5*f*z - 32*A^3*b*c^7*d^ \\
& 4*f^2*z - B^3*a^2*b^6*d*f^5*z - 128*A^3*a^4*b*c^3*f^6*z - 24*A^3*a^2*b^5*c \\
& f^6*z + 64*A^2*B*a^5*c^3*f^6*z - A^2*B*a^2*b^6*f^6*z + A^2*B*b^8*d*f^5*z + \\
& 2*A^3*a*b^7*f^6*z + B^3*b^8*d^2*f^4*z + 32*A^3*B*a*b*c^4*d*f^4 - 18*A^2*B^2
\end{aligned}$$

$$\begin{aligned}
& *a*b^2*c^3*d*f^4 + 32*A*B^3*a*b*c^4*d^2*f^3 - 28*A*B^3*a^2*b*c^3*d*f^4 + 6* \\
& A*B^3*a*b^3*c^2*d*f^4 - 10*A^3*B*b^3*c^3*d*f^4 - 4*A^3*B*b*c^5*d^2*f^3 - 4* \\
& A*B^3*b*c^5*d^3*f^2 - 28*A^3*B*a^2*b*c^3*f^5 + 6*A^3*B*a*b^3*c^2*f^5 + 9*A^ \\
& 2*B^2*b^2*c^4*d^2*f^3 - 3*A^2*B^2*a^2*b^2*c^2*f^5 - 10*B^4*a*b^2*c^3*d^2*f^ \\
& 3 - 3*B^4*a^2*b^2*c^2*d*f^4 - 10*A*B^3*b^3*c^3*d^2*f^3 + 3*A^2*B^2*b^4*c^2* \\
& d*f^4 + 36*A^2*B^2*a^2*c^4*d*f^4 - 24*A^2*B^2*a*c^5*d^2*f^3 + 4*A^2*B^2*c^6 \\
& *d^3*f^2 + 16*A^2*B^2*a^3*c^3*f^5 + 16*B^4*a^3*c^3*d*f^4 + 8*A^4*b^2*c^4*d* \\
& f^4 - 8*A^4*a*b^2*c^3*f^5 - 24*A^4*a*c^5*d*f^4 + 3*B^4*b^4*c^2*d^2*f^3 + 4* \\
& A^4*c^6*d^2*f^3 + 36*A^4*a^2*c^4*f^5 + B^4*b^2*c^4*d^3*f^2, z, k)*(root(256 \\
& 0*a^3*b^2*c^9*d^8*f*z^4 - 1152*a^2*b^4*c^8*d^8*f*z^4 + 384*a^5*b^8*c*d^3*f^ \\
& 6*z^4 + 384*a*b^8*c^5*d^7*f^2*z^4 + 288*a^3*b^10*c*d^4*f^5*z^4 + 288*a*b^10 \\
& *c^3*d^6*f^3*z^4 + 224*a^7*b^6*c*d^2*f^7*z^4 - 192*a^10*b^2*c^2*d*f^8*z^4 + \\
& 224*a*b^6*c^7*d^8*f*z^4 + 80*a*b^12*c*d^5*f^4*z^4 + 48*a^9*b^4*c*d*f^8*z^4 \\
& - 33920*a^6*b^2*c^6*d^5*f^4*z^4 + 27936*a^5*b^4*c^5*d^5*f^4*z^4 + 26112*a^ \\
& 7*b^2*c^5*d^4*f^5*z^4 + 26112*a^5*b^2*c^7*d^6*f^3*z^4 - 20352*a^6*b^4*c^4*d \\
& ^4*f^5*z^4 - 20352*a^4*b^4*c^6*d^6*f^3*z^4 - 13080*a^4*b^6*c^4*d^5*f^4*z^4 \\
& - 11520*a^8*b^2*c^4*d^3*f^6*z^4 - 11520*a^4*b^2*c^8*d^7*f^2*z^4 + 8736*a^5* \\
& b^6*c^3*d^4*f^5*z^4 + 8736*a^3*b^6*c^5*d^6*f^3*z^4 + 7488*a^7*b^4*c^3*d^3*f \\
& ^6*z^4 + 7488*a^3*b^4*c^7*d^7*f^2*z^4 + 3840*a^3*b^8*c^3*d^5*f^4*z^4 + 2560 \\
& *a^9*b^2*c^3*d^2*f^7*z^4 - 2416*a^6*b^6*c^2*d^3*f^6*z^4 - 2416*a^2*b^6*c^6* \\
& d^7*f^2*z^4 - 2160*a^4*b^8*c^2*d^4*f^5*z^4 - 2160*a^2*b^8*c^4*d^6*f^3*z^4 - \\
& 1152*a^8*b^4*c^2*d^2*f^7*z^4 - 720*a^2*b^10*c^2*d^5*f^4*z^4 - 16*b^8*c^6*d \\
& ^8*f*z^4 - 2048*a^4*c^10*d^8*f*z^4 + 256*a^11*c^3*d*f^8*z^4 - 4*a^8*b^6*d*f \\
& ^8*z^4 + 48*a*b^4*c^9*d^9*z^4 - 24*b^10*c^4*d^7*f^2*z^4 - 16*b^12*c^2*d^6*f \\
& ^3*z^4 + 17920*a^7*c^7*d^5*f^4*z^4 - 14336*a^8*c^6*d^4*f^5*z^4 - 14336*a^6* \\
& c^8*d^6*f^3*z^4 + 7168*a^9*c^5*d^3*f^6*z^4 + 7168*a^5*c^9*d^7*f^2*z^4 - 204 \\
& 8*a^10*c^4*d^2*f^7*z^4 - 24*a^4*b^10*d^3*f^6*z^4 - 16*a^6*b^8*d^2*f^7*z^4 - \\
& 16*a^2*b^12*d^4*f^5*z^4 - 192*a^2*b^2*c^10*d^9*z^4 - 4*b^14*d^5*f^4*z^4 - \\
& 4*b^6*c^8*d^9*z^4 + 256*a^3*c^11*d^9*z^4 + 912*A*B*a^6*b*c^3*d*f^6*z^2 + 19 \\
& 2*A*B*a^4*b^5*c*d*f^6*z^2 + 920*A*B*a^4*b^3*c^3*d^2*f^5*z^2 - 480*A*B*a^2*b \\
& ^5*c^3*d^3*f^4*z^2 - 336*A*B*a^2*b^3*c^5*d^4*f^3*z^2 - 272*A*B*a^3*b^3*c^4* \\
& d^3*f^4*z^2 + 240*A*B*a^3*b^5*c^2*d^2*f^5*z^2 + 192*A*B*a*b*c^8*d^6*f*z^2 - \\
& 2496*A*B*a^5*b*c^4*d^2*f^5*z^2 + 1872*A*B*a^4*b*c^5*d^3*f^4*z^2 - 744*A*B* \\
& a^5*b^3*c^2*d*f^6*z^2 - 720*A*B*a^2*b*c^7*d^5*f^2*z^2 + 504*A*B*a*b^3*c^6*d \\
& ^5*f^2*z^2 + 256*A*B*a^3*b*c^6*d^4*f^3*z^2 + 168*A*B*a*b^7*c^2*d^3*f^4*z^2 \\
& - 144*A*B*a^2*b^7*c*d^2*f^5*z^2 + 144*A*B*a*b^5*c^4*d^4*f^3*z^2 - 56*B^2*a* \\
& b^2*c^7*d^6*f*z^2 - 36*B^2*a^5*b^4*c*d*f^6*z^2 - 16*B^2*a*b^8*c*d^3*f^4*z^2 \\
& - 164*A^2*a^3*b^6*c*d*f^6*z^2 - 16*A^2*a*b^8*c*d^2*f^5*z^2 - 96*A*B*b^5*c^ \\
& 5*d^5*f^2*z^2 - 24*A*B*b^7*c^3*d^4*f^3*z^2 - 580*B^2*a^4*b^2*c^4*d^3*f^4*z^ \\
& 2 + 536*B^2*a^3*b^4*c^3*d^3*f^4*z^2 - 348*B^2*a^4*b^4*c^2*d^2*f^5*z^2 + 316 \\
& *B^2*a^2*b^2*c^6*d^5*f^2*z^2 + 200*B^2*a^5*b^2*c^3*d^2*f^5*z^2 - 120*B^2*a^ \\
& 2*b^4*c^4*d^4*f^3*z^2 - 66*B^2*a^2*b^6*c^2*d^3*f^4*z^2 - 16*B^2*a^3*b^2*c^5 \\
& *d^4*f^3*z^2 + 1952*A^2*a^4*b^2*c^4*d^2*f^5*z^2 - 1792*A^2*a^3*b^2*c^5*d^3* \\
& f^4*z^2 - 1272*A^2*a^3*b^4*c^3*d^2*f^5*z^2 + 976*A^2*a^2*b^2*c^6*d^4*f^3*z^ \\
& 2 + 960*A^2*a^2*b^4*c^4*d^3*f^4*z^2 + 282*A^2*a^2*b^6*c^2*d^2*f^5*z^2 - 72* \\
& A*B*b^3*c^7*d^6*f*z^2 - 16*A*B*b^9*c*d^3*f^4*z^2 - 16*A*B*a^3*b^7*d*f^6*z^2 \\
& + 16*A*B*a*b^9*d^2*f^5*z^2 - 180*B^2*a*b^4*c^5*d^5*f^2*z^2 + 132*B^2*a^6*b \\
& ^2*c^2*d*f^6*z^2 + 108*B^2*a^3*b^6*c*d^2*f^5*z^2 + 20*B^2*a*b^6*c^3*d^4*f^3 \\
& *z^2 - 736*A^2*a^5*b^2*c^3*d*f^6*z^2 + 624*A^2*a^4*b^4*c^2*d*f^6*z^2 - 416* \\
& A^2*a*b^2*c^7*d^5*f^2*z^2 - 276*A^2*a*b^4*c^5*d^4*f^3*z^2 - 196*A^2*a*b^6*c \\
& ^3*d^3*f^4*z^2 + 31*B^2*b^6*c^4*d^5*f^2*z^2 + 2*B^2*b^8*c^2*d^4*f^3*z^2 - 7 \\
& 68*B^2*a^5*c^5*d^3*f^4*z^2 + 512*B^2*a^6*c^4*d^2*f^5*z^2 + 512*B^2*a^4*c^6* \\
& d^4*f^3*z^2 - 128*B^2*a^3*c^7*d^5*f^2*z^2 + 80*A^2*b^4*c^6*d^5*f^2*z^2 + 31 \\
& *A^2*b^6*c^4*d^4*f^3*z^2 + 14*A^2*b^8*c^2*d^3*f^4*z^2 - 1152*A^2*a^3*c^7*d^ \\
& 4*f^3*z^2 + 1008*A^2*a^4*c^6*d^3*f^4*z^2 + 624*A^2*a^2*c^8*d^5*f^2*z^2 - 28 \\
& 8*A^2*a^5*c^5*d^2*f^5*z^2 - 10*B^2*a^2*b^8*d^2*f^5*z^2 - 48*A^2*a^6*b^2*c^2 \\
& *f^7*z^2 - 16*A*B*b*c^9*d^7*z^2 + 20*B^2*b^4*c^6*d^6*f*z^2 - 128*B^2*a^7*c^ \\
& 3*d*f^6*z^2 + 64*A^2*b^2*c^8*d^6*f*z^2 - 112*A^2*a^6*c^4*d*f^6*z^2 + 3*B^2* \\
& a^4*b^6*d*f^6*z^2 + 14*A^2*a^2*b^8*d*f^6*z^2 + 12*A^2*a^5*b^4*c*f^7*z^2 - 1
\end{aligned}$$

$$\begin{aligned}
& 60A^2a^9d^6f^2z^2 + 3B^2b^{10}d^3f^4z^2 - A^2b^{10}d^2f^5z^2 + 64 \\
& *A^2a^7c^3f^7z^2 + 4B^2b^2c^8d^7z^2 - A^2a^4b^6f^7z^2 + 16A^2 \\
& *c^{10}d^7z^2 - 160AB^2a^2b^6c^6d^4f^2z + 112AB^2a^4b^6c^3d^5f^5z - \\
& 24AB^2a^2b^5c^4d^2f^5z + 480A^2B^2a^2b^2c^4d^2f^4z - 176AB^2a \\
& ^2b^3c^3d^2f^4z - 10A^2B^2a^2b^6c^4d^2f^5z + 384AB^2a^2b^6c^5d^3f \\
& ^3z - 352AB^2a^3b^6c^4d^2f^4z - 288A^2B^2a^2b^2c^5d^3f^3z - 160 \\
& A^2B^2a^3b^2c^3d^5f^5z - 148A^2B^2a^2b^4c^3d^2f^4z + 112AB^2a^2b^3 \\
& *c^4d^3f^3z + 72A^2B^2a^2b^4c^2d^5f^5z + 72AB^2a^2b^5c^2d^2f^4z \\
& z + 48AB^2a^3b^3c^2d^5f^5z + 48B^3a^2b^2c^5d^4f^2z - 36B^3a^4 \\
& b^2c^2d^5f^5z - 4B^3a^2b^4c^3d^3f^3z - 480A^3a^2b^6c^5d^2f^4z - \\
& 160A^3a^2b^3c^3d^5f^5z + 128A^3a^2b^3c^4d^2f^4z + 112A^2B^2b^4 \\
& c^4d^3f^3z - 64AB^2b^5c^3d^3f^3z + 16A^2B^2b^2c^6d^4f^2z + 1 \\
& 6AB^2b^3c^5d^4f^2z - A^2B^2b^6c^2d^2f^4z + 448A^2B^2a^3c^5d^2 \\
& *f^4z - 352A^2B^2a^2c^6d^3f^3z - 48A^2B^2a^4b^2c^2f^6z + 12B^3 \\
& a^3b^4c^4d^5f^5z - 10B^3a^2b^6c^4d^2f^4z + 416A^3a^3b^6c^4d^5f^5z + \\
& 224A^3a^2b^6c^6d^3f^3z + 24A^3a^2b^5c^2d^5f^5z - 2AB^2b^7c^4d^2f^ \\
& 4z - 272A^2B^2a^4c^4d^5f^5z + 128A^2B^2a^7d^4f^2z + 12A^2B^2a^3 \\
& b^4c^4f^6z - 120B^3a^2b^2c^4d^3f^3z + 112B^3a^3b^2c^3d^2f^4z \\
& + 16AB^2b^6c^7d^5f^5z + 2AB^2a^2b^7d^5f^5z - 2A^3b^7c^4d^5f^5z - 1 \\
& 6A^2B^2c^8d^5f^5z + 11B^3b^6c^2d^3f^3z - 8B^3b^4c^4d^4f^2z - \\
& 64A^3b^3c^5d^3f^3z + 96A^3a^3b^3c^2f^6z - 4B^3b^2c^6d^5f^5z \\
& - 32A^3b^6c^7d^4f^2z - B^3a^2b^6d^5f^5z - 128A^3a^4b^6c^3f^6z - \\
& 24A^3a^2b^5c^4f^6z + 64A^2B^2a^5c^3f^6z - A^2B^2a^2b^6f^6z + A^ \\
& 2B^2b^8d^5f^5z + 2A^3a^2b^7f^6z + B^3b^8d^2f^4z + 32A^3B^2a^2b^6c^4 \\
& d^5f^4 - 18A^2B^2a^2b^2c^3d^5f^4 + 32AB^3a^2b^6c^4d^2f^3 - 28AB^3a^ \\
& 2b^6c^3d^5f^4 + 6AB^3a^2b^3c^2d^5f^4 - 10A^3B^2b^3c^3d^5f^4 - 4A^3B^ \\
& b^6c^5d^2f^3 - 4AB^3b^6c^5d^3f^2 - 28A^3B^2a^2b^6c^3f^5 + 6A^3B^2a^ \\
& b^3c^2f^5 + 9A^2B^2b^2c^4d^2f^3 - 3A^2B^2a^2b^2c^2f^5 - 10B^ \\
& 4a^2b^2c^3d^2f^3 - 3B^4a^2b^2c^2d^5f^4 - 10AB^3b^3c^3d^2f^3 + \\
& 3A^2B^2b^4c^2d^5f^4 + 36A^2B^2a^2c^4d^5f^4 - 24A^2B^2a^2c^5d^2f \\
& ^3 + 4A^2B^2c^6d^3f^2 + 16A^2B^2a^3c^3f^5 + 16B^4a^3c^3d^5f^4 \\
& + 8A^4b^2c^4d^5f^4 - 8A^4a^2b^2c^3f^5 - 24A^4a^2c^5d^5f^4 + 3B^4b^ \\
& 4c^2d^2f^3 + 4A^4c^6d^2f^3 + 36A^4a^2c^4f^5 + B^4b^2c^4d^3f^ \\
& 2, z, k) * ((4b^5c^8d^7f^2 + 4b^7c^6d^6f^3 - 4b^9c^4d^5f^4 - 4b^ \\
& 11c^2d^4f^5 - 612a^2b^5c^6d^5f^4 - 712a^2b^7c^4d^4f^5 - 132a^ \\
& 2b^9c^2d^3f^6 + 1696a^3b^3c^7d^5f^4 + 2736a^3b^5c^5d^4f^5 + 8 \\
& 96a^3b^7c^3d^3f^6 - 5120a^4b^3c^6d^4f^5 - 3140a^4b^5c^4d^3f^ \\
& 6 - 220a^4b^7c^2d^2f^7 + 5664a^5b^3c^5d^3f^6 + 1128a^5b^5c^3d \\
& ^2f^7 - 2560a^6b^3c^4d^2f^7 + 8a^2b^11c^4d^3f^6 + 8a^5b^7c^4d^5f^8 \\
& - 448a^8b^6c^4d^5f^8 - 32a^2b^3c^9d^7f^2 - 24a^2b^5c^7d^6f^3 + 88a^ \\
& b^7c^5d^5f^4 + 88a^2b^9c^3d^4f^5 + 64a^2b^6c^10d^7f^2 + 128a^3b^ \\
& c^9d^6f^3 + 16a^3b^9c^4d^2f^7 - 1600a^4b^6c^8d^5f^4 + 3840a^5b^6c^ \\
& 7d^4f^5 - 4160a^6b^6c^6d^3f^6 - 92a^6b^5c^2d^5f^8 + 2176a^7b^6c^5 \\
& d^2f^7 + 352a^7b^3c^3d^5f^8) / (16a^2c^6d^4 + a^4b^4f^4 + b^4c^4d^ \\
& 4 + 16a^6c^2f^4 + b^8d^2f^2 - 8a^2b^2c^5d^4 - 8a^5b^2c^4f^4 + 2a^ \\
& 2b^6d^5f^3 - 64a^3c^5d^3f - 64a^5c^3d^5f^3 + 2b^6c^2d^3f + 96a^ \\
& 4c^4d^2f^2 + 54a^2b^4c^2d^2f^2 - 112a^3b^2c^3d^2f^2 - 20a^2b^4 \\
& *c^3d^3f - 12a^2b^6c^4d^2f^2 - 20a^3b^4c^4d^3f + 64a^2b^2c^4d^3f \\
& + 64a^4b^2c^2d^5f^3) + (x*(128a^9c^4f^9 - 2a^6b^6c^4f^9 - 640a^8 \\
& c^5d^8f^8 + 6b^12c^4d^3f^6 + 24a^7b^4c^2f^9 - 96a^8b^2c^3f^9 + 12 \\
& 8a^2c^11d^7f^2 - 640a^3c^10d^6f^3 + 1152a^4c^9d^5f^4 - 640a^5 \\
& c^8d^4f^5 - 640a^6c^7d^3f^6 + 1152a^7c^6d^2f^7 + 8b^4c^9d^7f^ \\
& 2 + 22b^6c^7d^6f^3 + 26b^8c^5d^5f^4 + 18b^10c^3d^4f^5 + 672a^2 \\
& *b^2c^9d^6f^3 + 1224a^2b^4c^7d^5f^4 + 1202a^2b^6c^5d^4f^5 + 56 \\
& 4a^2b^8c^3d^3f^6 - 2048a^3b^2c^8d^5f^4 - 2744a^3b^4c^6d^4f^5 \\
& - 1736a^3b^6c^4d^3f^6 - 128a^3b^8c^2d^2f^7 + 2656a^4b^2c^7d^ \\
& 4f^5 + 2648a^4b^4c^5d^3f^6 + 570a^4b^6c^3d^2f^7 - 1344a^5b^2c^ \\
& ^6d^3f^6 - 904a^5b^4c^4d^2f^7 - 160a^6b^2c^5d^2f^7 + 2a^4b^8 \\
& *c^4d^8f^8 - 64a^2b^2c^10d^7f^2 - 216a^2b^4c^8d^6f^3 - 300a^2b^6c^6d^5
\end{aligned}$$

$$\begin{aligned}
& *f^4 - 240*a*b^8*c^4*d^4*f^5 - 92*a*b^{10}*c^2*d^3*f^6 + 10*a^2*b^{10}*c*d^2*f^7 \\
& - 12*a^5*b^6*c^2*d*f^8 - 40*a^6*b^4*c^3*d*f^8 + 384*a^7*b^2*c^4*d*f^8)/(\\
& 16*a^2*c^6*d^4 + a^4*b^4*f^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - \\
& 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 64*a^3*c^5*d^3*f - 6 \\
& 4*a^5*c^3*d*f^3 + 2*b^6*c^2*d^3*f + 96*a^4*c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2 \\
& *f^2 - 112*a^3*b^2*c^3*d^2*f^2 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - \\
& 20*a^3*b^4*c*d*f^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3)) + (A*a^4 \\
& *b^6*c*f^8 - 64*A*a^7*c^4*f^8 - 32*A*a*c^{10}*d^6*f^2 + 352*A*a^6*c^5*d*f^7 + \\
& A*b^{10}*c*d^2*f^6 - 12*A*a^5*b^4*c^2*f^8 + 48*A*a^6*b^2*c^3*f^8 + 224*A*a^2 \\
& *c^9*d^5*f^3 - 640*A*a^3*c^8*d^4*f^4 + 960*A*a^4*c^7*d^3*f^5 - 800*A*a^5*c^6 \\
& *d^2*f^6 + 8*A*b^2*c^9*d^6*f^2 + 16*A*b^4*c^7*d^5*f^3 + A*b^6*c^5*d^4*f^4 \\
& - 6*A*b^8*c^3*d^3*f^5 - 4*B*b^3*c^8*d^6*f^2 - 12*B*b^5*c^6*d^5*f^3 - 4*B*b^7 \\
& *c^4*d^4*f^4 + 4*B*b^9*c^2*d^3*f^5 - 120*A*a*b^2*c^8*d^5*f^3 - 60*A*a*b^4* \\
& c^6*d^4*f^4 + 36*A*a*b^6*c^4*d^3*f^5 - 8*A*a*b^8*c^2*d^2*f^6 - 20*A*a^3*b^6 \\
& *c^2*d*f^7 + 80*A*a^4*b^4*c^3*d*f^7 - 216*A*a^5*b^2*c^4*d*f^7 + 92*B*a*b^3* \\
& c^7*d^5*f^3 + 72*B*a*b^5*c^5*d^4*f^4 - 20*B*a*b^7*c^3*d^3*f^5 - 176*B*a^2*b \\
& *c^8*d^5*f^3 + 544*B*a^3*b*c^7*d^4*f^4 - 736*B*a^4*b*c^6*d^3*f^5 - 4*B*a^4* \\
& b^5*c^2*d*f^7 + 464*B*a^5*b*c^5*d^2*f^6 + 44*B*a^5*b^3*c^3*d*f^7 + 384*A*a^ \\
& 2*b^2*c^7*d^4*f^4 + 32*A*a^2*b^4*c^5*d^3*f^5 + 14*A*a^2*b^6*c^3*d^2*f^6 - 5 \\
& 60*A*a^3*b^2*c^6*d^3*f^5 - 56*A*a^3*b^4*c^4*d^2*f^6 + 456*A*a^4*b^2*c^5*d^2 \\
& *f^6 - 360*B*a^2*b^3*c^6*d^4*f^4 - 64*B*a^2*b^5*c^4*d^3*f^5 + 504*B*a^3*b^3 \\
& *c^5*d^3*f^5 + 40*B*a^3*b^5*c^3*d^2*f^6 - 276*B*a^4*b^3*c^4*d^2*f^6 + 2*A*a \\
& ^2*b^8*c*d*f^7 + 16*B*a*b*c^9*d^6*f^2 - 112*B*a^6*b*c^4*d*f^7)/(16*a^2*c^6* \\
& d^4 + a^4*b^4*f^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5 \\
& *d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d \\
& *f^3 + 2*b^6*c^2*d^3*f + 96*a^4*c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2*f^2 - 112* \\
& a^3*b^2*c^3*d^2*f^2 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4* \\
& c*d*f^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3) + (x*(64*B*a^7*c^4*f \\
& ^8 + 4*A*a^3*b^7*c*f^8 - 256*A*a^6*b*c^4*f^8 - B*a^4*b^6*c*f^8 - 320*B*a^6* \\
& c^5*d*f^7 + 3*B*b^{10}*c*d^2*f^6 - 48*A*a^4*b^5*c^2*f^8 + 192*A*a^5*b^3*c^3*f \\
& ^8 + 12*B*a^5*b^4*c^2*f^8 - 48*B*a^6*b^2*c^3*f^8 - 16*A*b^3*c^8*d^5*f^3 - 4 \\
& 8*A*b^5*c^6*d^4*f^4 - 36*A*b^7*c^4*d^3*f^5 - 4*A*b^9*c^2*d^2*f^6 - 64*B*a^2 \\
& *c^9*d^5*f^3 + 320*B*a^3*c^8*d^4*f^4 - 640*B*a^4*c^7*d^3*f^5 + 640*B*a^5*c^6 \\
& *d^2*f^6 + 4*B*b^4*c^7*d^5*f^3 + 23*B*b^6*c^5*d^4*f^4 + 22*B*b^8*c^3*d^3*f \\
& ^5 + 320*A*a*b^3*c^7*d^4*f^4 + 352*A*a*b^5*c^5*d^3*f^5 + 76*A*a*b^7*c^3*d^2 \\
& *f^6 - 512*A*a^2*b*c^8*d^4*f^4 - 60*A*a^2*b^7*c^2*d*f^7 + 1408*A*a^3*b*c^7* \\
& d^3*f^5 + 352*A*a^3*b^5*c^3*d*f^7 - 1792*A*a^4*b*c^6*d^2*f^6 - 976*A*a^4*b^ \\
& 3*c^4*d*f^7 - 132*B*a*b^4*c^6*d^4*f^4 - 196*B*a*b^6*c^4*d^3*f^5 - 40*B*a*b^ \\
& 8*c^2*d^2*f^6 - 20*B*a^3*b^6*c^2*d*f^7 + 52*B*a^4*b^4*c^3*d*f^7 + 64*B*a^5* \\
& b^2*c^4*d*f^7 + 4*A*a*b^9*c*d*f^7 - 1184*A*a^2*b^3*c^6*d^3*f^5 - 544*A*a^2* \\
& b^5*c^4*d^2*f^6 + 1664*A*a^3*b^3*c^5*d^2*f^6 + 80*B*a^2*b^2*c^7*d^4*f^4 + 5 \\
& 20*B*a^2*b^4*c^5*d^3*f^5 + 210*B*a^2*b^6*c^3*d^2*f^6 - 192*B*a^3*b^2*c^6*d^ \\
& 3*f^5 - 456*B*a^3*b^4*c^4*d^2*f^6 + 96*B*a^4*b^2*c^5*d^2*f^6 + 64*A*a*b*c^9 \\
& *d^5*f^3 + 1088*A*a^5*b*c^5*d*f^7 + 2*B*a^2*b^8*c*d*f^7))/(16*a^2*c^6*d^4 + \\
& a^4*b^4*f^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 \\
& - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 \\
& + 2*b^6*c^2*d^3*f + 96*a^4*c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b \\
& ^2*c^3*d^2*f^2 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f \\
& ^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3)) + (13*A^2*a^2*b^5*c^2*f^ \\
& 7 - 56*A^2*a^3*b^3*c^3*f^7 + 16*A^2*b^3*c^6*d^3*f^4 + A^2*b^5*c^4*d^2*f^5 + \\
& 8*B^2*b^3*c^6*d^4*f^3 + 9*B^2*b^5*c^4*d^3*f^4 - 64*A*B*a^5*c^4*f^7 - A^2*a \\
& *b^7*c*f^7 + 80*A^2*a^4*b*c^4*f^7 + 16*A^2*b*c^8*d^4*f^3 + 2*A^2*b^7*c^2*d* \\
& f^6 - 48*A^2*a*b*c^7*d^3*f^4 - 22*A^2*a*b^5*c^3*d*f^6 - 48*A^2*a^3*b*c^5*d* \\
& f^6 - 16*B^2*a*b*c^7*d^4*f^3 - 64*B^2*a^4*b*c^4*d*f^6 - A*B*b^8*c*d*f^6 - 8 \\
& *A^2*a*b^3*c^5*d^2*f^5 + 64*A^2*a^2*b^3*c^4*d*f^6 - 56*B^2*a*b^3*c^5*d^3*f^ \\
& 4 + 2*B^2*a*b^5*c^3*d^2*f^5 + 96*B^2*a^2*b*c^6*d^3*f^4 - 11*B^2*a^2*b^5*c^2 \\
& *d*f^6 - 16*B^2*a^3*b*c^5*d^2*f^5 + 40*B^2*a^3*b^3*c^3*d*f^6 + A*B*a^2*b^6* \\
& c*f^7 + 32*A*B*a*c^8*d^4*f^3 + 32*A*B*a^4*c^5*d*f^6 + B^2*a*b^7*c*d*f^6 - 8 \\
& *B^2*a^2*b^3*c^4*d^2*f^5 - 12*A*B*a^3*b^4*c^2*f^7 + 48*A*B*a^4*b^2*c^3*f^7
\end{aligned}$$

$$\begin{aligned}
& - 160*A*B*a^2*c^7*d^3*f^4 + 160*A*B*a^3*c^6*d^2*f^5 - 24*A*B*b^2*c^7*d^4*f^3 \\
& - 24*A*B*b^4*c^5*d^3*f^4 + A*B*b^6*c^3*d^2*f^5 + 120*A*B*a*b^2*c^6*d^3*f^4 \\
& - 4*A*B*a*b^4*c^4*d^2*f^5 - 24*A*B*a^2*b^4*c^3*d*f^6 + 8*A*B*a^3*b^2*c^4*d*f^6 \\
& - 24*A*B*a^2*b^2*c^5*d^2*f^5 + 10*A*B*a*b^6*c^2*d*f^6)/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + b^4*c^4*d^4 \\
& + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 \\
& + 2*b^6*c^2*d^3*f + 96*a^4*c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 \\
& + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3) + (x*(104*A^2*a^4*c^5*f^7 - 32*B^2*a^5*c^4*f^7 + 8*A^2*c^9*d^4*f^3 + A^2*b^8*c*f^7 + 50*A^2*a^2*b^4*c^3*f^7 \\
& - 96*A^2*a^3*b^2*c^4*f^7 - 12*B^2*a^3*b^4*c^2*f^7 + 42*B^2*a^4*b^2*c^3*f^7 + 208*A^2*a^2*c^7*d^2*f^5 + 8*A^2*b^2*c^7*d^3*f^4 + 18*A^2*b^4*c^5*d^2*f^5 - 32*B^2*a^2*c^7*d^3*f^4 \\
& + 32*B^2*a^3*c^6*d^2*f^5 + 2*B^2*b^2*c^7*d^4*f^3 - 6*B^2*b^4*c^5*d^3*f^4 + 9*B^2*b^6*c^3*d^2*f^5 - 12*A^2*a*b^6*c^2*f^7 + B^2*a^2*b^6*c*f^7 - 64*A^2*a*c^8*d^3*f^4 - 256*A^2*a^3*c^6*d*f^6 + 2*A^2*b^6*c^3*d*f^6 \\
& + 32*B^2*a^4*c^5*d*f^6 - 36*A^2*a*b^4*c^4*d*f^6 - 2*B^2*a*b^6*c^2*d*f^6 - 2*A*B*a*b^7*c*f^7 - 144*A^2*a*b^2*c^6*d^2*f^5 + 168*A^2*a^2*b^2*c^5*d*f^6 + 24*B^2*a*b^2*c^6*d^3*f^4 - 64*B^2*a*b^4*c^4*d^2*f^5 + 26*B^2*a^2*b^4*c^3*d*f^6 - 88*B^2*a^3*b^2*c^4*d*f^6 + 72*A*B*a^4*b*c^4*f^7 - 8*A*B*b*c^8*d^4*f^3 + 2*A*B*b^7*c^2*d*f^6 + 84*B^2*a^2*b^2*c^5*d^2*f^5 + 24*A*B*a^2*b^5*c^2*f^7 - 84*A*B*a^3*b^3*c^3*f^7 + 4*A*B*b^3*c^6*d^3*f^4 - 20*A*B*b^5*c^4*d^2*f^5 + 148*A*B*a*b^3*c^5*d^2*f^5 - 192*A*B*a^2*b*c^6*d^2*f^5 - 4*A*B*a^2*b^3*c^4*d*f^6 + 16*A*B*a*b*c^7*d^3*f^4 - 12*A*B*a*b^5*c^3*d*f^6 + 112*A*B*a^3*b*c^5*d*f^6))/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 + 2*b^6*c^2*d^3*f + 96*a^4*c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3)) - (16*A^3*a*c^6*d*f^5 - 4*A^3*c^7*d^2*f^4 - B^3*b^3*c^4*d^2*f^4 - 12*A^3*a^2*c^5*f^6 - 16*A*B^2*a^3*c^4*f^6 + 2*A^3*a*b^2*c^4*f^6 - 6*A^3*b^2*c^5*d*f^5 + 4*B^3*a*b*c^5*d^2*f^4 + 3*B^3*a*b^3*c^3*d*f^5 - 12*B^3*a^2*b*c^4*d*f^5 + 3*A*B^2*a^2*b^2*c^3*f^6 + A*B^2*b^2*c^5*d^2*f^4 - 3*A^2*B*a*b^3*c^3*f^6 + 16*A^2*B*a^2*b*c^4*f^6 - 8*A*B^2*a*c^6*d^2*f^4 + 24*A*B^2*a^2*c^5*d*f^5 - 3*A*B^2*b^4*c^3*d*f^5 + 4*A^2*B*b*c^6*d^2*f^4 + 9*A^2*B*b^3*c^4*d*f^5 + 4*A*B^2*a*b^2*c^4*d*f^5 - 28*A^2*B*a*b*c^5*d*f^5)/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 + 2*b^6*c^2*d^3*f + 96*a^4*c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3))*root(2560*a^3*b^2*c^9*d^8*f*z^4 - 1152*a^2*b^4*c^8*d^8*f*z^4 + 384*a^5*b^8*c*d^3*f^6*z^4 + 384*a*b^8*c^5*d^7*f^2*z^4 + 288*a^3*b^10*c*d^4*f^5*z^4 + 288*a*b^10*c^3*d^6*f^3*z^4 + 224*a^7*b^6*c*d^2*f^7*z^4 - 192*a^10*b^2*c^2*d*f^8*z^4 + 224*a*b^6*c^7*d^8*f*z^4 + 80*a*b^12*c*d^5*f^4*z^4 + 48*a^9*b^4*c*d*f^8*z^4 - 33920*a^6*b^2*c^6*d^5*f^4*z^4 + 27936*a^5*b^4*c^5*d^5*f^4*z^4 + 26112*a^7*b^2*c^5*d^4*f^5*z^4 + 26112*a^5*b^2*c^7*d^6*f^3*z^4 - 20352*a^6*b^4*c^4*d^4*f^5*z^4 - 20352*a^4*b^4*c^6*d^6*f^3*z^4 - 13080*a^4*b^6*c^4*d^5*f^4*z^4 - 11520*a^8*b^2*c^4*d^3*f^6*z^4 - 11520*a^4*b^2*c^8*d^7*f^2*z^4 + 8736*a^5*b^6*c^3*d^4*f^5*z^4 + 8736*a^3*b^6*c^5*d^6*f^3*z^4 + 7488*a^7*b^4*c^3*d^3*f^6*z^4 + 7488*a^3*b^4*c^7*d^7*f^2*z^4 + 3840*a^3*b^8*c^3*d^5*f^4*z^4 + 2560*a^9*b^2*c^3*d^2*f^7*z^4 - 2416*a^6*b^6*c^2*d^3*f^6*z^4 - 2416*a^2*b^6*c^6*d^7*f^2*z^4 - 2160*a^4*b^8*c^2*d^4*f^5*z^4 - 2160*a^2*b^8*c^4*d^6*f^3*z^4 - 1152*a^8*b^4*c^2*d^2*f^7*z^4 - 720*a^2*b^10*c^2*d^5*f^4*z^4 - 16*b^8*c^6*d^8*f*z^4 - 2048*a^4*c^10*d^8*f*z^4 + 256*a^11*c^3*d*f^8*z^4 - 4*a^8*b^6*d*f^8*z^4 + 48*a*b^4*c^9*d^9*z^4 - 24*b^10*c^4*d^7*f^2*z^4 - 16*b^12*c^2*d^6*f^3*z^4 + 17920*a^7*c^7*d^5*f^4*z^4 - 14336*a^8*c^6*d^4*f^5*z^4 - 14336*a^6*c^8*d^6*f^3*z^4 + 7168*a^9*c^5*d^3*f^6*z^4 + 7168*a^5*c^9*d^7*f^2*z^4 - 2048*a^10*c^4*d^2*f^7*z^4 - 24*a^4*b^10*d^3*f^6*z^4 - 16*a^6*b^8*d^2*f^7*z^4 - 16*a^2*b^12*d^4*f^5*z^4 - 192*a^2*b^2*c^10*d^9*z^4 - 4*b^14*d^5*f^4*z^4 - 4*b
\end{aligned}$$

$$\begin{aligned}
& \sim 6c^8d^9z^4 + 256a^3c^{11}d^9z^4 + 912A^2B^2a^6b^3c^3d^2f^6z^2 + 192A^2 \\
& B^2a^4b^5c^3d^2f^6z^2 + 920A^2B^2a^4b^3c^3d^2f^5z^2 - 480A^2B^2a^2b^5c^3 \\
& d^3f^4z^2 - 336A^2B^2a^2b^3c^5d^4f^3z^2 - 272A^2B^2a^3b^3c^4d^3 \\
& f^4z^2 + 240A^2B^2a^3b^5c^2d^2f^5z^2 + 192A^2B^2a^3b^3c^8d^6f^2z^2 - 24 \\
& 96A^2B^2a^5b^3c^4d^2f^5z^2 + 1872A^2B^2a^4b^3c^5d^3f^4z^2 - 744A^2B^2a^5 \\
& b^3c^2d^2f^6z^2 - 720A^2B^2a^2b^3c^7d^5f^2z^2 + 504A^2B^2a^3b^3c^6d^5 \\
& f^2z^2 + 256A^2B^2a^3b^3c^6d^4f^3z^2 + 168A^2B^2a^3b^7c^2d^3f^4z^2 - 1 \\
& 44A^2B^2a^2b^7c^2d^2f^5z^2 + 144A^2B^2a^3b^5c^4d^4f^3z^2 - 56B^2a^2b^2 \\
& c^7d^6f^2z^2 - 36B^2a^5b^4c^2d^2f^6z^2 - 16B^2a^2b^8c^2d^3f^4z^2 - \\
& 164A^2a^2b^3c^6d^2f^6z^2 - 16A^2a^2b^8c^2d^2f^5z^2 - 96A^2B^2a^5c^5d \\
& ^5f^2z^2 - 24A^2B^2a^7c^3d^4f^3z^2 - 580B^2a^4b^2c^4d^3f^4z^2 + \\
& 536B^2a^3b^4c^3d^3f^4z^2 - 348B^2a^4b^4c^2d^2f^5z^2 + 316B^2 \\
& a^2b^2c^6d^5f^2z^2 + 200B^2a^5b^2c^3d^2f^5z^2 - 120B^2a^2b^4 \\
& c^4d^4f^3z^2 - 66B^2a^2b^6c^2d^3f^4z^2 - 16B^2a^3b^2c^5d^4 \\
& f^3z^2 + 1952A^2a^4b^2c^4d^2f^5z^2 - 1792A^2a^3b^2c^5d^3f^4 \\
& z^2 - 1272A^2a^3b^4c^3d^2f^5z^2 + 976A^2a^2b^2c^6d^4f^3z^2 + \\
& 960A^2a^2b^4c^4d^3f^4z^2 + 282A^2a^2b^6c^2d^2f^5z^2 - 72A^2B^2 \\
& b^3c^7d^6f^2z^2 - 16A^2B^2a^9c^2d^3f^4z^2 - 16A^2B^2a^3b^7d^2f^6z^2 + \\
& 16A^2B^2a^3b^9d^2f^5z^2 - 180B^2a^2b^4c^5d^5f^2z^2 + 132B^2a^6b^2c^2 \\
& d^2f^6z^2 + 108B^2a^3b^6c^2d^2f^5z^2 + 20B^2a^2b^6c^3d^4f^3z^2 \\
& - 736A^2a^5b^2c^3d^2f^6z^2 + 624A^2a^4b^4c^2d^2f^6z^2 - 416A^2 \\
& a^2b^2c^7d^5f^2z^2 - 276A^2a^2b^4c^5d^4f^3z^2 - 196A^2a^2b^6c^3d^3 \\
& f^4z^2 + 31B^2a^6c^4d^5f^2z^2 + 2B^2a^8c^2d^4f^3z^2 - 768B^2 \\
& a^5c^5d^3f^4z^2 + 512B^2a^6c^4d^2f^5z^2 + 512B^2a^4c^6d^4 \\
& f^3z^2 - 128B^2a^3c^7d^5f^2z^2 + 80A^2b^4c^6d^5f^2z^2 + 31A^2 \\
& b^6c^4d^4f^3z^2 + 14A^2b^8c^2d^3f^4z^2 - 1152A^2a^3c^7d^4f^3 \\
& z^2 + 1008A^2a^4c^6d^3f^4z^2 + 624A^2a^2c^8d^5f^2z^2 - 288A^2 \\
& a^5c^5d^2f^5z^2 - 10B^2a^2b^8d^2f^5z^2 - 48A^2a^6b^2c^2f^7 \\
& z^2 - 16A^2B^2a^9d^7z^2 + 20B^2a^4c^6d^6f^2z^2 - 128B^2a^7c^3d \\
& f^6z^2 + 64A^2b^2c^8d^6f^2z^2 - 112A^2a^6c^4d^2f^6z^2 + 3B^2a^4 \\
& b^6d^2f^6z^2 + 14A^2a^2b^8d^2f^6z^2 + 12A^2a^5b^4c^2f^7z^2 - 160A^2 \\
& a^2c^9d^6f^2z^2 + 3B^2a^10d^3f^4z^2 - A^2b^10d^2f^5z^2 + 64A^2 \\
& a^7c^3f^7z^2 + 4B^2a^2b^8d^7z^2 - A^2a^4b^6f^7z^2 + 16A^2c^10 \\
& d^7z^2 - 160A^2B^2a^2b^6c^6d^4f^2z^2 + 112A^2B^2a^4b^3c^3d^2f^5z^2 - 24 \\
& A^2B^2a^2b^5c^3d^2f^5z^2 + 480A^2B^2a^2b^2c^4d^2f^4z^2 - 176A^2B^2a^2 \\
& b^3c^3d^2f^4z^2 - 10A^2B^2a^2b^6c^2d^2f^5z^2 + 384A^2B^2a^2b^3c^5d^3 \\
& f^3z^2 - 352A^2B^2a^3b^3c^4d^2f^4z^2 - 288A^2B^2a^2b^2c^5d^3f^3z^2 - 160A^2 \\
& B^2a^3b^2c^3d^2f^5z^2 - 148A^2B^2a^2b^4c^3d^2f^4z^2 + 112A^2B^2a^2b^3c^4 \\
& d^3f^3z^2 + 72A^2B^2a^2b^4c^2d^2f^5z^2 + 72A^2B^2a^2b^5c^2d^2f^4z^2 + \\
& 48A^2B^2a^3b^3c^2d^2f^5z^2 + 48B^3a^2b^2c^5d^4f^2z^2 - 36B^3a^4b^2 \\
& c^2d^2f^5z^2 - 4B^3a^2b^4c^3d^3f^3z^2 - 480A^3a^2b^3c^5d^2f^4z^2 - 16 \\
& 0A^3a^2b^3c^3d^2f^5z^2 + 128A^3a^2b^3c^4d^2f^4z^2 + 112A^2B^2b^4c^4 \\
& d^3f^3z^2 - 64A^2B^2b^5c^3d^3f^3z^2 + 16A^2B^2b^2c^6d^4f^2z^2 + 16A^2 \\
& B^2b^3c^5d^4f^2z^2 - A^2B^2b^6c^2d^2f^4z^2 + 448A^2B^2a^3c^5d^2f^4 \\
& z^2 - 352A^2B^2a^2c^6d^3f^3z^2 - 48A^2B^2a^4b^2c^2f^6z^2 + 12B^3a^3 \\
& b^4c^2d^2f^5z^2 - 10B^3a^2b^6c^2d^2f^4z^2 + 416A^3a^3b^3c^4d^2f^5z^2 + 224 \\
& A^3a^2b^3c^6d^3f^3z^2 + 24A^3a^2b^5c^2d^2f^5z^2 - 2A^2B^2b^7c^2d^2f^4z^2 \\
& - 272A^2B^2a^4c^4d^2f^5z^2 + 128A^2B^2a^3c^7d^4f^2z^2 + 12A^2B^2a^3b^4 \\
& c^2f^6z^2 - 120B^3a^2b^2c^4d^3f^3z^2 + 112B^3a^3b^2c^3d^2f^4z^2 + \\
& 16A^2B^2b^3c^7d^5f^2z^2 + 2A^2B^2a^2b^7d^2f^5z^2 - 2A^3b^7c^2d^2f^5z^2 - 16A^2 \\
& B^2c^8d^5f^2z^2 + 11B^3b^6c^2d^3f^3z^2 - 8B^3b^4c^4d^4f^2z^2 - 64A^3 \\
& b^3c^5d^3f^3z^2 + 96A^3a^3b^3c^2f^6z^2 - 4B^3b^2c^6d^5f^2z^2 - 32A^3 \\
& b^3c^7d^4f^2z^2 - B^3a^2b^6d^2f^5z^2 - 128A^3a^4b^3c^3f^6z^2 - 24 \\
& A^3a^2b^5c^2f^6z^2 + 64A^2B^2a^5c^3f^6z^2 - A^2B^2a^2b^6f^6z^2 + A^2B^2 \\
& b^8d^2f^5z^2 + 2A^3a^2b^7f^6z^2 + B^3b^8d^2f^4z^2 + 32A^3B^2a^2b^3c^4 \\
& d^2f^4z^2 - 18A^2B^2a^2b^2c^3d^2f^4z^2 + 32A^2B^3a^2b^3c^4d^2f^3z^2 - 28A^2B^3 \\
& a^2b^2c^3d^2f^4z^2 + 6A^2B^3a^2b^3c^2d^2f^4z^2 - 10A^3B^2b^3c^3d^2f^4z^2 - 4A^3 \\
& B^2b^3c^5d^2f^3z^2 - 4A^2B^3b^3c^5d^3f^2z^2 - 28A^3B^2a^2b^3c^3f^5z^2 + 6A^3 \\
& B^2a^2b^3c^2f^5z^2 + 9A^2B^2b^2c^4d^2f^3z^2 - 3A^2B^2a^2b^2c^2f^5z^2 - 10B^4a^2
\end{aligned}$$

```
*b^2*c^3*d^2*f^3 - 3*B^4*a^2*b^2*c^2*d*f^4 - 10*A*B^3*b^3*c^3*d^2*f^3 + 3*A
^2*B^2*b^4*c^2*d*f^4 + 36*A^2*B^2*a^2*c^4*d*f^4 - 24*A^2*B^2*a*c^5*d^2*f^3
+ 4*A^2*B^2*c^6*d^3*f^2 + 16*A^2*B^2*a^3*c^3*f^5 + 16*B^4*a^3*c^3*d*f^4 + 8
*A^4*b^2*c^4*d*f^4 - 8*A^4*a*b^2*c^3*f^5 - 24*A^4*a*c^5*d*f^4 + 3*B^4*b^4*c
^2*d^2*f^3 + 4*A^4*c^6*d^2*f^3 + 36*A^4*a^2*c^4*f^5 + B^4*b^2*c^4*d^3*f^2,
z, k), k, 1, 4)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)**2/(f*x**2+d),x)
```

```
[Out] Timed out
```

$$3.6 \quad \int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d-fx^2} dx$$

Optimal. Leaf size=331

$$\frac{(B\sqrt{d} - A\sqrt{f})\sqrt{af + b(-\sqrt{d})}\sqrt{f} + cd \tanh^{-1}\left(\frac{-2a\sqrt{f} + x(2c\sqrt{d} - b\sqrt{f}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f} + cd}\right) (A\sqrt{f} + B\sqrt{d})\sqrt{af + b\sqrt{d}}\sqrt{f}}{2\sqrt{d} f^{3/2}} + \dots$$

Rubi [A] time = 0.60, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1021, 1078, 621, 206, 1033, 724}

$$\frac{(B\sqrt{d} - A\sqrt{f})\sqrt{af + b(-\sqrt{d})}\sqrt{f} + cd \tanh^{-1}\left(\frac{-2a\sqrt{f} + x(2c\sqrt{d} - b\sqrt{f}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f} + cd}\right)}{2\sqrt{d} f^{3/2}} + \frac{(A\sqrt{f} + B\sqrt{d})\sqrt{af + b\sqrt{d}}\sqrt{f} + cd \tanh^{-1}\left(\frac{2a\sqrt{f} + x(b\sqrt{f} + 2c\sqrt{d}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}}\sqrt{f} + cd}\right)}{2\sqrt{d} f^{3/2}} - \frac{(2Ac + bB) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c} f} - \frac{B\sqrt{a + bx + cx^2}}{f}$$

Antiderivative was successfully verified.

[In] Int[((A + B*x)*Sqrt[a + b*x + c*x^2])/(d - f*x^2), x]

[Out] -((B*Sqrt[a + b*x + c*x^2])/f) - ((b*B + 2*A*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(2*Sqrt[c]*f) - ((B*Sqrt[d] - A*Sqrt[f])*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2]))/(2*Sqrt[d]*f^(3/2)) + ((B*Sqrt[d] + A*Sqrt[f])*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2]))/(2*Sqrt[d]*f^(3/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1021

Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q + 1))*x^2, x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(A + Bx)\sqrt{a + bx + cx^2}}{d - fx^2} dx &= -\frac{B\sqrt{a + bx + cx^2}}{f} + \frac{\int \frac{\frac{1}{2}(bBd + 2aAf) + (Bcd + Abf + aBf)x + \frac{1}{2}(bB + 2Ac)fx^2}{\sqrt{a + bx + cx^2}(d - fx^2)} dx}{f} \\ &= -\frac{B\sqrt{a + bx + cx^2}}{f} - \frac{\int \frac{-\frac{1}{2}(bB + 2Ac)df - \frac{1}{2}f(bBd + 2aAf) - f(Bcd + Abf + aBf)x}{\sqrt{a + bx + cx^2}(d - fx^2)} dx}{f^2} \quad (bB + 2Ac) \\ &= -\frac{B\sqrt{a + bx + cx^2}}{f} - \frac{(bB + 2Ac) \operatorname{Subst}\left(\int \frac{1}{4c - x^2} dx, x, \frac{b + 2cx}{\sqrt{a + bx + cx^2}}\right)}{f} + \frac{((B\sqrt{d} - A\sqrt{f}))}{f} \\ &= -\frac{B\sqrt{a + bx + cx^2}}{f} - \frac{(bB + 2Ac) \tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right)}{2\sqrt{c}f} - \frac{((B\sqrt{d} - A\sqrt{f}))}{f} \\ &= -\frac{B\sqrt{a + bx + cx^2}}{f} - \frac{(bB + 2Ac) \tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right)}{2\sqrt{c}f} - \frac{(B\sqrt{d} - A\sqrt{f})}{f} \end{aligned}$$

Mathematica [A] time = 0.80, size = 322, normalized size = 0.97

$$\frac{(A\sqrt{f} + B\sqrt{d})\sqrt{af + b\sqrt{d}\sqrt{f} + cd} \tanh^{-1}\left(\frac{2a\sqrt{f} + b\sqrt{d} + b\sqrt{f}\sqrt{a + 2c\sqrt{d}x}}{2\sqrt{a + x(b + cx)}\sqrt{af + b\sqrt{d}\sqrt{f} + cd}}\right) - (B\sqrt{d} - A\sqrt{f})\sqrt{af + b(-\sqrt{d})\sqrt{f} + cd} \tanh^{-1}\left(\frac{-2a\sqrt{f} + b(-\sqrt{d}) + 2c\sqrt{d}x}{2\sqrt{a + x(b + cx)}\sqrt{af + b(-\sqrt{d})\sqrt{f} + cd}}\right) - 2B\sqrt{d}\sqrt{f}\sqrt{a + x(b + cx)} - (2Ac + bB) \tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + x(b + cx)}}\right)}{2\sqrt{d}f^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*Sqrt[a + b*x + c*x^2])/(d - f*x^2), x]
```

```
[Out] -1/2*((b*B + 2*A*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]) / (Sqrt[c]*f) + (-2*B*Sqrt[d]*Sqrt[f]*Sqrt[a + x*(b + c*x)] + (B*Sqrt[d] + A*Sqrt[f])*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] - (B*Sqrt[d] - A*Sqrt[f])*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/(2*Sqrt[d]*f^(3/2))
```

IntegrateAlgebraic [C] time = 0.78, size = 523, normalized size = 1.58

$$\frac{B\sqrt{d}\sqrt{f}\sqrt{a + x(b + cx)} + (A\sqrt{f} + B\sqrt{d})\sqrt{af + b\sqrt{d}\sqrt{f} + cd} \tanh^{-1}\left(\frac{2a\sqrt{f} + b\sqrt{d} + b\sqrt{f}\sqrt{a + 2c\sqrt{d}x}}{2\sqrt{a + x(b + cx)}\sqrt{af + b\sqrt{d}\sqrt{f} + cd}}\right) - (B\sqrt{d} - A\sqrt{f})\sqrt{af + b(-\sqrt{d})\sqrt{f} + cd} \tanh^{-1}\left(\frac{-2a\sqrt{f} + b(-\sqrt{d}) + 2c\sqrt{d}x}{2\sqrt{a + x(b + cx)}\sqrt{af + b(-\sqrt{d})\sqrt{f} + cd}}\right) - 2B\sqrt{d}\sqrt{f}\sqrt{a + x(b + cx)} - (2Ac + bB) \tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + x(b + cx)}}\right)}{2\sqrt{d}f^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B*x)*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]

[Out] $-\left(\frac{B\sqrt{a+bx+cx^2}}{f}\right) + \left(\frac{(bB + 2Ac)\log[bf + 2cfx - 2\sqrt{c}f\sqrt{a+bx+cx^2}]}{2\sqrt{c}f}\right) - \text{RootSum}[b^2d - a^2f - 4b\sqrt{c}d\sqrt{1 + 4cd\sqrt{1^2 + 2af\sqrt{1^2 - f\sqrt{1^4}}}} \& , (b^2Bd\log[-(\sqrt{c}x) + \sqrt{a+bx+cx^2} - \sqrt{1}] + A*b*c*d\log[-(\sqrt{c}x) + \sqrt{a+bx+cx^2} - \sqrt{1}] - a^2*B*f*\log[-(\sqrt{c}x) + \sqrt{a+bx+cx^2} - \sqrt{1}] - 2*b*B*\sqrt{c}*d*\log[-(\sqrt{c}x) + \sqrt{a+bx+cx^2} - \sqrt{1}]*\sqrt{1} - 2*A*c^{3/2}*d*\log[-(\sqrt{c}x) + \sqrt{a+bx+cx^2} - \sqrt{1}]*\sqrt{1} - 2*a*A*\sqrt{c}*f*\log[-(\sqrt{c}x) + \sqrt{a+bx+cx^2} - \sqrt{1}]*\sqrt{1} + B*c*d*\log[-(\sqrt{c}x) + \sqrt{a+bx+cx^2} - \sqrt{1}]*\sqrt{1^2} + A*b*f*\log[-(\sqrt{c}x) + \sqrt{a+bx+cx^2} - \sqrt{1}]*\sqrt{1^2} + a*B*f*\log[-(\sqrt{c}x) + \sqrt{a+bx+cx^2} - \sqrt{1}]*\sqrt{1^2})/(b\sqrt{c}*d - 2*c*d*\sqrt{1} - a*f*\sqrt{1} + f*\sqrt{1^3}) \&]/(2*f)$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type

maple [B] time = 0.08, size = 3358, normalized size = 10.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x+A)*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)

[Out] $\frac{1}{2}(\sqrt{df})^{-1/2} * ((x + (\sqrt{df})^{1/2}/f)^{2c+1} * f * (-2c * (\sqrt{df})^{1/2} + b * f) * (x + (\sqrt{df})^{1/2}/f) + 1/f * (-b * (\sqrt{df})^{1/2} + a * f + c * d))^{1/2} * A - 1/2 * f * ((x + (\sqrt{df})^{1/2}/f)^{2c+1} * f * (-2c * (\sqrt{df})^{1/2} + b * f) * (x + (\sqrt{df})^{1/2}/f) + 1/f * (-b * (\sqrt{df})^{1/2} + a * f + c * d))^{1/2} * B - 1/2 * f * \ln((1/2 * f * (-2c * (\sqrt{df})^{1/2} + b * f) + c * (x + (\sqrt{df})^{1/2}/f)) / c^{1/2}) + ((x + (\sqrt{df})^{1/2}/f)^{2c+1} * f * (-2c * (\sqrt{df})^{1/2} + b * f) * (x + (\sqrt{df})^{1/2}/f) + 1/f * (-b * (\sqrt{df})^{1/2} + a * f + c * d))^{1/2}) * c^{1/2} * A + 1/2 * (\sqrt{df})^{1/2} / f^2 * \ln((1/2 * f * (-2c * (\sqrt{df})^{1/2} + b * f) + c * (x + (\sqrt{df})^{1/2}/f)) / c^{1/2}) + ((x + (\sqrt{df})^{1/2}/f)^{2c+1} * f * (-2c * (\sqrt{df})^{1/2} + b * f) * (x + (\sqrt{df})^{1/2}/f) + 1/f * (-b * (\sqrt{df})^{1/2} + a * f + c * d))^{1/2}) * c^{1/2} * B + 1/4 / (\sqrt{df})^{1/2} * \ln((1/2 * f * (-2c * (\sqrt{df})^{1/2} + b * f) + c * (x + (\sqrt{df})^{1/2}/f)) / c^{1/2}) + ((x + (\sqrt{df})^{1/2}/f)^{2c+1} * f * (-2c * (\sqrt{df})^{1/2} + b * f) * (x + (\sqrt{df})^{1/2}/f) + 1/f * (-b * (\sqrt{df})^{1/2} + a * f + c * d))^{1/2}) / c^{1/2} * b * A - 1/4 * f * \ln((1/2 * f * (-2c * (\sqrt{df})^{1/2} + b * f) + c * (x + (\sqrt{df})^{1/2}/f)) / c^{1/2}) + ((x + (\sqrt{df})^{1/2}/f)^{2c+1} * f * (-2c * (\sqrt{df})^{1/2} + b * f) * (x + (\sqrt{df})^{1/2}/f) + 1/f * (-b * (\sqrt{df})^{1/2} + a * f + c * d))^{1/2}) / c^{1/2} * b * B + 1/2 * f / (1/f * (-b * (\sqrt{df})^{1/2} + a * f + c * d))^{1/2} * \ln((2 * f * (-b * (\sqrt{df})^{1/2} + a * f + c * d) + 1/f * (-2c * (\sqrt{df})^{1/2} + b * f) * (x + (\sqrt{df})^{1/2}/f) + 2 * (1/f * (-b * (\sqrt{df})^{1/2} + a * f + c * d))^{1/2}) * ((x + (\sqrt{df})^{1/2}/f)^{2c+1} * f * (-2c * (\sqrt{df})^{1/2} + b * f) * (x + (\sqrt{df})^{1/2}/f) + 1/f * (-b * (\sqrt{df})^{1/2} + a * f + c * d))^{1/2}) / (x + (\sqrt{df})^{1/2}/f)) * b * A - 1/2 * (\sqrt{df})^{1/2} / f^2 / (1/f * (-b * (\sqrt{df})^{1/2} + a * f + c * d))^{1/2} * \ln((2 * f * (-b * (\sqrt{df})^{1/2} + a * f + c * d) + 1/f * (-2c * (\sqrt{df})^{1/2} + b * f) * (x + (\sqrt{df})^{1/2}/f) + 2 * (1/f * (-b * (\sqrt{df})^{1/2} + a * f + c * d))^{1/2}) * ((x + (\sqrt{df})^{1/2}/f)^{2c+1} * f * (-2c * (\sqrt{df})^{1/2} + b * f) * (x + (\sqrt{df})^{1/2}/f) + 1/f * (-b * (\sqrt{df})^{1/2} + a * f + c * d))^{1/2}) / (x + (\sqrt{df})^{1/2}/f))$

$$\begin{aligned}
& -b*(d*f)^{(1/2)+a*f+c*d})^{(1/2)}*((x+(d*f)^{(1/2)}/f)^{2*c+1}/f*(-2*c*(d*f)^{(1/2)} \\
& +b*f)*(x+(d*f)^{(1/2)}/f)+1/f*(-b*(d*f)^{(1/2)+a*f+c*d})^{(1/2)})/(x+(d*f)^{(1/2)} \\
& /f))*b*B-1/2/(d*f)^{(1/2)}/(1/f*(-b*(d*f)^{(1/2)+a*f+c*d})^{(1/2)}*\ln((2/f*(-b*(\\
& d*f)^{(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^{(1/2)+b*f)*(x+(d*f)^{(1/2)}/f)+2*(1/f*(-b \\
& *(d*f)^{(1/2)+a*f+c*d})^{(1/2)}*((x+(d*f)^{(1/2)}/f)^{2*c+1}/f*(-2*c*(d*f)^{(1/2)+b \\
& *f)*(x+(d*f)^{(1/2)}/f)+1/f*(-b*(d*f)^{(1/2)+a*f+c*d})^{(1/2)})/(x+(d*f)^{(1/2)}/f \\
&))*a*A+1/2/f/(1/f*(-b*(d*f)^{(1/2)+a*f+c*d})^{(1/2)}*\ln((2/f*(-b*(d*f)^{(1/2)+a \\
& *f+c*d)+1/f*(-2*c*(d*f)^{(1/2)+b*f)*(x+(d*f)^{(1/2)}/f)+2*(1/f*(-b*(d*f)^{(1/2) \\
& +a*f+c*d})^{(1/2)}*((x+(d*f)^{(1/2)}/f)^{2*c+1}/f*(-2*c*(d*f)^{(1/2)+b*f)*(x+(d*f) \\
& ^{(1/2)}/f)+1/f*(-b*(d*f)^{(1/2)+a*f+c*d})^{(1/2)})/(x+(d*f)^{(1/2)}/f))*a*B-1/2/(\\
& d*f)^{(1/2)}/f/(1/f*(-b*(d*f)^{(1/2)+a*f+c*d})^{(1/2)}*\ln((2/f*(-b*(d*f)^{(1/2)+a \\
& *f+c*d)+1/f*(-2*c*(d*f)^{(1/2)+b*f)*(x+(d*f)^{(1/2)}/f)+2*(1/f*(-b*(d*f)^{(1/2) \\
& +a*f+c*d})^{(1/2)}*((x+(d*f)^{(1/2)}/f)^{2*c+1}/f*(-2*c*(d*f)^{(1/2)+b*f)*(x+(d*f) \\
& ^{(1/2)}/f)+1/f*(-b*(d*f)^{(1/2)+a*f+c*d})^{(1/2)})/(x+(d*f)^{(1/2)}/f))*c*d*A+1/2 \\
& /f^2/(1/f*(-b*(d*f)^{(1/2)+a*f+c*d})^{(1/2)}*\ln((2/f*(-b*(d*f)^{(1/2)+a*f+c*d)+ \\
& 1/f*(-2*c*(d*f)^{(1/2)+b*f)*(x+(d*f)^{(1/2)}/f)+2*(1/f*(-b*(d*f)^{(1/2)+a*f+c*d} \\
&))^{(1/2)}*((x+(d*f)^{(1/2)}/f)^{2*c+1}/f*(-2*c*(d*f)^{(1/2)+b*f)*(x+(d*f)^{(1/2)}/f \\
&)+1/f*(-b*(d*f)^{(1/2)+a*f+c*d})^{(1/2)})/(x+(d*f)^{(1/2)}/f))*c*d*B-1/2/(d*f)^{(\\
& 1/2)}*((x-(d*f)^{(1/2)}/f)^{2*c+(2*c*(d*f)^{(1/2)+b*f)/f*(x-(d*f)^{(1/2)}/f)+(b*(d \\
& *f)^{(1/2)+a*f+c*d)/f)^{(1/2)}*A-1/2/f*((x-(d*f)^{(1/2)}/f)^{2*c+(2*c*(d*f)^{(1/2) \\
& +b*f)/f*(x-(d*f)^{(1/2)}/f)+(b*(d*f)^{(1/2)+a*f+c*d)/f)^{(1/2)}*B-1/2/f*\ln((1/2* \\
& (2*c*(d*f)^{(1/2)+b*f)/f+c*(x-(d*f)^{(1/2)}/f))/c^{(1/2)}+((x-(d*f)^{(1/2)}/f)^{2*c \\
& +(2*c*(d*f)^{(1/2)+b*f)/f*(x-(d*f)^{(1/2)}/f)+(b*(d*f)^{(1/2)+a*f+c*d)/f)^{(1/2) \\
&)}*c^{(1/2)}*A-1/2*(d*f)^{(1/2)}/f^2*\ln((1/2*(2*c*(d*f)^{(1/2)+b*f)/f+c*(x-(d*f)^{(\\
& 1/2)}/f))/c^{(1/2)}+((x-(d*f)^{(1/2)}/f)^{2*c+(2*c*(d*f)^{(1/2)+b*f)/f*(x-(d*f)^{(\\
& 1/2)}/f)+(b*(d*f)^{(1/2)+a*f+c*d)/f)^{(1/2)})*c^{(1/2)}*B-1/4/(d*f)^{(1/2)}*\ln((1/2 \\
& *(2*c*(d*f)^{(1/2)+b*f)/f+c*(x-(d*f)^{(1/2)}/f))/c^{(1/2)}+((x-(d*f)^{(1/2)}/f)^{2*c \\
& c+(2*c*(d*f)^{(1/2)+b*f)/f*(x-(d*f)^{(1/2)}/f)+(b*(d*f)^{(1/2)+a*f+c*d)/f)^{(1/2) \\
&))/c^{(1/2)}*b*A-1/4/f*\ln((1/2*(2*c*(d*f)^{(1/2)+b*f)/f+c*(x-(d*f)^{(1/2)}/f))/c \\
& ^{(1/2)}+((x-(d*f)^{(1/2)}/f)^{2*c+(2*c*(d*f)^{(1/2)+b*f)/f*(x-(d*f)^{(1/2)}/f)+(b* \\
& (d*f)^{(1/2)+a*f+c*d)/f)^{(1/2)}))/c^{(1/2)}*b*B+1/2/f/((b*(d*f)^{(1/2)+a*f+c*d)/f \\
&)^{(1/2)}*\ln((2*(b*(d*f)^{(1/2)+a*f+c*d)/f+(2*c*(d*f)^{(1/2)+b*f)/f*(x-(d*f)^{(1 \\
& /2)}/f)+2*((b*(d*f)^{(1/2)+a*f+c*d)/f)^{(1/2)}*((x-(d*f)^{(1/2)}/f)^{2*c+(2*c*(d*f) \\
&)^{(1/2)+b*f)/f*(x-(d*f)^{(1/2)}/f)+(b*(d*f)^{(1/2)+a*f+c*d)/f)^{(1/2)})/(x-(d*f) \\
& ^{(1/2)}/f))*b*A+1/2*(d*f)^{(1/2)}/f^2/((b*(d*f)^{(1/2)+a*f+c*d)/f)^{(1/2)}*\ln((2* \\
& (b*(d*f)^{(1/2)+a*f+c*d)/f+(2*c*(d*f)^{(1/2)+b*f)/f*(x-(d*f)^{(1/2)}/f)+2*((b*(\\
& d*f)^{(1/2)+a*f+c*d)/f)^{(1/2)}*((x-(d*f)^{(1/2)}/f)^{2*c+(2*c*(d*f)^{(1/2)+b*f)/f \\
& *(x-(d*f)^{(1/2)}/f)+(b*(d*f)^{(1/2)+a*f+c*d)/f)^{(1/2)})/(x-(d*f)^{(1/2)}/f))*b*B \\
& +1/2/(d*f)^{(1/2)}/((b*(d*f)^{(1/2)+a*f+c*d)/f)^{(1/2)}*\ln((2*(b*(d*f)^{(1/2)+a*f \\
& +c*d)/f+(2*c*(d*f)^{(1/2)+b*f)/f*(x-(d*f)^{(1/2)}/f)+2*((b*(d*f)^{(1/2)+a*f+c*d} \\
&)/f)^{(1/2)}*((x-(d*f)^{(1/2)}/f)^{2*c+(2*c*(d*f)^{(1/2)+b*f)/f*(x-(d*f)^{(1/2)}/f) \\
& +(b*(d*f)^{(1/2)+a*f+c*d)/f)^{(1/2)})/(x-(d*f)^{(1/2)}/f))*a*A+1/2/f/((b*(d*f)^{(\\
& 1/2)+a*f+c*d)/f)^{(1/2)}*\ln((2*(b*(d*f)^{(1/2)+a*f+c*d)/f+(2*c*(d*f)^{(1/2)+b*f \\
&)/f*(x-(d*f)^{(1/2)}/f)+2*((b*(d*f)^{(1/2)+a*f+c*d)/f)^{(1/2)}*((x-(d*f)^{(1/2)}/f \\
&)^{2*c+(2*c*(d*f)^{(1/2)+b*f)/f*(x-(d*f)^{(1/2)}/f)+(b*(d*f)^{(1/2)+a*f+c*d)/f)^{(\\
& 1/2)})/(x-(d*f)^{(1/2)}/f))*a*B+1/2/(d*f)^{(1/2)}/f/((b*(d*f)^{(1/2)+a*f+c*d)/f) \\
& ^{(1/2)}*\ln((2*(b*(d*f)^{(1/2)+a*f+c*d)/f+(2*c*(d*f)^{(1/2)+b*f)/f*(x-(d*f)^{(1/ \\
& 2)}/f)+2*((b*(d*f)^{(1/2)+a*f+c*d)/f)^{(1/2)}*((x-(d*f)^{(1/2)}/f)^{2*c+(2*c*(d*f) \\
& ^{(1/2)+b*f)/f*(x-(d*f)^{(1/2)}/f)+(b*(d*f)^{(1/2)+a*f+c*d)/f)^{(1/2)})/(x-(d*f) \\
& ^{(1/2)}/f))*c*d*A+1/2/f^2/((b*(d*f)^{(1/2)+a*f+c*d)/f)^{(1/2)}*\ln((2*(b*(d*f)^{(1 \\
& /2)+a*f+c*d)/f+(2*c*(d*f)^{(1/2)+b*f)/f*(x-(d*f)^{(1/2)}/f)+2*((b*(d*f)^{(1/2)+ \\
& a*f+c*d)/f)^{(1/2)}*((x-(d*f)^{(1/2)}/f)^{2*c+(2*c*(d*f)^{(1/2)+b*f)/f*(x-(d*f)^{(\\
& 1/2)}/f)+(b*(d*f)^{(1/2)+a*f+c*d)/f)^{(1/2)})/(x-(d*f)^{(1/2)}/f))*c*d*B
\end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see 'assume?' for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2 - (c*((b*sqrt(4*d*f)) / (2*f) + (c*d)/f+a)) /f^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + Bx) \sqrt{cx^2 + bx + a}}{d - fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/(d - f*x^2), x)

[Out] int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/(d - f*x^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{A\sqrt{a + bx + cx^2}}{-d + fx^2} dx - \int \frac{Bx\sqrt{a + bx + cx^2}}{-d + fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d), x)

[Out] -Integral(A*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(B*x*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)

$$3.7 \quad \int \frac{A+Bx}{\sqrt{a+bx+cx^2}(d-fx^2)} dx$$

Optimal. Leaf size=249

$$\frac{\left(\frac{A\sqrt{f}}{\sqrt{d}} + B\right) \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}}\sqrt{f+cd}}\right)}{2\sqrt{f}\sqrt{af+b\sqrt{d}}\sqrt{f+cd}} - \frac{\left(B - \frac{A\sqrt{f}}{\sqrt{d}}\right) \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f+cd}}\right)}{2\sqrt{f}\sqrt{af+b(-\sqrt{d})}\sqrt{f+cd}}$$

Rubi [A] time = 0.20, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1033, 724, 206}

$$\frac{\left(\frac{A\sqrt{f}}{\sqrt{d}} + B\right) \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}}\sqrt{f+cd}}\right)}{2\sqrt{f}\sqrt{af+b\sqrt{d}}\sqrt{f+cd}} - \frac{\left(B - \frac{A\sqrt{f}}{\sqrt{d}}\right) \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f+cd}}\right)}{2\sqrt{f}\sqrt{af+b(-\sqrt{d})}\sqrt{f+cd}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x)/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)), x]

[Out] -((B - (A*Sqrt[f])/Sqrt[d])*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + b*x + c*x^2]])/(2*Sqrt[f]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f) + ((B + (A*Sqrt[f])/Sqrt[d])*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + b*x + c*x^2]])/(2*Sqrt[f]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1033

Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]

Rubi steps

$$\begin{aligned} \int \frac{A+Bx}{\sqrt{a+bx+cx^2}(d-fx^2)} dx &= \frac{1}{2} \left(B - \frac{A\sqrt{f}}{\sqrt{d}} \right) \int \frac{1}{(-\sqrt{d}\sqrt{f}-fx)\sqrt{a+bx+cx^2}} dx + \frac{1}{2} \left(B + \frac{A\sqrt{f}}{\sqrt{d}} \right) \int \frac{1}{(\sqrt{d}\sqrt{f}-fx)\sqrt{a+bx+cx^2}} dx \\ &= \left(-B - \frac{A\sqrt{f}}{\sqrt{d}} \right) \text{Subst} \left(\int \frac{1}{4cdf + 4b\sqrt{d}f^{3/2} + 4af^2 - x^2} dx, x, \frac{-b\sqrt{d}\sqrt{f} - 2af}{\sqrt{a+bx+cx^2}} \right) \\ &\quad + \left(B + \frac{A\sqrt{f}}{\sqrt{d}} \right) \text{Subst} \left(\int \frac{1}{4cdf + 4b\sqrt{d}f^{3/2} + 4af^2 - x^2} dx, x, \frac{-b\sqrt{d}\sqrt{f} + 2af}{\sqrt{a+bx+cx^2}} \right) \\ &= -\frac{\left(B - \frac{A\sqrt{f}}{\sqrt{d}} \right) \tanh^{-1} \left(\frac{b\sqrt{d}-2a\sqrt{f}+(2c\sqrt{d}-b\sqrt{f})x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}} \right)}{2\sqrt{f}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} + \frac{\left(B + \frac{A\sqrt{f}}{\sqrt{d}} \right) \tanh^{-1} \left(\frac{b\sqrt{d}+2a\sqrt{f}+(2c\sqrt{d}+b\sqrt{f})x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}} \right)}{2\sqrt{f}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}} \end{aligned}$$

Mathematica [A] time = 0.24, size = 249, normalized size = 1.00

$$\frac{\frac{(B\sqrt{d}-A\sqrt{f}) \tanh^{-1} \left(\frac{-2a\sqrt{f}+b(\sqrt{d}-\sqrt{f}x)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} \right)}{\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} - \frac{(A\sqrt{f}+B\sqrt{d}) \tanh^{-1} \left(\frac{-2(a\sqrt{f}+c\sqrt{d}x)-b(\sqrt{d}+\sqrt{f}x)}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} \right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x)/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)), x]

[Out] (-(((B*Sqrt[d] - A*Sqrt[f])*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + x*(b + c*x)])))/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f] - ((B*Sqrt[d] + A*Sqrt[f])*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + x*(b + c*x)])))/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]/(2*Sqrt[d]*Sqrt[f])

IntegrateAlgebraic [C] time = 0.43, size = 219, normalized size = 0.88

$$\frac{1}{2} \text{RootSum} \left[\#1^4(-f) + 2\#1^2af + 4\#1^2cd - 4\#1b\sqrt{cd} - a^2f + b^2d \&, \frac{\#1^2B \log(-\#1 + \sqrt{a+bx+cx^2} - \sqrt{cx}) - 2\#1A\sqrt{c} \log(-\#1 + \sqrt{a+bx+cx^2} - \sqrt{cx}) + Ab \log(-\#1 + \sqrt{a+bx+cx^2} - \sqrt{cx}) - aB \log(-\#1 + \sqrt{a+bx+cx^2} - \sqrt{cx})}{\#1^3f - \#1af - 2\#1cd + b\sqrt{cd}} \& \right]$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B*x)/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)), x]

[Out] -1/2*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 &, (A*b*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*B*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*A*Sqrt[c]*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + B*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) &]

fricas [B] time = 52.20, size = 6113, normalized size = 24.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d), x, algorithm="fricas")

[Out] 1/4*sqrt((B^2*c*d^2 + A^2*a*f^2 + (B^2*a - 2*A*B*b + A^2*c)*d*f + (c^2*d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2))/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))/(c^2*d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2)*log(-((B^4*b^2 - 2*A*B^3*b*c)*d^2 - 2*(A*B^3*a*b - A^3*B*b*c)*d*f + (


```

g(-((B^4*b^2 - 2*A*B^3*b*c)*d^2 - 2*(A*B^3*a*b - A^3*B*b*c)*d*f + (2*A^3*B*
a*b - A^4*b^2)*f^2 + 2*((2*A^3*B*a - A^4*b)*c*f^2 + (B^4*b*c - 2*A*B^3*c^2)
*d^2 - 2*(A*B^3*a*c - A^3*B*c^2)*d*f)*x + 2*((B^3*b^2 - 3*A*B^2*b*c + 2*A^2
*B*c^2)*d^2*f - (3*A*B^2*a*b - A^2*B*b^2 - (4*A^2*B*a - A^3*b)*c)*d*f^2 + (
2*A^2*B*a^2 - A^3*a*b)*f^3 + (B*c^3*d^4*f - (B*b^2*c - (3*B*a - A*b)*c^2)*d
^3*f^2 - (B*a*b^2 - A*b^3 - (3*B*a^2 - 2*A*a*b)*c)*d^2*f^3 + (B*a^3 - A*a^2
*b)*d*f^4)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a
*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^
3*B*a*b + A^4*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*
f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4
)))*sqrt(c*x^2 + b*x + a)*sqrt((B^2*c*d^2 + A^2*a*f^2 + (B^2*a - 2*A*B*b +
A^2*c)*d*f - (c^2*d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2))*sqrt(((B^4*b^2
- 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A
^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/(
c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*c +
6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))/(c^2*d^3*f + a^2*d*f^
3 - (b^2 - 2*a*c)*d^2*f^2)) + (2*B^2*a*c^2*d^3*f - 2*A^2*a^3*f^4 - 2*(B^2*a
*b^2 - 2*B^2*a^2*c + A^2*a*c^2)*d^2*f^2 + 2*(B^2*a^3 + A^2*a*b^2 - 2*A^2*a^
2*c)*d*f^3 + (B^2*b*c^2*d^3*f - A^2*a^2*b*f^4 - (B^2*b^3 - 2*B^2*a*b*c + A^
2*b*c^2)*d^2*f^2 + (B^2*a^2*b + A^2*b^3 - 2*A^2*a*b*c)*d*f^3)*x)*sqrt(((B^4
*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*
(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^
2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*
c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))/x - 1/4*sqrt((B^
2*c*d^2 + A^2*a*f^2 + (B^2*a - 2*A*B*b + A^2*c)*d*f - (c^2*d^3*f + a^2*d*f^
3 - (b^2 - 2*a*c)*d^2*f^2))*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^
2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^
2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2
- 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 -
2*a^3*c)*d^2*f^4)))/(c^2*d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2))*log(-
(B^4*b^2 - 2*A*B^3*b*c)*d^2 - 2*(A*B^3*a*b - A^3*B*b*c)*d*f + (2*A^3*B*a*b
- A^4*b^2)*f^2 + 2*((2*A^3*B*a - A^4*b)*c*f^2 + (B^4*b*c - 2*A*B^3*c^2)*d^2
- 2*(A*B^3*a*c - A^3*B*c^2)*d*f)*x - 2*((B^3*b^2 - 3*A*B^2*b*c + 2*A^2*B*c
^2)*d^2*f - (3*A*B^2*a*b - A^2*B*b^2 - (4*A^2*B*a - A^3*b)*c)*d*f^2 + (2*A^
2*B*a^2 - A^3*a*b)*f^3 + (B*c^3*d^4*f - (B*b^2*c - (3*B*a - A*b)*c^2)*d^3*f
^2 - (B*a*b^2 - A*b^3 - (3*B*a^2 - 2*A*a*b)*c)*d^2*f^3 + (B*a^3 - A*a^2*b)*
d*f^4)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b
- A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B
*a*b + A^4*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2
+ (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))*
sqrt(c*x^2 + b*x + a)*sqrt((B^2*c*d^2 + A^2*a*f^2 + (B^2*a - 2*A*B*b + A^2*
c)*d*f - (c^2*d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2))*sqrt(((B^4*b^2 - 4
*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B
^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/(c^4*
d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*c + 6*a^
2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))/(c^2*d^3*f + a^2*d*f^3 -
(b^2 - 2*a*c)*d^2*f^2)) + (2*B^2*a*c^2*d^3*f - 2*A^2*a^3*f^4 - 2*(B^2*a*b^2
- 2*B^2*a^2*c + A^2*a*c^2)*d^2*f^2 + 2*(B^2*a^3 + A^2*a*b^2 - 2*A^2*a^2*c)
*d*f^3 + (B^2*b*c^2*d^3*f - A^2*a^2*b*f^4 - (B^2*b^3 - 2*B^2*a*b*c + A^2*b*
c^2)*d^2*f^2 + (B^2*a^2*b + A^2*b^3 - 2*A^2*a*b*c)*d*f^3)*x)*sqrt(((B^4*b^2
- 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A
^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/(
c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*c +
6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))/x)

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument ValueConj Error: Bad Argument Typ
 e

maple [B] time = 0.02, size = 714, normalized size = 2.87

$$A \ln \left(\frac{\sqrt{c x^2 + b x + a} \sqrt{-f x^2 + d}}{\sqrt{c x^2 + b x + a} \sqrt{-f x^2 + d}} \right) + A \ln \left(\frac{\sqrt{c x^2 + b x + a} \sqrt{-f x^2 + d}}{\sqrt{c x^2 + b x + a} \sqrt{-f x^2 + d}} \right) + B \ln \left(\frac{\sqrt{c x^2 + b x + a} \sqrt{-f x^2 + d}}{\sqrt{c x^2 + b x + a} \sqrt{-f x^2 + d}} \right) + B \ln \left(\frac{\sqrt{c x^2 + b x + a} \sqrt{-f x^2 + d}}{\sqrt{c x^2 + b x + a} \sqrt{-f x^2 + d}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x+A)/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)

[Out]
$$-1/2/(d*f)^{(1/2)}/(1/f*(-b*(d*f)^{(1/2)}+a*f+c*d))^{(1/2)}*\ln((2/f*(-b*(d*f)^{(1/2)}+a*f+c*d)+1/f*(-2*c*(d*f)^{(1/2)}+b*f)*(x+(d*f)^{(1/2)}/f)+2*(1/f*(-b*(d*f)^{(1/2)}+a*f+c*d))^{(1/2)}*((x+(d*f)^{(1/2)}/f)^2*c+1/f*(-2*c*(d*f)^{(1/2)}+b*f)*(x+(d*f)^{(1/2)}/f)+1/f*(-b*(d*f)^{(1/2)}+a*f+c*d))^{(1/2)})/(x+(d*f)^{(1/2)}/f))*A+1/2/f/(1/f*(-b*(d*f)^{(1/2)}+a*f+c*d))^{(1/2)}*\ln((2/f*(-b*(d*f)^{(1/2)}+a*f+c*d)+1/f*(-2*c*(d*f)^{(1/2)}+b*f)*(x+(d*f)^{(1/2)}/f)+2*(1/f*(-b*(d*f)^{(1/2)}+a*f+c*d))^{(1/2)}*((x+(d*f)^{(1/2)}/f)^2*c+1/f*(-2*c*(d*f)^{(1/2)}+b*f)*(x+(d*f)^{(1/2)}/f)+1/f*(-b*(d*f)^{(1/2)}+a*f+c*d))^{(1/2)})/(x+(d*f)^{(1/2)}/f))*B+1/2/(d*f)^{(1/2)}/((b*(d*f)^{(1/2)}+a*f+c*d)/f)^{(1/2)}*\ln((2*(b*(d*f)^{(1/2)}+a*f+c*d)/f+(2*c*(d*f)^{(1/2)}+b*f)/f*(x-(d*f)^{(1/2)}/f)+2*((b*(d*f)^{(1/2)}+a*f+c*d)/f)^{(1/2)}*((x-(d*f)^{(1/2)}/f)^2*c+(2*c*(d*f)^{(1/2)}+b*f)/f*(x-(d*f)^{(1/2)}/f)+(b*(d*f)^{(1/2)}+a*f+c*d)/f)^{(1/2)})/(x-(d*f)^{(1/2)}/f))*A+1/2/f/((b*(d*f)^{(1/2)}+a*f+c*d)/f)^{(1/2)}*\ln((2*(b*(d*f)^{(1/2)}+a*f+c*d)/f+(2*c*(d*f)^{(1/2)}+b*f)/f*(x-(d*f)^{(1/2)}/f)+2*((b*(d*f)^{(1/2)}+a*f+c*d)/f)^{(1/2)}*((x-(d*f)^{(1/2)}/f)^2*c+(2*c*(d*f)^{(1/2)}+b*f)/f*(x-(d*f)^{(1/2)}/f)+(b*(d*f)^{(1/2)}+a*f+c*d)/f)^{(1/2)})/(x-(d*f)^{(1/2)}/f))*B$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2 -c*((b*sqrt(4*d*f))/(2*f) + (c*d)/f+a) /f^2 zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + Bx}{(d - fx^2) \sqrt{cx^2 + bx + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)

[Out] int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{A}{-d\sqrt{a + bx + cx^2} + fx^2\sqrt{a + bx + cx^2}} dx - \int \frac{Bx}{-d\sqrt{a + bx + cx^2} + fx^2\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(A/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)), x)
- Integral(B*x/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2))
, x)
```


$$3.8 \quad \int \frac{A+Bx}{(a+bx+cx^2)^{3/2}(d-fx^2)} dx$$

Optimal. Leaf size=381

$$\frac{2\left(A\left(b^3f - bc(3af + cd)\right) + cx\left(-2Ac(af + cd) + bB(cd - af) + Ab^2f\right) + aB\left(2acf + b^2(-f) + 2c^2d\right)\right) \sqrt{f}}{(b^2 - 4ac) \sqrt{a + bx + cx^2} (b^2df - (af + cd)^2)}$$

Rubi [A] time = 0.80, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, number of rules / integrand size = 0.133, Rules used = {1018, 1033, 724, 206}

$$\frac{2(cx(-2Ac(af + cd) + bB(cd - af) + Ab^2f) - Abc(3af + cd) + aB(2acf + b^2(-f) + 2c^2d) + Ab^2f)}{(b^2 - 4ac) \sqrt{a + bx + cx^2} (b^2df - (af + cd)^2)} - \frac{\sqrt{f}(B\sqrt{d} - A\sqrt{f}) \tanh^{-1}\left(\frac{-2a\sqrt{f} + (2\sqrt{d} - \sqrt{f}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2\sqrt{d}(af + b(-\sqrt{d})\sqrt{f} + cd)^{3/2}} + \frac{\sqrt{f}(A\sqrt{f} + B\sqrt{d}) \tanh^{-1}\left(\frac{2a\sqrt{f} + (b\sqrt{f} + 2c\sqrt{d}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}(af + b\sqrt{d}\sqrt{f} + cd)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x)/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)), x]

[Out] (-2*(A*b^3*f - A*b*c*(c*d + 3*a*f) + a*B*(2*c^2*d - b^2*f + 2*a*c*f) + c*(A*b^2*f + b*B*(c*d - a*f) - 2*A*c*(c*d + a*f))*x)/((b^2 - 4*a*c)*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) - ((B*Sqrt[d] - A*Sqrt[f])*Sqrt[f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]/(2*Sqrt[d]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + ((B*Sqrt[d] + A*Sqrt[f])*Sqrt[f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]/(2*Sqrt[d]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1018

Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1)*((g*c)*(-(b*(c*d + a*f)))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p + q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] && !(IntegerQ[p] && ILtQ[q, -1])

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{A + Bx}{(a + bx + cx^2)^{3/2} (d - fx^2)} dx = -\frac{2(Ab^3f - Abc(cd + 3af) + aB(2c^2d - b^2f + 2acf) + c(Ab^2f + bB(cd - a))}{(b^2 - 4ac)(b^2df - (cd + af)^2)\sqrt{a + bx + cx^2}}$$

$$= -\frac{2(Ab^3f - Abc(cd + 3af) + aB(2c^2d - b^2f + 2acf) + c(Ab^2f + bB(cd - a))}{(b^2 - 4ac)(b^2df - (cd + af)^2)\sqrt{a + bx + cx^2}}$$

$$= -\frac{2(Ab^3f - Abc(cd + 3af) + aB(2c^2d - b^2f + 2acf) + c(Ab^2f + bB(cd - a))}{(b^2 - 4ac)(b^2df - (cd + af)^2)\sqrt{a + bx + cx^2}}$$

$$= -\frac{2(Ab^3f - Abc(cd + 3af) + aB(2c^2d - b^2f + 2acf) + c(Ab^2f + bB(cd - a))}{(b^2 - 4ac)(b^2df - (cd + af)^2)\sqrt{a + bx + cx^2}}$$

Mathematica [A] time = 0.65, size = 440, normalized size = 1.15

$$2 \left(\frac{B(2a^2cf + a(l^2(-f) - bcfx + 2c^2d) + bc^2dx) + A(-bc(3af + cd) - 2c^2x(af + cd) + b^2f + b^2cfx)}{\sqrt{a+x(b+cx)}} + \frac{\sqrt{f}(b^2-4ac)(A\sqrt{f-B\sqrt{d}})(af+b\sqrt{d}\sqrt{f+cd})\tanh^{-1}\left(\frac{-2a\sqrt{f}+b(\sqrt{d}-\sqrt{f}x)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f+cd}}}\right) + \sqrt{f}(4ac-b^2)(A\sqrt{f+B\sqrt{d}})(af+b(-\sqrt{d})\sqrt{f+cd})\tanh^{-1}\left(\frac{-2(a\sqrt{f}+b\sqrt{d}x)-b(\sqrt{d}+\sqrt{f}x)}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f+cd}}}\right)}{4\sqrt{d}\sqrt{af+b\sqrt{d}\sqrt{f+cd}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)), x]
[Out] (2*((A*(b^3*f - b*c*(c*d + 3*a*f) + b^2*c*f*x - 2*c^2*(c*d + a*f)*x) + B*(2*a^2*c*f + b*c^2*d*x + a*(2*c^2*d - b^2*f - b*c*f*x)))/Sqrt[a + x*(b + c*x)] + ((b^2 - 4*a*c)*(-B*Sqrt[d] + A*Sqrt[f])*Sqrt[f]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/(4*Sqrt[d]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f) + ((-b^2 + 4*a*c)*(B*Sqrt[d] + A*Sqrt[f])*Sqrt[f]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/(4*Sqrt[d]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f)))/(b^2 - 4*a*c)*(-b^2*d*f + (c*d + a*f)^2))
```

IntegrateAlgebraic [C] time = 1.50, size = 658, normalized size = 1.73

$$\frac{2 \left(\frac{B(2a^2cf + a(l^2(-f) - bcfx + 2c^2d) + bc^2dx) + A(-bc(3af + cd) - 2c^2x(af + cd) + b^2f + b^2cfx)}{\sqrt{a+x(b+cx)}} + \frac{\sqrt{f}(b^2-4ac)(A\sqrt{f-B\sqrt{d}})(af+b\sqrt{d}\sqrt{f+cd})\tanh^{-1}\left(\frac{-2a\sqrt{f}+b(\sqrt{d}-\sqrt{f}x)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f+cd}}}\right) + \sqrt{f}(4ac-b^2)(A\sqrt{f+B\sqrt{d}})(af+b(-\sqrt{d})\sqrt{f+cd})\tanh^{-1}\left(\frac{-2(a\sqrt{f}+b\sqrt{d}x)-b(\sqrt{d}+\sqrt{f}x)}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f+cd}}}\right)}{4\sqrt{d}\sqrt{af+b\sqrt{d}\sqrt{f+cd}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x)/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)), x]
[Out] (-2*(-(A*b*c^2*d) + 2*a*B*c^2*d + A*b^3*f - a*b^2*B*f - 3*a*A*b*c*f + 2*a^2*B*c*f + b*B*c^2*d*x - 2*A*c^3*d*x + A*b^2*c*f*x - a*b*B*c*f*x - 2*a*A*c^2*
```

$$\frac{f*x)}{((b^2 - 4*a*c)*(-(c^2*d^2) + b^2*d*f - 2*a*c*d*f - a^2*f^2)*\text{Sqrt}[a + b*x + c*x^2]) + (f*\text{RootSum}[b^2*d - a^2*f - 4*b*\text{Sqrt}[c]*d*\#1 + 4*c*d*\#1^2 + 2*a*f*\#1^2 - f*\#1^4 \& , (b^2*B*d*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] - A*b*c*d*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] + a*B*c*d*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] - 2*a*A*b*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] + a^2*B*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] - 2*b*B*\text{Sqrt}[c]*d*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1 + 2*A*c^(3/2)*d*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1 + 2*a*A*\text{Sqrt}[c]*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1 - B*c*d*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1^2 + A*b*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1^2 - a*B*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1^2)/(b*\text{Sqrt}[c]*d - 2*c*d*\#1 - a*f*\#1 + f*\#1^3) \&])/(2*(c^2*d^2 - b^2*d*f + 2*a*c*d*f + a^2*f^2))$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 10.66Conj E rror: Bad Argument Type

maple [B] time = 0.03, size = 2758, normalized size = 7.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x+A)/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)

[Out] $\frac{1/2/(d*f)^{(1/2)}*f/(a*f+c*d-(d*f)^{(1/2)}*b)/((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)}*c)*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)}*A-1/2/(a*f+c*d-(d*f)^{(1/2)}*b)/((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)}*c)*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)}*B+2/(a*f+c*d-(d*f)^{(1/2)}*b)/(4*a*c-b^2)/((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)}*c)*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)}*x*c^2*A-2*(d*f)^{(1/2)}/f/(a*f+c*d-(d*f)^{(1/2)}*b)/(4*a*c-b^2)/((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)}*c)*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)}*x*c^2*B-1/(d*f)^{(1/2)}*f/(a*f+c*d-(d*f)^{(1/2)}*b)/(4*a*c-b^2)/((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)}*c)*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)}*x*b*c*A+1/(a*f+c*d-(d*f)^{(1/2)}*b)/(4*a*c-b^2)/((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)}*c)*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)}*b*c*B-1/2/(d*f)^{(1/2)}*f/(a*f+c*d-(d*f)^{(1/2)}*b)/(4*a*c-b^2)/((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)}*c)*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)}*b^2*A+1/2/(a*f+c*d-(d*f)^{(1/2)}}$

$$\begin{aligned}
& 2) * b) / (4 * a * c - b^2) / ((x + (d * f)^{(1/2)} / f)^2 * c + (b * f - 2 * (d * f)^{(1/2)} * c) * (x + (d * f)^{(1/2)} / f) / f + (a * f + c * d - (d * f)^{(1/2)} * b) / f)^{(1/2)} * b^2 * B - 1/2 / (d * f)^{(1/2)} * f / (a * f + c * d - (d * f)^{(1/2)} * b) / ((a * f + c * d - (d * f)^{(1/2)} * b) / f)^{(1/2)} * \ln((2 * (a * f + c * d - (d * f)^{(1/2)} * b) / f + (b * f - 2 * (d * f)^{(1/2)} * c) * (x + (d * f)^{(1/2)} / f) / f + 2 * ((a * f + c * d - (d * f)^{(1/2)} * b) / f)^{(1/2)} * ((x + (d * f)^{(1/2)} / f)^2 * c + (b * f - 2 * (d * f)^{(1/2)} * c) * (x + (d * f)^{(1/2)} / f) / f + (a * f + c * d - (d * f)^{(1/2)} * b) / f)^{(1/2)}) / (x + (d * f)^{(1/2)} / f) * A + 1/2 / (a * f + c * d - (d * f)^{(1/2)} * b) / ((a * f + c * d - (d * f)^{(1/2)} * b) / f)^{(1/2)} * \ln((2 * (a * f + c * d - (d * f)^{(1/2)} * b) / f + (b * f - 2 * (d * f)^{(1/2)} * c) * (x + (d * f)^{(1/2)} / f) / f + 2 * ((a * f + c * d - (d * f)^{(1/2)} * b) / f)^{(1/2)} * ((x + (d * f)^{(1/2)} / f)^2 * c + (b * f - 2 * (d * f)^{(1/2)} * c) * (x + (d * f)^{(1/2)} / f) / f + (a * f + c * d - (d * f)^{(1/2)} * b) / f)^{(1/2)}) / (x + (d * f)^{(1/2)} / f) * B - 1/2 / (d * f)^{(1/2)} * f / (a * f + c * d + (d * f)^{(1/2)} * b) / ((x - (d * f)^{(1/2)} / f)^2 * c + (b * f + 2 * (d * f)^{(1/2)} * c) * (x - (d * f)^{(1/2)} / f) / f + (a * f + c * d + (d * f)^{(1/2)} * b) / f)^{(1/2)} * A - 1/2 / (a * f + c * d + (d * f)^{(1/2)} * b) / ((x - (d * f)^{(1/2)} / f)^2 * c + (b * f + 2 * (d * f)^{(1/2)} * c) * (x - (d * f)^{(1/2)} / f) / f + (a * f + c * d + (d * f)^{(1/2)} * b) / f)^{(1/2)} * B + 2 / (a * f + c * d + (d * f)^{(1/2)} * b) / (4 * a * c - b^2) / ((x - (d * f)^{(1/2)} / f)^2 * c + (b * f + 2 * (d * f)^{(1/2)} * c) * (x - (d * f)^{(1/2)} / f) / f + (a * f + c * d + (d * f)^{(1/2)} * b) / f)^{(1/2)} * x * c^2 * A + 2 * (d * f)^{(1/2)} / f / (a * f + c * d + (d * f)^{(1/2)} * b) / (4 * a * c - b^2) / ((x - (d * f)^{(1/2)} / f)^2 * c + (b * f + 2 * (d * f)^{(1/2)} * c) * (x - (d * f)^{(1/2)} / f) / f + (a * f + c * d + (d * f)^{(1/2)} * b) / f)^{(1/2)} * x * c^2 * B + 1 / (d * f)^{(1/2)} * f / (a * f + c * d + (d * f)^{(1/2)} * b) / (4 * a * c - b^2) / ((x - (d * f)^{(1/2)} / f)^2 * c + (b * f + 2 * (d * f)^{(1/2)} * c) * (x - (d * f)^{(1/2)} / f) / f + (a * f + c * d + (d * f)^{(1/2)} * b) / f)^{(1/2)} * x * b * c * A + 1 / (a * f + c * d + (d * f)^{(1/2)} * b) / (4 * a * c - b^2) / ((x - (d * f)^{(1/2)} / f)^2 * c + (b * f + 2 * (d * f)^{(1/2)} * c) * (x - (d * f)^{(1/2)} / f) / f + (a * f + c * d + (d * f)^{(1/2)} * b) / f)^{(1/2)} * x * b * c * B + 1 / (a * f + c * d + (d * f)^{(1/2)} * b) / (4 * a * c - b^2) / ((x - (d * f)^{(1/2)} / f)^2 * c + (b * f + 2 * (d * f)^{(1/2)} * c) * (x - (d * f)^{(1/2)} / f) / f + (a * f + c * d + (d * f)^{(1/2)} * b) / f)^{(1/2)} * b * c * A + (d * f)^{(1/2)} / f / (a * f + c * d + (d * f)^{(1/2)} * b) / (4 * a * c - b^2) / ((x - (d * f)^{(1/2)} / f)^2 * c + (b * f + 2 * (d * f)^{(1/2)} * c) * (x - (d * f)^{(1/2)} / f) / f + (a * f + c * d + (d * f)^{(1/2)} * b) / f)^{(1/2)} * b * c * B + 1/2 / (d * f)^{(1/2)} * f / (a * f + c * d + (d * f)^{(1/2)} * b) / (4 * a * c - b^2) / ((x - (d * f)^{(1/2)} / f)^2 * c + (b * f + 2 * (d * f)^{(1/2)} * c) * (x - (d * f)^{(1/2)} / f) / f + (a * f + c * d + (d * f)^{(1/2)} * b) / f)^{(1/2)} * b^2 * A + 1/2 / (a * f + c * d + (d * f)^{(1/2)} * b) / (4 * a * c - b^2) / ((x - (d * f)^{(1/2)} / f)^2 * c + (b * f + 2 * (d * f)^{(1/2)} * c) * (x - (d * f)^{(1/2)} / f) / f + (a * f + c * d + (d * f)^{(1/2)} * b) / f)^{(1/2)} * b^2 * B + 1/2 / (d * f)^{(1/2)} * f / (a * f + c * d + (d * f)^{(1/2)} * b) / ((a * f + c * d + (d * f)^{(1/2)} * b) / f)^{(1/2)} * \ln((2 * (a * f + c * d + (d * f)^{(1/2)} * b) / f + (b * f + 2 * (d * f)^{(1/2)} * c) * (x - (d * f)^{(1/2)} / f) / f + 2 * ((a * f + c * d + (d * f)^{(1/2)} * b) / f)^{(1/2)} * ((x - (d * f)^{(1/2)} / f)^2 * c + (b * f + 2 * (d * f)^{(1/2)} * c) * (x - (d * f)^{(1/2)} / f) / f + (a * f + c * d + (d * f)^{(1/2)} * b) / f)^{(1/2)}) / (x - (d * f)^{(1/2)} / f) * A + 1/2 / (a * f + c * d + (d * f)^{(1/2)} * b) / ((a * f + c * d + (d * f)^{(1/2)} * b) / f)^{(1/2)} * \ln((2 * (a * f + c * d + (d * f)^{(1/2)} * b) / f + (b * f + 2 * (d * f)^{(1/2)} * c) * (x - (d * f)^{(1/2)} / f) / f + 2 * ((a * f + c * d + (d * f)^{(1/2)} * b) / f)^{(1/2)} * ((x - (d * f)^{(1/2)} / f)^2 * c + (b * f + 2 * (d * f)^{(1/2)} * c) * (x - (d * f)^{(1/2)} / f) / f + (a * f + c * d + (d * f)^{(1/2)} * b) / f)^{(1/2)}) / (x - (d * f)^{(1/2)} / f) * B
\end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d), x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2 - (c*((b*sqrt(4*d*f)) / (2*f) + (c*d)/f+a)) / f^2 zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + Bx}{(d - fx^2)(cx^2 + bx + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)
[Out] int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
[Out] Timed out
```

$$3.9 \quad \int \frac{A+Bx}{(a+bx+cx^2)^{5/2}(d-fx^2)} dx$$

Optimal. Leaf size=797

$$\frac{(B\sqrt{d} - A\sqrt{f}) \tanh^{-1}\left(\frac{-2\sqrt{f}a + (2c\sqrt{d} - b\sqrt{f})x + b\sqrt{d}}{2\sqrt{-\sqrt{d}\sqrt{f}b + cd + af}\sqrt{cx^2 + bx + a}}\right) f^{3/2}}{2\sqrt{d}(-\sqrt{d}\sqrt{f}b + cd + af)^{5/2}} + \frac{(\sqrt{f}A + B\sqrt{d}) \tanh^{-1}\left(\frac{2\sqrt{f}a + (\sqrt{f}b + 2c\sqrt{d})x + b\sqrt{d}}{2\sqrt{\sqrt{d}\sqrt{f}b + cd + af}\sqrt{cx^2 + bx + a}}\right) f^{3/2}}{2\sqrt{d}(\sqrt{d}\sqrt{f}b + cd + af)^{5/2}}$$

Rubi [A] time = 1.87, antiderivative size = 796, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1018, 1064, 1033, 724, 206}

([1] A. R. Forster, "The Risch Algorithm", Springer, 1993) [2] R. E. Bryant, "Symbolic Boolean Algebra with Decomposition", Kluwer, 1992) [3] R. E. Bryant, "Symbolic Boolean Algebra with Decomposition", Kluwer, 1992) [4] R. E. Bryant, "Symbolic Boolean Algebra with Decomposition", Kluwer, 1992) [5] R. E. Bryant, "Symbolic Boolean Algebra with Decomposition", Kluwer, 1992) [6] R. E. Bryant, "Symbolic Boolean Algebra with Decomposition", Kluwer, 1992) [7] R. E. Bryant, "Symbolic Boolean Algebra with Decomposition", Kluwer, 1992) [8] R. E. Bryant, "Symbolic Boolean Algebra with Decomposition", Kluwer, 1992) [9] R. E. Bryant, "Symbolic Boolean Algebra with Decomposition", Kluwer, 1992) [10] R. E. Bryant, "Symbolic Boolean Algebra with Decomposition", Kluwer, 1992) [11] R. E. Bryant, "Symbolic Boolean Algebra with Decomposition", Kluwer, 1992) [12] R. E. Bryant, "Symbolic Boolean Algebra with Decomposition", Kluwer, 1992) [13] R. E. 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Antiderivative was successfully verified.

[In] Int[(A + B*x)/((a + b*x + c*x^2)^(5/2)*(d - f*x^2)), x]

[Out] (-2*(A*b^3*f - A*b*c*(c*d + 3*a*f) + a*B*(2*c^2*d - b^2*f + 2*a*c*f) + c*(A*b^2*f + b*B*(c*d - a*f) - 2*A*c*(c*d + a*f))*x)/(3*(b^2 - 4*a*c)*(b^2*d*f - (c*d + a*f)^2*(a + b*x + c*x^2)^(3/2)) - (2*(3*b^6*B*d*f^2 + 24*a^2*B*c^2*f*(c*d + a*f)^2 - A*b^5*f^2*(7*c*d + 6*a*f) - b^4*B*f*(7*c^2*d^2 + 14*a*c*d*f - 3*a^2*f^2) + A*b^3*c*f*(15*c^2*d^2 + 46*a*c*d*f + 43*a^2*f^2) + 2*b^2*B*c*(2*c^3*d^3 + 5*a*c^2*d^2*f + 4*a^2*c*d*f^2 - 11*a^3*f^3) - 4*A*b*c^2*(2*c^3*d^3 + 9*a*c^2*d^2*f + 24*a^2*c*d*f^2 + 17*a^3*f^3) + c*(3*b^5*B*d*f^2 - 2*A*b^4*f^2*(4*c*d + 3*a*f) - 8*A*c^2*(c*d + a*f)^2*(2*c*d + 5*a*f) - b^3*B*f*(17*c^2*d^2 + 10*a*c*d*f - 3*a^2*f^2) + 2*A*b^2*c*f*(15*c^2*d^2 + 2*2*a*c*d*f + 19*a^2*f^2) + 4*b*B*c*(2*c^3*d^3 + 11*a*c^2*d^2*f + 4*a^2*c*d*f^2 - 5*a^3*f^3))*x)/(3*(b^2 - 4*a*c)^2*(c^2*d^2 + 2*a*c*d*f - f*(b^2*d - a^2*f))^2*sqrt[a + b*x + c*x^2]) - ((B*sqrt[d] - A*sqrt[f])*f^(3/2)*ArcTanh[(b*sqrt[d] - 2*a*sqrt[f] + (2*c*sqrt[d] - b*sqrt[f])*x)/(2*sqrt[c*d - b*sqrt[d]*sqrt[f] + a*f]*sqrt[a + b*x + c*x^2])])/(2*sqrt[d]*(c*d - b*sqrt[d]*sqrt[f] + a*f)^(5/2)) + ((B*sqrt[d] + A*sqrt[f])*f^(3/2)*ArcTanh[(b*sqrt[d] + 2*a*sqrt[f] + (2*c*sqrt[d] + b*sqrt[f])*x)/(2*sqrt[c*d + b*sqrt[d]*sqrt[f] + a*f]*sqrt[a + b*x + c*x^2])])/(2*sqrt[d]*(c*d + b*sqrt[d]*sqrt[f] + a*f)^(5/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1018

Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si

```

mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
+ 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] && !( !IntegerQ[p
] && ILtQ[q, -1])

```

Rule 1033

```

Int[((g_) + (h_)*(x_))/((a_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (
f_)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]

```

Rule 1064

```

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x_
)^2)*((d_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p +
1)*(d + f*x^2)^(q + 1)*((A*c - a*C)*(-(b*(c*d + a*f))) + (A*b - a*B)*(2*c^2
*d + b^2*f - c*(2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(2*a*f)) - B*(b*c*d + a
*b*f) + C*(b^2*d - 2*a*(c*d - a*f))*x))/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a
*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p +
1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)
*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + a
*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)
) - (2*f*((A*c - a*C)*(-(b*(c*d + a*f))) + (A*b - a*B)*(2*c^2*d + b^2*f - c
*(2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + a*B*f) + 2*(A*c*(c*d
- a*f) - a*(c*C*d - a*C*f)))*(b*f*(p + 1)))*x - c*f*(b^2*(C*d + A*f) - b*(
B*c*d + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(2*p + 2*q + 5)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] && !( !IntegerQ[p] &&
ILtQ[q, -1]) && !IGtQ[q, 0]

```

Rubi steps

$$\int \frac{A + Bx}{(a + bx + cx^2)^{5/2} (d - fx^2)} dx = -\frac{2(Ab^3f - Abc(cd + 3af) + aB(2c^2d - b^2f + 2acf) + c(Ab^2f + bB(cd - a))}{3(b^2 - 4ac)(b^2df - (cd + af)^2)(a + bx + cx^2)^{3/2}}$$

$$= -\frac{2(Ab^3f - Abc(cd + 3af) + aB(2c^2d - b^2f + 2acf) + c(Ab^2f + bB(cd - a))}{3(b^2 - 4ac)(b^2df - (cd + af)^2)(a + bx + cx^2)^{3/2}}$$

$$= -\frac{2(Ab^3f - Abc(cd + 3af) + aB(2c^2d - b^2f + 2acf) + c(Ab^2f + bB(cd - a))}{3(b^2 - 4ac)(b^2df - (cd + af)^2)(a + bx + cx^2)^{3/2}}$$

$$= -\frac{2(Ab^3f - Abc(cd + 3af) + aB(2c^2d - b^2f + 2acf) + c(Ab^2f + bB(cd - a))}{3(b^2 - 4ac)(b^2df - (cd + af)^2)(a + bx + cx^2)^{3/2}}$$

Mathematica [A] time = 4.06, size = 674, normalized size = 0.85

$$\frac{\frac{2(Ab^3f - Abc(cd + 3af) + aB(2c^2d - b^2f + 2acf) + c(Ab^2f + bB(cd - a))}{3(b^2 - 4ac)(b^2df - (cd + af)^2)(a + bx + cx^2)^{3/2}}}{3(b^2 - 4ac)(b^2df - (cd + af)^2)(a + bx + cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/((a + b*x + c*x^2)^(5/2)*(d - f*x^2)), x]
[Out] (2*((4*c*(-(A*b^2*f) + b*B*(-(c*d) + a*f) + 2*A*c*(c*d + a*f))*(b + 2*c*x))
/(b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]) - (3*f*(b^4*B*d*f + 2*c*(c*d + a*f)^
2*(-(a*B) + A*c*x) + b^3*f*(-(A*(c*d + 2*a*f)) + B*c*d*x) + b*c*(c*d + a*f)
*(A*c*d + 5*a*A*f - 3*B*c*d*x + a*B*f*x) - b^2*(B*(c^2*d^2 + 2*a*c*d*f - a^
2*f^2) + 2*a*A*c*f^2*x))/((c^2*d^2 + 2*a*c*d*f + f*(-(b^2*d) + a^2*f))*Sqr
t[a + x*(b + c*x)]) + (A*(b^3*f - b*c*(c*d + 3*a*f) + b^2*c*f*x - 2*c^2*(c*
d + a*f)*x) + B*(2*a^2*c*f + b*c^2*d*x + a*(2*c^2*d - b^2*f - b*c*f*x)))/(a
+ x*(b + c*x))^(3/2) + (3*(b^2 - 4*a*c)*f^(3/2)*(((B*Sqrt[d]) + A*Sqrt[f
])*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^2*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x
+ b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a +
x*(b + c*x)])]/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f) - ((B*Sqrt[d] + A*Sqrt[f
])*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^2*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x)
- b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a +
x*(b + c*x)])]/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f))/(4*Sqrt[d]*(-(b^2*d*f
+ (c*d + a*f)^2)))/(3*(b^2 - 4*a*c)*(-(b^2*d*f) + (c*d + a*f)^2))
```

IntegrateAlgebraic [F] time = 180.08, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x)/((a + b*x + c*x^2)^(5/2)*(d - f*x^2)), x]
[Out] $Aborted
```


fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(5/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(5/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 5.84Conj Er
ror: Bad Argument Type
```

maple [B] time = 0.03, size = 6422, normalized size = 8.06

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+b*x+a)^(5/2)/(-f*x^2+d),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(5/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2
-(c*((b*sqrt(4*d*f)) / (2*f) + (c*d)/f+a))
/f^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + Bx}{(d - fx^2)(cx^2 + bx + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(5/2)),x)
```

```
[Out] int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)**(5/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

$$3.10 \quad \int \frac{1+2x}{(-1+x^2)\sqrt{-1+x+x^2}} dx$$

Optimal. Leaf size=47

$$\frac{3}{2} \tanh^{-1}\left(\frac{1-3x}{2\sqrt{x^2+x-1}}\right) - \frac{1}{2} \tan^{-1}\left(\frac{x+3}{2\sqrt{x^2+x-1}}\right)$$

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1033, 724, 206, 204}

$$\frac{3}{2} \tanh^{-1}\left(\frac{1-3x}{2\sqrt{x^2+x-1}}\right) - \frac{1}{2} \tan^{-1}\left(\frac{x+3}{2\sqrt{x^2+x-1}}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x)/((-1 + x^2)*Sqrt[-1 + x + x^2]),x]

[Out] -ArcTan[(3 + x)/(2*Sqrt[-1 + x + x^2])]/2 + (3*ArcTanh[(1 - 3*x)/(2*Sqrt[-1 + x + x^2])])/2

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1033

Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]

Rubi steps

$$\begin{aligned} \int \frac{1+2x}{(-1+x^2)\sqrt{-1+x+x^2}} dx &= \frac{1}{2} \int \frac{1}{(1+x)\sqrt{-1+x+x^2}} dx + \frac{3}{2} \int \frac{1}{(-1+x)\sqrt{-1+x+x^2}} dx \\ &= -\left(3 \operatorname{Subst}\left(\int \frac{1}{4-x^2} dx, x, \frac{-1+3x}{\sqrt{-1+x+x^2}}\right)\right) - \operatorname{Subst}\left(\int \frac{1}{-4-x^2} dx, x, \frac{-3}{\sqrt{-1+x+x^2}}\right) \\ &= \frac{1}{2} \tan^{-1}\left(\frac{-3-x}{2\sqrt{-1+x+x^2}}\right) + \frac{3}{2} \tanh^{-1}\left(\frac{1-3x}{2\sqrt{-1+x+x^2}}\right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.04

$$\frac{1}{2} \tan^{-1} \left(\frac{-x-3}{2\sqrt{x^2+x-1}} \right) - \frac{3}{2} \tanh^{-1} \left(\frac{3x-1}{2\sqrt{x^2+x-1}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x)/((-1 + x^2)*Sqrt[-1 + x + x^2]),x]

[Out] ArcTan[(-3 - x)/(2*Sqrt[-1 + x + x^2])]/2 - (3*ArcTanh[(-1 + 3*x)/(2*Sqrt[-1 + x + x^2])])/2

IntegrateAlgebraic [A] time = 0.22, size = 37, normalized size = 0.79

$$-\tan^{-1} \left(-\sqrt{x^2+x-1} + x + 1 \right) - 3 \tanh^{-1} \left(\sqrt{x^2+x-1} - x + 1 \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 2*x)/((-1 + x^2)*Sqrt[-1 + x + x^2]),x]

[Out] -ArcTan[1 + x - Sqrt[-1 + x + x^2]] - 3*ArcTanh[1 - x + Sqrt[-1 + x + x^2]]

fricas [A] time = 0.63, size = 46, normalized size = 0.98

$$\arctan \left(-x + \sqrt{x^2+x-1} - 1 \right) - \frac{3}{2} \log \left(-x + \sqrt{x^2+x-1} + 2 \right) + \frac{3}{2} \log \left(-x + \sqrt{x^2+x-1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x^2-1)/(x^2+x-1)^(1/2),x, algorithm="fricas")

[Out] arctan(-x + sqrt(x^2 + x - 1) - 1) - 3/2*log(-x + sqrt(x^2 + x - 1) + 2) + 3/2*log(-x + sqrt(x^2 + x - 1))

giac [A] time = 0.25, size = 48, normalized size = 1.02

$$\arctan \left(-x + \sqrt{x^2+x-1} - 1 \right) - \frac{3}{2} \log \left(\left| -x + \sqrt{x^2+x-1} + 2 \right| \right) + \frac{3}{2} \log \left(\left| -x + \sqrt{x^2+x-1} \right| \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x^2-1)/(x^2+x-1)^(1/2),x, algorithm="giac")

[Out] arctan(-x + sqrt(x^2 + x - 1) - 1) - 3/2*log(abs(-x + sqrt(x^2 + x - 1) + 2)) + 3/2*log(abs(-x + sqrt(x^2 + x - 1)))

maple [A] time = 0.02, size = 46, normalized size = 0.98

$$\frac{3 \operatorname{arctanh} \left(\frac{3x-1}{2\sqrt{3x+(x-1)^2-2}} \right)}{2} + \frac{\operatorname{arctan} \left(\frac{-x-3}{2\sqrt{-x+(x+1)^2-2}} \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+2*x)/(x^2-1)/(x^2+x-1)^(1/2),x)

[Out] -3/2*arctanh(1/2*(-1+3*x)/((x-1)^2+3*x-2)^(1/2))+1/2*arctan(1/2*(-3-x)/((1+x)^2-x-2)^(1/2))

maxima [A] time = 0.97, size = 65, normalized size = 1.38

$$-\frac{1}{2} \arcsin \left(\frac{2\sqrt{5}x}{5|2x+2|} + \frac{6\sqrt{5}}{5|2x+2|} \right) - \frac{3}{2} \log \left(\frac{2\sqrt{x^2+x-1}}{|2x-2|} + \frac{2}{|2x-2|} + \frac{3}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x^2-1)/(x^2+x-1)^(1/2),x, algorithm="maxima")

[Out] -1/2*arcsin(2/5*sqrt(5)*x/abs(2*x + 2) + 6/5*sqrt(5)/abs(2*x + 2)) - 3/2*log(2*sqrt(x^2 + x - 1)/abs(2*x - 2) + 2/abs(2*x - 2) + 3/2)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{2x+1}{(x^2-1)\sqrt{x^2+x-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x + 1)/((x^2 - 1)*(x + x^2 - 1)^(1/2)),x)

[Out] int((2*x + 1)/((x^2 - 1)*(x + x^2 - 1)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+1}{(x-1)(x+1)\sqrt{x^2+x-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x**2-1)/(x**2+x-1)**(1/2),x)

[Out] Integral((2*x + 1)/((x - 1)*(x + 1)*sqrt(x**2 + x - 1)), x)

$$3.11 \quad \int \frac{1+2x}{(1+x^2)\sqrt{-1+x+x^2}} dx$$

Optimal. Leaf size=117

$$\sqrt{\frac{1}{2}}(\sqrt{5}-2) \tanh^{-1}\left(\frac{\sqrt{5}x-2\sqrt{5}+5}{\sqrt{10}(\sqrt{5}-2)\sqrt{x^2+x-1}}\right) - \sqrt{\frac{1}{2}}(2+\sqrt{5}) \tan^{-1}\left(\frac{-\sqrt{5}x+2\sqrt{5}+5}{\sqrt{10}(2+\sqrt{5})\sqrt{x^2+x-1}}\right)$$

Rubi [A] time = 0.17, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1036, 1030, 207, 203}

$$\sqrt{\frac{1}{2}}(\sqrt{5}-2) \tanh^{-1}\left(\frac{\sqrt{5}x-2\sqrt{5}+5}{\sqrt{10}(\sqrt{5}-2)\sqrt{x^2+x-1}}\right) - \sqrt{\frac{1}{2}}(2+\sqrt{5}) \tan^{-1}\left(\frac{-\sqrt{5}x+2\sqrt{5}+5}{\sqrt{10}(2+\sqrt{5})\sqrt{x^2+x-1}}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x)/((1 + x^2)*Sqrt[-1 + x + x^2]),x]

[Out] -(Sqrt[(2 + Sqrt[5])/2]*ArcTan[(5 + 2*Sqrt[5] - Sqrt[5]*x)/(Sqrt[10*(2 + Sqrt[5])]*Sqrt[-1 + x + x^2])]) + Sqrt[(-2 + Sqrt[5])/2]*ArcTanh[(5 - 2*Sqrt[5] + Sqrt[5]*x)/(Sqrt[10*(-2 + Sqrt[5])]*Sqrt[-1 + x + x^2])])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1030

Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]

Rule 1036

Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist[1/(2*q), Int[Simp[-(a*h*e) - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[-(a*h*e) - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && NegQ[-(a*c)]

Rubi steps

$$\begin{aligned} \int \frac{1+2x}{(1+x^2)\sqrt{-1+x+x^2}} dx &= -\frac{\int \frac{-\sqrt{5}+(-5-2\sqrt{5})x}{(1+x^2)\sqrt{-1+x+x^2}} dx}{2\sqrt{5}} + \frac{\int \frac{\sqrt{5}+(-5+2\sqrt{5})x}{(1+x^2)\sqrt{-1+x+x^2}} dx}{2\sqrt{5}} \\ &= -\left((-5+2\sqrt{5}) \operatorname{Subst}\left(\int \frac{1}{10(2-\sqrt{5})+x^2} dx, x, \frac{-5+2\sqrt{5}-\sqrt{5}x}{\sqrt{-1+x+x^2}}\right)\right) + (5+2\sqrt{5}) \\ &= -\sqrt{\frac{1}{2}}(2+\sqrt{5}) \tan^{-1}\left(\frac{5+2\sqrt{5}-\sqrt{5}x}{\sqrt{10(2+\sqrt{5})}\sqrt{-1+x+x^2}}\right) + \sqrt{\frac{1}{2}}(-2+\sqrt{5}) \tanh^{-1} \end{aligned}$$

Mathematica [C] time = 0.03, size = 78, normalized size = 0.67

$$-\frac{1}{2}i \left(\sqrt{2+i} \tanh^{-1}\left(\frac{\sqrt{2+i}(x-i)}{2\sqrt{x^2+x-1}}\right) - \sqrt{2-i} \tanh^{-1}\left(\frac{\sqrt{2-i}(x+i)}{2\sqrt{x^2+x-1}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x)/((1 + x^2)*Sqrt[-1 + x + x^2]), x]

[Out] (-1/2*I)*(Sqrt[2 + I]*ArcTanh[(Sqrt[2 + I]*(-I + x))/(2*Sqrt[-1 + x + x^2])] - Sqrt[2 - I]*ArcTanh[(Sqrt[2 - I]*(I + x))/(2*Sqrt[-1 + x + x^2])])

IntegrateAlgebraic [C] time = 0.24, size = 106, normalized size = 0.91

$$\frac{1}{2} \operatorname{RootSum}\left[\#1^4 + 6\#1^2 - 4\#1 + 2\&, \frac{2\#1^2 \log(-\#1 + \sqrt{x^2 + x - 1} - x) - 2\#1 \log(-\#1 + \sqrt{x^2 + x - 1} - x) + 3 \log(-\#1 + \sqrt{x^2 + x - 1} - x)}{\#1^3 + 3\#1 - 1} \&\right]$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 2*x)/((1 + x^2)*Sqrt[-1 + x + x^2]), x]

[Out] RootSum[2 - 4*#1 + 6*#1^2 + #1^4 &, (3*Log[-x + Sqrt[-1 + x + x^2] - #1] - 2*Log[-x + Sqrt[-1 + x + x^2] - #1]*#1 + 2*Log[-x + Sqrt[-1 + x + x^2] - #1]*#1^2)/(-1 + 3*#1 + #1^3) &]/2

fricas [B] time = 0.56, size = 758, normalized size = 6.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x^2+1)/(x^2+x-1)^(1/2), x, algorithm="fricas")

[Out] 1/20*5^(1/4)*sqrt(4*sqrt(5) + 10)*(2*sqrt(5) - 5)*log(2*x^2 - 2*sqrt(x^2 + x - 1)*x + 1/5*(5^(1/4)*sqrt(x^2 + x - 1)*(2*sqrt(5) - 5) - 5^(1/4)*(sqrt(5)*(2*x + 1) - 5*x))*sqrt(4*sqrt(5) + 10) + x + sqrt(5)) - 1/20*5^(1/4)*sqrt(4*sqrt(5) + 10)*(2*sqrt(5) - 5)*log(2*x^2 - 2*sqrt(x^2 + x - 1)*x - 1/5*(5^(1/4)*sqrt(x^2 + x - 1)*(2*sqrt(5) - 5) - 5^(1/4)*(sqrt(5)*(2*x + 1) - 5*x))*sqrt(4*sqrt(5) + 10) + x + sqrt(5)) - 1/5*5^(3/4)*sqrt(4*sqrt(5) + 10)*arctan(2/55*sqrt(5)*(sqrt(5)*(2*x - 1) + 3*x + 4) + 1/275*sqrt(10*x^2 - 10*sqrt(x^2 + x - 1)*x + (5^(1/4)*sqrt(x^2 + x - 1)*(2*sqrt(5) - 5) - 5^(1/4)*(sqrt(5)*(2*x + 1) - 5*x))*sqrt(4*sqrt(5) + 10) + 5*x + 5*sqrt(5)))*((5^(3/4)*(2*sqrt(5) + 3) + 2*5^(1/4)*(4*sqrt(5) - 5))*sqrt(4*sqrt(5) + 10) + 2*sqrt(5)*(3*sqrt(5) + 10) - 20*sqrt(5) + 80) - 2/55*sqrt(x^2 + x - 1)*(sqrt(5)*(2*sqrt(5) + 3) + 8*sqrt(5) - 10) + 1/55*sqrt(5)*(16*x + 3) + 1/275*(5^(3/4)*(sqrt(5)*(3*x + 4) + 10*x - 5) - sqrt(x^2 + x - 1)*(5^(3/4)*(3*sqrt(5) + 10) - 10*5^(1/4)*(sqrt(5) - 4)) - 10*5^(1/4)*(sqrt(5)*(x - 6) - 4*x + 13))*sqrt(4*sqrt(5) + 10) - 4/11*x + 2/11) - 1/5*5^(3/4)*sqrt(4*sqrt(5) + 10)*arc

$\tan(-2/55\sqrt{5}*(\sqrt{5}*(2*x - 1) + 3*x + 4) + 1/275\sqrt{10*x^2 - 10*\sqrt{x^2 + x - 1}}*x - (5^{1/4}*\sqrt{x^2 + x - 1}*(2*\sqrt{5} - 5) - 5^{1/4}*(\sqrt{5}*(2*x + 1) - 5*x))*\sqrt{4*\sqrt{5} + 10} + 5*x + 5*\sqrt{5}))*((5^{3/4}*(2*\sqrt{5} + 3) + 2*5^{1/4}*(4*\sqrt{5} - 5))*\sqrt{4*\sqrt{5} + 10} - 2*\sqrt{5}*(3*\sqrt{5} + 10) + 20*\sqrt{5} - 80) + 2/55\sqrt{x^2 + x - 1}*(\sqrt{5}*(2*\sqrt{5} + 3) + 8*\sqrt{5} - 10) - 1/55\sqrt{5}*(16*x + 3) + 1/275*(5^{3/4}*(\sqrt{5}*(3*x + 4) + 10*x - 5) - \sqrt{x^2 + x - 1}*(5^{3/4}*(3*\sqrt{5} + 10) - 10*5^{1/4}*(\sqrt{5} - 4)) - 10*5^{1/4}*(\sqrt{5}*(x - 6) - 4*x + 13))*\sqrt{4*\sqrt{5} + 10} + 4/11*x - 2/11)$

giac [B] time = 0.38, size = 457, normalized size = 3.91

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x^2+1)/(x^2+x-1)^(1/2),x, algorithm="giac")

[Out] $1/4*\sqrt{2*\sqrt{5} - 4}*\log(16*(15*\sqrt{5}*(x - \sqrt{x^2 + x - 1}) + 33*x + 5*\sqrt{5} - 33*\sqrt{x^2 + x - 1} + 2*\sqrt{5*\sqrt{5} + 11} + 11)^2 + 16*(5*\sqrt{5}*(x - \sqrt{x^2 + x - 1}) + 11*x - 5*\sqrt{5}*\sqrt{5*\sqrt{5} + 11} - 15*\sqrt{5} - 11*\sqrt{x^2 + x - 1} - 11*\sqrt{5*\sqrt{5} + 11} - 33)^2) - 1/4*\sqrt{2*\sqrt{5} - 4}*\log(16*(15*\sqrt{5}*(x - \sqrt{x^2 + x - 1}) + 33*x + 5*\sqrt{5} - 33*\sqrt{x^2 + x - 1} - 2*\sqrt{5*\sqrt{5} + 11} + 11)^2 + 16*(5*\sqrt{5}*(x - \sqrt{x^2 + x - 1}) + 11*x + 5*\sqrt{5}*\sqrt{5*\sqrt{5} + 11} - 15*\sqrt{5} - 11*\sqrt{x^2 + x - 1} + 11*\sqrt{5*\sqrt{5} + 11} - 33)^2) + 1/2*\sqrt{2*\sqrt{5} - 4}*(\arctan(3) + \arctan(1/10*(x - \sqrt{x^2 + x - 1}))*(\sqrt{5}*\sqrt{5*\sqrt{5} + 11} + 4*\sqrt{5} - 5*\sqrt{5*\sqrt{5} + 11})) - 7/10*\sqrt{5}*\sqrt{5*\sqrt{5} + 11} + 1/5*\sqrt{5} + 3/2*\sqrt{5*\sqrt{5} + 11}))/(\sqrt{5} - 2) - 1/2*\sqrt{2*\sqrt{5} - 4}*(\arctan(3) + \arctan(-1/10*(x - \sqrt{x^2 + x - 1}))*(\sqrt{5}*\sqrt{5*\sqrt{5} + 11} - 4*\sqrt{5} - 5*\sqrt{5*\sqrt{5} + 11})) + 7/10*\sqrt{5}*\sqrt{5*\sqrt{5} + 11} + 1/5*\sqrt{5} - 3/2*\sqrt{5*\sqrt{5} + 11}))/(\sqrt{5} - 2)$

maple [B] time = 0.18, size = 637, normalized size = 5.44

$$\sqrt{\frac{10(-\sqrt{5}-2)}{(-1-\sqrt{5})^2} - \frac{5\sqrt{5}(-\sqrt{5}-2)}{(-1-\sqrt{5})^2} + 10 + 5\sqrt{5}}{\sqrt{5}}} \left(\operatorname{arctanh}\left(\frac{\frac{10(-\sqrt{5}-2)}{(-1-\sqrt{5})^2} - \frac{5\sqrt{5}(-\sqrt{5}-2)}{(-1-\sqrt{5})^2} + 10 + 5\sqrt{5}}{\sqrt{20+10\sqrt{5}}}}{\sqrt{\frac{10(-\sqrt{5}-2)}{(-1-\sqrt{5})^2} - \frac{5\sqrt{5}(-\sqrt{5}-2)}{(-1-\sqrt{5})^2} + 10 + 5\sqrt{5}}}}\right) + \sqrt{5} \operatorname{arctan}\left(\frac{\sqrt{5}\left(\sqrt{5}-2\right)\left(\frac{10(-\sqrt{5}-2)}{(-1-\sqrt{5})^2} + \sqrt{5}+9\right) + \sqrt{20+10\sqrt{5}}\left(\frac{\sqrt{5}(-\sqrt{5}-2)}{(-1-\sqrt{5})^2} - \frac{5\sqrt{5}(-\sqrt{5}-2)}{(-1-\sqrt{5})^2} + \sqrt{5}+2\right)\sqrt{5}-2\right)\sqrt{5}-2}}{5(-1-\sqrt{5})^2}\right) + 2 \operatorname{arctan}\left(\frac{\sqrt{5}\left(\sqrt{5}-2\right)\left(\frac{10(-\sqrt{5}-2)}{(-1-\sqrt{5})^2} + \sqrt{5}+9\right) + \sqrt{20+10\sqrt{5}}\left(\frac{\sqrt{5}(-\sqrt{5}-2)}{(-1-\sqrt{5})^2} - \frac{5\sqrt{5}(-\sqrt{5}-2)}{(-1-\sqrt{5})^2} + \sqrt{5}+2\right)\sqrt{5}-2\right)\sqrt{5}-2}}{5(-1-\sqrt{5})^2}\right) \right)$$

$$\sqrt{\frac{\frac{5\sqrt{5}(-\sqrt{5}-2)}{(-1-\sqrt{5})^2} - \frac{5\sqrt{5}(-\sqrt{5}-2)}{(-1-\sqrt{5})^2} + 10 + 5\sqrt{5}}{(-1-\sqrt{5})^2}}{\sqrt{20+10\sqrt{5}}}} \left(\frac{2\sqrt{5}(-\sqrt{5}-2)}{(-1-\sqrt{5})^2} + 1 \right) \sqrt{20+10\sqrt{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+2*x)/(x^2+1)/(x^2+x-1)^(1/2),x)

[Out] $(10*(-5^{1/2}-2+x)^2/(-5^{1/2}+2-x)^2-5*5^{1/2}*(-5^{1/2}-2+x)^2/(-5^{1/2}+2-x)^2+10+5*5^{1/2})^{1/2}*5^{1/2}*(\arctan(1/5*5^{1/2}*((5^{1/2}-2)*(-5^{1/2}-2+x)^2/(-5^{1/2}+2-x)^2+4*5^{1/2}+9))^{1/2}*(20+10*5^{1/2})^{1/2}*(5^{1/2}*(-5^{1/2}-2+x)^2/(-5^{1/2}+2-x)^2+2*(-5^{1/2}-2+x)^2/(-5^{1/2}+2-x)^2-5^{1/2}+2)*(-5^{1/2}-2+x)/(-5^{1/2}+2-x)*(5^{1/2}-2)/((-5^{1/2}-2+x)^4/(-5^{1/2}+2-x)^4-18*(-5^{1/2}-2+x)^2/(-5^{1/2}+2-x)^2+1))*5^{1/2}+\operatorname{arctanh}((10*(-5^{1/2}-2+x)^2/(-5^{1/2}+2-x)^2-5*5^{1/2}*(-5^{1/2}-2+x)^2/(-5^{1/2}+2-x)^2+10+5*5^{1/2})^{1/2}/(20+10*5^{1/2})^{1/2})+2*\operatorname{arctan}(1/5*5^{1/2}*((5^{1/2}-2)*(-5^{1/2}-2+x)^2/(-5^{1/2}+2-x)^2+4*5^{1/2}+9))^{1/2}*(20+10*5^{1/2})^{1/2}*(5^{1/2}*(-5^{1/2}-2+x)^2/(-5^{1/2}+2-x)^2+2*(-5^{1/2}-2+x)^2/(-5^{1/2}+2-x)^2-5^{1/2}+2)*(-5^{1/2}-2+x)/(-5^{1/2}+2-x)*(5^{1/2}-2)/((-5^{1/2}-2+x)^4/(-5^{1/2}+2-x)^4-18*(-5^{1/2}-2+x)^2/(-5^{1/2}+2-x)^2+1)))/(-5*(5^{1/2}*(-5^{1/2}-2+x)^2/(-5^{1/2}+2-x)^2-2*(-5^{1/2}-2+x)^2/(-5^{1/2}+2-x)^2-5^{1/2}-2)/((-5^{1/2}-2+x)/(-5^{1/2}+2-x)+1)^2)^{1/2}/((-5^{1/2}-2+x)/(-5^{1/2}+2-x)+1)/(20+10*5^{1/2})^{1/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+1}{\sqrt{x^2+x-1}(x^2+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x^2+1)/(x^2+x-1)^(1/2),x, algorithm="maxima")

[Out] integrate((2*x + 1)/(sqrt(x^2 + x - 1)*(x^2 + 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2x+1}{(x^2+1)\sqrt{x^2+x-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x + 1)/((x^2 + 1)*(x + x^2 - 1)^(1/2)),x)

[Out] int((2*x + 1)/((x^2 + 1)*(x + x^2 - 1)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+1}{(x^2+1)\sqrt{x^2+x-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x**2+1)/(x**2+x-1)**(1/2),x)

[Out] Integral((2*x + 1)/((x**2 + 1)*sqrt(x**2 + x - 1)), x)

$$3.12 \quad \int \frac{a-c+bx}{(1+x^2)\sqrt{a+bx+cx^2}} dx$$

Optimal. Leaf size=484

$$\frac{\sqrt{-a\left(2c - \sqrt{a^2 - 2ac + b^2 + c^2}\right) + c\left(c - \sqrt{a^2 - 2ac + b^2 + c^2}\right) + a^2 + b^2} \tan^{-1}\left(\frac{b\sqrt{a^2 - 2ac + b^2 + c^2}}{\sqrt{2}\sqrt[4]{a^2 - 2ac + b^2 + c^2}}\sqrt{-a\left(2c - \sqrt{a^2 - 2ac + b^2 + c^2}\right)}\right)}{\sqrt{2}\sqrt[4]{a^2 - 2ac + b^2 + c^2}}$$

Rubi [A] time = 23.58, antiderivative size = 484, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1036, 1030, 208, 205}

$$\frac{\sqrt{-a\left(2c - \sqrt{a^2 - 2ac + b^2 + c^2}\right) + c\left(c - \sqrt{a^2 - 2ac + b^2 + c^2}\right) + a^2 + b^2} \tan^{-1}\left(\frac{b\sqrt{a^2 - 2ac + b^2 + c^2}}{\sqrt{2}\sqrt[4]{a^2 - 2ac + b^2 + c^2}}\sqrt{-a\left(2c - \sqrt{a^2 - 2ac + b^2 + c^2}\right)}\right)}{\sqrt{2}\sqrt[4]{a^2 - 2ac + b^2 + c^2}}$$

Antiderivative was successfully verified.

[In] Int[(a - c + b*x)/((1 + x^2)*Sqrt[a + b*x + c*x^2]), x]

[Out] -((Sqrt[a^2 + b^2 + c*(c - Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c - Sqrt[a^2 + b^2 - 2*a*c + c^2])])*ArcTan[(b*Sqrt[a^2 + b^2 - 2*a*c + c^2] - (b^2 + (a - c)*(a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2]))*x)/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*Sqrt[a^2 + b^2 + c*(c - Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c - Sqrt[a^2 + b^2 - 2*a*c + c^2])]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4))) - (Sqrt[a^2 + b^2 + c*(c + Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c + Sqrt[a^2 + b^2 - 2*a*c + c^2])])*ArcTanh[(b*Sqrt[a^2 + b^2 - 2*a*c + c^2] + (b^2 + (a - c)*(a - c - Sqrt[a^2 + b^2 - 2*a*c + c^2]))*x)/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*Sqrt[a^2 + b^2 + c*(c + Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c + Sqrt[a^2 + b^2 - 2*a*c + c^2])]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1030

Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2], x] /; FreeQ[{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]

Rule 1036

Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist[1/(2*q), Int[Simp[-(a*h*e) - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[-(a*h*e) - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && NegQ[-(a*c)]

Rubi steps

$$\int \frac{a-c+bx}{(1+x^2)\sqrt{a+bx+cx^2}} dx = -\frac{\int \frac{-b^2-(a-c)(a-c+\sqrt{a^2+b^2-2ac+c^2})-b\sqrt{a^2+b^2-2ac+c^2}x}{(1+x^2)\sqrt{a+bx+cx^2}} dx}{2\sqrt{a^2+b^2-2ac+c^2}} + \frac{\int \frac{-b^2-(a-c)(a-c-\sqrt{a^2+b^2-2ac+c^2})-b\sqrt{a^2+b^2-2ac+c^2}x}{(1+x^2)\sqrt{a+bx+cx^2}} dx}{2\sqrt{a^2+b^2-2ac+c^2}}$$

$$= \left(b \left(b^2 + (a-c) \left(a-c - \sqrt{a^2+b^2-2ac+c^2} \right) \right) \right) \text{Subst} \left(\int \frac{\sqrt{a^2+b^2+c \left(c - \sqrt{a^2+b^2-2ac+c^2} \right) - a \left(2c - \sqrt{a^2+b^2-2ac+c^2} \right)} \tan^{-1} \left(\frac{x}{\sqrt{2} \sqrt{a^2+b^2-2ac+c^2}} \right)}{-2b\sqrt{a^2+b^2-2ac+c^2}} dx \right)$$

Mathematica [C] time = 0.08, size = 136, normalized size = 0.28

$$\frac{1}{2}i \left(\sqrt{a+ib-c} \tanh^{-1} \left(\frac{2a+b(x+i)+2icx}{2\sqrt{a+ib-c}\sqrt{a+x(b+cx)}} \right) - \sqrt{a-ib-c} \tanh^{-1} \left(\frac{2a+b(x-i)-2icx}{2\sqrt{a-ib-c}\sqrt{a+x(b+cx)}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a - c + b*x)/((1 + x^2)*Sqrt[a + b*x + c*x^2]), x]

[Out] (I/2)*(-(Sqrt[a - I*b - c]*ArcTanh[(2*a - (2*I)*c*x + b*(-I + x))/(2*Sqrt[a - I*b - c]*Sqrt[a + x*(b + c*x)])]) + Sqrt[a + I*b - c]*ArcTanh[(2*a + (2*I)*c*x + b*(I + x))/(2*Sqrt[a + I*b - c]*Sqrt[a + x*(b + c*x)])])

IntegrateAlgebraic [C] time = 0.42, size = 210, normalized size = 0.43

$$\frac{1}{2} \text{RootSum} \left[\#1^4 - 2\#1^2 a + 4\#1^2 c - 4\#1 b \sqrt{c} + a^2 + b^2 \&, \frac{\#1^2(-b) \log(-\#1 + \sqrt{a+bx+cx^2} - \sqrt{cx}) - 2\#1 c^{3/2} \log(-\#1 + \sqrt{a+bx+cx^2} - \sqrt{cx}) + 2\#1 a \sqrt{c} \log(-\#1 + \sqrt{a+bx+cx^2} - \sqrt{cx}) + bc \log(-\#1 + \sqrt{a+bx+cx^2} - \sqrt{cx})}{-\#1^3 + \#1 a - 2\#1 c + b\sqrt{c}} \right]$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a - c + b*x)/((1 + x^2)*Sqrt[a + b*x + c*x^2]), x]

[Out] RootSum[a^2 + b^2 - 4*b*Sqrt[c]*#1 - 2*a*#1^2 + 4*c*#1^2 + #1^4 &, (b*c*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*a*Sqrt[c]*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*c^(3/2)*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - b*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c] + a*#1 - 2*c*#1 - #1^3) &]/2

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a-c)/(x^2+1)/(c*x^2+b*x+a)^(1/2), x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a-c)/(x^2+1)/(c*x^2+b*x+a)^(1/2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the root
 of a polynomial with parameters. This might be wrong.The choice was done
 assuming [a,b,c]=[42,62,91]Warning, choosing root of [1,0,%%{-2,[2,0,0]%%
 }+%%{4,[1,0,1]%%}+%%{4,[1,0,0]%%}+%%{-2,[0,2,0]%%}+%%{-2,[0,0,2]%%}
 +%%{-4,[0,0,1]%%},%%{-8,[2,0,0]%%}+%%{16,[1,0,1]%%}+%%{-8,[0,2,0]%%
 }+%%{-8,[0,0,2]%%},%%{1,[4,0,0]%%}+%%{-4,[3,0,1]%%}+%%{4,[3,0,0]%%}
 +%%{2,[2,2,0]%%}+%%{6,[2,0,2]%%}+%%{-12,[2,0,1]%%}+%%{-4,[1,2,1]%%}
 +%%{4,[1,2,0]%%}+%%{-4,[1,0,3]%%}+%%{12,[1,0,2]%%}+%%{1,[0,4,0]%%}+
 %%{2,[0,2,2]%%}+%%{-4,[0,2,1]%%}+%%{-4,[0,2,0]%%}+%%{1,[0,0,4]%%}+
 %%{-4,[0,0,3]%%}] at parameters values [-27,26,-89]Warning, need to choose
 a branch for the root of a polynomial with parameters. This might be wrong
 .The choice was done assuming [a,b]=[-66,-52]Evaluation time: 7.83Unable to
 convert to real %%{1.0,[2]%%}+%%{132.0,[1]%%}+%%{7060.0,[0]%%} Error
 : Bad Argument Value

maple [B] time = 0.52, size = 6871419, normalized size = 14197.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a-c)/(x^2+1)/(c*x^2+b*x+a)^(1/2),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx + a - c}{\sqrt{cx^2 + bx + a}(x^2 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a-c)/(x^2+1)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")

[Out] integrate((b*x + a - c)/(sqrt(c*x^2 + b*x + a)*(x^2 + 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a - c + bx}{(x^2 + 1) \sqrt{cx^2 + bx + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - c + b*x)/((x^2 + 1)*(a + b*x + c*x^2)^(1/2)),x)

[Out] int((a - c + b*x)/((x^2 + 1)*(a + b*x + c*x^2)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx - c}{(x^2 + 1) \sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a-c)/(x**2+1)/(c*x**2+b*x+a)**(1/2),x)

[Out] Integral((a + b*x - c)/((x**2 + 1)*sqrt(a + b*x + c*x**2)), x)

$$3.13 \quad \int \frac{(A+Bx)(a+bx+cx^2)}{d+ex+fx^2} dx$$

Optimal. Leaf size=184

$$\frac{\log(d+ex+fx^2)(Af(ce-bf)-B(af^2-bef-cdf+ce^2))}{2f^3} - \frac{\tanh^{-1}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)(Af(2af^2-bef-2cdf+ce^2))}{f^3\sqrt{e^2-4df}}$$

Rubi [A] time = 0.35, antiderivative size = 182, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1628, 634, 618, 206, 628}

$$\frac{\log(d+ex+fx^2)(Bf(be-af)+Af(ce-bf)-Bc(e^2-df))}{2f^3} - \frac{\tanh^{-1}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)(Af(2af^2-bef-2cdf+ce^2)+Bf(-aef-2bdf+be^2)-Bc(e^3-3def))}{f^3\sqrt{e^2-4df}} - \frac{x(-Acf-bBf+Bce)}{f^2} + \frac{Bcx^2}{2f}$$

Antiderivative was successfully verified.

[In] Int[((A + B*x)*(a + b*x + c*x^2))/(d + e*x + f*x^2), x]

[Out] -(((B*c*e - b*B*f - A*c*f)*x)/f^2) + (B*c*x^2)/(2*f) - ((B*f*(b*e^2 - 2*b*d*f - a*e*f) - B*c*(e^3 - 3*d*e*f) + A*f*(c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2))*ArcTanh[(e + 2*f*x)/Sqrt[e^2 - 4*d*f]])/(f^3*Sqrt[e^2 - 4*d*f]) - ((B*f*(b*e - a*f) + A*f*(c*e - b*f) - B*c*(e^2 - d*f))*Log[d + e*x + f*x^2])/(2*f^3)

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1628

Int[(Pq)*((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx)(a+bx+cx^2)}{d+ex+fx^2} dx &= \int \left(-\frac{Bce-bBf-Acf}{f^2} + \frac{Bcx}{f} + \frac{-Af(cd-af)+Bd(ce-bf)-(Bf(be-af)+Ae^2-df))}{f^2(d+ex+fx^2)} \right) dx \\
&= -\frac{(Bce-bBf-Acf)x}{f^2} + \frac{Bcx^2}{2f} + \frac{\int \frac{-Af(cd-af)+Bd(ce-bf)-(Bf(be-af)+Ae^2-df))}{d+ex+fx^2}}{f^2} \\
&= -\frac{(Bce-bBf-Acf)x}{f^2} + \frac{Bcx^2}{2f} + \frac{(-Bf(be-af)-Af(ce-bf)+Bc(e^2-df))}{2f^3} \\
&= -\frac{(Bce-bBf-Acf)x}{f^2} + \frac{Bcx^2}{2f} - \frac{(Bf(be-af)+Af(ce-bf)-Bc(e^2-df))}{2f^3} \\
&= -\frac{(Bce-bBf-Acf)x}{f^2} + \frac{Bcx^2}{2f} - \frac{(Bf(be^2-2bdf-ae^2)-Bc(e^3-3def))}{f^3}
\end{aligned}$$

Mathematica [A] time = 0.21, size = 175, normalized size = 0.95

$$\frac{\log(d+x(e+fx))(Bf(af-be)+Af(bf-ce)+Bc(e^2-df)) - \frac{2 \tan^{-1}\left(\frac{e+2fx}{\sqrt{4df-e^2}}\right) (Af(-2af^2+bf+2cdf-ce^2)+Bf(aef+2bdf-be^2)+Bc(e^3-3def))}{\sqrt{4df-e^2}} + 2fx(Acf+bBf-Bce) + Bcf^2x^2}{2f^3}$$

Antiderivative was successfully verified.

[In] Integrate[((A+B*x)*(a+b*x+c*x^2))/(d+e*x+f*x^2),x]

[Out] (2*f*(-(B*c*e) + b*B*f + A*c*f)*x + B*c*f^2*x^2 - (2*(B*f*(-(b*e^2) + 2*b*d*f + a*e*f) + B*c*(e^3 - 3*d*e*f) + A*f*(-(c*e^2) + 2*c*d*f + b*e*f - 2*a*f^2))*ArcTan[(e + 2*f*x)/Sqrt[-e^2 + 4*d*f]])/Sqrt[-e^2 + 4*d*f] + (B*f*(-(b*e) + a*f) + A*f*(-(c*e) + b*f) + B*c*(e^2 - d*f))*Log[d + x*(e + f*x)]/(2*f^3)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+Bx)(a+bx+cx^2)}{d+ex+fx^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A+B*x)*(a+b*x+c*x^2))/(d+e*x+f*x^2),x]

[Out] IntegrateAlgebraic[((A+B*x)*(a+b*x+c*x^2))/(d+e*x+f*x^2), x]

fricas [A] time = 0.44, size = 583, normalized size = 3.17

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Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] [1/2*((B*c*e^2*f^2 - 4*B*c*d*f^3)*x^2 - (B*c*e^3 - 2*A*a*f^3 + (2*(B*b + A*c)*d + (B*a + A*b)*e)*f^2 - (3*B*c*d*e + (B*b + A*c)*e^2)*f)*sqrt(e^2 - 4*d*f)*log((2*f^2*x^2 + 2*e*f*x + e^2 - 2*d*f - sqrt(e^2 - 4*d*f)*(2*f*x + e))/(f*x^2 + e*x + d)) - 2*(B*c*e^3*f + 4*(B*b + A*c)*d*f^3 - (4*B*c*d*e + (B*b + A*c)*e^2)*f^2)*x + (B*c*e^4 - 4*(B*a + A*b)*d*f^3 + (4*B*c*d^2 + 4*(B*b + A*c)*d*e + (B*a + A*b)*e^2)*f^2 - (5*B*c*d*e^2 + (B*b + A*c)*e^3)*f)*log(f*x^2 + e*x + d)/(e^2*f^3 - 4*d*f^4), 1/2*((B*c*e^2*f^2 - 4*B*c*d*f^3)*x^2

$2 + 2*(B*c*e^3 - 2*A*a*f^3 + (2*(B*b + A*c)*d + (B*a + A*b)*e)*f^2 - (3*B*c*d*e + (B*b + A*c)*e^2)*f)*\sqrt{-e^2 + 4*d*f}*\arctan(-\sqrt{-e^2 + 4*d*f}*(2*f*x + e)/(e^2 - 4*d*f)) - 2*(B*c*e^3*f + 4*(B*b + A*c)*d*f^3 - (4*B*c*d*e + (B*b + A*c)*e^2)*f^2)*x + (B*c*e^4 - 4*(B*a + A*b)*d*f^3 + (4*B*c*d^2 + 4*(B*b + A*c)*d*e + (B*a + A*b)*e^2)*f^2 - (5*B*c*d*e^2 + (B*b + A*c)*e^3)*f)*\log(f*x^2 + e*x + d)/(e^2*f^3 - 4*d*f^4]$

giac [A] time = 0.17, size = 191, normalized size = 1.04

$$\frac{Bcfx^2 + 2Bbfx + 2Acfx - 2Bcxe}{2f^2} - \frac{(Bcdf - Baf^2 - Abf^2 + Bbfe + Acfe - Bce^2)\log(fx^2 + xe + d)}{2f^3} - \frac{(2Bbd^2 + 2Acdf^2 - 2Aaf^3 - 3Bcdf + Baf^2e + Abf^2e - Bbfe^2 - Acfe^2 + Bce^3)\arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)}{\sqrt{4df-e^2}f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] $1/2*(B*c*f*x^2 + 2*B*b*f*x + 2*A*c*f*x - 2*B*c*x*e)/f^2 - 1/2*(B*c*d*f - B*a*f^2 - A*b*f^2 + B*b*f*e + A*c*f*e - B*c*e^2)*\log(f*x^2 + x*e + d)/f^3 - (2*B*b*d*f^2 + 2*A*c*d*f^2 - 2*A*a*f^3 - 3*B*c*d*f*e + B*a*f^2*e + A*b*f^2*e - B*b*f*e^2 - A*c*f*e^2 + B*c*e^3)*\arctan((2*f*x + e)/\sqrt{4*d*f - e^2})/(\sqrt{4*d*f - e^2})*f^3$

maple [B] time = 0.01, size = 510, normalized size = 2.77

$$\frac{2Aa \arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)}{\sqrt{4df-e^2}} - \frac{Ab \arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)}{\sqrt{4df-e^2}} + \frac{2Ad \arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)}{\sqrt{4df-e^2}} + \frac{Ac^2 \arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)}{\sqrt{4df-e^2}} - \frac{Bb \arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)}{\sqrt{4df-e^2}} + \frac{Bc \arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)}{\sqrt{4df-e^2}} + \frac{2Aaf^3 - 3Bcdf + Baf^2e + Abf^2e - Bbfe^2 - Acfe^2 + Bce^3}{2f^3} \arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right) + \frac{Bce^2}{2f} \frac{A \ln(f^2 + ex + d)}{f} + \frac{A \ln(f^2 + ex + d)}{2f} + \frac{Bb \ln(f^2 + ex + d)}{2f} + \frac{Bc \ln(f^2 + ex + d)}{2f} + \frac{A \ln(f^2 + ex + d)}{2f} + \frac{Bb \ln(f^2 + ex + d)}{2f} + \frac{Bc \ln(f^2 + ex + d)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x+A)*(c*x^2+b*x+a)/(f*x^2+e*x+d),x)

[Out] $1/2*B*c/f*x^2+A*c/f*x+B*b/f*x-1/f^2*B*c*e*x+1/2/f*\ln(f*x^2+e*x+d)*A*b-1/2/f^2*\ln(f*x^2+e*x+d)*A*c*e+1/2/f*\ln(f*x^2+e*x+d)*B*a-1/2/f^2*\ln(f*x^2+e*x+d)*B*b*e-1/2/f^2*\ln(f*x^2+e*x+d)*B*c*d+1/2/f^3*\ln(f*x^2+e*x+d)*B*c*e^2+2/(4*d*f-e^2)^(1/2)*\arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*A*a-2/f/(4*d*f-e^2)^(1/2)*\arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*A*c*d-2/f/(4*d*f-e^2)^(1/2)*\arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*B*b*d+3/f^2/(4*d*f-e^2)^(1/2)*\arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*B*c*d*e-1/f/(4*d*f-e^2)^(1/2)*\arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e*A*b+1/f^2/(4*d*f-e^2)^(1/2)*\arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^2*A*c-1/f/(4*d*f-e^2)^(1/2)*\arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e*B*a+1/f^2/(4*d*f-e^2)^(1/2)*\arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^2*B*b-1/f^3/(4*d*f-e^2)^(1/2)*\arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^3*B*c$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details)Is 4*d*f-e^2 positive or negative?

mupad [B] time = 3.85, size = 273, normalized size = 1.48

$$x \left(\frac{Ac + Bd}{f} - \frac{Bce}{f^2} \right) \cdot \ln(fx^2 + ex + d) + \frac{(Bce^2 - 4Abdf^2 - 4Bbd^2 - Acd^2 - Bbdf^2 + Abef^2 + Baef^2 + 4Bcd^2 + 4Acdf^2 + 4Bbde^2 - 5Bcd^2)f}{2(4df^2 - e^2)} - \frac{\arctan\left(\frac{e}{\sqrt{4df-e^2}} + \frac{2fx}{\sqrt{4df-e^2}}\right)}{f \sqrt{4df-e^2}} (Bce^2 - 2Aaf^2 + Abef^2 + 2Acdf^2 + Bae^2 + 2Bbd^2 - Acd^2 - Bbdf^2 - 3Bcd^2) + \frac{Bce^2}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x)*(a + b*x + c*x^2))/(d + e*x + f*x^2),x)

```
[Out] x*((A*c + B*b)/f - (B*c*e)/f^2) - (log(d + e*x + f*x^2)*(B*c*e^4 - 4*A*b*d*
f^3 - 4*B*a*d*f^3 - A*c*e^3*f - B*b*e^3*f + A*b*e^2*f^2 + B*a*e^2*f^2 + 4*B
*c*d^2*f^2 + 4*A*c*d*e*f^2 + 4*B*b*d*e*f^2 - 5*B*c*d*e^2*f))/(2*(4*d*f^4 -
e^2*f^3)) - (atan(e/(4*d*f - e^2)^(1/2) + (2*f*x)/(4*d*f - e^2)^(1/2))*(B*c
*e^3 - 2*A*a*f^3 + A*b*e*f^2 + 2*A*c*d*f^2 + B*a*e*f^2 + 2*B*b*d*f^2 - A*c*
e^2*f - B*b*e^2*f - 3*B*c*d*e*f))/(f^3*(4*d*f - e^2)^(1/2)) + (B*c*x^2)/(2*
f)
```

sympy [B] time = 16.65, size = 1260, normalized size = 6.85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x**2+b*x+a)/(f*x**2+e*x+d),x)
```

```
[Out] B*c*x**2/(2*f) + x*(A*c/f + B*b/f - B*c*e/f**2) + (-sqrt(-4*d*f + e**2))*(-2
*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 + 2*B*b*d*f
**2 - B*b*e**2*f - 3*B*c*d*e*f + B*c*e**3)/(2*f**3*(4*d*f - e**2)) + (A*b*f
**2 - A*c*e*f + B*a*f**2 - B*b*e*f - B*c*d*f + B*c*e**2)/(2*f**3))*log(x +
(-A*a*e*f**2 + 2*A*b*d*f**2 - A*c*d*e*f + 2*B*a*d*f**2 - B*b*d*e*f - 2*B*c
*d**2*f + B*c*d*e**2 - 4*d*f**3*(-sqrt(-4*d*f + e**2))*(-2*A*a*f**3 + A*b*e*f
**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 + 2*B*b*d*f**2 - B*b*e**2*f -
3*B*c*d*e*f + B*c*e**3)/(2*f**3*(4*d*f - e**2)) + (A*b*f**2 - A*c*e*f + B*a
*f**2 - B*b*e*f - B*c*d*f + B*c*e**2)/(2*f**3)) + e**2*f**2*(-sqrt(-4*d*f +
e**2))*(-2*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 +
2*B*b*d*f**2 - B*b*e**2*f - 3*B*c*d*e*f + B*c*e**3)/(2*f**3*(4*d*f - e**2))
) + (A*b*f**2 - A*c*e*f + B*a*f**2 - B*b*e*f - B*c*d*f + B*c*e**2)/(2*f**3)
))/(-2*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 + 2*B
*b*d*f**2 - B*b*e**2*f - 3*B*c*d*e*f + B*c*e**3)) + (sqrt(-4*d*f + e**2))*(-
2*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 + 2*B*b*d*
f**2 - B*b*e**2*f - 3*B*c*d*e*f + B*c*e**3)/(2*f**3*(4*d*f - e**2)) + (A*b*
f**2 - A*c*e*f + B*a*f**2 - B*b*e*f - B*c*d*f + B*c*e**2)/(2*f**3))*log(x +
(-A*a*e*f**2 + 2*A*b*d*f**2 - A*c*d*e*f + 2*B*a*d*f**2 - B*b*d*e*f - 2*B*c
*d**2*f + B*c*d*e**2 - 4*d*f**3*(sqrt(-4*d*f + e**2))*(-2*A*a*f**3 + A*b*e*f
**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 + 2*B*b*d*f**2 - B*b*e**2*f -
3*B*c*d*e*f + B*c*e**3)/(2*f**3*(4*d*f - e**2)) + (A*b*f**2 - A*c*e*f + B*a
*f**2 - B*b*e*f - B*c*d*f + B*c*e**2)/(2*f**3)) + e**2*f**2*(sqrt(-4*d*f +
e**2))*(-2*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 +
2*B*b*d*f**2 - B*b*e**2*f - 3*B*c*d*e*f + B*c*e**3)/(2*f**3*(4*d*f - e**2))
+ (A*b*f**2 - A*c*e*f + B*a*f**2 - B*b*e*f - B*c*d*f + B*c*e**2)/(2*f**3)
))/(-2*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 + 2*B
*b*d*f**2 - B*b*e**2*f - 3*B*c*d*e*f + B*c*e**3))
```

$$3.14 \quad \int \frac{(A+Bx)(a+bx+cx^2)^2}{d+ex+fx^2} dx$$

Optimal. Leaf size=542

$$\frac{\log(d+ex+fx^2) \left(B(-f^2(-a^2f^2+2abef - (b^2(e^2-df))) + 2cf(af(e^2-df) - b(e^3-2def)) + c^2(d^2f^2 - 3def)) \right)}{2f^5}$$

Rubi [A] time = 1.10, antiderivative size = 542, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1011, 634, 618, 206, 628}

Antiderivative was successfully verified.

Antiderivative was successfully verified.

[In] Int[((A + B*x)*(a + b*x + c*x^2)^2)/(d + e*x + f*x^2), x]

[Out] ((B*(c*e - b*f)*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) + A*f*(b^2*f^2 - 2*c*f*(b*e - a*f) + c^2*(e^2 - d*f)))*x)/f^4 - ((A*c*f*(c*e - 2*b*f) - B*(b^2*f^2 - 2*c*f*(b*e - a*f) + c^2*(e^2 - d*f)))*x^2)/(2*f^3) - (c*(B*c*e - 2*b*B*f - A*c*f)*x^3)/(3*f^2) + (B*c^2*x^4)/(4*f) - ((A*f*(c^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2) - f^2*(2*a*b*e*f - 2*a^2*f^2 - b^2*(e^2 - 2*d*f)) + 2*c*f*(a*f*(e^2 - 2*d*f) - b*(e^3 - 3*d*e*f))) - B*(c^2*(e^5 - 5*d*e^3*f + 5*d^2*e*f^2) + f^2*(a^2*e*f^2 - 2*a*b*f*(e^2 - 2*d*f) + b^2*(e^3 - 3*d*e*f)) + 2*c*f*(a*e*f*(e^2 - 3*d*f) - b*(e^4 - 4*d*e^2*f + 2*d^2*f^2)))*ArcTanh[(e + 2*f*x)/Sqrt[e^2 - 4*d*f]])/(f^5*Sqrt[e^2 - 4*d*f]) + ((A*f*(c*e - b*f)*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) + B*(c^2*(e^4 - 3*d*e^2*f + d^2*f^2) - f^2*(2*a*b*e*f - a^2*f^2 - b^2*(e^2 - d*f)) + 2*c*f*(a*f*(e^2 - d*f) - b*(e^3 - 2*d*e*f))))*Log[d + e*x + f*x^2]/(2*f^5)

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1011

Int[((g_) + (h_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Int[ExpandIntegrand[(a + b*x +

$c*x^2)^p*(d + e*x + f*x^2)^q*(g + h*x), x]$, $x]$ /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && IGtQ[p, 0] && IntegerQ[q]

Rubi steps

$$\begin{aligned} \int \frac{(A + Bx)(a + bx + cx^2)^2}{d + ex + fx^2} dx &= \int \frac{(B(ce - bf)(f(be - 2af) - c(e^2 - 2df)) + Af(b^2f^2 - 2cf(be - af) + c^2(e^2 - 2df)))}{f^4} \\ &= \frac{(B(ce - bf)(f(be - 2af) - c(e^2 - 2df)) + Af(b^2f^2 - 2cf(be - af) + c^2(e^2 - 2df)))}{f^4} \\ &= \frac{(B(ce - bf)(f(be - 2af) - c(e^2 - 2df)) + Af(b^2f^2 - 2cf(be - af) + c^2(e^2 - 2df)))}{f^4} \\ &= \frac{(B(ce - bf)(f(be - 2af) - c(e^2 - 2df)) + Af(b^2f^2 - 2cf(be - af) + c^2(e^2 - 2df)))}{f^4} \\ &= \frac{(B(ce - bf)(f(be - 2af) - c(e^2 - 2df)) + Af(b^2f^2 - 2cf(be - af) + c^2(e^2 - 2df)))}{f^4} \end{aligned}$$

Mathematica [A] time = 0.60, size = 535, normalized size = 0.99

Integrate[(A + B*x)*(a + b*x + c*x^2)^2/(d + e*x + f*x^2), x] // FullSimplify

Antiderivative was successfully verified.

[In] Integrate[((A + B*x)*(a + b*x + c*x^2)^2)/(d + e*x + f*x^2), x]

[Out] (12*f*(-(B*(c*e - b*f)*(f*(-(b*e) + 2*a*f) + c*(e^2 - 2*d*f))) + A*f*(b^2*f^2 + 2*c*f*(-(b*e) + a*f) + c^2*(e^2 - d*f)))*x + 6*f^2*(A*c*f*(-(c*e) + 2*b*f) + B*(b^2*f^2 + 2*c*f*(-(b*e) + a*f) + c^2*(e^2 - d*f)))*x^2 + 4*c*f^3*(-(B*c*e) + 2*b*B*f + A*c*f)*x^3 + 3*B*c^2*f^4*x^4 - (12*(-(A*f*(c^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2) + f^2*(-2*a*b*e*f + 2*a^2*f^2 + b^2*(e^2 - 2*d*f)) + 2*c*f*(a*f*(e^2 - 2*d*f) - b*(e^3 - 3*d*e*f)))) + B*(c^2*(e^5 - 5*d*e^3*f + 5*d^2*e*f^2) + f^2*(a^2*e*f^2 + 2*a*b*f*(-e^2 + 2*d*f) + b^2*(e^3 - 3*d*e*f) - 2*c*f*(-(a*e*f*(e^2 - 3*d*f)) + b*(e^4 - 4*d*e^2*f + 2*d^2*f^2)))))*ArcTan[(e + 2*f*x)/Sqrt[-e^2 + 4*d*f]]/Sqrt[-e^2 + 4*d*f] + 6*(A*f*(-(c*e) + b*f)*(f*(-(b*e) + 2*a*f) + c*(e^2 - 2*d*f)) + B*(c^2*(e^4 - 3*d*e^2*f + d^2*f^2) + f^2*(-2*a*b*e*f + a^2*f^2 + b^2*(e^2 - d*f)) - 2*c*f*(a*f*(-e^2 + d*f) + b*(e^3 - 2*d*e*f))))*Log[d + x*(e + f*x)]/(12*f^5)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx)(a + bx + cx^2)^2}{d + ex + fx^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B*x)*(a + b*x + c*x^2)^2)/(d + e*x + f*x^2), x]

[Out] IntegrateAlgebraic[((A + B*x)*(a + b*x + c*x^2)^2)/(d + e*x + f*x^2), x]

fricas [A] time = 0.61, size = 1837, normalized size = 3.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/12*(3*(B*c^2*e^2*f^4 - 4*B*c^2*d*f^5)*x^4 - 4*(B*c^2*e^3*f^3 + 4*(2*B*b*c + A*c^2)*d*f^5 - (4*B*c^2*d*e + (2*B*b*c + A*c^2)*e^2)*f^4)*x^3 + 6*(B*c^2*e^4*f^2 - 4*(B*b^2 + 2*(B*a + A*b)*c)*d*f^5 + (4*B*c^2*d^2 + 4*(2*B*b*c + A*c^2)*d*e + (B*b^2 + 2*(B*a + A*b)*c)*e^2)*f^4 - (5*B*c^2*d*e^2 + (2*B*b*c + A*c^2)*e^3)*f^3)*x^2 - 6*(B*c^2*e^5 - 2*A*a^2*f^5 + (2*(2*B*a*b + A*b^2 + 2*A*a*c)*d + (B*a^2 + 2*A*a*b)*e)*f^4 - (2*(2*B*b*c + A*c^2)*d^2 + 3*(B*b^2 + 2*(B*a + A*b)*c)*d*e + (2*B*a*b + A*b^2 + 2*A*a*c)*e^2)*f^3 + (5*B*c^2*d^2*e + 4*(2*B*b*c + A*c^2)*d*e^2 + (B*b^2 + 2*(B*a + A*b)*c)*e^3)*f^2 - (5*B*c^2*d*e^3 + (2*B*b*c + A*c^2)*e^4)*f)*sqrt(e^2 - 4*d*f)*log((2*f^2*x^2 + 2*e*f*x + e^2 - 2*d*f - sqrt(e^2 - 4*d*f)*(2*f*x + e))/(f*x^2 + e*x + d)) - 12*(B*c^2*e^5*f + 4*(2*B*a*b + A*b^2 + 2*A*a*c)*d*f^5 - (4*(2*B*b*c + A*c^2)*d^2 + 4*(B*b^2 + 2*(B*a + A*b)*c)*d*e + (2*B*a*b + A*b^2 + 2*A*a*c)*e^2)*f^4 + (8*B*c^2*d^2*e + 5*(2*B*b*c + A*c^2)*d*e^2 + (B*b^2 + 2*(B*a + A*b)*c)*e^3)*f^3 - (6*B*c^2*d*e^3 + (2*B*b*c + A*c^2)*e^4)*f^2)*x + 6*(B*c^2*e^6 - 4*(B*a^2 + 2*A*a*b)*d*f^5 + (4*(B*b^2 + 2*(B*a + A*b)*c)*d^2 + 4*(2*B*a*b + A*b^2 + 2*A*a*c)*d*e + (B*a^2 + 2*A*a*b)*e^2)*f^4 - (4*B*c^2*d^3 + 8*(2*B*b*c + A*c^2)*d^2*e + 5*(B*b^2 + 2*(B*a + A*b)*c)*d*e^2 + (2*B*a*b + A*b^2 + 2*A*a*c)*e^3)*f^3 + (13*B*c^2*d^2*e^2 + 6*(2*B*b*c + A*c^2)*d*e^3 + (B*b^2 + 2*(B*a + A*b)*c)*e^4)*f^2 - (7*B*c^2*d*e^4 + (2*B*b*c + A*c^2)*e^5)*f)*log(f*x^2 + e*x + d)/(e^2*f^5 - 4*d*f^6), 1/12*(3*(B*c^2*e^2*f^4 - 4*B*c^2*d*f^5)*x^4 - 4*(B*c^2*e^3*f^3 + 4*(2*B*b*c + A*c^2)*d*f^5 - (4*B*c^2*d*e + (2*B*b*c + A*c^2)*e^2)*f^4)*x^3 + 6*(B*c^2*e^4*f^2 - 4*(B*b^2 + 2*(B*a + A*b)*c)*d*f^5 + (4*B*c^2*d^2 + 4*(2*B*b*c + A*c^2)*d*e + (B*b^2 + 2*(B*a + A*b)*c)*e^2)*f^4 - (5*B*c^2*d*e^2 + (2*B*b*c + A*c^2)*e^3)*f^3)*x^2 + 12*(B*c^2*e^5 - 2*A*a^2*f^5 + (2*(2*B*a*b + A*b^2 + 2*A*a*c)*d + (B*a^2 + 2*A*a*b)*e)*f^4 - (2*(2*B*b*c + A*c^2)*d^2 + 3*(B*b^2 + 2*(B*a + A*b)*c)*d*e + (2*B*a*b + A*b^2 + 2*A*a*c)*e^2)*f^3 + (5*B*c^2*d^2*e + 4*(2*B*b*c + A*c^2)*d*e^2 + (B*b^2 + 2*(B*a + A*b)*c)*e^3)*f^2 - (5*B*c^2*d*e^3 + (2*B*b*c + A*c^2)*e^4)*f)*sqrt(-e^2 + 4*d*f)*arctan(-sqrt(-e^2 + 4*d*f)*(2*f*x + e)/(e^2 - 4*d*f)) - 12*(B*c^2*e^5*f + 4*(2*B*a*b + A*b^2 + 2*A*a*c)*d*f^5 - (4*(2*B*b*c + A*c^2)*d^2 + 4*(B*b^2 + 2*(B*a + A*b)*c)*d*e + (2*B*a*b + A*b^2 + 2*A*a*c)*e^2)*f^4 + (8*B*c^2*d^2*e + 5*(2*B*b*c + A*c^2)*d*e^2 + (B*b^2 + 2*(B*a + A*b)*c)*e^3)*f^3 - (6*B*c^2*d*e^3 + (2*B*b*c + A*c^2)*e^4)*f^2)*x + 6*(B*c^2*e^6 - 4*(B*a^2 + 2*A*a*b)*d*f^5 + (4*(B*b^2 + 2*(B*a + A*b)*c)*d^2 + 4*(2*B*a*b + A*b^2 + 2*A*a*c)*d*e + (B*a^2 + 2*A*a*b)*e^2)*f^4 - (4*B*c^2*d^3 + 8*(2*B*b*c + A*c^2)*d^2*e + 5*(B*b^2 + 2*(B*a + A*b)*c)*d*e^2 + (2*B*a*b + A*b^2 + 2*A*a*c)*e^3)*f^3 + (13*B*c^2*d^2*e^2 + 6*(2*B*b*c + A*c^2)*d*e^3 + (B*b^2 + 2*(B*a + A*b)*c)*e^4)*f^2 - (7*B*c^2*d*e^4 + (2*B*b*c + A*c^2)*e^5)*f)*log(f*x^2 + e*x + d)/(e^2*f^5 - 4*d*f^6)] \end{aligned}$$

giac [A] time = 0.25, size = 738, normalized size = 1.36

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="giac")

[Out]
$$\begin{aligned} & 1/12*(3*B*c^2*f^3*x^4 + 8*B*b*c*f^3*x^3 + 4*A*c^2*f^3*x^3 - 4*B*c^2*f^2*x^3 *e - 6*B*c^2*d*f^2*x^2 + 6*B*b^2*f^3*x^2 + 12*B*a*c*f^3*x^2 + 12*A*b*c*f^3*x^2 - 12*B*b*c*f^2*x^2*e - 6*A*c^2*f^2*x^2*e - 24*B*b*c*d*f^2*x - 12*A*c^2*d*f^2*x + 24*B*a*b*f^3*x + 12*A*b^2*f^3*x + 24*A*a*c*f^3*x + 6*B*c^2*f*x^2*e^2 + 24*B*c^2*d*f*x*e - 12*B*b^2*f^2*x*e - 24*B*a*c*f^2*x*e - 24*A*b*c*f^2*x*e + 24*B*b*c*f*x*e^2 + 12*A*c^2*f*x*e^2 - 12*B*c^2*x*e^3)/f^4 + 1/2*(B*c^2*d^2*f^2 - B*b^2*d*f^3 - 2*B*a*c*d*f^3 - 2*A*b*c*d*f^3 + B*a^2*f^4 + 2*A*a*b*f^4 + 4*B*b*c*d*f^2*e + 2*A*c^2*d*f^2*e - 2*B*a*b*f^3*e - A*b^2*f^3*e - 2*A*a*c*f^3*e - 3*B*c^2*d*f*e^2 + B*b^2*f^2*e^2 + 2*B*a*c*f^2*e^2 + 2*A*b* \end{aligned}$$

$$c*f^2*e^2 - 2*B*b*c*f*e^3 - A*c^2*f*e^3 + B*c^2*e^4)*\log(f*x^2 + x*e + d)/f^5 + (4*B*b*c*d^2*f^3 + 2*A*c^2*d^2*f^3 - 4*B*a*b*d*f^4 - 2*A*b^2*d*f^4 - 4*A*a*c*d*f^4 + 2*A*a^2*f^5 - 5*B*c^2*d^2*f^2*e + 3*B*b^2*d*f^3*e + 6*B*a*c*d*f^3*e + 6*A*b*c*d*f^3*e - B*a^2*f^4*e - 2*A*a*b*f^4*e - 8*B*b*c*d*f^2*e^2 - 4*A*c^2*d*f^2*e^2 + 2*B*a*b*f^3*e^2 + A*b^2*f^3*e^2 + 2*A*a*c*f^3*e^2 + 5*B*c^2*d*f*e^3 - B*b^2*f^2*e^3 - 2*B*a*c*f^2*e^3 - 2*A*b*c*f^2*e^3 + 2*B*b*c*f*e^4 + A*c^2*f*e^4 - B*c^2*e^5)*\arctan((2*f*x + e)/\sqrt{4*d*f - e^2})/(\sqrt{4*d*f - e^2})*f^5)$$

maple [B] time = 0.01, size = 1672, normalized size = 3.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x)

[Out] $\frac{1}{4}B*c^2/f*x^4 - 2*B*b*c*d/f^2*x + 1/2/f*\ln(f*x^2+e*x+d)*B*a^2 + 2/(4*d*f-e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f-e^2)^{(1/2)})*A*a^2 + 1/3*A*c^2/f*x^3 + 1/2*B*b^2/f*x^2 + A*b^2/f*x - A*c^2*d/f^2*x + 2*B*a*b/f*x + B*a*c/f*x^2 - 1/2*B*c^2*d/f^2*x^2 + 2*A*a*c/f*x + 2/3*B*b*c/f*x^3 + A*b*c/f*x^2 - 8/f^3/(4*d*f-e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f-e^2)^{(1/2)})*B*b*c*d*e^2 + 6/f^2/(4*d*f-e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f-e^2)^{(1/2)})*A*b*c*d*e + 6/f^2/(4*d*f-e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f-e^2)^{(1/2)})*B*a*c*d*e + 1/f^3*A*c^2*e^2*x - 1/f^2*B*b^2*e*x - 1/f^4*B*c^2*e^3*x - 1/2/f^2*A*x^2*c^2*e + 1/2/f^3*B*x^2*c^2*e^2 - 1/3/f^2*B*x^3*c^2*e - 1/2/f^2*\ln(f*x^2+e*x+d)*A*b^2*e - 1/2/f^4*\ln(f*x^2+e*x+d)*A*c^2*e^3 - 1/2/f^2*\ln(f*x^2+e*x+d)*B*b^2*d + 1/2/f^3*\ln(f*x^2+e*x+d)*B*b^2*e^2 + 1/2/f^3*\ln(f*x^2+e*x+d)*B*c^2*d^2 + 1/2/f^5*\ln(f*x^2+e*x+d)*B*c^2*e^4 + 1/f*\ln(f*x^2+e*x+d)*A*a*b - 1/f^2*B*x^2*b*c*e - 2/f^2*A*b*c*e*x - 2/f^2*B*a*c*e*x + 2/f^3*B*b*c*e^2*x + 2/f^3*B*c^2*d*e*x + 1/f^2/(4*d*f-e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f-e^2)^{(1/2)})*e^2*A*b^2 - 1/f/(4*d*f-e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f-e^2)^{(1/2)})*e*B*a^2 - 1/f^3/(4*d*f-e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f-e^2)^{(1/2)})*e^3*B*b^2 - 1/f^5/(4*d*f-e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f-e^2)^{(1/2)})*e^5*B*c^2 - 2/f/(4*d*f-e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f-e^2)^{(1/2)})*A*b^2*d + 2/f^2/(4*d*f-e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f-e^2)^{(1/2)})*A*c^2*d^2 - 1/f^2*\ln(f*x^2+e*x+d)*A*a*c*e - 1/f^2*\ln(f*x^2+e*x+d)*A*b*c*d + 1/f^3*\ln(f*x^2+e*x+d)*A*b*c*e^2 + 1/f^3*\ln(f*x^2+e*x+d)*A*c^2*d*e - 1/f^2*\ln(f*x^2+e*x+d)*B*a*b*e - 1/f^2*\ln(f*x^2+e*x+d)*B*a*c*d + 1/f^3*\ln(f*x^2+e*x+d)*B*a*c*e^2 - 1/f^4*\ln(f*x^2+e*x+d)*B*b*c*e^3 - 3/2/f^4*\ln(f*x^2+e*x+d)*B*c^2*d*e^2 + 1/f^4/(4*d*f-e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f-e^2)^{(1/2)})*e^4*A*c^2 - 2/f^3/(4*d*f-e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f-e^2)^{(1/2)})*e^3*A*b*c + 2/f^2/(4*d*f-e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f-e^2)^{(1/2)})*e^2*B*a*b - 2/f^3/(4*d*f-e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f-e^2)^{(1/2)})*e^3*B*a*c + 2/f^3*\ln(f*x^2+e*x+d)*B*b*c*d*e - 4/f/(4*d*f-e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f-e^2)^{(1/2)})*A*a*c*d - 4/f^3/(4*d*f-e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f-e^2)^{(1/2)})*A*c^2*d*e^2 - 4/f/(4*d*f-e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f-e^2)^{(1/2)})*B*a*b*d + 3/f^2/(4*d*f-e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f-e^2)^{(1/2)})*B*b^2*d*e + 4/f^2/(4*d*f-e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f-e^2)^{(1/2)})*B*b*c*d^2 - 5/f^3/(4*d*f-e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f-e^2)^{(1/2)})*B*c^2*d^2*e - 2/f/(4*d*f-e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f-e^2)^{(1/2)})*e*A*a*b + 5/f^4/(4*d*f-e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f-e^2)^{(1/2)})*B*c^2*d*e^3 + 2/f^4/(4*d*f-e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f-e^2)^{(1/2)})*e^4*B*b*c + 2/f^2/(4*d*f-e^2)^{(1/2)}*\arctan((2*f*x+e)/(4*d*f-e^2)^{(1/2)})*e^2*A*a*c$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* h

elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details) Is $4*d*f - e^2$ positive or negative?

mupad [B] time = 4.85, size = 893, normalized size = 1.65

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(((A + B*x)*(a + b*x + c*x^2)^2)/(d + e*x + f*x^2), x)$

[Out] $x^3 * ((A*c^2 + 2*B*b*c)/(3*f) - (B*c^2*e)/(3*f^2)) + x * ((A*b^2 + 2*A*a*c + 2*B*a*b)/f - (d * ((A*c^2 + 2*B*b*c)/f - (B*c^2*e)/f^2))/f + (e * ((e * ((A*c^2 + 2*B*b*c)/f - (B*c^2*e)/f^2))/f - (B*b^2 + 2*A*b*c + 2*B*a*c)/f + (B*c^2*d)/f^2))/f - x^2 * ((e * ((A*c^2 + 2*B*b*c)/f - (B*c^2*e)/f^2))/(2*f) - (B*b^2 + 2*A*b*c + 2*B*a*c)/(2*f) + (B*c^2*d)/(2*f^2)) - (\log(d + e*x + f*x^2) * (B*c^2*e^6 - 4*B*a^2*d*f^5 - A*c^2*e^5*f - A*b^2*e^3*f^3 + B*a^2*e^2*f^4 + 4*B*b^2*d^2*f^4 + B*b^2*e^4*f^2 - 4*B*c^2*d^3*f^3 + 6*A*c^2*d*e^3*f^2 - 8*A*c^2*d^2*e*f^3 - 5*B*b^2*d*e^2*f^3 - 8*A*a*b*d*f^5 - 2*B*b*c*e^5*f + 13*B*c^2*d^2*e^2*f^2 + 2*A*a*b*e^2*f^4 - 2*A*a*c*e^3*f^3 + 8*A*b*c*d^2*f^4 - 2*B*a*b*e^3*f^3 + 8*B*a*c*d^2*f^4 + 2*A*b*c*e^4*f^2 + 2*B*a*c*e^4*f^2 + 4*A*b^2*d*e*f^4 - 7*B*c^2*d*e^4*f - 10*A*b*c*d*e^2*f^3 - 10*B*a*c*d*e^2*f^3 + 12*B*b*c*d*e^3*f^2 - 16*B*b*c*d^2*e*f^3 + 8*A*a*c*d*e*f^4 + 8*B*a*b*d*e*f^4)) / (2 * (4*d*f^6 - e^2*f^5)) + (B*c^2*x^4)/(4*f) + (\text{atan}(e/(4*d*f - e^2))^{1/2} + (2*f*x)/(4*d*f - e^2))^{1/2} * (2*A*a^2*f^5 - B*c^2*e^5 - 2*A*b^2*d*f^4 - B*a^2*e*f^4 + A*c^2*e^4*f + A*b^2*e^2*f^3 + 2*A*c^2*d^2*f^3 - B*b^2*e^3*f^2 - 4*A*c^2*d*e^2*f^2 - 5*B*c^2*d^2*e*f^2 - 2*A*a*b*e*f^4 - 4*A*a*c*d*f^4 - 4*B*a*b*d*f^4 + 2*B*b*c*e^4*f + 2*A*a*c*e^2*f^3 + 2*B*a*b*e^2*f^3 - 2*A*b*c*e^3*f^2 - 2*B*a*c*e^3*f^2 + 4*B*b*c*d^2*f^3 + 3*B*b^2*d*e*f^3 + 5*B*c^2*d*e^3*f - 8*B*b*c*d*e^2*f^2 + 6*A*b*c*d*e*f^3 + 6*B*a*c*d*e*f^3)) / (f^5 * (4*d*f - e^2)^{1/2})$

sympy [B] time = 145.64, size = 4663, normalized size = 8.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((B*x+A)*(c*x**2+b*x+a)**2/(f*x**2+e*x+d), x)$

[Out] $B*c**2*x**4/(4*f) + x**3*(A*c**2/(3*f) + 2*B*b*c/(3*f) - B*c**2*e/(3*f**2)) + x**2*(A*b*c/f - A*c**2*e/(2*f**2) + B*a*c/f + B*b**2/(2*f) - B*b*c*e/f**2 - B*c**2*d/(2*f**2) + B*c**2*e**2/(2*f**3)) + x*(2*A*a*c/f + A*b**2/f - 2*A*b*c*e/f**2 - A*c**2*d/f**2 + A*c**2*e**2/f**3 + 2*B*a*b/f - 2*B*a*c*e/f**2 - B*b**2*e/f**2 - 2*B*b*c*d/f**2 + 2*B*b*c*e**2/f**3 + 2*B*c**2*d*e/f**3 - B*c**2*e**3/f**4) + (-\sqrt{-4*d*f + e**2}) * (-2*A*a**2*f**5 + 2*A*a*b*e*f**4 + 4*A*a*c*d*f**4 - 2*A*a*c*e**2*f**3 + 2*A*b**2*d*f**4 - A*b**2*e**2*f**3 - 6*A*b*c*d*e*f**3 + 2*A*b*c*e**3*f**2 - 2*A*c**2*d**2*f**3 + 4*A*c**2*d*e**2*f**2 - A*c**2*e**4*f + B*a**2*e*f**4 + 4*B*a*b*d*f**4 - 2*B*a*b*e**2*f**3 - 6*B*a*c*d*e*f**3 + 2*B*a*c*e**3*f**2 - 3*B*b**2*d*e*f**3 + B*b**2*e**3*f**2 - 4*B*b*c*d**2*f**3 + 8*B*b*c*d*e**2*f**2 - 2*B*b*c*e**4*f + 5*B*c**2*d**2*e*f**2 - 5*B*c**2*d*e**3*f + B*c**2*e**5)/(2*f**5*(4*d*f - e**2)) + (2*A*a*b*f**4 - 2*A*a*c*e*f**3 - A*b**2*e*f**3 - 2*A*b*c*d*f**3 + 2*A*b*c*e**2*f**2 + 2*A*c**2*d*e*f**2 - A*c**2*e**3*f + B*a**2*f**4 - 2*B*a*b*e*f**3 - 2*B*a*c*d*f**3 + 2*B*a*c*e**2*f**2 - B*b**2*d*f**3 + B*b**2*e**2*f**2 + 4*B*b*c*d*e*f**2 - 2*B*b*c*e**3*f + B*c**2*d**2*f**2 - 3*B*c**2*d*e**2*f + B*c**2*e**4)/(2*f**5)) * \log(x + (-A*a**2*e*f**4 + 4*A*a*b*d*f**4 - 2*A*a*c*d*e*f**3 - A*b**2*d*e*f**3 - 4*A*b*c*d**2*f**3 + 2*A*b*c*d*e**2*f**2 + 3*A*c**2*d**2*e*f**2 - A*c**2*d*e**3*f + 2*B*a**2*d*f**4 - 2*B*a*b*d*e*f**3 - 4*B*a*c*d**2*f**3 + 2*B*a*c*d*e**2*f**2 - 2*B*b**2*d**2*f**3 + B*b**2*d*e**2*f**2 + 6*B*b*c*d**2*e*f**2 - 2*B*b*c*d*e**3*f + 2*B*c**2*d**3*f**2 - 4*B*c**2*d**2*e**2*f + B*c**2*d*e**4 - 4*d*f**5 * (-\sqrt{-4*d*f + e**2}) * (-2*A*a**2$

$$\begin{aligned}
& f^{**5} + 2*A*a*b*e*f^{**4} + 4*A*a*c*d*f^{**4} - 2*A*a*c*e^{**2}*f^{**3} + 2*A*b^{**2}*d*f^{**4} \\
& - A*b^{**2}*e^{**2}*f^{**3} - 6*A*b*c*d*e*f^{**3} + 2*A*b*c*e^{**3}*f^{**2} - 2*A*c^{**2}*d^{**2} \\
& *f^{**3} + 4*A*c^{**2}*d*e^{**2}*f^{**2} - A*c^{**2}*e^{**4}*f + B*a^{**2}*e*f^{**4} + 4*B*a*b*d*f^{**4} \\
& - 2*B*a*b*e^{**2}*f^{**3} - 6*B*a*c*d*e*f^{**3} + 2*B*a*c*e^{**3}*f^{**2} - 3*B*b^{**2}*d* \\
& e*f^{**3} + B*b^{**2}*e^{**3}*f^{**2} - 4*B*b*c*d^{**2}*f^{**3} + 8*B*b*c*d*e^{**2}*f^{**2} - 2*B*b \\
& *c*e^{**4}*f + 5*B*c^{**2}*d^{**2}*e*f^{**2} - 5*B*c^{**2}*d*e^{**3}*f + B*c^{**2}*e^{**5})/(2*f^{**5} \\
& *(4*d*f - e^{**2})) + (2*A*a*b*f^{**4} - 2*A*a*c*e*f^{**3} - A*b^{**2}*e*f^{**3} - 2*A*b*c \\
& *d*f^{**3} + 2*A*b*c*e^{**2}*f^{**2} + 2*A*c^{**2}*d*e*f^{**2} - A*c^{**2}*e^{**3}*f + B*a^{**2}*f \\
& **4 - 2*B*a*b*e*f^{**3} - 2*B*a*c*d*f^{**3} + 2*B*a*c*e^{**2}*f^{**2} - B*b^{**2}*d*f^{**3} + \\
& B*b^{**2}*e^{**2}*f^{**2} + 4*B*b*c*d*e*f^{**2} - 2*B*b*c*e^{**3}*f + B*c^{**2}*d^{**2}*f^{**2} - 3 \\
& *B*c^{**2}*d*e^{**2}*f + B*c^{**2}*e^{**4})/(2*f^{**5})) + e^{**2}*f^{**4}*(-sqrt(-4*d*f + e^{**2}) \\
& *(-2*A*a^{**2}*f^{**5} + 2*A*a*b*e*f^{**4} + 4*A*a*c*d*f^{**4} - 2*A*a*c*e^{**2}*f^{**3} + 2* \\
& A*b^{**2}*d*f^{**4} - A*b^{**2}*e^{**2}*f^{**3} - 6*A*b*c*d*e*f^{**3} + 2*A*b*c*e^{**3}*f^{**2} - 2 \\
& *A*c^{**2}*d^{**2}*f^{**3} + 4*A*c^{**2}*d*e^{**2}*f^{**2} - A*c^{**2}*e^{**4}*f + B*a^{**2}*e*f^{**4} + \\
& 4*B*a*b*d*f^{**4} - 2*B*a*b*e^{**2}*f^{**3} - 6*B*a*c*d*e*f^{**3} + 2*B*a*c*e^{**3}*f^{**2} - \\
& 3*B*b^{**2}*d*e*f^{**3} + B*b^{**2}*e^{**3}*f^{**2} - 4*B*b*c*d^{**2}*f^{**3} + 8*B*b*c*d*e^{**2}* \\
& f^{**2} - 2*B*b*c*e^{**4}*f + 5*B*c^{**2}*d^{**2}*e*f^{**2} - 5*B*c^{**2}*d*e^{**3}*f + B*c^{**2}*e \\
& **5)/(2*f^{**5}*(4*d*f - e^{**2})) + (2*A*a*b*f^{**4} - 2*A*a*c*e*f^{**3} - A*b^{**2}*e*f \\
& **3 - 2*A*b*c*d*f^{**3} + 2*A*b*c*e^{**2}*f^{**2} + 2*A*c^{**2}*d*e*f^{**2} - A*c^{**2}*e^{**3}*f \\
& + B*a^{**2}*f^{**4} - 2*B*a*b*e*f^{**3} - 2*B*a*c*d*f^{**3} + 2*B*a*c*e^{**2}*f^{**2} - B*b \\
& **2*d*f^{**3} + B*b^{**2}*e^{**2}*f^{**2} + 4*B*b*c*d*e*f^{**2} - 2*B*b*c*e^{**3}*f + B*c^{**2}*d \\
& **2*f^{**2} - 3*B*c^{**2}*d*e^{**2}*f + B*c^{**2}*e^{**4})/(2*f^{**5}))/(-2*A*a^{**2}*f^{**5} + 2* \\
& A*a*b*e*f^{**4} + 4*A*a*c*d*f^{**4} - 2*A*a*c*e^{**2}*f^{**3} + 2*A*b^{**2}*d*f^{**4} - A*b^{**2} \\
& *e^{**2}*f^{**3} - 6*A*b*c*d*e*f^{**3} + 2*A*b*c*e^{**3}*f^{**2} - 2*A*c^{**2}*d^{**2}*f^{**3} + 4 \\
& *A*c^{**2}*d*e^{**2}*f^{**2} - A*c^{**2}*e^{**4}*f + B*a^{**2}*e*f^{**4} + 4*B*a*b*d*f^{**4} - 2*B \\
& a*b*e^{**2}*f^{**3} - 6*B*a*c*d*e*f^{**3} + 2*B*a*c*e^{**3}*f^{**2} - 3*B*b^{**2}*d*e*f^{**3} + \\
& B*b^{**2}*e^{**3}*f^{**2} - 4*B*b*c*d^{**2}*f^{**3} + 8*B*b*c*d*e^{**2}*f^{**2} - 2*B*b*c*e^{**4}*f \\
& + 5*B*c^{**2}*d^{**2}*e*f^{**2} - 5*B*c^{**2}*d*e^{**3}*f + B*c^{**2}*e^{**5})) + (sqrt(-4*d*f \\
& + e^{**2})*(-2*A*a^{**2}*f^{**5} + 2*A*a*b*e*f^{**4} + 4*A*a*c*d*f^{**4} - 2*A*a*c*e^{**2}*f \\
& **3 + 2*A*b^{**2}*d*f^{**4} - A*b^{**2}*e^{**2}*f^{**3} - 6*A*b*c*d*e*f^{**3} + 2*A*b*c*e^{**3}*f \\
& **2 - 2*A*c^{**2}*d^{**2}*f^{**3} + 4*A*c^{**2}*d*e^{**2}*f^{**2} - A*c^{**2}*e^{**4}*f + B*a^{**2}*e \\
& f^{**4} + 4*B*a*b*d*f^{**4} - 2*B*a*b*e^{**2}*f^{**3} - 6*B*a*c*d*e*f^{**3} + 2*B*a*c*e^{**3} \\
& *f^{**2} - 3*B*b^{**2}*d*e*f^{**3} + B*b^{**2}*e^{**3}*f^{**2} - 4*B*b*c*d^{**2}*f^{**3} + 8*B*b*c \\
& d*e^{**2}*f^{**2} - 2*B*b*c*e^{**4}*f + 5*B*c^{**2}*d^{**2}*e*f^{**2} - 5*B*c^{**2}*d*e^{**3}*f + B \\
& *c^{**2}*e^{**5})/(2*f^{**5}*(4*d*f - e^{**2})) + (2*A*a*b*f^{**4} - 2*A*a*c*e*f^{**3} - A*b \\
& **2*e*f^{**3} - 2*A*b*c*d*f^{**3} + 2*A*b*c*e^{**2}*f^{**2} + 2*A*c^{**2}*d*e*f^{**2} - A*c^{**2} \\
& *e^{**3}*f + B*a^{**2}*f^{**4} - 2*B*a*b*e*f^{**3} - 2*B*a*c*d*f^{**3} + 2*B*a*c*e^{**2}*f^{**2} \\
& - B*b^{**2}*d*f^{**3} + B*b^{**2}*e^{**2}*f^{**2} + 4*B*b*c*d*e*f^{**2} - 2*B*b*c*e^{**3}*f + B \\
& *c^{**2}*d^{**2}*f^{**2} - 3*B*c^{**2}*d*e^{**2}*f + B*c^{**2}*e^{**4})/(2*f^{**5}))*log(x + (-A*a \\
& **2*e*f^{**4} + 4*A*a*b*d*f^{**4} - 2*A*a*c*d*e*f^{**3} - A*b^{**2}*d*e*f^{**3} - 4*A*b*c*d \\
& **2*f^{**3} + 2*A*b*c*d*e^{**2}*f^{**2} + 3*A*c^{**2}*d^{**2}*e*f^{**2} - A*c^{**2}*d*e^{**3}*f + 2 \\
& *B*a^{**2}*d*f^{**4} - 2*B*a*b*d*e*f^{**3} - 4*B*a*c*d^{**2}*f^{**3} + 2*B*a*c*d*e^{**2}*f^{**2} \\
& - 2*B*b^{**2}*d^{**2}*f^{**3} + B*b^{**2}*d*e^{**2}*f^{**2} + 6*B*b*c*d^{**2}*e*f^{**2} - 2*B*b*c \\
& d*e^{**3}*f + 2*B*c^{**2}*d^{**3}*f^{**2} - 4*B*c^{**2}*d^{**2}*e^{**2}*f + B*c^{**2}*d*e^{**4} - 4*d \\
& f^{**5}*(sqrt(-4*d*f + e^{**2})*(-2*A*a^{**2}*f^{**5} + 2*A*a*b*e*f^{**4} + 4*A*a*c*d*f^{**4} \\
& - 2*A*a*c*e^{**2}*f^{**3} + 2*A*b^{**2}*d*f^{**4} - A*b^{**2}*e^{**2}*f^{**3} - 6*A*b*c*d*e*f^{**3} \\
& + 2*A*b*c*e^{**3}*f^{**2} - 2*A*c^{**2}*d^{**2}*f^{**3} + 4*A*c^{**2}*d*e^{**2}*f^{**2} - A*c^{**2} \\
& e^{**4}*f + B*a^{**2}*e*f^{**4} + 4*B*a*b*d*f^{**4} - 2*B*a*b*e^{**2}*f^{**3} - 6*B*a*c*d*e*f \\
& **3 + 2*B*a*c*e^{**3}*f^{**2} - 3*B*b^{**2}*d*e*f^{**3} + B*b^{**2}*e^{**3}*f^{**2} - 4*B*b*c*d \\
& **2*f^{**3} + 8*B*b*c*d*e^{**2}*f^{**2} - 2*B*b*c*e^{**4}*f + 5*B*c^{**2}*d^{**2}*e*f^{**2} - 5*B \\
& *c^{**2}*d*e^{**3}*f + B*c^{**2}*e^{**5})/(2*f^{**5}*(4*d*f - e^{**2})) + (2*A*a*b*f^{**4} - 2*A \\
& *a*c*e*f^{**3} - A*b^{**2}*e*f^{**3} - 2*A*b*c*d*f^{**3} + 2*A*b*c*e^{**2}*f^{**2} + 2*A*c^{**2} \\
& *d*e*f^{**2} - A*c^{**2}*e^{**3}*f + B*a^{**2}*f^{**4} - 2*B*a*b*e*f^{**3} - 2*B*a*c*d*f^{**3} + \\
& 2*B*a*c*e^{**2}*f^{**2} - B*b^{**2}*d*f^{**3} + B*b^{**2}*e^{**2}*f^{**2} + 4*B*b*c*d*e*f^{**2} - \\
& 2*B*b*c*e^{**3}*f + B*c^{**2}*d^{**2}*f^{**2} - 3*B*c^{**2}*d*e^{**2}*f + B*c^{**2}*e^{**4})/(2*f^{**5} \\
& 5)) + e^{**2}*f^{**4}*(sqrt(-4*d*f + e^{**2})*(-2*A*a^{**2}*f^{**5} + 2*A*a*b*e*f^{**4} + 4*A \\
& *a*c*d*f^{**4} - 2*A*a*c*e^{**2}*f^{**3} + 2*A*b^{**2}*d*f^{**4} - A*b^{**2}*e^{**2}*f^{**3} - 6*A \\
& *b*c*d*e*f^{**3} + 2*A*b*c*e^{**3}*f^{**2} - 2*A*c^{**2}*d^{**2}*f^{**3} + 4*A*c^{**2}*d*e^{**2}*f^{**2} \\
& - A*c^{**2}*e^{**4}*f + B*a^{**2}*e*f^{**4} + 4*B*a*b*d*f^{**4} - 2*B*a*b*e^{**2}*f^{**3} - 6*
\end{aligned}$$

$$\begin{aligned}
& B*a*c*d*e*f**3 + 2*B*a*c*e**3*f**2 - 3*B*b**2*d*e*f**3 + B*b**2*e**3*f**2 - \\
& 4*B*b*c*d**2*f**3 + 8*B*b*c*d*e**2*f**2 - 2*B*b*c*e**4*f + 5*B*c**2*d**2*e \\
& *f**2 - 5*B*c**2*d*e**3*f + B*c**2*e**5)/(2*f**5*(4*d*f - e**2)) + (2*A*a*b \\
& *f**4 - 2*A*a*c*e*f**3 - A*b**2*e*f**3 - 2*A*b*c*d*f**3 + 2*A*b*c*e**2*f**2 \\
& + 2*A*c**2*d*e*f**2 - A*c**2*e**3*f + B*a**2*f**4 - 2*B*a*b*e*f**3 - 2*B*a \\
& *c*d*f**3 + 2*B*a*c*e**2*f**2 - B*b**2*d*f**3 + B*b**2*e**2*f**2 + 4*B*b*c* \\
& d*e*f**2 - 2*B*b*c*e**3*f + B*c**2*d**2*f**2 - 3*B*c**2*d*e**2*f + B*c**2*e \\
& **4)/(2*f**5)))/(-2*A*a**2*f**5 + 2*A*a*b*e*f**4 + 4*A*a*c*d*f**4 - 2*A*a*c \\
& *e**2*f**3 + 2*A*b**2*d*f**4 - A*b**2*e**2*f**3 - 6*A*b*c*d*e*f**3 + 2*A*b* \\
& c*e**3*f**2 - 2*A*c**2*d**2*f**3 + 4*A*c**2*d*e**2*f**2 - A*c**2*e**4*f + B \\
& *a**2*e*f**4 + 4*B*a*b*d*f**4 - 2*B*a*b*e**2*f**3 - 6*B*a*c*d*e*f**3 + 2*B* \\
& a*c*e**3*f**2 - 3*B*b**2*d*e*f**3 + B*b**2*e**3*f**2 - 4*B*b*c*d**2*f**3 + \\
& 8*B*b*c*d*e**2*f**2 - 2*B*b*c*e**4*f + 5*B*c**2*d**2*e*f**2 - 5*B*c**2*d*e* \\
& *3*f + B*c**2*e**5)
\end{aligned}$$

$$3.15 \quad \int \frac{A+Bx}{(a+bx+cx^2)(d+ex+fx^2)} dx$$

Optimal. Leaf size=406

$$\frac{\tanh^{-1}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)\left(B(aef-2bdf+cde)-A(2af^2-bef-2cdf+ce^2)\right)}{\sqrt{e^2-4df}\left(f(a^2f-abe+b^2d)-c(bde-a(e^2-2df))+c^2d^2\right)} + \frac{\log(a+bx+cx^2)(-aBf+Abf)}{2\left(f(a^2f-abe+b^2d)-c(bde-a(e^2-2df))+c^2d^2\right)}$$

Rubi [A] time = 0.48, antiderivative size = 398, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 5, integrand size = 30, number of rules / integrand size = 0.167, Rules used = {1022, 634, 618, 206, 628}

$$\frac{\tanh^{-1}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)\left(B(aef-2bdf+cde)-A(2af^2-bef-2cdf+ce^2)\right)}{\sqrt{e^2-4df}\left(f(a^2f-abe+b^2d)+ac(e^2-2df)-bcde+c^2d^2\right)} + \frac{\log(a+bx+cx^2)(-aBf+Abf-Ace+Bcd)}{2\left(f(a^2f-abe+b^2d)+ac(e^2-2df)-bcde+c^2d^2\right)} - \frac{\log(d+ex+fx^2)(-aBf+Abf-Ace+Bcd)}{2\left(f(a^2f-abe+b^2d)+ac(e^2-2df)-bcde+c^2d^2\right)} - \frac{\tanh^{-1}\left(\frac{b+2ex}{\sqrt{b^2-4ac}}\right)\left(-b(aBf+Ace+Bcd)+2c(-aAf+aBe+AcD)+AB^2f\right)}{\sqrt{b^2-4ac}\left(f(a^2f-abe+b^2d)+ac(e^2-2df)-bcde+c^2d^2\right)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)),x]

[Out] -(((A*b^2*f + 2*c*(A*c*d + a*B*e - a*A*f) - b*(B*c*d + A*c*e + a*B*f))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))) + ((B*(c*d*e - 2*b*d*f + a*e*f) - A*(c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2))*ArcTanh[(e + 2*f*x)/Sqrt[e^2 - 4*d*f]])/(Sqrt[e^2 - 4*d*f]*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))) + ((B*c*d - A*c*e + A*b*f - a*B*f)*Log[a + b*x + c*x^2])/(2*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))) - ((B*c*d - A*c*e + A*b*f - a*B*f)*Log[d + e*x + f*x^2])/(2*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f)))

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1022

Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)), x_Symbol] := With[{q = Simplify[c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2]}, Dist[1/q, Int[Simp[g*c^2*d - g*b*c*e + a*h*c*e + g*b^2*f - a*b*h*f - a*g*c*f + c*(h*c*d - g*c*e +

$g*b*f - a*h*f)*x, x]/(a + b*x + c*x^2), x], x] + \text{Dist}[1/q, \text{Int}[\text{Simp}[-(h*c*d*e) + g*c*e^2 + b*h*d*f - g*c*d*f - g*b*e*f + a*g*f^2 - f*(h*c*d - g*c*e + g*b*f - a*h*f)*x, x]/(d + e*x + f*x^2), x], x] /; \text{NeQ}[q, 0]] /; \text{FreeQ}[\{a, b, c, d, e, f, g, h\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[e^2 - 4*d*f, 0]$

Rubi steps

$$\int \frac{A + Bx}{(a + bx + cx^2)(d + ex + fx^2)} dx = \int \frac{aB(ce-bf)+A(c^2d+b^2f-c(be+af))+c(Bcd-Ace+Abf-aBf)x}{c^2d^2 - bcde + f(b^2d - abe + a^2f) + ac(e^2 - 2df)} dx + \int \frac{-Af(be-af)+Ac(e^2-d)}{c^2d^2 - bcde + f(b^2d - abe + a^2f) + ac(e^2 - 2df)} dx$$

$$= \frac{(Bcd - Ace + Abf - aBf) \int \frac{b+2cx}{a+bx+cx^2} dx}{2(c^2d^2 - bcde + f(b^2d - abe + a^2f) + ac(e^2 - 2df))} - \frac{(Bcd - Ace + Abf - aBf) \log(a + bx + cx^2)}{2(c^2d^2 - bcde + f(b^2d - abe + a^2f) + ac(e^2 - 2df))} - \frac{(Bcd - Ace + Abf - aBf) \log(d + ex + fx^2)}{2(c^2d^2 - bcde + f(b^2d - abe + a^2f) + ac(e^2 - 2df))}$$

$$= -\frac{(Ab^2f + 2c(Acd + aBe - aAf) - b(Bcd + Ace + aBf)) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2 - 4ac} (c^2d^2 - bcde + f(b^2d - abe + a^2f) + ac(e^2 - 2df))}$$

Mathematica [A] time = 0.46, size = 267, normalized size = 0.66

$$\frac{2 \tan^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right) (-b(aBf + Ace + Bcd) + 2c(-aAf + aBe + Acd) + Ab^2f) - 2 \tan^{-1}\left(\frac{e+2fx}{\sqrt{4f^2-e^2}}\right) (A(-2af^2 + bef + 2cdf - ce^2) + B(af - 2bdf + cde)) + \log(a + x(b + cx))(-aBf + Abf - Ace + Bcd) + \log(d + x(e + fx))(aBf - Abf + Ace - Bcd)}{2(f(a^2f - abe + b^2d) + ac(e^2 - 2df) - bcde + c^2d^2)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)),x]

[Out] ((2*(A*b^2*f + 2*c*(A*c*d + a*B*e - a*A*f) - b*(B*c*d + A*c*e + a*B*f))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - (2*(B*(c*d*e - 2*b*d*f + a*e*f) + A*(-(c*e^2) + 2*c*d*f + b*e*f - 2*a*f^2))*ArcTan[(e + 2*f*x)/Sqrt[-e^2 + 4*d*f]])/Sqrt[-e^2 + 4*d*f] + (B*c*d - A*c*e + A*b*f - a*B*f)*Log[a + x*(b + c*x)] + (- (B*c*d) + A*c*e - A*b*f + a*B*f)*Log[d + x*(e + f*x)]/(2*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f)))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx}{(a + bx + cx^2)(d + ex + fx^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)),x]

[Out] IntegrateAlgebraic[(A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.17, size = 416, normalized size = 1.02

$$\frac{(Bcd - Baf + Abf - Ace) \log(cx^2 + bx + a)}{2(c^2d^2 + b^2df - 2acdf + a^2f^2 - bcde - abfe + ace^2)} - \frac{(Bcd - Baf + Abf - Ace) \log(fx^2 + xe + d)}{2(c^2d^2 + b^2df - 2acdf + a^2f^2 - bcde - abfe + ace^2)} - \frac{(Bbd - 2A^2d + Bbdf - Ab^2f + 2Aacf - 2Bbce + Abce) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(c^2d^2 + b^2df - 2acdf + a^2f^2 - bcde - abfe + ace^2)\sqrt{-b^2+4ac}} + \frac{(2Bbd - 2Aacdf + 2Aaf^2 - Bcde - Bafe - Abfe + Ace^2) \arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)}{(c^2d^2 + b^2df - 2acdf + a^2f^2 - bcde - abfe + ace^2)\sqrt{4df-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d), x, algorithm="giac")

[Out] 1/2*(B*c*d - B*a*f + A*b*f - A*c*e)*log(c*x^2 + b*x + a)/(c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2 - b*c*d*e - a*b*f*e + a*c*e^2) - 1/2*(B*c*d - B*a*f + A*b*f - A*c*e)*log(f*x^2 + x*e + d)/(c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2 - b*c*d*e - a*b*f*e + a*c*e^2) - (B*b*c*d - 2*A*c^2*d + B*a*b*f - A*b^2*f + 2*A*a*c*f - 2*B*a*c*e + A*b*c*e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2 - b*c*d*e - a*b*f*e + a*c*e^2)*sqrt(-b^2 + 4*a*c) + (2*B*b*d*f - 2*A*c*d*f + 2*A*a*f^2 - B*c*d*e - B*a*f*e - A*b*f*e + A*c*e^2)*arctan((2*f*x + e)/sqrt(4*d*f - e^2))/(c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2 - b*c*d*e - a*b*f*e + a*c*e^2)*sqrt(4*d*f - e^2)

maple [B] time = 0.01, size = 1698, normalized size = 4.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d), x)

[Out] 1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)*ln(c*x^2+b*x+a)*A*b*f-1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)*c*ln(c*x^2+b*x+a)*A*e-1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)*ln(c*x^2+b*x+a)*B*a*f+1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)*c*ln(c*x^2+b*x+a)*B*d-2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*A*a*c*f+1/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*A*b^2*f-1/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*A*b*c*e+2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*A*c^2*d-1/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*B*a*b*f+2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*B*a*c*e-1/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*B*b*c*d-1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)*f*ln(f*x^2+e*x+d)*A*b+1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)*f*ln(f*x^2+e*x+d)*A*c*e+1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)*f*ln(f*x^2+e*x+d)*B*a-1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)*ln(f*x^2+e*x+d)*B*c*d+2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*A*a*f^2-1/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*A*b*e*f-2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*A*c*d*f+1/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*A*c*e^2-1/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*B*a*e*f+2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*B*b*d*f-1/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*B*c*d*e

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details)Is 4*d*f-e^2 positive or negative?

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)),x)

[Out] \text{Hanged}

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+e*x+d),x)

[Out] Timed out

$$3.16 \quad \int \frac{A+Bx}{(a+bx+cx^2)^2(d+ex+fx^2)} dx$$

Optimal. Leaf size=1075

$$\frac{Ac(2ace - b(cd + af)) + (Ab - aB)(fb^2 + 2c^2d - c(be + 2af)) + c(Afb^2 - (Bcd + Ace + aBf)b + 2c(Acd + (b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))(cx^2 + bx + a))}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))(cx^2 + bx + a)}$$

Rubi [A] time = 4.18, antiderivative size = 1067, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1016, 1072, 634, 618, 206, 628}

Antiderivative was successfully verified.

[In] Int[(A + B*x)/((a + b*x + c*x^2)^2*(d + e*x + f*x^2)),x]

[Out] -((A*c*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*b^2*f + 2*c*(A*c*d + a*B*e - a*A*f) - b*(B*c*d + A*c*e + a*B*f))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(a + b*x + c*x^2)) - ((b^5*(B*d - A*e)*f^2 - 2*b^4*f*(B*c*d*e - a*A*f^2 - A*c*(e^2 - d*f)) - 4*c^2*(A*(c^3*d^3 - 3*a^3*f^3 - a^2*c*f*(e^2 - 7*d*f)) + a*c^2*d*(3*e^2 - 5*d*f)) - a*B*e*(c^2*d^2 - 3*a^2*f^2 - a*c*(e^2 - 2*d*f))) - 4*b^2*(B*c^3*d^2*e + A*c*f*(2*c^2*d^2 + 3*a^2*f^2 + 3*a*c*(e^2 - d*f))) + 2*b*c*(B*(c^3*d^3 + 3*a^3*f^3 + a*c^2*d*(e^2 - 7*d*f) + 3*a^2*c*f*(e^2 + d*f)) + A*c*e*(3*c^2*d^2 + 3*a^2*f^2 + a*c*(3*e^2 + 2*d*f))) - b^3*(A*c*e*(c*e^2 - 2*c*d*f - 4*a*f^2) + B*(4*a*c*d*f^2 + a^2*f^3 - c^2*d*(e^2 + 5*d*f)))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]/((b^2 - 4*a*c)^(3/2)*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))^2) + ((B*(c^2*d*e*(e^2 - 3*d*f) - 2*c*d*f*(b*e^2 - 2*b*d*f - a*e*f) + f^2*(b^2*d*e - 4*a*b*d*f + a^2*e*f)) - A*(c^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2) - f^2*(2*a*b*e*f - 2*a^2*f^2 - b^2*(e^2 - 2*d*f)) + 2*c*f*(a*f*(e^2 - 2*d*f) - b*(e^3 - 3*d*e*f))))*ArcTanh[(e + 2*f*x)/Sqrt[e^2 - 4*d*f]]/(Sqrt[e^2 - 4*d*f]*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))^2) + ((A*(c*e - b*f)*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) - B*(2*c*d*f*(b*e - a*f) - f^2*(b^2*d - a^2*f) - c^2*d*(e^2 - d*f)))*Log[a + b*x + c*x^2]/(2*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))^2) - ((A*(c*e - b*f)*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) - B*(2*c*d*f*(b*e - a*f) - f^2*(b^2*d - a^2*f) - c^2*d*(e^2 - d*f)))*Log[d + e*x + f*x^2]/(2*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))^2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d},

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1016

Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*c*d - 2*a*c*e + a*b*f))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1])

Rule 1072

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] := With[{q = c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

Rubi steps

$$\int \frac{A + Bx}{(a + bx + cx^2)^2 (d + ex + fx^2)} dx = -\frac{Ac(2ace - b(cd + af)) + (Ab - aB)(2c^2d + b^2f - c(be + 2af)) + c}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - ad))} + \dots$$

Mathematica [A] time = 6.70, size = 952, normalized size = 0.89

Antiderivative was successfully verified.

[In] Integrate[(A + B*x)/((a + b*x + c*x^2)^2*(d + e*x + f*x^2)),x]

[Out] $((-2*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))*(A*(b^3*f + b^2*c*(-e + f*x) + b*c*(-3*a*f + c*(d - e*x)) + 2*c^2*(c*d*x + a*(e - f*x))) + B*(2*a^2*c*f - b*c^2*d*x - a*(b^2*f + 2*c^2*(d - e*x) + b*c*(-e + f*x)))))/((b^2 - 4*a*c)*(a + x*(b + c*x))) - (2*(b^5*(B*d - A*e)*f^2 + 2*b^4*f*(-(B*c*d*e) + a*A*f^2 + A*c*(e^2 - d*f)) - 4*b^2*(B*c^3*d^2*e + A*c*f*(2*c^2*d^2 + 3*a^2*f^2 + 3*a*c*(e^2 - d*f))) + 2*b*c*(B*(c^3*d^3 + 3*a^3*f^3 + a*c^2*d*(e^2 - 7*d*f) + 3*a^2*c*f*(e^2 + d*f)) + A*c*e*(3*c^2*d^2 + 3*a^2*f^2 + a*c*(3*e^2 + 2*d*f))) + 4*c^2*(a*B*e*(c^2*d^2 - 3*a^2*f^2 - a*c*(e^2 - 2*d*f)) + A*(-(c^3*d^3) + 3*a^3*f^3 + a^2*c*f*(e^2 - 7*d*f) + a*c^2*d*(-3*e^2 + 5*d*f))) + b^3*(A*c*e*(-(c*e^2) + 2*c*d*f + 4*a*f^2) + B*(-4*a*c*d*f^2 - a^2*f^3 + c^2*d*(e^2 + 5*d*f)))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]]/(-b^2 + 4*a*c)^(3/2) + (2*(B*(c^2*d*e*(-e^2 + 3*d*f)) - 2*c*d*f*(-(b*e^2) + 2*b*d*f + a*e*f) + f^2*(-(b^2*d*e) + 4*a*b*d*f - a^2*e*f)) + A*(c^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2) + f^2*(-2*a*b*e*f + 2*a^2*f^2 + b^2*(e^2 - 2*d*f)) + 2*c*f*(a*f*(e^2 - 2*d*f) - b*(e^3 - 3*d*e*f)))*ArcTan[(e + 2*f*x)/Sqrt[-e^2 + 4*d*f]]/Sqrt[-e^2 + 4*d*f] - (A*(c*e - b*f)*(f*(-(b*e) + 2*a*f) + c*(e^2 - 2*d*f)) + B*(2*c*d*f*(b*e - a*f) + f^2*(-(b^2*d) + a^2*f) + c^2*d*(-e^2 + d*f))*Log[a + x*(b + c*x)] + (A*(c*e - b*f)*(f*(-(b*e) + 2*a*f) + c*(e^2 - 2*d*f)) + B*(2*c*d*f*(b*e - a*f) + f^2*(-(b^2*d) + a^2*f) + c^2*d*(-e^2 + d*f))*Log[d + x*(e + f*x)])/(2*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))^2)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx}{(a + bx + cx^2)^2 (d + ex + fx^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x)/((a + b*x + c*x^2)^2*(d + e*x + f*x^2)),x]

[Out] IntegrateAlgebraic[(A + B*x)/((a + b*x + c*x^2)^2*(d + e*x + f*x^2)), x]
fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [B] time = 0.45, size = 3226, normalized size = 3.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="giac")

[Out]
$$-1/2*(B*c^2*d^2*f - B*b^2*d*f^2 - 2*B*a*c*d*f^2 + 2*A*b*c*d*f^2 + B*a^2*f^3 - 2*A*a*b*f^3 + 2*B*b*c*d*f*e - 2*A*c^2*d*f*e + A*b^2*f^2*e + 2*A*a*c*f^2*e - B*c^2*d*e^2 - 2*A*b*c*f*e^2 + A*c^2*e^3)*\log(c*x^2 + b*x + a)/(c^4*d^4 + 2*b^2*c^2*d^3*f - 4*a*c^3*d^3*f + b^4*d^2*f^2 - 4*a*b^2*c*d^2*f^2 + 6*a^2*c^2*d^2*f^2 + 2*a^2*b^2*d*f^3 - 4*a^3*c*d*f^3 + a^4*f^4 - 2*b*c^3*d^3*e - 2*b^3*c*d^2*f*e + 2*a*b*c^2*d^2*f*e - 2*a*b^3*d*f^2*e + 2*a^2*b*c*d*f^2*e - 2*a^3*b*f^3*e + b^2*c^2*d^2*e^2 + 2*a*c^3*d^2*e^2 + 4*a*b^2*c*d*f*e^2 - 4*a^2*c^2*d*f*e^2 + a^2*b^2*f^2*e^2 + 2*a^3*c*f^2*e^2 - 2*a*b*c^2*d*e^3 - 2*a^2*b*c*f*e^3 + a^2*c^2*e^4) + 1/2*(B*c^2*d^2*f - B*b^2*d*f^2 - 2*B*a*c*d*f^2 + 2*A*b*c*d*f^2 + B*a^2*f^3 - 2*A*a*b*f^3 + 2*B*b*c*d*f*e - 2*A*c^2*d*f*e + A*b^2*f^2*e + 2*A*a*c*f^2*e - B*c^2*d*e^2 - 2*A*b*c*f*e^2 + A*c^2*e^3)*\log(f*x^2 + x*e + d)/(c^4*d^4 + 2*b^2*c^2*d^3*f - 4*a*c^3*d^3*f + b^4*d^2*f^2 - 4*a*b^2*c*d^2*f^2 + 6*a^2*c^2*d^2*f^2 + 2*a^2*b^2*d*f^3 - 4*a^3*c*d*f^3 + a^4*f^4 - 2*b*c^3*d^3*e - 2*b^3*c*d^2*f*e + 2*a*b*c^2*d^2*f*e - 2*a*b^3*d*f^2*e + 2*a^2*b*c*d*f^2*e - 2*a^3*b*f^3*e + b^2*c^2*d^2*e^2 + 2*a*c^3*d^2*e^2 + 4*a*b^2*c*d*f*e^2 - 4*a^2*c^2*d*f*e^2 + a^2*b^2*f^2*e^2 + 2*a^3*c*f^2*e^2 - 2*a*b*c^2*d*e^3 - 2*a^2*b*c*f*e^3 + a^2*c^2*e^4) + (2*B*b*c^4*d^3 - 4*A*c^5*d^3 + 5*B*b^3*c^2*d^2*f - 14*B*a*b*c^3*d^2*f - 8*A*b^2*c^3*d^2*f + 20*A*a*c^4*d^2*f + B*b^5*d*f^2 - 4*B*a*b^3*c*d*f^2 - 2*A*b^4*c*d*f^2 + 6*B*a^2*b*c^2*d*f^2 + 12*A*a*b^2*c^2*d*f^2 - 28*A*a^2*c^3*d*f^2 - B*a^2*b^3*f^3 + 2*A*a*b^4*f^3 + 6*B*a^3*b*c*f^3 - 12*A*a^2*b^2*c*f^3 + 12*A*a^3*c^2*f^3 - 4*B*b^2*c^3*d^2*e + 4*B*a*c^4*d^2*e + 6*A*b*c^4*d^2*e - 2*B*b^4*c*d*f*e + 2*A*b^3*c^2*d*f*e + 8*B*a^2*c^3*d*f*e + 4*A*a*b*c^3*d*f*e - A*b^5*f^2*e + 4*A*a*b^3*c*f^2*e - 12*B*a^3*c^2*f^2*e + 6*A*a^2*b*c^2*f^2*e + B*b^3*c^2*d*e^2 + 2*B*a*b*c^3*d*e^2 - 12*A*a*c^4*d*e^2 + 2*A*b^4*c*f*e^2 + 6*B*a^2*b*c^2*f*e^2 - 12*A*a*b^2*c^2*f*e^2 + 4*A*a^2*c^3*f*e^2 - A*b^3*c^2*e^3 - 4*B*a^2*c^3*e^3 + 6*A*a*b*c^3*e^3)*\arctan((2*c*x + b)/\sqrt{-b^2 + 4*a*c})/((b^2*c^4*d^4 - 4*a*c^5*d^4 + 2*b^4*c^2*d^3*f - 12*a*b^2*c^3*d^3*f + 16*a^2*c^4*d^3*f + b^6*d^2*f^2 - 8*a*b^4*c*d^2*f^2 + 22*a^2*b^2*c^2*d^2*f^2 - 24*a^3*c^3*d^2*f^2 + 2*a^2*b^4*d*f^3 - 12*a^3*b^2*c*d*f^3 + 16*a^4*c^2*d*f^3 + a^4*b^2*f^4 - 4*a^5*c*f^4 - 2*b^3*c^3*d^3*e + 8*a*b*c^4*d^3*e - 2*b^5*c*d^2*f*e + 10*a*b^3*c^2*d^2*f*e - 8*a^2*b*c^3*d^2*f*e - 2*a*b^5*d*f^2*e + 10*a^2*b^3*c*d*f^2*e - 8*a^3*b*c^2*d*f^2*e - 2*a^3*b^3*f^3*e + 8*a^4*b*c*f^3*e + b^4*c^2*d^2*e^2 - 2*a*b^2*c^3*d^2*e^2 - 8*a^2*c^4*d^2*e^2 + 4*a*b^4*c*d*f*e^2 - 20*a^2*b^2*c^2*d*f*e^2 + 16*a^3*c^3*d*f*e^2 + a^2*b^4*f^2*e^2 - 2*a^3*b^2*c*f^2*e^2 - 8*a^4*c^2*f^2*e^2 - 2*a*b^3*c^2*d*e^3 + 8*a^2*b*c^3*d*e^3 - 2*a^2*b^3*c*f*e^3 + 8*a^3*b*c^2*f*e^3 + a^2*b^2*c^2*e^4 - 4*a^3*c^3*e^4)*\sqrt{-b^2 + 4*a*c}) - (4*B*b*c*d^2*f^2 - 2*A*c^2*d^2*f^2 - 4*B*a*b*d*f^3 + 2*A*b^2*d*f^3 + 4*A*a*c*d*f^3 - 2*A*a^2*f^4 - 3*B*c^2*d^2*f*e + B*b^2*d*f^2*e + 2*B*a*c*d*f^2*e - 6*A*b*c*d*f^2*e + B*a^2*f^3*e + 2*A*a*b*f^3*e - 2*B*b*c*d*f*e^2 + 4*A*c^2*d*f*e^2 - A*b^2*f^2*e^2 - 2*A*a*c*f^2*e^2 + B*c^2*d*e^3 + 2*A*b*c*f*e^3 - A*c^2*e^4)*\arctan((2*f*x + e)/\sqrt{4*d*f - e^2})/((c^4*d^4 + 2*b^2*c^2*d^3*f - 4*a*c^3*d^3*f + b^4*d^2*f^2 - 4*a*b^2*c*d^2*f^2 + 6*a^2$$

```
*c^2*d^2*f^2 + 2*a^2*b^2*d*f^3 - 4*a^3*c*d*f^3 + a^4*f^4 - 2*b*c^3*d^3*e -
2*b^3*c*d^2*f*e + 2*a*b*c^2*d^2*f*e - 2*a*b^3*d*f^2*e + 2*a^2*b*c*d*f^2*e -
2*a^3*b*f^3*e + b^2*c^2*d^2*e^2 + 2*a*c^3*d^2*e^2 + 4*a*b^2*c*d*f*e^2 - 4*
a^2*c^2*d*f*e^2 + a^2*b^2*f^2*e^2 + 2*a^3*c*f^2*e^2 - 2*a*b*c^2*d*e^3 - 2*a
^2*b*c*f*e^3 + a^2*c^2*e^4)*sqrt(4*d*f - e^2)) + (2*B*a*c^4*d^3 - A*b*c^4*d
^3 + 3*B*a*b^2*c^2*d^2*f - 2*A*b^3*c^2*d^2*f - 6*B*a^2*c^3*d^2*f + 5*A*a*b*
c^3*d^2*f + B*a*b^4*d*f^2 - A*b^5*d*f^2 - 4*B*a^2*b^2*c*d*f^2 + 5*A*a*b^3*c
*d*f^2 + 6*B*a^3*c^2*d*f^2 - 7*A*a^2*b*c^2*d*f^2 + B*a^3*b^2*f^3 - A*a^2*b^
3*f^3 - 2*B*a^4*c*f^3 + 3*A*a^3*b*c*f^3 - 3*B*a*b*c^3*d^2*e + 2*A*b^2*c^3*d
^2*e - 2*A*a*c^4*d^2*e - 2*B*a*b^3*c*d*f*e + 2*A*b^4*c*d*f*e + 2*B*a^2*b*c^
2*d*f*e - 6*A*a*b^2*c^2*d*f*e + 4*A*a^2*c^3*d*f*e - B*a^2*b^3*f^2*e + A*a*b
^4*f^2*e + B*a^3*b*c*f^2*e - 2*A*a^2*b^2*c*f^2*e - 2*A*a^3*c^2*f^2*e + B*a*
b^2*c^2*d*e^2 - A*b^3*c^2*d*e^2 + 2*B*a^2*c^3*d*e^2 + A*a*b*c^3*d*e^2 + 2*B
*a^2*b^2*c*f*e^2 - 2*A*a*b^3*c*f*e^2 - 2*B*a^3*c^2*f*e^2 + 5*A*a^2*b*c^2*f*
e^2 - B*a^2*b*c^2*e^3 + A*a*b^2*c^2*e^3 - 2*A*a^2*c^3*e^3 + (B*b*c^4*d^3 -
2*A*c^5*d^3 + B*b^3*c^2*d^2*f - B*a*b*c^3*d^2*f - 3*A*b^2*c^3*d^2*f + 6*A*a
*c^4*d^2*f + B*a*b^3*c*d*f^2 - A*b^4*c*d*f^2 - B*a^2*b*c^2*d*f^2 + 4*A*a*b^
2*c^2*d*f^2 - 6*A*a^2*c^3*d*f^2 + B*a^3*b*c*f^3 - A*a^2*b^2*c*f^3 + 2*A*a^3
*c^2*f^3 - B*b^2*c^3*d^2*e - 2*B*a*c^4*d^2*e + 3*A*b*c^4*d^2*e - 4*B*a*b^2*
c^2*d*f*e + 2*A*b^3*c^2*d*f*e + 4*B*a^2*c^3*d*f*e - 2*A*a*b*c^3*d*f*e - B*a
^2*b^2*c*f^2*e + A*a*b^3*c*f^2*e - 2*B*a^3*c^2*f^2*e - A*a^2*b*c^2*f^2*e +
3*B*a*b*c^3*d*e^2 - A*b^2*c^3*d*e^2 - 2*A*a*c^4*d*e^2 + 3*B*a^2*b*c^2*f*e^2
- 2*A*a*b^2*c^2*f*e^2 + 2*A*a^2*c^3*f*e^2 - 2*B*a^2*c^3*e^3 + A*a*b*c^3*e^
3)*x)/((c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2 - b*c*d*e - a*b*f*e + a*c*e
^2)^2*(c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

maple [B] time = 0.05, size = 51470, normalized size = 47.88

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive or negative?
```

mupad [B] time = 30.31, size = 118429, normalized size = 110.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/((a + b*x + c*x^2)^2*(d + e*x + f*x^2)),x)
```

```
[Out] symsum(log((x*(4*A^3*b^3*c^4*f^6 + 16*B^3*a^3*c^4*f^6 - 3*B^3*a^2*b^2*c^3*f
^6 + 4*B^3*a^2*c^5*e^2*f^4 + B^3*b^2*c^5*d^2*f^4 - 16*A^3*a*b*c^5*f^6 + 16*
A^3*a*c^6*e*f^5 + 20*A^2*B*a^2*c^5*f^6 - 3*A^2*B*b^4*c^3*f^6 + 4*A^2*B*c^7*
d^2*f^4 - 16*B^3*a^2*c^5*d*f^5 - 4*A^3*b^2*c^5*e*f^5 + 6*B^3*a*b^2*c^4*d*f^
5 - 4*B^3*a^2*b*c^4*e*f^5 + A^2*B*b^2*c^5*e^2*f^4 - 24*A^2*B*a*c^6*d*f^5 +
6*A*B^2*a*b^3*c^3*f^6 - 28*A*B^2*a^2*b*c^4*f^6 + 8*A^2*B*a*b^2*c^4*f^6 - 4*
```

$$\begin{aligned}
& A^2B^2b^2c^6d^2f^4 + 8A^2B^2a^2c^5ef^5 - 6A^2B^2b^3c^4d^2f^5 + 8A^2 \\
& *B^2b^2c^5d^2f^5 + 2A^2B^2b^3c^4ef^5 - 4B^3a^2b^2c^5d^2ef^4 - 4A^2B^2 \\
& a^2b^2c^5ef^4 + 2A^2B^2a^2b^2c^4ef^5 + 2A^2B^2b^2c^5d^2ef^4 + 16A^2 \\
& B^2a^2b^2c^5d^2f^5 - 12A^2B^2a^2b^2c^5ef^5 + 8A^2B^2a^2c^6d^2ef^4 - 4A^2 \\
& B^2b^2c^6d^2ef^4) / (16a^2c^6d^4 + a^4b^4f^4 + 16a^4c^4e^4 + b^4c^4d^4 + 16a^6c^2f^4 + b^8d^2f^2 - 8a^2b^2c^5d^4 - 8a^5b^2c^2f^4 + 2 \\
& a^2b^6d^2f^3 - 2a^3b^5ef^3 - 64a^3c^5d^3f - 64a^5c^3d^2f^3 - 2b^5c^3d^3e + 2b^6c^2d^3f + a^2b^4c^2e^4 - 8a^3b^2c^3e^4 + 32a^3c^5d^2e^2 + a^2b^6e^2f^2 + 96a^4c^4d^2f^2 + b^6c^2d^2e^2 + 3 \\
& 2a^5c^3e^2f^2 - 2a^2b^7d^2ef^2 - 2b^7c^2d^2ef + 54a^2b^4c^2d^2f^2 - 112a^3b^2c^3d^2f^2 + 16a^2b^3c^4d^3e - 2a^2b^5c^2d^2e^3 - 32 \\
& a^2b^2c^5d^3e - 32a^3b^2c^4d^2e^3 - 20a^2b^4c^3d^3f - 12a^2b^6c^2d^2 \\
& f^2 - 20a^3b^4c^2d^2f^3 - 2a^2b^5c^2e^3f - 32a^4b^2c^3e^3f + 16a^4 \\
& b^3c^2ef^3 - 32a^5b^2c^2ef^3 - 64a^4c^4d^2ef^2 - 6a^2b^4c^3d^2e^2 + 16a^2b^3c^3d^2e^3 + 64a^2b^2c^4d^3f + 64a^4b^2c^2d^2f^3 + 16 \\
& a^3b^3c^2e^3f - 6a^3b^4c^2ef^2 - 48a^2b^3c^3d^2ef - 36a^2b^4c^2d^2ef + 96a^3b^2c^3d^2ef - 48a^3b^3c^2d^2ef^2 + 4a^2b^6 \\
& c^2d^2ef + 18a^2b^5c^2d^2ef + 18a^2b^5c^2d^2ef^2 + 32a^3b^2c^4d^2 \\
& ef + 32a^4b^2c^3d^2ef^2) - \text{root}(48416a^6b^2c^6d^4e^2f^4z^4 - 415 \\
& 44a^5b^4c^5d^4e^2f^4z^4 - 31872a^7b^2c^5d^3e^2f^5z^4 - 31872a^5b^2c^7d^5e^2f^3z^4 - 29184a^6b^2c^6d^3e^4f^3z^4 + 28800a^5 \\
& b^4c^5d^3e^4f^3z^4 + 21510a^4b^6c^4d^4e^2f^4z^4 + 21408a^6b^4c^4d^3e^2f^5z^4 + 21408a^4b^4c^6d^5e^2f^3z^4 - 18112a^7b^3c^4d^2e^3f^5z^4 - 18112a^4b^3c^7d^5e^3f^2z^4 - 15600a^5b^5c^4d^3e^3f^4z^4 - 15600a^4b^5c^5d^4e^3f^3z^4 + 15296a^6b^3c^5d^3e^3f^4z^4 + 15296a^5b^3c^6d^4e^3f^3z^4 + 14016a^7b^2c^5d^2e^4f^4z^4 + 14016a^5b^2c^7d^4e^4f^2z^4 - 13920a^4b^6c^4d^3e^4f^3z^4 - 11648a^6b^3c^5d^2e^5f^3z^4 - 11648a^5b^3c^6d^3e^5f^2z^4 + 10432a^6b^2c^6d^2e^6f^2z^4 + 9008a^6b^5c^3d^2e^3f^5z^4 + 9008a^3b^5c^6d^5e^3f^2z^4 + 8544a^5b^5c^4d^2e^5f^3z^4 + 8544a^4b^5c^5d^3e^5f^2z^4 - 8496a^5b^4c^5d^2e^6f^2z^4 + 7488a^8b^2c^4d^2e^2f^6z^4 + 7488a^4b^2c^8d^6e^2f^2z^4 + 7380a^4b^7c^3d^3e^3f^4z^4 + 7380a^3b^7c^4d^4e^3f^3z^4 - 6720a^3b^8c^3d^4e^2f^4z^4 - 5784a^5b^6c^3d^3e^2f^5z^4 - 5784a^3b^6c^5d^5e^2f^3z^4 - 3440a^6b^4c^4d^2e^4f^4z^4 - 3440a^4b^4c^6d^4e^4f^2z^4 + 3360a^3b^8c^3d^3e^4f^3z^4 + 3140a^4b^6c^4d^2e^6f^2z^4 - 2760a^4b^7c^3d^2e^5f^3z^4 - 2760a^3b^7c^4d^3e^5f^2z^4 - 1764a^5b^7c^2d^2e^3f^5z^4 - 1764a^2b^7c^5d^5e^3f^2z^4 - 1640a^3b^9c^2d^3e^3f^4z^4 - 1640a^2b^9c^3d^4e^3f^3z^4 - 1604a^6b^6c^2d^2e^2f^6z^4 - 1604a^2b^6c^6d^6e^2f^2z^4 - 1500a^5b^6c^3d^2e^4f^4z^4 - 1500a^3b^6c^5d^4e^4f^2z^4 + 1140a^2b^10c^2d^4e^2f^4z^4 + 810a^4b^8c^2d^2e^4f^4z^4 + 810a^2b^8c^4d^4e^4f^2z^4 - 544a^3b^8c^3d^2e^6f^2z^4 + 416a^3b^9c^2d^2e^5f^3z^4 + 416a^2b^9c^3d^3e^5f^2z^4 - 384a^2b^10c^2d^3e^4f^3z^4 + 180a^4b^8c^2d^3e^2f^5z^4 + 180a^2b^8c^4d^5e^2f^3z^4 + 48a^7b^4c^3d^2e^2f^6z^4 + 48a^3b^4c^7d^6e^2f^2z^4 + 36a^2b^10c^2d^2e^6f^2z^4 - 1024a^10b^2c^3d^2ef^8z^4 - 1024a^3b^2c^10d^8ef^8z^4 - 192a^8b^5c^2d^2ef^8z^4 - 192a^2b^5c^8d^8ef^8z^4 + 16128a^7b^3c^4d^3ef^6z^4 + 16128a^4b^3c^7d^6ef^3z^4 - 11712a^6b^5c^3d^3ef^6z^4 - 11712a^3b^5c^6d^6ef^3z^4 + 11520a^8b^2c^5d^2e^3f^5z^4 + 11520a^5b^2c^8d^5e^3f^2z^4 - 9984a^6b^3c^5d^4ef^5z^4 - 9984a^5b^3c^6d^5ef^4z^4 + 8640a^5b^5c^4d^4ef^5z^4 + 8640a^4b^5c^5d^5ef^4z^4 - 7424a^7b^2c^6d^3e^3f^4z^4 - 7424a^6b^2c^7d^4e^3f^3z^4 - 6912a^8b^3c^3d^2ef^7z^4 - 6912a^3b^3c^8d^7ef^2z^4 + 4800a^7b^3c^4d^2ef^5z^4 + 4800a^4b^3c^7d^4ef^5z^4 + 4608a^7b^2c^6d^2e^5f^3z^4 + 4608a^6b^2c^7d^3e^5f^2z^4 - 4560a^4b^7c^3d^4ef^5z^4 - 4560a^3b^7c^4d^5ef^4z^4 + 4176a^5b^7c^2d^3ef^6z^4 + 4176a^2b^7c^5d^6ef^3z^4 + 3264a^7b^5c^2d^2ef^7z^4 + 3264a^2b^5c^7d^7ef^2z^4 + 3008a^8b^3c^3d^2e^3f^6z^4 + 3008a^3b^3c^8
\end{aligned}$$

$$\begin{aligned}
& d^6 e^3 f^2 z^4 + 2880 a^6 b^3 c^5 d e^7 f^2 z^4 + 2880 a^5 b^3 c^6 d^2 e^7 f^2 z^4 - 2240 a^7 b^4 c^3 d e^4 f^5 z^4 - 2240 a^3 b^4 c^7 d^5 e^4 f^2 z^4 - 1488 a^5 b^5 c^4 d e^7 f^2 z^4 - 1488 a^4 b^5 c^5 d^2 e^7 f^2 z^4 + 1440 a^3 b^9 c^2 d^4 e f^5 z^4 + 1440 a^2 b^9 c^3 d^5 e f^4 z^4 - 1328 a^6 b^5 c^3 d e^5 f^4 z^4 - 1328 a^3 b^5 c^6 d^4 e^5 f^2 z^4 - 1152 a^7 b^2 c^5 d e^6 f^3 z^4 - 1152 a^5 b^2 c^7 d^3 e^6 f^2 z^4 - 1120 a^6 b^4 c^4 d e^6 f^3 z^4 - 1120 a^4 b^4 c^6 d^3 e^6 f^2 z^4 + 912 a^6 b^6 c^2 d e^4 f^5 z^4 + 912 a^2 b^6 c^6 d^5 e^4 f^2 z^4 + 872 a^5 b^6 c^3 d e^6 f^3 z^4 + 872 a^3 b^6 c^5 d^3 e^6 f^2 z^4 + 768 a^8 b^2 c^4 d e^4 f^5 z^4 + 768 a^4 b^2 c^8 d^5 e^4 f^2 z^4 - 672 a^8 b^4 c^2 d e^2 f^7 z^4 - 672 a^2 b^4 c^8 d^7 e^2 f^2 z^4 - 624 a^7 b^5 c^2 d e^3 f^6 z^4 - 624 a^2 b^5 c^7 d^6 e^3 f^2 z^4 + 480 a^5 b^8 c^2 d e^2 f^6 z^4 + 480 a^3 b^8 c^5 d^6 e^2 f^2 z^4 + 316 a^4 b^7 c^3 d e^7 f^2 z^4 + 316 a^3 b^7 c^4 d^2 e^7 f^2 z^4 - 204 a^4 b^8 c^2 d e^6 f^3 z^4 - 204 a^2 b^8 c^4 d^3 e^6 f^2 z^4 + 168 a^3 b^10 c^3 d^5 e^2 f^3 z^4 + 168 a^3 b^10 c^3 d^5 e^2 f^3 z^4 + 156 a^2 b^11 c^2 d^4 e^3 f^3 z^4 + 156 a^2 b^11 c^2 d^4 e^3 f^3 z^4 + 128 a^9 b^2 c^3 d e^2 f^7 z^4 + 128 a^3 b^2 c^9 d^7 e^2 f^2 z^4 - 124 a^3 b^10 c^2 d^2 e^4 f^4 z^4 - 124 a^3 b^10 c^3 d^4 e^4 f^2 z^4 + 100 a^4 b^9 c^2 d e^3 f^5 z^4 + 100 a^4 b^9 c^4 d^5 e^3 f^2 z^4 + 36 a^5 b^7 c^2 d e^5 f^4 z^4 + 36 a^2 b^7 c^5 d^4 e^5 f^2 z^4 - 24 a^3 b^9 c^2 d e^7 f^2 z^4 - 24 a^2 b^11 c^2 d^2 e^5 f^3 z^4 - 24 a^2 b^9 c^3 d^2 e^7 f^2 z^4 - 24 a^2 b^11 c^2 d^3 e^5 f^2 z^4 - 9216 a^8 b^3 c^5 d^3 e f^6 z^4 - 9216 a^5 b^3 c^8 d^6 e f^3 z^4 - 5376 a^8 b^3 c^5 d e^5 f^4 z^4 - 5376 a^5 b^3 c^8 d^4 e^5 f^2 z^4 + 5120 a^9 b^3 c^4 d^2 e f^7 z^4 + 5120 a^7 b^3 c^6 d^4 e f^5 z^4 + 5120 a^6 b^3 c^7 d^5 e f^4 z^4 + 5120 a^4 b^3 c^9 d^7 e f^2 z^4 - 4352 a^9 b^3 c^4 d e^3 f^6 z^4 - 4352 a^4 b^3 c^9 d^6 e^3 f^2 z^4 - 1792 a^7 b^3 c^6 d e^7 f^2 z^4 - 1792 a^6 b^3 c^7 d^2 e^7 f^2 z^4 - 1600 a^6 b^2 c^6 d e^8 f^2 z^4 + 912 a^5 b^4 c^5 d e^8 f^2 z^4 + 768 a^9 b^3 c^2 d e^2 f^8 z^4 + 768 a^2 b^3 c^9 d^8 e f^2 z^4 - 720 a^4 b^9 c^3 d e^3 f^6 z^4 - 720 a^2 b^9 c^4 d^6 e f^3 z^4 - 656 a^6 b^7 c^2 d e^2 f^7 z^4 - 656 a^2 b^7 c^6 d^7 e f^2 z^4 - 240 a^2 b^11 c^2 d^5 e f^4 z^4 - 240 a^2 b^11 c^2 d^5 e f^4 z^4 + 216 a^7 b^6 c^2 d e^2 f^7 z^4 + 216 a^2 b^6 c^7 d^7 e^2 f^2 z^4 - 204 a^4 b^6 c^4 d e^8 f^2 z^4 - 144 a^5 b^8 c^2 d e^4 f^5 z^4 - 144 a^2 b^8 c^5 d^5 e^4 f^2 z^4 - 84 a^2 b^12 c^2 d^4 e^2 f^4 z^4 + 36 a^4 b^9 c^2 d e^5 f^4 z^4 + 36 a^2 b^9 c^4 d^4 e^5 f^2 z^4 + 20 a^6 b^7 c^2 d e^3 f^6 z^4 + 20 a^2 b^7 c^6 d^6 e^3 f^2 z^4 + 16 a^3 b^10 c^2 d e^6 f^3 z^4 + 16 a^3 b^8 c^3 d e^8 f^2 z^4 + 16 a^2 b^12 c^2 d^3 e^4 f^3 z^4 + 16 a^2 b^10 c^3 d^3 e^6 f^2 z^4 + 48 b^11 c^3 d^6 e f^3 z^4 + 48 b^9 c^5 d^7 e f^2 z^4 - 20 b^8 c^6 d^7 e^2 f^2 z^4 + 8 b^10 c^4 d^5 e^4 f^2 z^4 - 4 b^13 c^2 d^4 e^3 f^3 z^4 - 4 b^11 c^3 d^4 e^5 f^2 z^4 + 4 b^9 c^5 d^6 e^3 f^2 z^4 + 3072 a^9 c^5 d e^4 f^5 z^4 + 3072 a^5 c^9 d^5 e^4 f^2 z^4 + 2560 a^8 c^6 d e^6 f^3 z^4 + 2560 a^6 c^8 d^3 e^6 f^2 z^4 + 1536 a^10 c^4 d e^2 f^7 z^4 + 1536 a^4 c^10 d^7 e^2 f^2 z^4 + 48 a^5 b^9 d^2 e f^7 z^4 + 48 a^3 b^11 d^3 e f^6 z^4 - 20 a^6 b^8 d e^2 f^7 z^4 + 8 a^4 b^10 d e^4 f^5 z^4 + 4 a^5 b^9 d e^3 f^6 z^4 - 4 a^3 b^11 d e^5 f^4 z^4 - 4 a^2 b^13 d^3 e^3 f^4 z^4 + 768 a^9 b^3 c^4 e^5 f^5 z^4 + 768 a^8 b^3 c^5 e^7 f^3 z^4 + 256 a^10 b^3 c^3 e^3 f^7 z^4 - 192 a^6 b^3 c^5 e^9 f^2 z^4 - 68 a^7 b^6 c^4 e^4 f^6 z^4 + 48 a^8 b^5 c^3 e^3 f^7 z^4 + 48 a^5 b^5 c^4 e^9 f^2 z^4 + 36 a^6 b^7 c^4 e^5 f^5 z^4 - 12 a^9 b^4 c^3 e^2 f^8 z^4 - 4 a^4 b^9 c^3 e^7 f^3 z^4 - 4 a^4 b^7 c^3 e^9 f^2 z^4 + 384 a^5 b^8 c^2 d^3 f^7 z^4 + 384 a^2 b^8 c^5 d^7 f^3 z^4 + 288 a^3 b^10 c^2 d^4 f^6 z^4 + 288 a^2 b^10 c^3 d^6 f^4 z^4 + 224 a^7 b^6 c^2 d^2 f^8 z^4 + 224 a^2 b^6 c^7 d^8 f^2 z^4 - 192 a^10 b^2 c^2 d^2 f^9 z^4 - 192 a^2 b^2 c^10 d^9 f^2 z^4 + 768 a^5 b^3 c^8 d^3 e^7 z^4 + 768 a^4 b^3 c^9 d^5 e^5 z^4 + 256 a^3 b^3 c^10 d^7 e^3 z^4 - 192 a^5 b^3 c^6 d e^9 z^4 - 68 a^2 b^6 c^7 d^6 e^4 z^4 + 48 a^4 b^5 c^5 d e^9 z^4 + 48 a^2 b^5 c^8 d^7 e^3 z^4 + 36 a^2 b^7 c^6 d^5 e^5 z^4 - 12 a^2 b^4 c^9 d^8 e^2 z^4 - 4 a^3 b^7 c^4 d e^9 z^4 - 4 a^2 b^9 c^4 d^3 e^7 z^4 + 16 b^13 c^2 d^5 e f^4 z^4 + 16 b^7 c^7 d^8 e f^2 z^4 + 768 a^7 c^7 d e^8 f^2 z^4 + 16 a^7 b^7 d e f^8 z^4 + 16 a^2 b^13 d^4 e f^5 z^4 + 256 a^7 b^3 c^6 e^9 f^2 z^4 + 80 a^2 b^12 c^2 d^5 f^5 z^4 + 48 a^9 b^4 c^2 d f^9 z^4 + 48 a^2 b^4 c^9 d^9 f^2 z^4 + 256 a^6 b^3 c^7 d e^9 z^4 - 42 b^10 c^4 d^6 e^2 f^2 z^4 - 20 b^12 c^2 d^5 e^2 f^3 z^4 + 6 b^12 c^2 d^4 e^4 f^2 z^4 + 4 b^11 c^3 d^5 e^3 f^2 z^4 - 24960 a^7 c^7 d^4 e^2 f^4 z^4 + 18944 a^8 c^6 d^3 e^2 f^5 z^4 + 18944 a^6 c
\end{aligned}$$

$$\begin{aligned}
& \sim 8d^5e^2f^3z^4 + 14336a^7c^7d^3e^4f^3z^4 - 9984a^8c^6d^2e^4f^4z^4 - 9984a^6c^8d^4e^4f^2z^4 - 7936a^9c^5d^2e^2f^6z^4 - 7936 \\
& *a^5c^9d^6e^2f^2z^4 - 4352a^7c^7d^2e^6f^2z^4 - 42a^4b^{10}d^2e^2f^6z^4 - 20a^2b^{12}d^3e^2f^5z^4 + 6a^2b^{12}d^2e^4f^4z^4 + 4a \\
& ^3b^{11}d^2e^3f^5z^4 - 480a^8b^2c^4e^6f^4z^4 + 440a^7b^4c^3e^6f^4z^4 - 320a^8b^3c^3e^5f^5z^4 - 320a^7b^3c^4e^7f^3z^4 + 240* \\
& a^8b^4c^2e^4f^6z^4 + 240a^6b^4c^4e^8f^2z^4 - 192a^9b^3c^2e^3f^7z^4 - 192a^9b^2c^3e^4f^6z^4 - 192a^7b^2c^5e^8f^2z^4 - 90a \\
& ^6b^6c^2e^6f^4z^4 - 68a^5b^6c^3e^8f^2z^4 + 48a^{10}b^2c^2e^2f^8z^4 - 48a^7b^5c^2e^5f^5z^4 - 48a^6b^5c^3e^7f^3z^4 + 36a^5b \\
& ^7c^2e^7f^3z^4 + 6a^4b^8c^2e^8f^2z^4 - 33920a^6b^2c^6d^5f^5z^4 + 27936a^5b^4c^5d^5f^5z^4 + 26112a^7b^2c^5d^4f^6z^4 + 26112 \\
& *a^5b^2c^7d^6f^4z^4 - 20352a^6b^4c^4d^4f^6z^4 - 20352a^4b^4c^6d^6f^4z^4 - 13080a^4b^6c^4d^5f^5z^4 - 11520a^8b^2c^4d^3f^7z \\
& ^4 - 11520a^4b^2c^8d^7f^3z^4 + 8736a^5b^6c^3d^4f^6z^4 + 8736a^3b^6c^5d^6f^4z^4 + 7488a^7b^4c^3d^3f^7z^4 + 7488a^3b^4c^7d^7 \\
& *f^3z^4 + 3840a^3b^8c^3d^5f^5z^4 + 2560a^9b^2c^3d^2f^8z^4 + 2560a^3b^2c^9d^8f^2z^4 - 2416a^6b^6c^2d^3f^7z^4 - 2416a^2b^6c^6 \\
& d^7f^3z^4 - 2160a^4b^8c^2d^4f^6z^4 - 2160a^2b^8c^4d^6f^4z^4 - 1152a^8b^4c^2d^2f^8z^4 - 1152a^2b^4c^8d^8f^2z^4 - 720a^2b^ \\
& ^{10}c^2d^5f^5z^4 - 480a^4b^2c^8d^4e^6z^4 + 440a^3b^4c^7d^4e^6z^4 - 320a^4b^3c^7d^3e^7z^4 - 320a^3b^3c^8d^5e^5z^4 + 240a^4b \\
& ^4c^6d^2e^8z^4 + 240a^2b^4c^8d^6e^4z^4 - 192a^5b^2c^7d^2e^8z^4 - 192a^3b^2c^9d^6e^4z^4 - 192a^2b^3c^9d^7e^3z^4 - 90a^2b^6 \\
& c^6d^4e^6z^4 - 68a^3b^6c^5d^2e^8z^4 - 48a^3b^5c^6d^3e^7z^4 - 48a^2b^5c^7d^5e^5z^4 + 48a^2b^2c^10d^8e^2z^4 + 36a^2b^7c^5 \\
& d^3e^7z^4 + 6a^2b^8c^4d^2e^8z^4 - 4b^6c^8d^9fz^4 + 256a^{11}c^3d^9fz^4 + 256a^3c^{11}d^9fz^4 - 4a^8b^6d^9fz^4 - 384a^9c^5e \\
& ^6f^4z^4 - 256a^{10}c^4e^4f^6z^4 - 256a^8c^6e^8f^2z^4 - 64a^{11}c^3e^2f^8z^4 - 24b^{10}c^4d^7f^3z^4 - 16b^{12}c^2d^6f^4z^4 - 16b^ \\
& ^8c^6d^8f^2z^4 + 17920a^7c^7d^5f^5z^4 - 14336a^8c^6d^4f^6z^4 - 14336a^6c^8d^6f^4z^4 + 7168a^9c^5d^3f^7z^4 + 7168a^5c^9d^7f^ \\
& ^3z^4 - 2048a^{10}c^4d^2f^8z^4 - 2048a^4c^{10}d^8f^2z^4 + 6b^8c^6d^6e^4z^4 + 6a^6b^8e^4f^6z^4 - 4b^9c^5d^5e^5z^4 - 4b^7c^7d^7e \\
& ^3z^4 - 4a^7b^7e^3f^7z^4 - 4a^5b^9e^5f^5z^4 - 384a^5c^9d^4e^6z^4 - 256a^6c^8d^2e^8z^4 - 256a^4c^{10}d^6e^4z^4 - 64a^3c^{11}d \\
& ^8e^2z^4 - 24a^4b^{10}d^3f^7z^4 - 16a^6b^8d^2f^8z^4 - 16a^2b^{12}d^4f^6z^4 + 48a^6b^2c^6e^{10}z^4 - 12a^5b^4c^5e^{10}z^4 - 4b^{14}d \\
& ^5f^5z^4 - 64a^7c^7e^{10}z^4 + b^{14}d^4e^2f^4z^4 + b^{10}c^4d^4e^6z^4 + b^6c^8d^8e^2z^4 + a^8b^6e^2f^8z^4 + a^4b^{10}e^6f^4z^4 + a^ \\
& ^4b^6c^4e^{10}z^4 - 4820A*B*a^4b*c^5d^2e^2f^4z^2 + 2976A*B*a^3b*c^6d^3e^2f^3z^2 - 2328A*B*a^3b*c^6d^2e^4f^2z^2 + 1848A*B*a^2b^4c \\
& ^4d^3e^ef^4z^2 - 1768A*B*a^3b^4c^3d^2e^ef^5z^2 + 1528A*B*a^4b^2c^4d^2e^ef^5z^2 - 1136A*B*a^3b^2c^5d^3e^ef^4z^2 - 974A*B*a^4b^3c^3 \\
& d^e^2f^5z^2 + 692A*B*a^2b*c^7d^4e^2f^2z^2 + 588A*B*a^b^6c^3d^2e^3f^3z^2 - 580A*B*a^3b^3c^4d^e^4f^3z^2 + 488A*B*a^3b^4c^3d^e^3f \\
& ^4z^2 - 444A*B*a^2b^2c^6d^2e^5f^z^2 - 412A*B*a^b^5c^4d^2e^4f^2z^2 + 366A*B*a^2b^6c^2d^2e^ef^5z^2 - 352A*B*a^2b^2c^6d^4e^ef^3z^ \\
& ^2 + 326A*B*a^2b^4c^4d^e^5f^2z^2 + 324A*B*a^b^5c^4d^3e^2f^3z^2 - 302A*B*a^b^3c^6d^4e^2f^2z^2 - 296A*B*a^b^7c^2d^2e^2f^4z^2 + 12 \\
& 2A*B*a^4b^2c^4d^e^3f^4z^2 - 122A*B*a^2b^6c^2d^e^3f^4z^2 - 84A*B*a^3b^2c^5d^e^5f^2z^2 + 72A*B*a^b^4c^5d^3e^3f^2z^2 - 64A*B*a^2 \\
& *b^5c^3d^e^4f^3z^2 + 60A*B*a^3b^5c^2d^e^2f^5z^2 + 1312A*B*a^5b*c^4d^e^2f^5z^2 + 1040A*B*a^4b*c^5d^e^4f^3z^2 - 500A*B*a^b^6c^3d^ \\
& ^3e^ef^4z^2 - 376A*B*a^b^2c^7d^5e^ef^2z^2 + 276A*B*a^4b^4c^2d^e^ef^6z^2 - 262A*B*a^2b^3c^5d^e^6f^z^2 + 238A*B*a^b^2c^7d^4e^3f^z^2 + \\
& 232A*B*a^5b^2c^3d^e^ef^6z^2 - 176A*B*a^2b^c^7d^3e^4f^z^2 - 120A*B*a^b^6c^3d^e^5f^2z^2 - 108A*B*a^b^4c^5d^4e^ef^3z^2 + 68A*B*a^b^7c \\
& ^2d^e^4f^3z^2 + 68A*B*a^b^4c^5d^2e^5f^z^2 + 46A*B*a^2b^7c^d^e^2*
\end{aligned}$$

$$\begin{aligned}
& f^5 z^2 - 36 A B a^2 b^3 c^6 d^3 e^4 f^2 z^2 - 1932 A B a^2 b^3 c^5 d^3 e^2 f^3 z^2 - 1818 A B a^2 b^4 c^4 d^2 e^3 f^3 z^2 + 1620 A B a^3 b^3 c^4 d^2 e^2 f^4 z^2 + 1560 A B a^2 b^3 c^5 d^2 e^4 f^2 z^2 + 1244 A B a^3 b^2 c^5 d^2 e^3 f^3 z^2 + 820 A B a^2 b^2 c^6 d^3 e^3 f^2 z^2 + 480 A B a^2 b^5 c^3 d^2 e^2 f^4 z^2 + 352 A B a^3 b^2 c^6 d^2 e^6 f^2 z^2 - 108 A B a^3 b^6 c^4 d^2 e^6 f^2 z^2 + 82 A B a^2 b^5 c^4 d^2 e^6 f^2 z^2 - 64 A B a^2 b^8 c^4 d^5 e^2 f^2 z^2 + 16 A B a^2 b^8 c^4 d^2 e^6 f^5 z^2 - 4 A B a^2 b^8 c^4 d^3 e^3 f^4 z^2 + 16 B^2 a^2 b^8 c^4 d^6 e^6 f^2 z^2 + 56 A B b^2 c^8 d^6 e^6 f^2 z^2 - 8 A B b^9 c^4 d^4 e^4 f^3 z^2 - 8 A B b^7 c^3 d^2 e^6 f^2 z^2 - 800 A B a^6 c^4 d^2 e^6 f^6 z^2 + 10 A B a^2 b^8 d^2 e^6 f^6 z^2 - 6 A B a^2 b^9 d^2 e^2 f^5 z^2 - 12 A B a^5 b^4 c^2 e^6 f^7 z^2 + 912 A B a^6 b^3 c^3 d^2 e^6 f^7 z^2 + 192 A B a^4 b^5 c^4 d^2 e^6 f^7 z^2 + 192 A B a^2 b^8 c^4 d^6 e^6 f^2 z^2 - 20 A B a^2 b^4 c^5 d^2 e^7 z^2 + 4 A B a^2 b^8 c^4 d^4 e^4 z^2 + 2144 B^2 a^4 b^3 c^5 d^3 e^6 f^4 z^2 - 1120 B^2 a^3 b^3 c^6 d^4 e^6 f^3 z^2 - 688 B^2 a^5 b^3 c^4 d^2 e^6 f^5 z^2 - 256 B^2 a^3 b^3 c^6 d^2 e^5 f^5 z^2 + 152 B^2 a^2 b^3 c^6 d^5 e^6 f^2 z^2 + 120 B^2 a^5 b^3 c^2 d^2 e^6 f^6 z^2 - 116 B^2 a^5 b^3 c^4 d^2 e^3 f^4 z^2 + 110 B^2 a^2 b^7 c^2 d^3 e^6 f^4 z^2 - 80 B^2 a^2 b^7 c^2 d^5 e^6 f^2 z^2 - 72 B^2 a^2 b^5 c^4 d^4 e^6 f^3 z^2 - 48 B^2 a^4 b^3 c^5 d^2 e^5 f^2 z^2 - 46 B^2 a^2 b^3 c^6 d^4 e^3 f^2 z^2 - 44 B^2 a^2 b^4 c^5 d^3 e^4 f^2 z^2 - 34 B^2 a^2 b^5 c^4 d^2 e^5 f^2 z^2 + 20 B^2 a^2 b^2 c^7 d^4 e^3 f^2 z^2 - 10 B^2 a^3 b^6 c^4 d^2 e^2 f^5 z^2 - 10 B^2 a^2 b^7 c^4 d^2 e^2 f^5 z^2 - 10 B^2 a^2 b^2 c^7 d^5 e^2 f^2 z^2 - 7 B^2 a^2 b^4 c^4 d^2 e^6 f^2 z^2 - 6 B^2 a^3 b^2 c^5 d^2 e^6 f^2 z^2 + 4 B^2 a^2 b^8 c^4 d^2 e^2 f^4 z^2 - 2 B^2 a^2 b^7 c^4 d^2 e^3 f^4 z^2 + 3196 A^2 a^4 b^3 c^5 d^2 e^3 f^4 z^2 - 3184 A^2 a^4 b^3 c^5 d^2 e^6 f^5 z^2 + 1568 A^2 a^3 b^3 c^6 d^3 e^6 f^4 z^2 + 1504 A^2 a^3 b^3 c^6 d^2 e^5 f^2 z^2 - 656 A^2 a^4 b^3 c^3 d^2 e^6 f^6 z^2 - 400 A^2 a^2 b^6 c^3 d^2 e^4 f^3 z^2 + 314 A^2 a^2 b^5 c^4 d^2 e^5 f^2 z^2 - 264 A^2 a^3 b^5 c^2 d^2 e^6 f^2 z^2 + 240 A^2 a^2 b^2 c^6 d^2 e^6 f^2 z^2 - 224 A^2 a^2 b^7 c^4 d^4 e^6 f^3 z^2 + 216 A^2 a^2 b^5 c^4 d^3 e^6 f^4 z^2 - 192 A^2 a^2 b^7 c^4 d^2 e^5 f^5 z^2 + 178 A^2 a^2 b^7 c^2 d^2 e^3 f^4 z^2 - 154 A^2 a^2 b^7 c^2 d^2 e^6 f^5 z^2 + 128 A^2 a^2 b^3 c^6 d^4 e^6 f^3 z^2 + 106 A^2 a^2 b^3 c^6 d^2 e^5 f^5 z^2 - 12 A^2 a^2 b^2 c^7 d^3 e^4 f^2 z^2 - 58 A B b^8 c^2 d^2 e^3 f^3 z^2 + 40 A B b^7 c^3 d^2 e^4 f^2 z^2 - 28 A B b^7 c^3 d^3 e^2 f^3 z^2 - 24 A B b^5 c^5 d^4 e^2 f^2 z^2 - 20 A B b^6 c^4 d^3 e^3 f^2 z^2 + 2768 A B a^4 c^6 d^2 e^3 f^3 z^2 - 1712 A B a^3 c^7 d^3 e^3 f^2 z^2 - 156 A B a^4 b^2 c^4 e^5 f^3 z^2 + 146 A B a^4 b^3 c^3 e^4 f^4 z^2 - 106 A B a^5 b^2 c^3 e^3 f^5 z^2 + 90 A B a^5 b^3 c^2 e^2 f^6 z^2 + 38 A B a^3 b^3 c^4 e^6 f^2 z^2 - 36 A B a^3 b^5 c^2 e^4 f^4 z^2 + 16 A B a^3 b^4 c^3 e^5 f^3 z^2 - 9 A B a^4 b^4 c^2 e^3 f^5 z^2 - 8 A B a^2 b^5 c^3 e^6 f^2 z^2 + 2 A B a^2 b^6 c^2 e^5 f^3 z^2 + 920 A B a^4 b^3 c^3 d^2 e^2 f^6 z^2 - 480 A B a^2 b^5 c^3 d^3 e^5 f^5 z^2 - 336 A B a^2 b^3 c^5 d^4 e^4 f^4 z^2 - 272 A B a^3 b^3 c^4 d^3 e^5 f^5 z^2 + 240 A B a^3 b^5 c^2 d^2 e^6 f^6 z^2 - 32 A B a^2 c^9 d^6 e^6 f^2 z^2 - 792 B^2 a^2 b^3 c^5 d^3 e^3 f^2 z^2 + 714 B^2 a^2 b^4 c^4 d^3 e^2 f^3 z^2 - 572 B^2 a^3 b^2 c^5 d^3 e^2 f^3 z^2 - 475 B^2 a^2 b^2 c^6 d^4 e^2 f^2 z^2 + 265 B^2 a^4 b^2 c^4 d^2 e^2 f^4 z^2 + 260 B^2 a^3 b^3 c^4 d^2 e^3 f^3 z^2 - 212 B^2 a^3 b^4 c^3 d^2 e^2 f^4 z^2 + 180 B^2 a^3 b^2 c^5 d^2 e^4 f^2 z^2 - 158 B^2 a^2 b^4 c^4 d^2 e^4 f^2 z^2 + 47 B^2 a^2 b^6 c^2 d^2 e^2 f^4 z^2 + 16 B^2 a^2 b^5 c^3 d^2 e^3 f^3 z^2 + 2752 A^2 a^3 b^2 c^5 d^2 e^2 f^4 z^2 - 2148 A^2 a^2 b^4 c^4 d^2 e^2 f^4 z^2 + 206 4 A^2 a^2 b^3 c^5 d^2 e^3 f^3 z^2 - 424 A^2 a^2 b^2 c^6 d^3 e^2 f^3 z^2 - 1 98 A^2 a^2 b^2 c^6 d^2 e^4 f^2 z^2 - 272 B^2 a^6 b^3 c^3 d^2 e^6 f^6 z^2 - 24 B^2 a^4 b^5 c^4 d^2 e^6 f^6 z^2 + 1808 A^2 a^5 b^3 c^4 d^2 e^6 f^6 z^2 - 244 A^2 a^2 b^8 c^8 d^4 e^3 f^2 z^2 + 208 A^2 a^2 b^8 c^8 d^5 e^6 f^2 z^2 + 134 A^2 a^2 b^7 c^4 d^2 e^6 f^6 z^2 - 76 A^2 a^2 b^4 c^5 d^2 e^6 f^2 z^2 + 4 A^2 a^2 b^8 c^4 d^2 e^2 f^5 z^2 + 148 A B b^4 c^6 d^5 e^6 f^2 z^2 + 65 A B b^6 c^4 d^4 e^6 f^3 z^2 + 46 A B b^8 c^2 d^3 e^6 f^4 z^2 - 38 A B b^3 c^7 d^5 e^2 f^2 z^2 + 34 A B b^9 c^4 d^2 e^2 f^4 z^2 - 29 A B b^4 c^6 d^4 e^3 f^2 z^2 + 20 A B b^5 c^5 d^3 e^4 f^2 z^2 + 12 A B b^8 c^2 d^2 e^5 f^2 z^2 - 7 A B b^6 c^4 d^2 e^5 f^2 z^2 - 2880 A B a^4 c^6 d^3 e^6 f^4 z^2 + 2784 A B a^5 c^5 d^2 e^6 f^5 z^2 - 1112 A B a^5 c^5 d^2 e^3 f^4 z^2 + 896 A B a^3 c^7 d^4 e^6 f^3 z^2 + 848 A B a^3 c^7 d^2 e^5 f^5 z^2 - 560 A B a^4 c^6 d^2 e^5 f^2 z^2 + 96 A B a^2 c^8 d^5 e^6 f^2 z^2 - 88 A B a^2 c^8 d^4 e^3 f^2 z^2 - 100 A B a^6 b^3 c^3 e^2 f^6 z^2 - 76 A B a^5 b^3 c^4 e^4 f^4 z^2 + 48 A B a^6 b^3 c^3 e^2 f^6 z^2
\end{aligned}$$

$$\begin{aligned}
& b^2c^2ef^7z^2 - 42ABa^3b^2c^5e^7fz^2 + 36ABa^4b^3c^5e^6f^2z^2 - 24ABa^4b^5c^2e^2f^6z^2 + 10ABa^3b^6c^3e^3f^5z^2 + 7ABa^2b^4c^4e^7fz^2 + 2ABa^2b^7c^4e^4f^4z^2 - 2496ABa^5b^3c^4d^2f^6z^2 + 1872ABa^4b^3c^5d^3f^5z^2 - 744ABa^5b^3c^2d^3f^7z^2 - 720ABa^2b^3c^7d^5f^3z^2 + 504ABa^3b^3c^6d^5f^3z^2 + 256ABa^3b^3c^6d^4f^4z^2 + 168ABa^3b^7c^2d^3f^5z^2 - 144ABa^2b^7c^2d^2f^6z^2 + 144ABa^3b^5c^4d^4f^4z^2 + 66ABa^2b^2c^6d^5e^7z^2 - 36ABa^3b^2c^7d^3e^5z^2 + 20ABa^3b^3c^6d^2e^6z^2 + 12ABa^2b^3c^7d^2e^6z^2 + 1208B^2a^3b^3c^6d^3e^3f^2z^2 - 848B^2a^3b^3c^4d^3e^3f^4z^2 + 672B^2a^2b^3c^5d^4e^3f^3z^2 - 632B^2a^4b^3c^5d^2e^3f^3z^2 + 432B^2a^4b^3c^3d^2e^5f^5z^2 + 276B^2a^2b^2c^6d^3e^4f^4z^2 - 196B^2a^3b^6c^3d^3e^2f^3z^2 - 168B^2a^2b^5c^3d^3e^3f^4z^2 + 154B^2a^2b^3c^5d^2e^5f^5z^2 + 148B^2a^3b^5c^4d^3e^3f^2z^2 + 96B^2a^3b^4c^5d^4e^2f^2z^2 - 72B^2a^3b^5c^2d^2e^5f^5z^2 + 70B^2a^5b^2c^3d^2e^2f^5z^2 - 60B^2a^4b^3c^3d^2e^3f^4z^2 + 52B^2a^3b^6c^3d^2e^4f^2z^2 + 36B^2a^4b^2c^4d^2e^4f^3z^2 - 32B^2a^3b^7c^2d^2e^3f^3z^2 + 24B^2a^3b^5c^2d^2e^3f^4z^2 + 15B^2a^4b^4c^2d^2e^2f^5z^2 - 8B^2a^3b^4c^3d^2e^4f^3z^2 + 8B^2a^2b^5c^3d^2e^5f^2z^2 - 2B^2a^3b^3c^4d^2e^5f^2z^2 - 2B^2a^2b^6c^2d^2e^4f^3z^2 - 3176A^2a^3b^3c^6d^2e^3f^3z^2 - 2252A^2a^4b^2c^4d^2e^2f^5z^2 + 1952A^2a^3b^4c^3d^2e^2f^5z^2 - 1496A^2a^3b^3c^4d^2e^3f^4z^2 + 1378A^2a^2b^4c^4d^2e^4f^3z^2 + 1184A^2a^3b^3c^4d^2e^5f^5z^2 - 1166A^2a^2b^3c^5d^2e^5f^2z^2 - 1164A^2a^3b^2c^5d^2e^4f^3z^2 - 1152A^2a^2b^3c^5d^3e^3f^4z^2 + 578A^2a^3b^6c^3d^2e^2f^4z^2 - 548A^2a^3b^5c^4d^2e^3f^3z^2 + 440A^2a^3b^2c^7d^4e^2f^2z^2 - 412A^2a^2b^6c^2d^2e^2f^5z^2 - 360A^2a^3b^3c^6d^3e^3f^2z^2 + 312A^2a^3b^4c^5d^3e^2f^3z^2 + 248A^2a^2b^3c^7d^3e^3f^2z^2 - 224A^2a^2b^5c^3d^2e^3f^4z^2 + 216A^2a^2b^5c^3d^2e^5f^5z^2 + 52A^2a^3b^4c^5d^2e^4f^2z^2 - 16B^2b^3c^7d^6e^6fz^2 - 14B^2b^9c^3d^3e^3f^4z^2 + 32B^2a^4c^6d^2e^6fz^2 - 20A^2b^9c^3d^3e^3f^4z^2 + 18A^2b^9c^3d^2e^5f^5z^2 + 8A^2b^6c^4d^2e^6fz^2 - 360A^2a^3c^7d^2e^6fz^2 + 136A^2a^3c^9d^5e^2fz^2 + 2B^2a^3b^7d^2e^6fz^2 + 2B^2a^3b^9d^2e^5f^5z^2 + 12B^2a^4b^3c^5e^7fz^2 - 204A^2a^3b^3c^6e^7fz^2 - 128A^2a^6b^3c^3e^3f^7z^2 - 48A^2a^3b^5c^4e^7fz^2 - 36B^2a^5b^4c^3d^2f^7z^2 - 24A^2a^4b^5c^3e^3f^7z^2 - 16B^2a^3b^8c^3d^3f^5z^2 - 164A^2a^3b^6c^3d^2f^7z^2 - 16A^2a^3b^8c^3d^2f^6z^2 + 4B^2a^3b^3c^6d^2e^7z^2 - 4B^2a^3b^3c^8d^5e^3z^2 + 48A^2a^3b^3c^8d^3e^5z^2 + 36A^2a^2b^3c^7d^2e^7z^2 - 6A^2a^3b^3c^6d^2e^7z^2 + 136ABa^6c^4e^3f^5z^2 - 96ABa^5b^5c^5d^5f^3z^2 + 80ABa^5c^5e^5f^3z^2 - 72ABa^3b^3c^7d^6f^2z^2 - 24ABa^3b^7c^3d^4f^4z^2 + 14ABa^3b^3c^7d^4e^4z^2 - 14ABa^2b^2c^8d^5e^3z^2 - 2ABa^5b^5c^5d^2e^6z^2 - 2ABa^4b^4c^6d^3e^5z^2 + 2ABa^3b^7e^2f^6z^2 - ABa^2b^8e^3f^5z^2 + 16ABa^2c^8d^3e^5z^2 - 2ABa^2b^3c^5e^8z^2 + 22B^2b^8c^2d^3e^2f^3z^2 - 12B^2b^7c^3d^3e^3f^2z^2 + 12B^2b^6c^4d^4e^2f^2z^2 - 6B^2b^8c^2d^2e^4f^2z^2 - 864B^2a^4c^6d^3e^2f^3z^2 + 496B^2a^3c^7d^4e^2f^2z^2 + 224B^2a^5c^5d^2e^2f^4z^2 + 136B^2a^4c^6d^2e^4f^2z^2 - 53A^2b^8c^2d^2e^2f^4z^2 + 52A^2b^7c^3d^2e^3f^3z^2 + 52A^2b^5c^5d^3e^3f^2z^2 - 36A^2b^6c^4d^3e^2f^3z^2 - 12A^2b^4c^6d^4e^2f^2z^2 - 9A^2b^6c^4d^2e^4f^2z^2 + 836A^2a^4c^6d^2e^2f^4z^2 - 668A^2a^2c^8d^4e^2f^2z^2 + 656A^2a^3c^7d^2e^4f^2z^2 + 368A^2a^3c^7d^3e^2f^3z^2 - 45B^2a^6b^2c^2e^2f^6z^2 - 18B^2a^5b^2c^3e^4f^4z^2 - 9B^2a^4b^2c^4e^6f^2z^2 - 6B^2a^5b^3c^2e^3f^5z^2 + 3B^2a^4b^4c^2e^4f^4z^2 - 2B^2a^4b^3c^3e^5f^3z^2 - 580B^2a^4b^2c^4d^3f^5z^2 + 536B^2a^3b^4c^3d^3f^5z^2 + 471A^2a^4b^2c^4e^4f^4z^2 - 436A^2a^3b^4c^3e^4f^4z^2 - 348B^2a^4b^4c^2d^2f^6z^2 + 316B^2a^2b^2c^6d^5f^3z^2 + 310A^2a^3b^3c^4e^5f^3z^2 + 232A^2a^5b^2c^3e^2f^6z^2 - 229A^2a^2b^4c^4e^6f^2z^2 - 216A^2a^4b^4c^2e^2f^6z^2 + 204A^2a^4b^3c^3e^3f^5z^2 + 200B^2a^5b^2c^3d^2f^6z^2 + 150A^2a^3b^2c^5e^6f^2z^2 - 1
\end{aligned}$$

$$\begin{aligned}
& 20*B^2*a^2*b^4*c^4*d^4*f^4*z^2 + 91*A^2*a^2*b^6*c^2*e^4*f^4*z^2 + 72*A^2*a^3*b^5*c^2*e^3*f^5*z^2 - 66*B^2*a^2*b^6*c^2*d^3*f^5*z^2 + 44*A^2*a^2*b^5*c^3*e^5*f^3*z^2 - 16*B^2*a^3*b^2*c^5*d^4*f^4*z^2 + 1952*A^2*a^4*b^2*c^4*d^2*f^6*z^2 - 1792*A^2*a^3*b^2*c^5*d^3*f^5*z^2 - 1272*A^2*a^3*b^4*c^3*d^2*f^6*z^2 \\
& + 976*A^2*a^2*b^2*c^6*d^4*f^4*z^2 + 960*A^2*a^2*b^4*c^4*d^3*f^5*z^2 + 282*A^2*a^2*b^6*c^2*d^2*f^6*z^2 - 45*B^2*a^2*b^2*c^6*d^2*e^6*z^2 - 48*A^2*b^6*c^9*d^6*e*f*z^2 - 14*A^2*a*b^9*d*e*f^6*z^2 - 7*A*B*b^10*d^2*e*f^5*z^2 + 2*A*B*b^10*d*e^3*f^4*z^2 - 64*A*B*a^7*c^3*e*f^7*z^2 - 16*A*B*b^9*c*d^3*f^5*z^2 + 8*A*B*a^4*c^6*e^7*f*z^2 + 4*A*B*b^9*d^6*e^2*z^2 + 2*A*B*b^6*c^4*d*e^7*z^2 - 120*A*B*a^3*c^7*d*e^7*z^2 - 16*A*B*a^3*b^7*d*f^7*z^2 + 16*A*B*a*b^9*d^2*f^6*z^2 + 8*A*B*a*c^9*d^5*e^3*z^2 + 12*A*B*a^3*b*c^6*e^8*z^2 - 48*B^2*b^5*c^5*d^5*e*f^2*z^2 + 15*B^2*b^4*c^6*d^5*e^2*f*z^2 - 14*B^2*b^7*c^3*d^4*e*f^3*z^2 + 4*B^2*b^9*c*d^2*e^3*f^3*z^2 + 4*B^2*b^7*c^3*d^2*e^5*f*z^2 + 4*B^2*b^5*c^5*d^4*e^3*f*z^2 - B^2*b^6*c^4*d^3*e^4*f*z^2 - 336*B^2*a^3*c^7*d^3*e^4*f*z^2 + 112*B^2*a^5*c^5*d*e^4*f^3*z^2 - 112*A^2*b^3*c^7*d^5*e*f^2*z^2 + 80*B^2*a^6*c^4*d*e^2*f^5*z^2 - 48*A^2*b^5*c^5*d^4*e*f^3*z^2 + 36*A^2*b^8*c^2*d*e^4*f^3*z^2 + 36*A^2*b^3*c^7*d^4*e^3*f*z^2 - 28*A^2*b^7*c^3*d*e^5*f^2*z^2 + 20*A^2*b^2*c^8*d^5*e^2*f*z^2 + 16*B^2*a^2*c^8*d^5*e^2*f*z^2 - 14*A^2*b^7*c^3*d^3*e*f^4*z^2 - 14*A^2*b^4*c^6*d^3*e^4*f*z^2 - 10*A^2*b^5*c^5*d^2*e^5*f*z^2 - 1008*A^2*a^4*c^6*d*e^4*f^3*z^2 - 760*A^2*a^5*c^5*d*e^2*f^5*z^2 + 272*A^2*a^2*c^8*d^3*e^4*f*z^2 + 48*B^2*a^5*b^4*c^4*e^5*f^3*z^2 + 36*B^2*a^6*b^3*c^3*e^3*f^5*z^2 + 12*B^2*a^5*b^4*c^2*f^6*z^2 - 624*A^2*a^4*b^3*c^5*e^5*f^3*z^2 - 548*A^2*a^5*b^3*c^4*e^3*f^5*z^2 + 182*A^2*a^2*b^3*c^5*e^7*f*z^2 - 180*B^2*a*b^4*c^5*d^5*f^3*z^2 + 132*B^2*a^6*b^2*c^2*d*f^7*z^2 + 108*B^2*a^3*b^6*c*d^2*f^6*z^2 + 96*A^2*a^5*b^3*c^2*e*f^7*z^2 + 68*A^2*a*b^6*c^3*e^6*f^2*z^2 + 58*A^2*a^3*b^6*c*e^2*f^6*z^2 - 56*B^2*a*b^2*c^7*d^6*f^2*z^2 - 38*A^2*a^2*b^7*c*e^3*f^5*z^2 - 36*A^2*a*b^7*c^2*e^5*f^3*z^2 + 20*B^2*a*b^6*c^3*d^4*f^4*z^2 - 736*A^2*a^5*b^2*c^3*d*f^7*z^2 + 624*A^2*a^4*b^4*c^2*d*f^7*z^2 - 416*A^2*a*b^2*c^7*d^5*f^3*z^2 - 276*A^2*a*b^4*c^5*d^4*f^4*z^2 - 196*A^2*a*b^6*c^3*d^3*f^5*z^2 + 8*B^2*a*b^4*c^5*d^2*e^6*z^2 + 6*B^2*a*b^2*c^7*d^4*e^4*z^2 + 2*B^2*a^2*b^3*c^5*d*e^7*z^2 + 2*B^2*a*b^3*c^6*d^3*e^5*z^2 - 18*A^2*a*b^2*c^7*d^2*e^6*z^2 - 16*A*B*b^9*d^7*f*z^2 - B^2*b^10*d^2*e^2*f^4*z^2 + 48*B^2*a^7*c^3*e^2*f^6*z^2 - 36*B^2*a^6*c^4*e^4*f^4*z^2 + 31*B^2*b^6*c^4*d^5*f^3*z^2 - 24*B^2*a^5*c^5*e^6*f^2*z^2 + 20*B^2*b^4*c^6*d^6*f^2*z^2 - 6*A^2*b^8*c^2*e^6*f^2*z^2 + 2*B^2*b^8*c^2*d^4*f^4*z^2 - 768*B^2*a^5*c^5*d^3*f^5*z^2 + 512*B^2*a^6*c^4*d^2*f^6*z^2 + 512*B^2*a^4*c^6*d^4*f^4*z^2 + 232*A^2*a^5*c^5*e^4*f^4*z^2 + 188*A^2*a^4*c^6*e^6*f^2*z^2 - 128*B^2*a^3*c^7*d^5*f^3*z^2 + 92*A^2*a^6*c^4*e^2*f^6*z^2 + 80*A^2*b^4*c^6*d^5*f^3*z^2 + 64*A^2*b^2*c^8*d^6*f^2*z^2 + 31*A^2*b^6*c^4*d^4*f^4*z^2 + 14*A^2*b^8*c^2*d^3*f^5*z^2 - 5*B^2*b^4*c^6*d^4*e^4*z^2 + 4*B^2*b^3*c^7*d^5*e^3*z^2 + 2*B^2*b^5*c^5*d^3*e^5*z^2 - B^2*b^6*c^4*d^2*e^6*z^2 - B^2*b^2*c^8*d^6*e^2*z^2 - B^2*a^4*b^6*e^2*f^6*z^2 - 1152*A^2*a^3*c^7*d^4*f^4*z^2 + 1008*A^2*a^4*c^6*d^3*f^5*z^2 + 624*A^2*a^2*c^8*d^5*f^3*z^2 - 288*A^2*a^5*c^5*d^2*f^6*z^2 + 56*B^2*a^3*c^7*d^2*e^6*z^2 - 10*B^2*a^2*b^8*d^2*f^6*z^2 - 9*A^2*b^2*c^8*d^4*e^4*z^2 - 5*A^2*a^2*b^8*e^2*f^6*z^2 - 4*B^2*a^2*c^8*d^4*e^4*z^2 + 3*A^2*b^4*c^6*d^2*e^6*z^2 - 2*A^2*b^3*c^7*d^3*e^5*z^2 - 36*A^2*a^2*c^8*d^2*e^6*z^2 - 48*A^2*a^6*b^2*c^2*f^8*z^2 - 45*A^2*a^2*b^2*c^6*e^8*z^2 + 4*A^2*b^10*d*e^2*f^5*z^2 + 4*B^2*b^2*c^8*d^7*f*z^2 + 4*A^2*b^9*c^5*f^3*z^2 + 4*A^2*b^7*c^3*e^7*f*z^2 - 128*B^2*a^7*c^3*d*f^7*z^2 - 160*A^2*a*c^9*d^6*f^2*z^2 - 112*A^2*a^6*c^4*d*f^7*z^2 + 12*A^2*b^9*d^5*e^3*z^2 + 4*A^2*a*b^9*e^3*f^5*z^2 + 3*B^2*a^4*b^6*d*f^7*z^2 + 2*A^2*a^3*b^7*e*f^7*z^2 - 24*A^2*a*c^9*d^4*e^4*z^2 + 14*A^2*a^2*b^8*d*f^7*z^2 + 12*A^2*a^5*b^4*c^2*f^8*z^2 + 12*A^2*a*b^4*c^5*e^8*z^2 + A*B*a^4*b^6*e*f^7*z^2 + B^2*a^2*b^8*d*e^2*f^5*z^2 + 16*A^2*c^10*d^7*f*z^2 + 3*B^2*b^10*d^3*f^5*z^2 - A^2*b^10*e^4*f^4*z^2 - 4*A^2*c^10*d^6*e^2*z^2 - A^2*b^10*d^2*f^6*z^2 + 64*A^2*a^7*c^3*f^8*z^2 - 4*B^2*a^4*c^6*e^8*z^2 - A^2*b^6*c^4*e^8*z^2 + 48*A^2*a^3*c^7*e^8*z^2 - A^2*a^4*b^6*f^8*z^2 + 720*A^2*B*a*b^2*c^5*d^2*e^2*f^3*z - 600*A^2*B*a^2*b^2*c^4*d*e^2*f^4*z + 576*A*B^2*a^2*b^2*c^4*d^2*e*f^4*z + 348*A*B^2*a*b^2*c^5*d^2*e^3*f^2*z - 336*A*B^2*a^2*b^2*c^5*d^2*e^2*f^3*z - 260*A*B^2*a*b^3*c^4*d^2*e^2*f^3*z - 240*A*B^2*a^2*b^2*c^4*d*e^3*f^
\end{aligned}$$

$$\begin{aligned}
& 3*z + 196*A*B^2*a^2*b^3*c^3*d*e^2*f^4*z + 172*A^2*B*a*b*c^6*d*e^5*f*z + 20* \\
& A*B^2*a*b^6*c*d*e*f^5*z - 912*A^2*B*a^2*b*c^5*d^2*e*f^4*z - 644*A^2*B*a*b*c \\
& ^6*d^2*e^3*f^2*z - 432*A*B^2*a*b^2*c^5*d^3*e*f^3*z + 372*A^2*B*a^2*b*c^5*d* \\
& e^3*f^3*z - 330*A^2*B*a*b^2*c^5*d*e^4*f^2*z + 312*A*B^2*a*b*c^6*d^3*e^2*f^2 \\
& *z - 208*A*B^2*a^3*b^2*c^3*d*e*f^5*z + 192*A^2*B*a^2*b^3*c^3*d*e*f^5*z + 17 \\
& 2*A^2*B*a*b^3*c^4*d*e^3*f^3*z + 108*A*B^2*a^2*b*c^5*d*e^4*f^2*z + 104*A*B^2 \\
& *a^3*b*c^4*d*e^2*f^4*z - 80*A^2*B*a*b^3*c^4*d^2*e*f^4*z + 68*A^2*B*a*b^4*c^ \\
& 3*d*e^2*f^4*z - 60*A*B^2*a*b^5*c^2*d*e^2*f^4*z + 58*A*B^2*a*b^3*c^4*d*e^4*f \\
& ^2*z - 36*A*B^2*a*b^4*c^3*d^2*e*f^4*z - 24*A*B^2*a^2*b^4*c^2*d*e*f^5*z + 24 \\
& *A*B^2*a*b^4*c^3*d*e^3*f^3*z + 592*A^2*B*a*b*c^6*d^3*e*f^3*z + 240*A^2*B*a^ \\
& 3*b*c^4*d*e*f^5*z - 132*A*B^2*a*b*c^6*d^2*e^4*f*z - 60*A*B^2*a*b^2*c^5*d*e^ \\
& 5*f*z - 48*A^2*B*a*b^5*c^2*d*e*f^5*z + 20*B^3*a*b*c^6*d^3*e^3*f*z + 16*B^3* \\
& a^4*b*c^3*d*e*f^5*z - 16*B^3*a*b*c^6*d^4*e*f^2*z + 12*B^3*a^2*b*c^5*d*e^5*f \\
& *z + 320*A^3*a*b*c^6*d*e^4*f^2*z + 40*A^3*a*b^4*c^3*d*e*f^5*z - 48*A^2*B*b* \\
& c^7*d^4*e*f^2*z - 44*A^2*B*b^3*c^5*d*e^5*f*z - 20*A*B^2*b*c^7*d^4*e^2*f*z + \\
& 14*A*B^2*b^4*c^4*d*e^5*f*z + 12*A^2*B*b*c^7*d^3*e^3*f*z + 4*A*B^2*b^7*c*d* \\
& e^2*f^4*z + 160*A*B^2*a^4*c^4*d*e*f^5*z + 152*A^2*B*a*c^7*d^2*e^4*f*z - 40* \\
& A*B^2*a*c^7*d^3*e^3*f*z + 32*A*B^2*a*c^7*d^4*e*f^2*z - 16*A*B^2*a^2*c^6*d*e \\
& ^5*f*z + 128*A^2*B*a^4*b*c^3*e*f^6*z + 42*A^2*B*a*b^2*c^5*e^6*f*z + 24*A^2* \\
& B*a^2*b^5*c*e*f^6*z - 12*A*B^2*a^3*b^4*c*e*f^6*z - 12*A*B^2*a^2*b*c^5*e^6*f \\
& *z - 10*A^2*B*a*b^6*c*e^2*f^5*z - 160*A*B^2*a*b*c^6*d^4*f^3*z + 112*A*B^2*a \\
& ^4*b*c^3*d*f^6*z - 24*A*B^2*a^2*b^5*c*d*f^6*z - 84*B^3*a*b^2*c^5*d^3*e^2*f^ \\
& 2*z - 80*B^3*a^2*b^3*c^3*d^2*e*f^4*z - 60*B^3*a^2*b*c^5*d^2*e^3*f^2*z - 20* \\
& B^3*a^3*b^2*c^3*d*e^2*f^4*z - 20*B^3*a*b^3*c^4*d^2*e^3*f^2*z - 9*B^3*a^2*b^ \\
& 2*c^4*d*e^4*f^2*z - 8*B^3*a*b^4*c^3*d^2*e^2*f^3*z + 6*B^3*a^2*b^4*c^2*d*e^2 \\
& *f^4*z - 4*B^3*a^2*b^3*c^3*d*e^3*f^3*z - 216*A^2*B*b^4*c^4*d^2*e^2*f^3*z + \\
& 196*A^2*B*b^3*c^5*d^2*e^3*f^2*z - 108*A*B^2*b^3*c^5*d^3*e^2*f^2*z - 94*A*B^ \\
& 2*b^4*c^4*d^2*e^3*f^2*z + 88*A^2*B*b^2*c^6*d^3*e^2*f^2*z + 80*A*B^2*b^5*c^3 \\
& *d^2*e^2*f^3*z + 360*A^2*B*a^2*c^6*d^2*e^2*f^3*z + 8*A*B^2*a^2*c^6*d^2*e^3* \\
& f^2*z + 153*A^2*B*a^2*b^2*c^4*e^4*f^3*z - 144*A^2*B*a^2*b^3*c^3*e^3*f^4*z + \\
& 80*A^2*B*a^3*b^2*c^3*e^2*f^5*z + 36*A*B^2*a^3*b^2*c^3*e^3*f^4*z + 12*A^2*B \\
& *a^2*b^4*c^2*e^2*f^5*z + 12*A*B^2*a^3*b^3*c^2*e^2*f^5*z + 9*A*B^2*a^2*b^2*c \\
& ^4*e^5*f^2*z - 6*A*B^2*a^2*b^4*c^2*e^3*f^4*z + 4*A*B^2*a^2*b^3*c^3*e^4*f^3* \\
& z + 480*A^2*B*a^2*b^2*c^4*d^2*f^5*z - 176*A*B^2*a^2*b^3*c^3*d^2*f^5*z - 10* \\
& A^2*B*a*b^6*c*d*f^6*z + 16*A*B^2*a*b*c^6*d*e^6*z + 80*B^3*a*b^3*c^4*d^3*e*f \\
& ^3*z - 48*B^3*a^3*b*c^4*d^2*e*f^4*z + 48*B^3*a^2*b*c^5*d^3*e*f^3*z + 44*B^3 \\
& *a^3*b*c^4*d*e^3*f^3*z + 24*B^3*a*b^5*c^2*d^2*e*f^4*z + 18*B^3*a*b^2*c^5*d^ \\
& 2*e^4*f*z + 696*A^3*a^2*b*c^5*d*e^2*f^4*z - 504*A^3*a*b*c^6*d^2*e^2*f^3*z - \\
& 192*A^3*a*b^2*c^5*d*e^3*f^3*z - 144*A^3*a^2*b^2*c^4*d*e*f^5*z + 96*A^3*a*b \\
& ^2*c^5*d^2*e*f^4*z - 72*A^3*a*b^3*c^4*d*e^2*f^4*z - 208*A^2*B*b^3*c^5*d^3*e \\
& *f^3*z + 152*A*B^2*b^4*c^4*d^3*e*f^3*z + 80*A^2*B*b^5*c^3*d^2*e*f^4*z + 75* \\
& A^2*B*b^4*c^4*d*e^4*f^2*z - 59*A^2*B*b^2*c^6*d^2*e^4*f*z - 52*A^2*B*b^5*c^3 \\
& *d*e^3*f^3*z + 42*A*B^2*b^3*c^5*d^2*e^4*f*z - 21*A*B^2*b^6*c^2*d^2*e*f^4*z \\
& - 16*A*B^2*b^5*c^3*d*e^4*f^2*z + 16*A*B^2*b^2*c^6*d^4*e*f^2*z + 16*A*B^2*b^ \\
& 2*c^6*d^3*e^3*f*z + 11*A^2*B*b^6*c^2*d*e^2*f^4*z + 4*A*B^2*b^6*c^2*d*e^3*f^ \\
& 3*z - 256*A^2*B*a*c^7*d^3*e^2*f^2*z - 96*A*B^2*a^3*c^5*d^2*e*f^4*z - 36*A^2 \\
& *B*a^2*c^6*d*e^4*f^2*z - 32*A^2*B*a^3*c^5*d*e^2*f^4*z - 32*A*B^2*a^2*c^6*d^ \\
& 3*e*f^3*z + 8*A*B^2*a^3*c^5*d*e^3*f^3*z - 96*A^2*B*a^3*b^3*c^2*e*f^6*z + 68 \\
& *A^2*B*a^3*b*c^4*e^3*f^4*z - 60*A*B^2*a^4*b*c^3*e^2*f^5*z - 60*A*B^2*a^3*b* \\
& c^4*e^4*f^3*z + 48*A*B^2*a^4*b^2*c^2*e*f^6*z - 38*A^2*B*a*b^3*c^4*e^5*f^2*z \\
& - 36*A^2*B*a^2*b*c^5*e^5*f^2*z + 36*A^2*B*a*b^5*c^2*e^3*f^4*z - 16*A^2*B*a \\
& *b^4*c^3*e^4*f^3*z + 384*A*B^2*a^2*b*c^5*d^3*f^4*z - 352*A*B^2*a^3*b*c^4*d^ \\
& 2*f^5*z - 288*A^2*B*a*b^2*c^5*d^3*f^4*z - 160*A^2*B*a^3*b^2*c^3*d*f^6*z - 1 \\
& 48*A^2*B*a*b^4*c^3*d^2*f^5*z + 112*A*B^2*a*b^3*c^4*d^3*f^4*z + 72*A^2*B*a^2 \\
& *b^4*c^2*d*f^6*z + 72*A*B^2*a*b^5*c^2*d^2*f^5*z + 48*A*B^2*a^3*b^3*c^2*d*f^ \\
& 6*z + 102*B^3*a^2*b^2*c^4*d^2*e^2*f^3*z - 32*B^3*b^5*c^3*d^3*e*f^3*z - 8*B^ \\
& 3*b^3*c^5*d^3*e^3*f*z - 7*B^3*b^4*c^4*d^2*e^4*f*z + 5*B^3*b^2*c^6*d^4*e^2*f \\
& *z + 80*A^3*b^2*c^6*d^3*e*f^3*z - 74*A^3*b^3*c^5*d*e^4*f^2*z - 64*A^3*b^4*c \\
& ^4*d^2*e*f^4*z + 60*A^3*b^4*c^4*d*e^3*f^3*z - 48*B^3*a^4*c^4*d*e^2*f^4*z -
\end{aligned}$$

$$\begin{aligned}
& 24*B^3*a^3*c^5*d*e^4*f^2*z + 20*B^3*a^2*c^6*d^2*e^4*f*z - 16*A^3*b^5*c^3*d* \\
& e^2*f^4*z + 8*A^3*b*c^7*d^3*e^2*f^2*z + 480*A^3*a^2*c^6*d^2*e*f^4*z - 392*A \\
& ^3*a^2*c^6*d*e^3*f^3*z + 280*A^3*a*c^7*d^2*e^3*f^2*z - 4*B^3*a^4*b*c^3*e^3* \\
& f^4*z - 200*A^3*a^3*b*c^4*e^2*f^5*z - 144*A^3*a^2*b*c^5*e^4*f^3*z + 48*B^3* \\
& a*b^2*c^5*d^4*f^3*z + 42*A^3*a*b^2*c^5*e^5*f^2*z - 36*B^3*a^4*b^2*c^2*d*f^6 \\
& *z - 32*A^3*a^3*b^2*c^3*e*f^6*z - 24*A^3*a^2*b^4*c^2*e*f^6*z - 24*A^3*a*b^5 \\
& *c^2*e^2*f^5*z + 10*A^3*a*b^3*c^4*e^4*f^3*z - 4*B^3*a*b^4*c^3*d^3*f^4*z - 4 \\
& *A^3*a*b^4*c^3*e^3*f^4*z - 480*A^3*a^2*b*c^5*d^2*f^5*z - 160*A^3*a^2*b^3*c^ \\
& 3*d*f^6*z + 128*A^3*a*b^3*c^4*d^2*f^5*z + 8*A^2*B*b^5*c^3*e^5*f^2*z - 2*A^2 \\
& *B*b^6*c^2*e^4*f^3*z + 112*A^2*B*b^4*c^4*d^3*f^4*z - 92*A^2*B*a^4*c^4*e^2*f \\
& ^5*z - 64*A^2*B*a^3*c^5*e^4*f^3*z - 64*A*B^2*b^5*c^3*d^3*f^4*z + 24*A*B^2*a \\
& ^4*c^4*e^3*f^4*z + 24*A*B^2*a^3*c^5*e^5*f^2*z + 16*A^2*B*b^2*c^6*d^4*f^3*z \\
& + 16*A*B^2*b^3*c^5*d^4*f^3*z - A^2*B*b^6*c^2*d^2*f^5*z + 448*A^2*B*a^3*c^5* \\
& d^2*f^5*z - 352*A^2*B*a^2*c^6*d^3*f^4*z - 5*A*B^2*b^2*c^6*d^2*e^5*z - 48*A^ \\
& 2*B*a^4*b^2*c^2*f^7*z - 2*B^3*b^7*c*d^2*e*f^4*z + 34*A^3*b^2*c^6*d*e^5*f*z \\
& + 16*A^3*b*c^7*d^2*e^4*f*z + 2*A^3*b^6*c^2*d*e*f^5*z - 416*A^3*a^3*c^5*d*e* \\
& f^5*z - 224*A^3*a*c^7*d^3*e*f^3*z + 12*B^3*a^3*b^4*c*d*f^6*z - 10*B^3*a*b^6 \\
& *c*d^2*f^5*z + 416*A^3*a^3*b*c^4*d*f^6*z + 224*A^3*a*b*c^6*d^3*f^4*z + 24*A \\
& ^3*a*b^5*c^2*d*f^6*z - 4*B^3*a*b*c^6*d^2*e^5*z + 20*A^2*B*c^8*d^4*e^2*f*z - \\
& 7*A^2*B*b^4*c^4*e^6*f*z - 2*A^2*B*b^7*c*e^3*f^4*z - 64*A*B^2*a^5*c^3*e*f^6 \\
& *z + 16*A*B^2*b*c^7*d^5*f^2*z - 8*A^2*B*a^2*c^6*e^6*f*z - 2*A*B^2*b^7*c*d^2 \\
& *f^5*z - 272*A^2*B*a^4*c^4*d*f^6*z + 128*A^2*B*a*c^7*d^4*f^3*z + 9*A^2*B*b^ \\
& 2*c^6*d*e^6*z - 4*A*B^2*b^3*c^5*d*e^6*z + 4*A*B^2*b*c^7*d^3*e^4*z + 8*A*B^2 \\
& *a*c^7*d^2*e^5*z + 12*A^2*B*a^3*b^4*c*f^7*z + 30*B^3*b^4*c^4*d^3*e^2*f^2*z \\
& + 8*B^3*b^5*c^3*d^2*e^3*f^2*z - 2*B^3*b^6*c^2*d^2*e^2*f^3*z + 152*A^3*b^3*c \\
& ^5*d^2*e^2*f^3*z - 108*A^3*b^2*c^6*d^2*e^3*f^2*z + 48*B^3*a^3*c^5*d^2*e^2*f \\
& ^3*z - 16*B^3*a^2*c^6*d^3*e^2*f^2*z - 3*B^3*a^4*b^2*c^2*e^2*f^5*z - 120*B^3 \\
& *a^2*b^2*c^4*d^3*f^4*z + 112*B^3*a^3*b^2*c^3*d^2*f^5*z + 112*A^3*a^2*b^3*c^ \\
& 3*e^2*f^5*z + 12*A^3*a^2*b^2*c^4*e^3*f^4*z - 120*A^3*a*c^7*d*e^5*f*z - 52*A \\
& ^3*a*b*c^6*e^6*f*z + 10*A^3*a*b^6*c*e*f^6*z - 2*A*B^2*b^8*d*e*f^5*z - 2*A^2 \\
& *B*a*b^7*e*f^6*z - 24*A^2*B*a*c^7*d*e^6*z + 2*A*B^2*a*b^7*d*f^6*z - 12*A^2* \\
& B*a*b*c^6*e^7*z - 2*A^3*b^7*c*d*f^6*z - 4*A^3*b*c^7*d*e^6*z + 16*B^3*a^5*c^ \\
& 3*e^2*f^5*z + 11*B^3*b^6*c^2*d^3*f^4*z - 11*A^3*b^4*c^4*e^5*f^2*z - 8*B^3*b \\
& ^4*c^4*d^4*f^3*z - 4*B^3*b^2*c^6*d^5*f^2*z + 4*B^3*a^4*c^4*e^4*f^3*z + 4*A^ \\
& 3*b^5*c^3*e^4*f^3*z - A^3*b^6*c^2*e^3*f^4*z + 136*A^3*a^3*c^5*e^3*f^4*z + 6 \\
& 8*A^3*a^2*c^6*e^5*f^2*z - 64*A^3*b^3*c^5*d^3*f^4*z + 2*B^3*b^3*c^5*d^2*e^5* \\
& z - B^3*b^2*c^6*d^3*e^4*z + 96*A^3*a^3*b^3*c^2*f^7*z + A*B^2*a^2*b^6*e*f^6* \\
& z + 32*A^3*c^8*d^4*e*f^2*z - 24*A^3*c^8*d^3*e^3*f*z + 10*A^3*b^3*c^5*e^6*f* \\
& z + 2*A^3*b^7*c*e^2*f^5*z + 128*A^3*a^4*c^4*e*f^6*z - 32*A^3*b*c^7*d^4*f^3* \\
& z - 4*B^3*a^2*c^6*d*e^6*z - B^3*a^2*b^6*d*f^6*z - 128*A^3*a^4*b*c^3*f^7*z - \\
& 24*A^3*a^2*b^5*c*f^7*z - 16*A^2*B*c^8*d^5*f^2*z - 4*A^2*B*c^8*d^3*e^4*z + \\
& 64*A^2*B*a^5*c^3*f^7*z + 2*A^2*B*b^3*c^5*e^7*z + 4*A*B^2*a^2*c^6*e^7*z - A^ \\
& 2*B*a^2*b^6*f^7*z + 4*A^3*c^8*d^2*e^5*z - 3*A^3*b^2*c^6*e^7*z + A^2*B*b^8*d \\
& *f^6*z - A^3*b^8*e*f^6*z + 16*A^3*a*c^7*e^7*z + 2*A^3*a*b^7*f^7*z + A^2*B*b \\
& ^8*e^2*f^5*z + B^3*b^8*d^2*f^5*z - 48*A^2*B^2*a*b*c^4*d*e*f^4 + 28*A*B^3*a* \\
& b^2*c^3*d*e*f^4 - 16*A*B^3*a*b*c^4*d*e^2*f^3 + 16*A^3*B*a*c^5*d*e*f^4 + 32* \\
& A^3*B*a*b*c^4*d*f^5 + 12*A^2*B^2*b^3*c^3*d*e*f^4 + 5*A*B^3*b^2*c^4*d^2*e*f^ \\
& 3 + 4*A*B^3*b^3*c^3*d*e^2*f^3 + 24*A^2*B^2*a*c^5*d*e^2*f^3 + 24*A^2*B^2*a^2 \\
& *b*c^3*e*f^5 + 12*A^2*B^2*a*b*c^4*e^3*f^3 - 6*A^2*B^2*a*b^3*c^2*e*f^5 + 4*A \\
& *B^3*a^2*b*c^3*e^2*f^4 + 3*A*B^3*a^2*b^2*c^2*e*f^5 - 18*A^2*B^2*a*b^2*c^3*d \\
& *f^5 - 4*B^4*a^2*b*c^3*d*e*f^4 + 4*B^4*a*b*c^4*d^2*e*f^3 - 6*A*B^3*b^4*c^2* \\
& d*e*f^4 + 4*A^3*B*b*c^5*d*e^2*f^3 - 2*A^3*B*b^2*c^4*d*e*f^4 - 8*A*B^3*a^2*c \\
& ^4*d*e*f^4 - 8*A*B^3*a*c^5*d^2*e*f^3 + 26*A^3*B*a*b^2*c^3*e*f^5 + 8*A^3*B*a \\
& *b*c^4*e^2*f^4 + 32*A*B^3*a*b*c^4*d^2*f^4 - 28*A*B^3*a^2*b*c^3*d*f^5 + 6*A* \\
& B^3*a*b^3*c^2*d*f^5 - 9*A^2*B^2*b^2*c^4*d*e^2*f^3 - 18*A^2*B^2*a*b^2*c^3*e^ \\
& 2*f^4 - 4*A^3*B*c^6*d^2*e*f^3 - 3*A^3*B*b^4*c^2*e*f^5 - 44*A^3*B*a^2*c^4*e* \\
& f^5 - 16*A^3*B*a*c^5*e^3*f^3 - 16*A*B^3*a^3*c^3*e*f^5 - 10*A^3*B*b^3*c^3*d* \\
& f^5 - 4*A^3*B*b*c^5*d^2*f^4 - 4*A*B^3*b*c^5*d^3*f^3 - 28*A^3*B*a^2*b*c^3*f^ \\
& 6 + 6*A^3*B*a*b^3*c^2*f^6 - 4*A^4*b*c^5*d*e*f^4 - 20*A^4*a*b*c^4*e*f^5 + 3*
\end{aligned}$$

$$\begin{aligned}
& A^2B^2b^4c^2e^2f^4 - 2A^2B^2b^3c^3e^3f^3 + 12A^2B^2a^2c^4e^2f^4 + 9A^2B^2b^2c^4d^2f^4 - 3A^2B^2a^2b^2c^2f^6 - 2B^4b^3c^3d^2e^2f^3 + 4B^4a^2c^4d^2e^2f^3 - 10B^4a^2b^2c^3d^2f^4 - 3B^4a^2b^2c^2d^2f^5 + 3A^3B^3b^2c^4e^3f^3 - 2A^3B^3b^3c^3e^2f^4 - 10A^3B^3b^3c^3d^2f^4 - 4A^3B^3a^2c^4e^3f^3 + 3A^2B^2b^4c^2d^2f^5 + 36A^2B^2a^2c^4d^2f^5 - 24A^2B^2a^2c^5d^2f^4 + 4A^2B^2c^6d^3f^3 + 16A^2B^2a^3c^3f^6 + 4A^4b^3c^3e^2f^5 + 16B^4a^3c^3d^2f^5 + 16A^4a^2c^5e^2f^4 + 8A^4b^2c^4d^2f^5 - 8A^4a^2b^2c^3f^6 - 24A^4a^2c^5d^2f^5 + 3B^4b^4c^2d^2f^4 - 3A^4b^2c^4e^2f^4 + 4A^4c^6d^2f^4 + 36A^4a^2c^4f^6 + B^4b^2c^4d^3f^3, z, k) \cdot (\text{root}(48416a^6b^2c^6d^4e^2f^4z^4 - 41544a^5b^4c^5d^4e^2f^4z^4 - 31872a^7b^2c^5d^3e^2f^5z^4 - 31872a^5b^2c^7d^5e^2f^3z^4 - 29184a^6b^2c^6d^3e^4f^3z^4 + 28800a^5b^4c^5d^3e^4f^3z^4 + 21510a^4b^6c^4d^4e^2f^4z^4 + 21408a^6b^4c^4d^3e^2f^5z^4 + 21408a^4b^4c^6d^5e^2f^3z^4 - 18112a^7b^3c^4d^2e^3f^5z^4 - 18112a^4b^3c^7d^5e^3f^2z^4 - 15600a^5b^5c^4d^3e^3f^4z^4 - 15600a^4b^5c^5d^4e^3f^3z^4 + 15296a^6b^3c^5d^3e^3f^4z^4 + 15296a^5b^3c^6d^4e^3f^3z^4 + 14016a^7b^2c^5d^2e^4f^4z^4 + 14016a^5b^2c^7d^4e^4f^2z^4 - 13920a^4b^6c^4d^3e^4f^3z^4 - 11648a^6b^3c^5d^2e^5f^3z^4 - 11648a^5b^3c^6d^3e^5f^2z^4 + 10432a^6b^2c^6d^2e^6f^2z^4 + 9008a^6b^5c^3d^2e^3f^5z^4 + 9008a^3b^5c^6d^5e^3f^2z^4 + 8544a^5b^5c^4d^2e^5f^3z^4 + 8544a^4b^5c^5d^3e^5f^2z^4 - 8496a^5b^4c^5d^2e^6f^2z^4 + 7488a^8b^2c^4d^2e^2f^6z^4 + 7488a^4b^2c^8d^6e^2f^2z^4 + 7380a^4b^7c^3d^3e^3f^4z^4 + 7380a^3b^7c^4d^4e^3f^3z^4 - 6720a^3b^8c^3d^4e^2f^4z^4 - 5784a^5b^6c^3d^3e^2f^5z^4 - 5784a^3b^6c^5d^5e^2f^3z^4 - 3440a^6b^4c^4d^2e^4f^4z^4 - 3440a^4b^4c^6d^4e^4f^2z^4 + 3360a^3b^8c^3d^3e^4f^3z^4 + 3140a^4b^6c^4d^2e^6f^2z^4 - 2760a^4b^7c^3d^2e^5f^3z^4 - 2760a^3b^7c^4d^3e^5f^2z^4 - 1764a^5b^7c^2d^2e^3f^5z^4 - 1764a^2b^7c^5d^5e^3f^2z^4 - 1640a^3b^9c^2d^3e^3f^4z^4 - 1640a^2b^9c^3d^4e^3f^3z^4 - 1604a^6b^6c^2d^2e^2f^6z^4 - 1604a^2b^6c^6d^6e^2f^2z^4 - 1500a^5b^6c^3d^2e^4f^4z^4 - 1500a^3b^6c^5d^4e^4f^2z^4 + 140a^2b^10c^2d^4e^2f^4z^4 + 810a^4b^8c^2d^2e^4f^4z^4 + 810a^2b^8c^4d^4e^4f^2z^4 - 544a^3b^8c^3d^2e^6f^2z^4 + 416a^3b^9c^2d^2e^5f^3z^4 + 416a^2b^9c^3d^3e^5f^2z^4 - 384a^2b^10c^2d^3e^4f^3z^4 + 180a^4b^8c^2d^3e^2f^5z^4 + 180a^2b^8c^4d^5e^2f^3z^4 + 48a^7b^4c^3d^2e^2f^6z^4 + 48a^3b^4c^7d^6e^2f^2z^4 + 36a^2b^10c^2d^2e^6f^2z^4 - 1024a^10b^3c^3d^2e^8f^8z^4 - 1024a^3b^3c^10d^8e^8f^8z^4 - 192a^8b^5c^3d^2e^8f^8z^4 - 192a^3b^5c^8d^8e^8f^8z^4 + 16128a^7b^3c^4d^3e^6f^6z^4 + 16128a^4b^3c^7d^6e^6f^3z^4 - 11712a^6b^5c^3d^3e^6f^6z^4 - 11712a^3b^5c^6d^6e^6f^3z^4 + 11520a^8b^3c^5d^2e^3f^5z^4 + 11520a^5b^3c^8d^5e^3f^2z^4 - 9984a^6b^3c^5d^4e^5f^5z^4 - 9984a^5b^3c^6d^5e^5f^4z^4 + 8640a^5b^5c^4d^4e^5f^5z^4 + 8640a^4b^5c^5d^5e^5f^4z^4 - 7424a^7b^3c^6d^3e^3f^4z^4 - 7424a^6b^3c^7d^4e^3f^3z^4 - 6912a^8b^3c^3d^2e^7f^7z^4 - 6912a^3b^3c^8d^7e^7f^2z^4 + 4800a^7b^3c^4d^2e^5f^4z^4 + 4800a^4b^3c^7d^4e^5f^4z^4 + 4608a^7b^3c^6d^2e^5f^3z^4 + 4608a^6b^3c^7d^3e^5f^2z^4 - 4560a^4b^7c^3d^4e^5f^5z^4 - 4560a^3b^7c^4d^5e^5f^4z^4 + 4176a^5b^7c^2d^3e^6f^6z^4 + 4176a^2b^7c^5d^6e^6f^3z^4 + 3264a^7b^5c^2d^2e^6f^7z^4 + 3264a^2b^5c^7d^7e^6f^2z^4 + 3008a^8b^3c^3d^2e^3f^6z^4 + 3008a^3b^3c^8d^6e^3f^6z^4 + 2880a^6b^3c^5d^2e^7f^2z^4 + 2880a^5b^3c^6d^2e^7f^2z^4 - 2240a^7b^4c^3d^2e^4f^5z^4 - 2240a^3b^4c^7d^5e^4f^5z^4 - 1488a^5b^5c^4d^2e^7f^2z^4 - 1488a^4b^5c^5d^2e^7f^2z^4 + 1440a^3b^9c^2d^4e^5f^5z^4 + 1440a^2b^9c^3d^5e^5f^4z^4 - 1328a^6b^5c^3d^2e^5f^4z^4 - 1328a^3b^5c^6d^4e^5f^4z^4 - 1152a^7b^2c^5d^2e^6f^3z^4 - 1152a^5b^2c^7d^3e^6f^3z^4 - 1120a^6b^4c^4d^2e^6f^3z^4 - 1120a^4b^4c^6d^3e^6f^3z^4 + 912a^6b^6c^2d^2e^4f^5z^4 + 912a^2b^6c^6d^5e^4f^5z^4 + 872a^5b^6c^3d^2e^6f^3z^4 + 872a^3b^6c^5d^3e^6f^3z^4 + 768a^8b^2c^4d^2e^4f^5z^4 + 768a^4b^2c^8
\end{aligned}$$

$$\begin{aligned}
& *d^5e^4f^3z^4 - 672a^8b^4c^2d^2e^2f^7z^4 - 672a^2b^4c^8d^7e^2f^* \\
& z^4 - 624a^7b^5c^2d^2e^3f^6z^4 - 624a^2b^5c^7d^6e^3f^*z^4 + 480a \\
& ^5b^8c^d^2e^2f^6z^4 + 480a^*b^8c^5d^6e^2f^2z^4 + 316a^4b^7c^3 \\
& d^e^7f^2z^4 + 316a^3b^7c^4d^2e^7f^*z^4 - 204a^4b^8c^2d^2e^6f^3z \\
& ^4 - 204a^2b^8c^4d^3e^6f^*z^4 + 168a^3b^10c^d^3e^2f^5z^4 + 168a \\
& *b^10c^3d^5e^2f^3z^4 + 156a^2b^11c^d^3e^3f^4z^4 + 156a^*b^11c^2 \\
& d^4e^3f^3z^4 + 128a^9b^2c^3d^2e^2f^7z^4 + 128a^3b^2c^9d^7e^2 \\
& f^*z^4 - 124a^3b^10c^d^2e^4f^4z^4 - 124a^*b^10c^3d^4e^4f^2z^4 + 1 \\
& 00a^4b^9c^d^2e^3f^5z^4 + 100a^*b^9c^4d^5e^3f^2z^4 + 36a^5b^7c \\
& ^2d^2e^5f^4z^4 + 36a^2b^7c^5d^4e^5f^*z^4 - 24a^3b^9c^2d^2e^7f^2* \\
& z^4 - 24a^2b^11c^d^2e^5f^3z^4 - 24a^2b^9c^3d^2e^7f^*z^4 - 24a^*b \\
& ^11c^2d^3e^5f^2z^4 - 9216a^8b^c^5d^3e^f^6z^4 - 9216a^5b^c^8d^6 \\
& *e^f^3z^4 - 5376a^8b^c^5d^2e^5f^4z^4 - 5376a^5b^c^8d^4e^5f^*z^4 + \\
& 5120a^9b^c^4d^2e^f^7z^4 + 5120a^7b^c^6d^4e^f^5z^4 + 5120a^6b^c^ \\
& ^7d^5e^f^4z^4 + 5120a^4b^c^9d^7e^f^2z^4 - 4352a^9b^c^4d^2e^3f^6z \\
& ^4 - 4352a^4b^c^9d^6e^3f^*z^4 - 1792a^7b^c^6d^2e^7f^2z^4 - 1792a^6 \\
& *b^c^7d^2e^7f^*z^4 - 1600a^6b^2c^6d^2e^8f^*z^4 + 912a^5b^4c^5d^2e^8 \\
& *f^*z^4 + 768a^9b^3c^2d^2e^f^8z^4 + 768a^2b^3c^9d^8e^f^*z^4 - 720a^ \\
& 4b^9c^d^3e^f^6z^4 - 720a^*b^9c^4d^6e^f^3z^4 - 656a^6b^7c^d^2e^f \\
& ^7z^4 - 656a^*b^7c^6d^7e^f^2z^4 - 240a^2b^11c^d^4e^f^5z^4 - 240a \\
& *b^11c^2d^5e^f^4z^4 + 216a^7b^6c^d^2e^2f^7z^4 + 216a^*b^6c^7d^7e \\
& ^2f^*z^4 - 204a^4b^6c^4d^2e^8f^*z^4 - 144a^5b^8c^d^4e^f^5z^4 - 144 \\
& a^*b^8c^5d^5e^4f^*z^4 - 84a^*b^12c^d^4e^2f^4z^4 + 36a^4b^9c^d^2e^5 \\
& f^4z^4 + 36a^*b^9c^4d^4e^5f^*z^4 + 20a^6b^7c^d^3e^f^6z^4 + 20a^*b^ \\
& 7c^6d^6e^3f^*z^4 + 16a^3b^10c^d^2e^6f^3z^4 + 16a^3b^8c^3d^2e^8f^* \\
& z^4 + 16a^*b^12c^d^3e^4f^3z^4 + 16a^*b^10c^3d^3e^6f^*z^4 + 48b^11c \\
& ^3d^6e^f^3z^4 + 48b^9c^5d^7e^f^2z^4 - 20b^8c^6d^7e^2f^*z^4 + 8 \\
& b^10c^4d^5e^4f^*z^4 - 4b^13c^d^4e^3f^3z^4 - 4b^11c^3d^4e^5f^*z^ \\
& 4 + 4b^9c^5d^6e^3f^*z^4 + 3072a^9c^5d^2e^4f^5z^4 + 3072a^5c^9d^5 \\
& *e^4f^*z^4 + 2560a^8c^6d^2e^6f^3z^4 + 2560a^6c^8d^3e^6f^*z^4 + 1536 \\
& *a^10c^4d^2e^2f^7z^4 + 1536a^4c^10d^7e^2f^*z^4 + 48a^5b^9d^2e^f^ \\
& 7z^4 + 48a^3b^11d^3e^f^6z^4 - 20a^6b^8d^2e^2f^7z^4 + 8a^4b^10d \\
& *e^4f^5z^4 + 4a^5b^9d^2e^3f^6z^4 - 4a^3b^11d^2e^5f^4z^4 - 4a^*b^1 \\
& 3d^3e^3f^4z^4 + 768a^9b^c^4e^5f^5z^4 + 768a^8b^c^5e^7f^3z^4 + \\
& 256a^10b^c^3e^3f^7z^4 - 192a^6b^3c^5e^9f^*z^4 - 68a^7b^6c^e^4* \\
& f^6z^4 + 48a^8b^5c^e^3f^7z^4 + 48a^5b^5c^4e^9f^*z^4 + 36a^6b^7* \\
& c^e^5f^5z^4 - 12a^9b^4c^e^2f^8z^4 - 4a^4b^9c^e^7f^3z^4 - 4a^4* \\
& b^7c^3e^9f^*z^4 + 384a^5b^8c^d^3f^7z^4 + 384a^*b^8c^5d^7f^3z^4 + \\
& 288a^3b^10c^d^4f^6z^4 + 288a^*b^10c^3d^6f^4z^4 + 224a^7b^6c^d^ \\
& 2f^8z^4 + 224a^*b^6c^7d^8f^2z^4 - 192a^10b^2c^2d^f^9z^4 - 192a^ \\
& 2b^2c^10d^9f^*z^4 + 768a^5b^c^8d^3e^7z^4 + 768a^4b^c^9d^5e^5z^ \\
& 4 + 256a^3b^c^10d^7e^3z^4 - 192a^5b^3c^6d^2e^9z^4 - 68a^*b^6c^7d \\
& ^6e^4z^4 + 48a^4b^5c^5d^2e^9z^4 + 48a^*b^5c^8d^7e^3z^4 + 36a^*b^7 \\
& c^6d^5e^5z^4 - 12a^*b^4c^9d^8e^2z^4 - 4a^3b^7c^4d^2e^9z^4 - 4a \\
& *b^9c^4d^3e^7z^4 + 16b^13c^d^5e^f^4z^4 + 16b^7c^7d^8e^f^*z^4 + 7 \\
& 68a^7c^7d^2e^8f^*z^4 + 16a^7b^7d^2e^f^8z^4 + 16a^*b^13d^4e^f^5z^4 + \\
& 256a^7b^c^6e^9f^*z^4 + 80a^*b^12c^d^5f^5z^4 + 48a^9b^4c^d^f^9z^4 \\
& + 48a^*b^4c^9d^9f^*z^4 + 256a^6b^c^7d^2e^9z^4 - 42b^10c^4d^6e^2f \\
& ^2z^4 - 20b^12c^2d^5e^2f^3z^4 + 6b^12c^2d^4e^4f^2z^4 + 4b^11c \\
& ^3d^5e^3f^2z^4 - 24960a^7c^7d^4e^2f^4z^4 + 18944a^8c^6d^3e^2 \\
& *f^5z^4 + 18944a^6c^8d^5e^2f^3z^4 + 14336a^7c^7d^3e^4f^3z^4 - \\
& 9984a^8c^6d^2e^4f^4z^4 - 9984a^6c^8d^4e^4f^2z^4 - 7936a^9c^5* \\
& d^2e^2f^6z^4 - 7936a^5c^9d^6e^2f^2z^4 - 4352a^7c^7d^2e^6f^2z \\
& ^4 - 42a^4b^10d^2e^2f^6z^4 - 20a^2b^12d^3e^2f^5z^4 + 6a^2b^12 \\
& *d^2e^4f^4z^4 + 4a^3b^11d^2e^3f^5z^4 - 480a^8b^2c^4e^6f^4z^4 \\
& + 440a^7b^4c^3e^6f^4z^4 - 320a^8b^3c^3e^5f^5z^4 - 320a^7b^3c \\
& ^4e^7f^3z^4 + 240a^8b^4c^2e^4f^6z^4 + 240a^6b^4c^4e^8f^2z^4 \\
& - 192a^9b^3c^2e^3f^7z^4 - 192a^9b^2c^3e^4f^6z^4 - 192a^7b^2* \\
& c^5e^8f^2z^4 - 90a^6b^6c^2e^6f^4z^4 - 68a^5b^6c^3e^8f^2z^4 +
\end{aligned}$$

$$\begin{aligned}
& 48a^{10}b^2c^2e^2f^8z^4 - 48a^7b^5c^2e^5f^5z^4 - 48a^6b^5c^3e^7f^3z^4 + 36a^5b^7c^2e^7f^3z^4 + 6a^4b^8c^2e^8f^2z^4 - 3392 \\
& 0a^6b^2c^6d^5f^5z^4 + 27936a^5b^4c^5d^5f^5z^4 + 26112a^7b^2c^5d^4f^6z^4 + 26112a^5b^2c^7d^6f^4z^4 - 20352a^6b^4c^4d^4f^6z^4 \\
& - 20352a^4b^4c^6d^6f^4z^4 - 13080a^4b^6c^4d^5f^5z^4 - 11520a^8b^2c^4d^3f^7z^4 - 11520a^4b^2c^8d^7f^3z^4 + 8736a^5b^6c^3 \\
& d^4f^6z^4 + 8736a^3b^6c^5d^6f^4z^4 + 7488a^7b^4c^3d^3f^7z^4 + 7488a^3b^4c^7d^7f^3z^4 + 3840a^3b^8c^3d^5f^5z^4 + 2560a^9b^2 \\
& c^3d^2f^8z^4 + 2560a^3b^2c^9d^8f^2z^4 - 2416a^6b^6c^2d^3f^7z^4 - 2416a^2b^6c^6d^7f^3z^4 - 2160a^4b^8c^2d^4f^6z^4 - 2160a \\
& ^2b^8c^4d^6f^4z^4 - 1152a^8b^4c^2d^2f^8z^4 - 1152a^2b^4c^8d^8f^2z^4 - 720a^2b^10c^2d^5f^5z^4 - 480a^4b^2c^8d^4e^6z^4 + 44 \\
& 0a^3b^4c^7d^4e^6z^4 - 320a^4b^3c^7d^3e^7z^4 - 320a^3b^3c^8d^5e^5z^4 + 240a^4b^4c^6d^2e^8z^4 + 240a^2b^4c^8d^6e^4z^4 - 19 \\
& 2a^5b^2c^7d^2e^8z^4 - 192a^3b^2c^9d^6e^4z^4 - 192a^2b^3c^9d^7e^3z^4 - 90a^2b^6c^6d^4e^6z^4 - 68a^3b^6c^5d^2e^8z^4 - 48a \\
& ^3b^5c^6d^3e^7z^4 - 48a^2b^5c^7d^5e^5z^4 + 48a^2b^2c^10d^8e^2z^4 + 36a^2b^7c^5d^3e^7z^4 + 6a^2b^8c^4d^2e^8z^4 - 4b^6c^8 \\
& d^9f^z^4 + 256a^{11}c^3d^9f^z^4 + 256a^3c^{11}d^9f^z^4 - 4a^8b^6d^f^9z^4 - 384a^9c^5e^6f^4z^4 - 256a^{10}c^4e^4f^6z^4 - 256a^8c^6e^8f^2z^4 \\
& - 64a^{11}c^3e^2f^8z^4 - 24b^{10}c^4d^7f^3z^4 - 16b^{12}c^2d^6f^4z^4 - 16b^8c^6d^8f^2z^4 + 17920a^7c^7d^5f^5z^4 - 14336 \\
& a^8c^6d^4f^6z^4 - 14336a^6c^8d^6f^4z^4 + 7168a^9c^5d^3f^7z^4 + 7168a^5c^9d^7f^3z^4 - 2048a^{10}c^4d^2f^8z^4 - 2048a^4c^{10}d^8 \\
& f^2z^4 + 6b^8c^6d^6e^4z^4 + 6a^6b^8e^4f^6z^4 - 4b^9c^5d^5e^5z^4 - 4b^7c^7d^7e^3z^4 - 4a^7b^7e^3f^7z^4 - 4a^5b^9e^5f^5z^4 \\
& - 384a^5c^9d^4e^6z^4 - 256a^6c^8d^2e^8z^4 - 256a^4c^{10}d^6e^4z^4 - 64a^3c^{11}d^8e^2z^4 - 24a^4b^{10}d^3f^7z^4 - 16a^6b^8d^2 \\
& f^8z^4 - 16a^2b^{12}d^4f^6z^4 + 48a^6b^2c^6e^{10}z^4 - 12a^5b^4c^5e^{10}z^4 - 4b^{14}d^5f^5z^4 - 64a^7c^7e^{10}z^4 + b^{14}d^4e^2f^4z^4 \\
& + b^{10}c^4d^4e^6z^4 + b^6c^8d^8e^2z^4 + a^8b^6e^2f^8z^4 + a^4b^{10}e^6f^4z^4 + a^4b^6c^4e^{10}z^4 - 4820A^*B^*a^4b^c^5d^2e^2f^4z^2 \\
& ^2 + 2976A^*B^*a^3b^c^6d^3e^2f^3z^2 - 2328A^*B^*a^3b^c^6d^2e^4f^2z^2 + 1848A^*B^*a^2b^4c^4d^3e^f^4z^2 - 1768A^*B^*a^3b^4c^3d^2e^f^5z^2 \\
& + 1528A^*B^*a^4b^2c^4d^2e^f^5z^2 - 1136A^*B^*a^3b^2c^5d^3e^f^4z^2 - 974A^*B^*a^4b^3c^3d^e^2f^5z^2 + 692A^*B^*a^2b^c^7d^4e^2f^2z^2 + 5 \\
& 88A^*B^*a^b^6c^3d^2e^3f^3z^2 - 580A^*B^*a^3b^3c^4d^e^4f^3z^2 + 488A^*B^*a^3b^4c^3d^e^3f^4z^2 - 444A^*B^*a^2b^2c^6d^2e^5f^z^2 - 412A^*B \\
& *a^b^5c^4d^2e^4f^2z^2 + 366A^*B^*a^2b^6c^2d^2e^f^5z^2 - 352A^*B^*a^2b^2c^6d^4e^f^3z^2 + 326A^*B^*a^2b^4c^4d^e^5f^2z^2 + 324A^*B^*a^b^5 \\
& c^4d^3e^2f^3z^2 - 302A^*B^*a^b^3c^6d^4e^2f^2z^2 - 296A^*B^*a^b^7c^2d^2e^2f^4z^2 + 122A^*B^*a^4b^2c^4d^e^3f^4z^2 - 122A^*B^*a^2b^6c^2 \\
& d^e^3f^4z^2 - 84A^*B^*a^3b^2c^5d^e^5f^2z^2 + 72A^*B^*a^b^4c^5d^3e^3f^2z^2 - 64A^*B^*a^2b^5c^3d^e^4f^3z^2 + 60A^*B^*a^3b^5c^2d^e^2f^5 \\
& z^2 + 1312A^*B^*a^5b^c^4d^e^2f^5z^2 + 1040A^*B^*a^4b^c^5d^e^4f^3z^2 - 500A^*B^*a^b^6c^3d^3e^f^4z^2 - 376A^*B^*a^b^2c^7d^5e^f^2z^2 + 276A \\
& *B^*a^4b^4c^2d^e^f^6z^2 - 262A^*B^*a^2b^3c^5d^e^6f^z^2 + 238A^*B^*a^b^2c^7d^4e^3f^z^2 + 232A^*B^*a^5b^2c^3d^e^f^6z^2 - 176A^*B^*a^2b^c^7d^3e^4f^z^2 \\
& - 120A^*B^*a^b^6c^3d^e^5f^2z^2 - 108A^*B^*a^b^4c^5d^4e^f^3z^2 + 68A^*B^*a^b^7c^2d^e^4f^3z^2 + 68A^*B^*a^b^4c^5d^2e^5f^z^2 + 4 \\
& 6A^*B^*a^2b^7c^d^e^2f^5z^2 - 36A^*B^*a^b^3c^6d^3e^4f^z^2 - 1932A^*B^*a^2b^3c^5d^3e^2f^3z^2 - 1818A^*B^*a^2b^4c^4d^2e^3f^3z^2 + 1620A^* \\
& B^*a^3b^3c^4d^2e^2f^4z^2 + 1560A^*B^*a^2b^3c^5d^2e^4f^2z^2 + 1244A^*B^*a^3b^2c^5d^2e^3f^3z^2 + 820A^*B^*a^2b^2c^6d^3e^3f^2z^2 + 48 \\
& 0A^*B^*a^2b^5c^3d^2e^2f^4z^2 + 352A^*B^*a^3b^c^6d^e^6f^z^2 - 108A^*B^*a^3b^6c^d^e^f^6z^2 + 82A^*B^*a^b^5c^4d^e^6f^z^2 - 64A^*B^*a^b^c^8d^5e^2f^z^2 \\
& + 16A^*B^*a^b^8c^d^2e^f^5z^2 - 4A^*B^*a^b^8c^d^e^3f^4z^2 + 16A^*B^2a^b^c^8d^6e^f^z^2 + 56A^*B^*b^2c^8d^6e^f^z^2 - 8A^*B^*b^9c^d^e^4f^3z^2 - 8A^*B^*b^7c^3d^e^6f^z^2 \\
& - 800A^*B^*a^6c^4d^e^f^6z^2 + 10A^*B^*a
\end{aligned}$$

$$\begin{aligned}
& ^2b^8d^*e^f^6z^2 - 6*A*B*a^b^9d^*e^2f^5z^2 - 12*A*B*a^5b^4c^*e^f^7z^2 \\
& + 912*A*B*a^6b^c^3d^*f^7z^2 + 192*A*B*a^4b^5c^*d^*f^7z^2 + 192*A*B*a^b^* \\
& c^8d^6f^2z^2 - 20*A*B*a^b^4c^5d^*e^7z^2 + 4*A*B*a^b^*c^8d^4e^4z^2 + \\
& 2144*B^2a^4b^*c^5d^3e^*f^4z^2 - 1120*B^2a^3b^*c^6d^4e^*f^3z^2 - 688*B \\
& ^2a^5b^*c^4d^2e^*f^5z^2 - 256*B^2a^3b^*c^6d^2e^5f^*z^2 + 152*B^2a^*b^ \\
& 3c^6d^5e^*f^2z^2 + 120*B^2a^5b^3c^2d^*e^*f^6z^2 - 116*B^2a^5b^*c^4d \\
& *e^3f^4z^2 + 110*B^2a^*b^7c^2d^3e^*f^4z^2 - 80*B^2a^2b^*c^7d^5e^*f^2 \\
& *z^2 - 72*B^2a^*b^5c^4d^4e^*f^3z^2 - 48*B^2a^4b^*c^5d^*e^5f^2z^2 - 46 \\
& *B^2a^*b^3c^6d^4e^3f^*z^2 - 44*B^2a^*b^4c^5d^3e^4f^*z^2 - 34*B^2a^*b^ \\
& 5c^4d^2e^5f^*z^2 + 20*B^2a^2b^*c^7d^4e^3f^*z^2 - 10*B^2a^3b^6c^*d^*e \\
& ^2f^5z^2 - 10*B^2a^2b^7c^*d^2e^*f^5z^2 - 10*B^2a^*b^2c^7d^5e^2f^*z^ \\
& 2 - 7*B^2a^2b^4c^4d^*e^6f^*z^2 - 6*B^2a^3b^2c^5d^*e^6f^*z^2 + 4*B^2a^ \\
& *b^8c^*d^2e^2f^4z^2 - 2*B^2a^2b^7c^*d^*e^3f^4z^2 + 3196*A^2a^4b^*c^5 \\
& *d^*e^3f^4z^2 - 3184*A^2a^4b^*c^5d^2e^*f^5z^2 + 1568*A^2a^3b^*c^6d^3* \\
& e^*f^4z^2 + 1504*A^2a^3b^*c^6d^*e^5f^2z^2 - 656*A^2a^4b^3c^3d^*e^*f^6* \\
& z^2 - 400*A^2a^*b^6c^3d^*e^4f^3z^2 + 314*A^2a^*b^5c^4d^*e^5f^2z^2 - 2 \\
& 64*A^2a^3b^5c^2d^*e^*f^6z^2 + 240*A^2a^2b^2c^6d^*e^6f^*z^2 - 224*A^2* \\
& a^2b^*c^7d^4e^*f^3z^2 + 216*A^2a^*b^5c^4d^3e^*f^4z^2 - 192*A^2a^2b^*c \\
& ^7d^2e^5f^*z^2 + 178*A^2a^*b^7c^2d^*e^3f^4z^2 - 154*A^2a^*b^7c^2d^2* \\
& e^*f^5z^2 + 128*A^2a^*b^3c^6d^4e^*f^3z^2 + 106*A^2a^*b^3c^6d^2e^5f^*z \\
& ^2 - 12*A^2a^*b^2c^7d^3e^4f^*z^2 - 58*A*B*b^8c^2d^2e^3f^3z^2 + 40*A \\
& *B*b^7c^3d^2e^4f^2z^2 - 28*A*B*b^7c^3d^3e^2f^3z^2 - 24*A*B*b^5c^ \\
& 5d^4e^2f^2z^2 - 20*A*B*b^6c^4d^3e^3f^2z^2 + 2768*A*B*a^4c^6d^2e \\
& ^3f^3z^2 - 1712*A*B*a^3c^7d^3e^3f^2z^2 - 156*A*B*a^4b^2c^4e^5f^3 \\
& *z^2 + 146*A*B*a^4b^3c^3e^4f^4z^2 - 106*A*B*a^5b^2c^3e^3f^5z^2 + \\
& 90*A*B*a^5b^3c^2e^2f^6z^2 + 38*A*B*a^3b^3c^4e^6f^2z^2 - 36*A*B*a^ \\
& 3b^5c^2e^4f^4z^2 + 16*A*B*a^3b^4c^3e^5f^3z^2 - 9*A*B*a^4b^4c^2* \\
& e^3f^5z^2 - 8*A*B*a^2b^5c^3e^6f^2z^2 + 2*A*B*a^2b^6c^2e^5f^3z^2 \\
& + 920*A*B*a^4b^3c^3d^2f^6z^2 - 480*A*B*a^2b^5c^3d^3f^5z^2 - 336* \\
& A*B*a^2b^3c^5d^4f^4z^2 - 272*A*B*a^3b^3c^4d^3f^5z^2 + 240*A*B*a^3 \\
& *b^5c^2d^2f^6z^2 - 32*A*B*a^c^9d^6e^*f^*z^2 - 792*B^2a^2b^3c^5d^3e \\
& ^3f^2z^2 + 714*B^2a^2b^4c^4d^3e^2f^3z^2 - 572*B^2a^3b^2c^5d^3* \\
& e^2f^3z^2 - 475*B^2a^2b^2c^6d^4e^2f^2z^2 + 265*B^2a^4b^2c^4d^2 \\
& *e^2f^4z^2 + 260*B^2a^3b^3c^4d^2e^3f^3z^2 - 212*B^2a^3b^4c^3d^ \\
& 2e^2f^4z^2 + 180*B^2a^3b^2c^5d^2e^4f^2z^2 - 158*B^2a^2b^4c^4d \\
& ^2e^4f^2z^2 + 47*B^2a^2b^6c^2d^2e^2f^4z^2 + 16*B^2a^2b^5c^3d^ \\
& 2e^3f^3z^2 + 2752*A^2a^3b^2c^5d^2e^2f^4z^2 - 2148*A^2a^2b^4c^4 \\
& *d^2e^2f^4z^2 + 2064*A^2a^2b^3c^5d^2e^3f^3z^2 - 424*A^2a^2b^2c^ \\
& ^6d^3e^2f^3z^2 - 198*A^2a^2b^2c^6d^2e^4f^2z^2 - 272*B^2a^6b^*c^ \\
& 3d^*e^*f^6z^2 - 24*B^2a^4b^5c^*d^*e^*f^6z^2 + 1808*A^2a^5b^*c^4d^*e^*f^6* \\
& z^2 - 244*A^2a^*b^*c^8d^4e^3f^*z^2 + 208*A^2a^*b^*c^8d^5e^*f^2z^2 + 134*A^ \\
& 2a^2b^7c^*d^*e^*f^6z^2 - 76*A^2a^*b^4c^5d^*e^6f^*z^2 + 4*A^2a^*b^8c^*d^*e^ \\
& 2f^5z^2 + 148*A*B*b^4c^6d^5e^*f^2z^2 + 65*A*B*b^6c^4d^4e^*f^3z^2 + \\
& 46*A*B*b^8c^2d^3e^*f^4z^2 - 38*A*B*b^3c^7d^5e^2f^*z^2 + 34*A*B*b^9c^* \\
& d^2e^2f^4z^2 - 29*A*B*b^4c^6d^4e^3f^*z^2 + 20*A*B*b^5c^5d^3e^4f^*z \\
& ^2 + 12*A*B*b^8c^2d^*e^5f^2z^2 - 7*A*B*b^6c^4d^2e^5f^*z^2 - 2880*A*B* \\
& a^4c^6d^3e^*f^4z^2 + 2784*A*B*a^5c^5d^2e^*f^5z^2 - 1112*A*B*a^5c^5d \\
& *e^3f^4z^2 + 896*A*B*a^3c^7d^4e^*f^3z^2 + 848*A*B*a^3c^7d^2e^5f^*z^ \\
& 2 - 560*A*B*a^4c^6d^*e^5f^2z^2 + 96*A*B*a^2c^8d^5e^*f^2z^2 - 88*A*B*a \\
& ^2c^8d^4e^3f^*z^2 - 100*A*B*a^6b^*c^3e^2f^6z^2 - 76*A*B*a^5b^*c^4e^4 \\
& *f^4z^2 + 48*A*B*a^6b^2c^2e^*f^7z^2 - 42*A*B*a^3b^2c^5e^7f^*z^2 + 36 \\
& *A*B*a^4b^*c^5e^6f^2z^2 - 24*A*B*a^4b^5c^*e^2f^6z^2 + 10*A*B*a^3b^6* \\
& c^*e^3f^5z^2 + 7*A*B*a^2b^4c^4e^7f^*z^2 + 2*A*B*a^2b^7c^*e^4f^4z^2 - \\
& 2496*A*B*a^5b^*c^4d^2f^6z^2 + 1872*A*B*a^4b^*c^5d^3f^5z^2 - 744*A*B* \\
& a^5b^3c^2d^*f^7z^2 - 720*A*B*a^2b^*c^7d^5f^3z^2 + 504*A*B*a^b^3c^6d \\
& ^5f^3z^2 + 256*A*B*a^3b^*c^6d^4f^4z^2 + 168*A*B*a^*b^7c^2d^3f^5z^2 \\
& - 144*A*B*a^2b^7c^*d^2f^6z^2 + 144*A*B*a^*b^5c^4d^4f^4z^2 + 66*A*B*a^ \\
& 2b^2c^6d^*e^7z^2 - 36*A*B*a^*b^2c^7d^3e^5z^2 + 20*A*B*a^*b^3c^6d^2e \\
& ^6z^2 + 12*A*B*a^2b^*c^7d^2e^6z^2 + 1208*B^2a^3b^*c^6d^3e^3f^2z^2
\end{aligned}$$

$$\begin{aligned}
& - 848*B^2*a^3*b^3*c^4*d^3*e*f^4*z^2 + 672*B^2*a^2*b^3*c^5*d^4*e*f^3*z^2 - 6 \\
& 32*B^2*a^4*b*c^5*d^2*e^3*f^3*z^2 + 432*B^2*a^4*b^3*c^3*d^2*e*f^5*z^2 + 276* \\
& B^2*a^2*b^2*c^6*d^3*e^4*f^3*z^2 - 196*B^2*a*b^6*c^3*d^3*e^2*f^3*z^2 - 168*B^2 \\
& *a^2*b^5*c^3*d^3*e*f^4*z^2 + 154*B^2*a^2*b^3*c^5*d^2*e^5*f^3*z^2 + 148*B^2*a* \\
& b^5*c^4*d^3*e^3*f^2*z^2 + 96*B^2*a*b^4*c^5*d^4*e^2*f^2*z^2 - 72*B^2*a^3*b^5 \\
& *c^2*d^2*e*f^5*z^2 + 70*B^2*a^5*b^2*c^3*d*e^2*f^5*z^2 - 60*B^2*a^4*b^3*c^3* \\
& d*e^3*f^4*z^2 + 52*B^2*a*b^6*c^3*d^2*e^4*f^2*z^2 + 36*B^2*a^4*b^2*c^4*d*e^4 \\
& *f^3*z^2 - 32*B^2*a*b^7*c^2*d^2*e^3*f^3*z^2 + 24*B^2*a^3*b^5*c^2*d*e^3*f^4* \\
& z^2 + 15*B^2*a^4*b^4*c^2*d*e^2*f^5*z^2 - 8*B^2*a^3*b^4*c^3*d*e^4*f^3*z^2 + \\
& 8*B^2*a^2*b^5*c^3*d*e^5*f^2*z^2 - 2*B^2*a^3*b^3*c^4*d*e^5*f^2*z^2 - 2*B^2*a \\
& ^2*b^6*c^2*d*e^4*f^3*z^2 - 3176*A^2*a^3*b*c^6*d^2*e^3*f^3*z^2 - 2252*A^2*a^ \\
& 4*b^2*c^4*d*e^2*f^5*z^2 + 1952*A^2*a^3*b^4*c^3*d*e^2*f^5*z^2 - 1496*A^2*a^3 \\
& *b^3*c^4*d*e^3*f^4*z^2 + 1378*A^2*a^2*b^4*c^4*d*e^4*f^3*z^2 + 1184*A^2*a^3* \\
& b^3*c^4*d^2*e*f^5*z^2 - 1166*A^2*a^2*b^3*c^5*d*e^5*f^2*z^2 - 1164*A^2*a^3*b \\
& ^2*c^5*d*e^4*f^3*z^2 - 1152*A^2*a^2*b^3*c^5*d^3*e*f^4*z^2 + 578*A^2*a*b^6*c \\
& ^3*d^2*e^2*f^4*z^2 - 548*A^2*a*b^5*c^4*d^2*e^3*f^3*z^2 + 440*A^2*a*b^2*c^7* \\
& d^4*e^2*f^2*z^2 - 412*A^2*a^2*b^6*c^2*d*e^2*f^5*z^2 - 360*A^2*a*b^3*c^6*d^3 \\
& *e^3*f^2*z^2 + 312*A^2*a*b^4*c^5*d^3*e^2*f^3*z^2 + 248*A^2*a^2*b*c^7*d^3*e^ \\
& 3*f^2*z^2 - 224*A^2*a^2*b^5*c^3*d*e^3*f^4*z^2 + 216*A^2*a^2*b^5*c^3*d^2*e*f \\
& ^5*z^2 + 52*A^2*a*b^4*c^5*d^2*e^4*f^2*z^2 - 16*B^2*b^3*c^7*d^6*e*f*z^2 - 14 \\
& *B^2*b^9*c*d^3*e*f^4*z^2 + 32*B^2*a^4*c^6*d*e^6*f*z^2 - 20*A^2*b^9*c*d*e^3* \\
& f^4*z^2 + 18*A^2*b^9*c*d^2*e*f^5*z^2 + 8*A^2*b^6*c^4*d*e^6*f*z^2 - 360*A^2* \\
& a^3*c^7*d*e^6*f*z^2 + 136*A^2*a*c^9*d^5*e^2*f*z^2 + 2*B^2*a^3*b^7*d*e*f^6*z \\
& ^2 + 2*B^2*a*b^9*d^2*e*f^5*z^2 + 12*B^2*a^4*b*c^5*e^7*f*z^2 - 204*A^2*a^3*b \\
& *c^6*e^7*f*z^2 - 128*A^2*a^6*b*c^3*e*f^7*z^2 - 48*A^2*a*b^5*c^4*e^7*f*z^2 - \\
& 36*B^2*a^5*b^4*c*d*f^7*z^2 - 24*A^2*a^4*b^5*c*e*f^7*z^2 - 16*B^2*a*b^8*c*d \\
& ^3*f^5*z^2 - 164*A^2*a^3*b^6*c*d*f^7*z^2 - 16*A^2*a*b^8*c*d^2*f^6*z^2 + 4*B \\
& ^2*a^3*b*c^6*d*e^7*z^2 - 4*B^2*a*b*c^8*d^5*e^3*z^2 + 48*A^2*a*b*c^8*d^3*e^5 \\
& *z^2 + 36*A^2*a^2*b*c^7*d*e^7*z^2 - 6*A^2*a*b^3*c^6*d*e^7*z^2 + 136*A*B*a^6 \\
& *c^4*e^3*f^5*z^2 - 96*A*B*b^5*c^5*d^5*f^3*z^2 + 80*A*B*a^5*c^5*e^5*f^3*z^2 \\
& - 72*A*B*b^3*c^7*d^6*f^2*z^2 - 24*A*B*b^7*c^3*d^4*f^4*z^2 + 14*A*B*b^3*c^7* \\
& d^4*e^4*z^2 - 14*A*B*b^2*c^8*d^5*e^3*z^2 - 2*A*B*b^5*c^5*d^2*e^6*z^2 - 2*A* \\
& B*b^4*c^6*d^3*e^5*z^2 + 2*A*B*a^3*b^7*e^2*f^6*z^2 - A*B*a^2*b^8*e^3*f^5*z^2 \\
& + 16*A*B*a^2*c^8*d^3*e^5*z^2 - 2*A*B*a^2*b^3*c^5*e^8*z^2 + 22*B^2*b^8*c^2* \\
& d^3*e^2*f^3*z^2 - 12*B^2*b^7*c^3*d^3*e^3*f^2*z^2 + 12*B^2*b^6*c^4*d^4*e^2*f \\
& ^2*z^2 - 6*B^2*b^8*c^2*d^2*e^4*f^2*z^2 - 864*B^2*a^4*c^6*d^3*e^2*f^3*z^2 + \\
& 496*B^2*a^3*c^7*d^4*e^2*f^2*z^2 + 224*B^2*a^5*c^5*d^2*e^2*f^4*z^2 + 136*B^2 \\
& *a^4*c^6*d^2*e^4*f^2*z^2 - 53*A^2*b^8*c^2*d^2*e^2*f^4*z^2 + 52*A^2*b^7*c^3* \\
& d^2*e^3*f^3*z^2 + 52*A^2*b^5*c^5*d^3*e^3*f^2*z^2 - 36*A^2*b^6*c^4*d^3*e^2*f \\
& ^3*z^2 - 12*A^2*b^4*c^6*d^4*e^2*f^2*z^2 - 9*A^2*b^6*c^4*d^2*e^4*f^2*z^2 + 8 \\
& 36*A^2*a^4*c^6*d^2*e^2*f^4*z^2 - 668*A^2*a^2*c^8*d^4*e^2*f^2*z^2 + 656*A^2* \\
& a^3*c^7*d^2*e^4*f^2*z^2 + 368*A^2*a^3*c^7*d^3*e^2*f^3*z^2 - 45*B^2*a^6*b^2* \\
& c^2*e^2*f^6*z^2 - 18*B^2*a^5*b^2*c^3*e^4*f^4*z^2 - 9*B^2*a^4*b^2*c^4*e^6*f^ \\
& 2*z^2 - 6*B^2*a^5*b^3*c^2*e^3*f^5*z^2 + 3*B^2*a^4*b^4*c^2*e^4*f^4*z^2 - 2*B \\
& ^2*a^4*b^3*c^3*e^5*f^3*z^2 - 580*B^2*a^4*b^2*c^4*d^3*f^5*z^2 + 536*B^2*a^3* \\
& b^4*c^3*d^3*f^5*z^2 + 471*A^2*a^4*b^2*c^4*e^4*f^4*z^2 - 436*A^2*a^3*b^4*c^3 \\
& *e^4*f^4*z^2 - 348*B^2*a^4*b^4*c^2*d^2*f^6*z^2 + 316*B^2*a^2*b^2*c^6*d^5*f^ \\
& 3*z^2 + 310*A^2*a^3*b^3*c^4*e^5*f^3*z^2 + 232*A^2*a^5*b^2*c^3*e^2*f^6*z^2 - \\
& 229*A^2*a^2*b^4*c^4*e^6*f^2*z^2 - 216*A^2*a^4*b^4*c^2*e^2*f^6*z^2 + 204*A^ \\
& 2*a^4*b^3*c^3*e^3*f^5*z^2 + 200*B^2*a^5*b^2*c^3*d^2*f^6*z^2 + 150*A^2*a^3*b \\
& ^2*c^5*e^6*f^2*z^2 - 120*B^2*a^2*b^4*c^4*d^4*f^4*z^2 + 91*A^2*a^2*b^6*c^2*e \\
& ^4*f^4*z^2 + 72*A^2*a^3*b^5*c^2*e^3*f^5*z^2 - 66*B^2*a^2*b^6*c^2*d^3*f^5*z^ \\
& 2 + 44*A^2*a^2*b^5*c^3*e^5*f^3*z^2 - 16*B^2*a^3*b^2*c^5*d^4*f^4*z^2 + 1952* \\
& A^2*a^4*b^2*c^4*d^2*f^6*z^2 - 1792*A^2*a^3*b^2*c^5*d^3*f^5*z^2 - 1272*A^2*a \\
& ^3*b^4*c^3*d^2*f^6*z^2 + 976*A^2*a^2*b^2*c^6*d^4*f^4*z^2 + 960*A^2*a^2*b^4* \\
& c^4*d^3*f^5*z^2 + 282*A^2*a^2*b^6*c^2*d^2*f^6*z^2 - 45*B^2*a^2*b^2*c^6*d^2* \\
& e^6*z^2 - 48*A^2*b*c^9*d^6*e*f*z^2 - 14*A^2*a*b^9*d*e*f^6*z^2 - 7*A*B*b^10* \\
& d^2*e*f^5*z^2 + 2*A*B*b^10*d*e^3*f^4*z^2 - 64*A*B*a^7*c^3*e*f^7*z^2 - 16*A* \\
& B*b^9*c*d^3*f^5*z^2 + 8*A*B*a^4*c^6*e^7*f*z^2 + 4*A*B*b*c^9*d^6*e^2*z^2 + 2
\end{aligned}$$

$$\begin{aligned}
& *A*B*b^6*c^4*d*e^7*z^2 - 120*A*B*a^3*c^7*d*e^7*z^2 - 16*A*B*a^3*b^7*d*f^7*z^2 \\
& + 16*A*B*a*b^9*d^2*f^6*z^2 + 8*A*B*a*c^9*d^5*e^3*z^2 + 12*A*B*a^3*b*c^6* \\
& e^8*z^2 - 48*B^2*b^5*c^5*d^5*e*f^2*z^2 + 15*B^2*b^4*c^6*d^5*e^2*f*z^2 - 14* \\
& B^2*b^7*c^3*d^4*e*f^3*z^2 + 4*B^2*b^9*c*d^2*e^3*f^3*z^2 + 4*B^2*b^7*c^3*d^2 \\
& *e^5*f*z^2 + 4*B^2*b^5*c^5*d^4*e^3*f*z^2 - B^2*b^6*c^4*d^3*e^4*f*z^2 - 336* \\
& B^2*a^3*c^7*d^3*e^4*f*z^2 + 112*B^2*a^5*c^5*d*e^4*f^3*z^2 - 112*A^2*b^3*c^7 \\
& *d^5*e*f^2*z^2 + 80*B^2*a^6*c^4*d*e^2*f^5*z^2 - 48*A^2*b^5*c^5*d^4*e*f^3*z^2 \\
& + 36*A^2*b^8*c^2*d*e^4*f^3*z^2 + 36*A^2*b^3*c^7*d^4*e^3*f*z^2 - 28*A^2*b^7 \\
& *c^3*d*e^5*f^2*z^2 + 20*A^2*b^2*c^8*d^5*e^2*f*z^2 + 16*B^2*a^2*c^8*d^5*e^2 \\
& *f*z^2 - 14*A^2*b^7*c^3*d^3*e*f^4*z^2 - 14*A^2*b^4*c^6*d^3*e^4*f*z^2 - 10*A \\
& ^2*b^5*c^5*d^2*e^5*f*z^2 - 1008*A^2*a^4*c^6*d*e^4*f^3*z^2 - 760*A^2*a^5*c^5 \\
& *d*e^2*f^5*z^2 + 272*A^2*a^2*c^8*d^3*e^4*f*z^2 + 48*B^2*a^5*b*c^4*e^5*f^3*z^2 \\
& + 36*B^2*a^6*b*c^3*e^3*f^5*z^2 + 12*B^2*a^5*b^4*c*e^2*f^6*z^2 - 624*A^2* \\
& a^4*b*c^5*e^5*f^3*z^2 - 548*A^2*a^5*b*c^4*e^3*f^5*z^2 + 182*A^2*a^2*b^3*c^5 \\
& *e^7*f*z^2 - 180*B^2*a*b^4*c^5*d^5*f^3*z^2 + 132*B^2*a^6*b^2*c^2*d*f^7*z^2 \\
& + 108*B^2*a^3*b^6*c*d^2*f^6*z^2 + 96*A^2*a^5*b^3*c^2*e*f^7*z^2 + 68*A^2*a*b \\
& ^6*c^3*e^6*f^2*z^2 + 58*A^2*a^3*b^6*c*e^2*f^6*z^2 - 56*B^2*a*b^2*c^7*d^6*f^2 \\
& *z^2 - 38*A^2*a^2*b^7*c*e^3*f^5*z^2 - 36*A^2*a*b^7*c^2*e^5*f^3*z^2 + 20*B^2 \\
& *a*b^6*c^3*d^4*f^4*z^2 - 736*A^2*a^5*b^2*c^3*d*f^7*z^2 + 624*A^2*a^4*b^4*c \\
& ^2*d*f^7*z^2 - 416*A^2*a*b^2*c^7*d^5*f^3*z^2 - 276*A^2*a*b^4*c^5*d^4*f^4*z^2 \\
& - 196*A^2*a*b^6*c^3*d^3*f^5*z^2 + 8*B^2*a*b^4*c^5*d^2*e^6*z^2 + 6*B^2*a*b \\
& ^2*c^7*d^4*e^4*z^2 + 2*B^2*a^2*b^3*c^5*d*e^7*z^2 + 2*B^2*a*b^3*c^6*d^3*e^5* \\
& z^2 - 18*A^2*a*b^2*c^7*d^2*e^6*z^2 - 16*A*B*b*c^9*d^7*f*z^2 - B^2*b^10*d^2* \\
& e^2*f^4*z^2 + 48*B^2*a^7*c^3*e^2*f^6*z^2 - 36*B^2*a^6*c^4*e^4*f^4*z^2 + 31* \\
& B^2*b^6*c^4*d^5*f^3*z^2 - 24*B^2*a^5*c^5*e^6*f^2*z^2 + 20*B^2*b^4*c^6*d^6*f^2 \\
& *z^2 - 6*A^2*b^8*c^2*e^6*f^2*z^2 + 2*B^2*b^8*c^2*d^4*f^4*z^2 - 768*B^2*a^5 \\
& *c^5*d^3*f^5*z^2 + 512*B^2*a^6*c^4*d^2*f^6*z^2 + 512*B^2*a^4*c^6*d^4*f^4*z^2 \\
& + 232*A^2*a^5*c^5*e^4*f^4*z^2 + 188*A^2*a^4*c^6*e^6*f^2*z^2 - 128*B^2*a^3 \\
& *c^7*d^5*f^3*z^2 + 92*A^2*a^6*c^4*e^2*f^6*z^2 + 80*A^2*b^4*c^6*d^5*f^3*z^2 \\
& + 64*A^2*b^2*c^8*d^6*f^2*z^2 + 31*A^2*b^6*c^4*d^4*f^4*z^2 + 14*A^2*b^8*c^2 \\
& *d^3*f^5*z^2 - 5*B^2*b^4*c^6*d^4*e^4*z^2 + 4*B^2*b^3*c^7*d^5*e^3*z^2 + 2*B^2 \\
& *b^5*c^5*d^3*e^5*z^2 - B^2*b^6*c^4*d^2*e^6*z^2 - B^2*b^2*c^8*d^6*e^2*z^2 - \\
& B^2*a^4*b^6*e^2*f^6*z^2 - 1152*A^2*a^3*c^7*d^4*f^4*z^2 + 1008*A^2*a^4*c^6* \\
& d^3*f^5*z^2 + 624*A^2*a^2*c^8*d^5*f^3*z^2 - 288*A^2*a^5*c^5*d^2*f^6*z^2 + 5 \\
& 6*B^2*a^3*c^7*d^2*e^6*z^2 - 10*B^2*a^2*b^8*d^2*f^6*z^2 - 9*A^2*b^2*c^8*d^4* \\
& e^4*z^2 - 5*A^2*a^2*b^8*e^2*f^6*z^2 - 4*B^2*a^2*c^8*d^4*e^4*z^2 + 3*A^2*b^4 \\
& *c^6*d^2*e^6*z^2 - 2*A^2*b^3*c^7*d^3*e^5*z^2 - 36*A^2*a^2*c^8*d^2*e^6*z^2 - \\
& 48*A^2*a^6*b^2*c^2*f^8*z^2 - 45*A^2*a^2*b^2*c^6*e^8*z^2 + 4*A^2*b^10*d*e^2 \\
& *f^5*z^2 + 4*B^2*b^2*c^8*d^7*f*z^2 + 4*A^2*b^9*c*e^5*f^3*z^2 + 4*A^2*b^7*c^3 \\
& *e^7*f*z^2 - 128*B^2*a^7*c^3*d*f^7*z^2 - 160*A^2*a*c^9*d^6*f^2*z^2 - 112*A \\
& ^2*a^6*c^4*d*f^7*z^2 + 12*A^2*b*c^9*d^5*e^3*z^2 + 4*A^2*a*b^9*e^3*f^5*z^2 + \\
& 3*B^2*a^4*b^6*d*f^7*z^2 + 2*A^2*a^3*b^7*e*f^7*z^2 - 24*A^2*a*c^9*d^4*e^4*z^2 \\
& + 14*A^2*a^2*b^8*d*f^7*z^2 + 12*A^2*a^5*b^4*c*f^8*z^2 + 12*A^2*a*b^4*c^5 \\
& *e^8*z^2 + A*B*a^4*b^6*e*f^7*z^2 + B^2*a^2*b^8*d*e^2*f^5*z^2 + 16*A^2*c^10* \\
& d^7*f*z^2 + 3*B^2*b^10*d^3*f^5*z^2 - A^2*b^10*e^4*f^4*z^2 - 4*A^2*c^10*d^6* \\
& e^2*z^2 - A^2*b^10*d^2*f^6*z^2 + 64*A^2*a^7*c^3*f^8*z^2 - 4*B^2*a^4*c^6*e^8 \\
& *z^2 - A^2*b^6*c^4*e^8*z^2 + 48*A^2*a^3*c^7*e^8*z^2 - A^2*a^4*b^6*f^8*z^2 + \\
& 720*A^2*B*a*b^2*c^5*d^2*e^2*f^3*z - 600*A^2*B*a^2*b^2*c^4*d*e^2*f^4*z + 57 \\
& 6*A*B^2*a^2*b^2*c^4*d^2*e*f^4*z + 348*A*B^2*a*b^2*c^5*d^2*e^3*f^2*z - 336*A \\
& *B^2*a^2*b*c^5*d^2*e^2*f^3*z - 260*A*B^2*a*b^3*c^4*d^2*e^2*f^3*z - 240*A*B^2 \\
& *a^2*b^2*c^4*d*e^3*f^3*z + 196*A*B^2*a^2*b^3*c^3*d*e^2*f^4*z + 172*A^2*B*a \\
& *b*c^6*d*e^5*f*z + 20*A*B^2*a*b^6*c*d*e*f^5*z - 912*A^2*B*a^2*b*c^5*d^2*e*f^4 \\
& *z - 644*A^2*B*a*b*c^6*d^2*e^3*f^2*z - 432*A*B^2*a*b^2*c^5*d^3*e*f^3*z + \\
& 372*A^2*B*a^2*b*c^5*d*e^3*f^3*z - 330*A^2*B*a*b^2*c^5*d*e^4*f^2*z + 312*A*B \\
& ^2*a*b*c^6*d^3*e^2*f^2*z - 208*A*B^2*a^3*b^2*c^3*d*e*f^5*z + 192*A^2*B*a^2* \\
& b^3*c^3*d*e*f^5*z + 172*A^2*B*a*b^3*c^4*d*e^3*f^3*z + 108*A*B^2*a^2*b*c^5*d \\
& *e^4*f^2*z + 104*A*B^2*a^3*b*c^4*d*e^2*f^4*z - 80*A^2*B*a*b^3*c^4*d^2*e*f^4 \\
& *z + 68*A^2*B*a*b^4*c^3*d*e^2*f^4*z - 60*A*B^2*a*b^5*c^2*d*e^2*f^4*z + 58*A \\
& *B^2*a*b^3*c^4*d*e^4*f^2*z - 36*A*B^2*a*b^4*c^3*d^2*e*f^4*z - 24*A*B^2*a^2*
\end{aligned}$$

$$\begin{aligned}
& b^4c^2de^5f^5z + 24A^2B^2a^2b^4c^3de^3f^3z + 592A^2B^2a^2b^4c^3de^3f^3z + 592A^2B^2a^2b^4c^3de^3f^3z + 592A^2B^2a^2b^4c^3de^3f^3z \\
& e^5f^3z + 240A^2B^2a^3b^2c^4de^5f^5z - 132A^2B^2a^2b^4c^6d^2e^4f^5z - 60A^2B^2a^2b^2c^5de^5f^5z - 48A^2B^2a^2b^5c^2de^5f^5z + 20B^3a^2b^2c^6d^3e^3f^5z \\
& + 16B^3a^4b^2c^3de^5f^5z - 16B^3a^2b^2c^6d^4e^5f^2z + 12B^3a^2b^2c^5de^5f^5z + 320A^3a^2b^2c^6de^4f^2z + 40A^3a^2b^4c^3de^5f^5z \\
& - 48A^2B^2b^7c^4de^2f^4z - 44A^2B^2b^3c^5de^5f^5z - 20A^2B^2b^2c^7d^4e^2f^5z + 14A^2B^2b^4c^4de^5f^5z + 12A^2B^2b^2c^7d^3e^3f^5z \\
& + 4A^2B^2b^7c^4de^2f^4z + 160A^2B^2a^4c^4de^5f^5z + 152A^2B^2a^2c^7d^2e^4f^5z - 40A^2B^2a^2c^7d^3e^3f^5z + 32A^2B^2a^2c^7d^4e^5f^2z \\
& - 16A^2B^2a^2c^6de^5f^5z + 128A^2B^2a^4b^2c^3e^6f^6z + 42A^2B^2a^2b^2c^5e^6f^6z + 24A^2B^2a^2b^5c^2e^6f^6z - 12A^2B^2a^3b^4c^2e^6f^6z - 12A^2B^2a^2b^2c^5e^6f^6z \\
& - 10A^2B^2a^2b^6c^2e^2f^5z - 160A^2B^2a^2b^2c^6d^4f^3z + 112A^2B^2a^4b^2c^3d^2f^6z - 24A^2B^2a^2b^5c^2d^2f^6z - 84B^3a^2b^2c^5d^3e^2f^2z \\
& - 80B^3a^2b^3c^3d^2e^2f^4z - 60B^3a^2b^2c^5d^2e^3f^2z - 20B^3a^3b^2c^3d^2e^2f^4z - 20B^3a^2b^3c^4d^2e^3f^2z - 9B^3a^2b^2c^4d^2e^4f^2z \\
& - 8B^3a^2b^4c^3d^2e^2f^3z + 6B^3a^2b^4c^2d^2e^2f^4z - 4B^3a^2b^3c^3d^2e^3f^3z - 216A^2B^2b^4c^4d^2e^2f^3z + 196A^2B^2b^3c^5d^2e^3f^2z \\
& - 108A^2B^2b^3c^5d^3e^2f^2z - 94A^2B^2b^4c^4d^2e^3f^2z + 88A^2B^2b^2c^6d^3e^2f^2z + 80A^2B^2b^5c^3d^2e^2f^3z + 360A^2B^2a^2c^6d^2e^2f^3z + 8A^2B^2a^2c^6d^2e^3f^2z \\
& + 153A^2B^2a^2b^2c^4e^4f^3z - 144A^2B^2a^2b^3c^3e^3f^4z + 80A^2B^2a^3b^2c^3e^2f^5z + 36A^2B^2a^3b^2c^3e^3f^4z + 12A^2B^2a^2b^4c^2e^2f^5z + 12A^2B^2a^3b^3c^2e^2f^5z \\
& + 9A^2B^2a^2b^2c^4e^5f^2z - 6A^2B^2a^2b^4c^2e^3f^4z + 4A^2B^2a^2b^3c^3e^4f^3z + 480A^2B^2a^2b^2c^4d^2f^5z - 176A^2B^2a^2b^3c^3d^2f^5z - 10A^2B^2a^2b^6c^2d^2f^6z \\
& + 16A^2B^2a^2b^2c^6d^2e^6z + 80B^3a^2b^3c^4d^3e^5f^3z - 48B^3a^3b^2c^4d^2e^5f^4z + 48B^3a^2b^2c^5d^3e^5f^3z + 44B^3a^3b^2c^4d^2e^3f^3z + 24B^3a^2b^5c^2d^2e^5f^4z \\
& + 18B^3a^2b^2c^5d^2e^4f^5z + 696A^3a^2b^2c^5d^2e^2f^4z - 504A^3a^2b^2c^6d^2e^2f^3z - 192A^3a^2b^2c^5d^2e^3f^3z - 144A^3a^2b^2c^4d^2e^5f^5z \\
& + 96A^3a^2b^2c^5d^2e^5f^4z - 72A^3a^2b^3c^4d^2e^2f^4z - 208A^2B^2b^3c^5d^3e^5f^3z + 152A^2B^2b^4c^4d^3e^5f^3z + 80A^2B^2b^5c^3d^2e^5f^4z + 75A^2B^2b^4c^4d^2e^4f^2z \\
& - 59A^2B^2b^2c^6d^2e^4f^5z - 52A^2B^2b^5c^3d^2e^3f^3z + 42A^2B^2b^3c^5d^2e^4f^5z - 21A^2B^2b^6c^2d^2e^5f^4z - 16A^2B^2b^5c^3d^2e^4f^2z + 16A^2B^2b^2c^6d^4e^5f^2z \\
& + 16A^2B^2b^2c^6d^3e^3f^5z + 11A^2B^2b^6c^2d^2e^2f^4z + 4A^2B^2b^6c^2d^2e^3f^3z - 256A^2B^2a^2c^7d^3e^2f^2z - 96A^2B^2a^3c^5d^2e^5f^4z - 36A^2B^2a^2c^6d^3e^5f^4z \\
& - 32A^2B^2a^3c^5d^2e^2f^4z - 32A^2B^2a^2c^6d^3e^5f^3z + 8A^2B^2a^3c^5d^2e^3f^3z - 96A^2B^2a^3b^3c^2e^5f^6z + 68A^2B^2a^3b^2c^4e^3f^4z - 60A^2B^2a^4b^2c^3e^2f^5z \\
& - 60A^2B^2a^3b^2c^4e^4f^3z + 48A^2B^2a^4b^2c^2e^5f^6z - 38A^2B^2a^2b^3c^4e^5f^2z - 36A^2B^2a^2b^2c^5e^5f^2z + 36A^2B^2a^2b^5c^2e^3f^4z - 16A^2B^2a^2b^4c^3e^4f^3z + 384A^2B^2a^2b^2c^5d^3f^4z \\
& - 352A^2B^2a^3b^2c^4d^2f^5z - 288A^2B^2a^2b^2c^5d^3f^4z - 160A^2B^2a^3b^2c^3d^2f^6z - 148A^2B^2a^2b^4c^3d^2f^5z + 112A^2B^2a^2b^3c^4d^3f^4z + 72A^2B^2a^2b^4c^2d^2f^6z + 72A^2B^2a^2b^5c^2d^2f^5z + 48A^2B^2a^3b^3c^2d^2f^6z \\
& + 102B^3a^2b^2c^4d^2e^2f^3z - 32B^3b^5c^3d^3e^5f^3z - 8B^3b^3c^5d^3e^3f^5z - 7B^3b^4c^4d^2e^4f^5z + 5B^3b^2c^6d^4e^2f^5z + 80A^3b^2c^6d^3e^5f^3z - 74A^3b^3c^5d^4e^4f^2z - 64A^3b^4c^4d^2e^5f^4z + 60A^3b^4c^4d^2e^3f^3z - 48B^3a^4c^4d^2e^2f^4z - 24B^3a^3c^5d^2e^4f^2z + 20B^3a^2c^6d^2e^4f^5z - 16A^3b^5c^3d^2e^2f^4z + 8A^3b^2c^7d^3e^2f^2z + 480A^3a^2c^6d^2e^5f^4z - 392A^3a^2c^6d^2e^3f^3z + 280A^3a^2c^7d^2e^3f^2z - 4B^3a^4b^2c^3e^3f^4z - 200A^3a^3b^2c^4e^2f^5z - 144A^3a^2b^2c^5e^4f^3z + 48B^3a^2b^2c^5d^4f^3z + 42A^3a^2b^2c^5e^5f^2z - 36B^3a^4b^2c^2d^2f^6z - 32A^3a^3b^2c^3e^5f^6z - 24A^3a^2b^4c^2e^5f^6z - 24A^3a^2b^5c^2e^2f^5z + 10A^3a^2b^3c^4e^4f^3z - 4B^3a^2b^4c^3d^3f^4z - 4A^3a^2b^4c^3e^3f^4z - 480A^3a^2b^2c^5d^2f^5z - 160A^3a^2b^3c^3d^2f^6z + 128A^3a^2b^3c^4d^2f^5z + 8A^2B^2b^5
\end{aligned}$$

$$\begin{aligned}
& *c^3e^5f^2z - 2A^2B*b^6c^2e^4f^3z + 112A^2B*b^4c^4d^3f^4z - \\
& 92A^2B*a^4c^4e^2f^5z - 64A^2B*a^3c^5e^4f^3z - 64A*B^2*b^5c^3* \\
& d^3f^4z + 24A*B^2*a^4c^4e^3f^4z + 24A*B^2*a^3c^5e^5f^2z + 16A^ \\
& 2*B*b^2c^6d^4f^3z + 16A*B^2*b^3c^5d^4f^3z - A^2*B*b^6c^2d^2f^5* \\
& z + 448A^2B*a^3c^5d^2f^5z - 352A^2B*a^2c^6d^3f^4z - 5A*B^2*b^2 \\
& *c^6d^2e^5z - 48A^2B*a^4*b^2c^2f^7z - 2B^3*b^7*c^d^2e^f^4z + 34* \\
& A^3*b^2c^6d^5e^fz + 16A^3*b^c^7d^2e^4fz + 2A^3*b^6c^2d^5e^f^5z \\
& - 416A^3*a^3c^5d^5e^f^5z - 224A^3*a^c^7d^3e^f^3z + 12B^3*a^3*b^4*c^ \\
& d^f^6z - 10B^3*a*b^6*c^d^2f^5z + 416A^3*a^3*b^c^4d^f^6z + 224A^3*a^* \\
& b^c^6d^3f^4z + 24A^3*a*b^5c^2d^f^6z - 4B^3*a*b^c^6d^2e^5z + 20A^ \\
& 2*B*c^8d^4e^2fz - 7A^2B*b^4c^4e^6fz - 2A^2B*b^7*c^e^3f^4z - \\
& 64A*B^2*a^5c^3e^f^6z + 16A*B^2*b^c^7d^5f^2z - 8A^2B*a^2c^6e^6f \\
& *z - 2A*B^2*b^7*c^d^2f^5z - 272A^2B*a^4c^4d^f^6z + 128A^2B*a^c^7* \\
& d^4f^3z + 9A^2B*b^2c^6d^5e^6z - 4A*B^2*b^3c^5d^5e^6z + 4A*B^2*b^c^ \\
& ^7d^3e^4z + 8A*B^2*a^c^7d^2e^5z + 12A^2B*a^3b^4*c^f^7z + 30B^3* \\
& b^4c^4d^3e^2f^2z + 8B^3*b^5c^3d^2e^3f^2z - 2B^3*b^6c^2d^2e^2 \\
& *f^3z + 152A^3*b^3c^5d^2e^2f^3z - 108A^3*b^2c^6d^2e^3f^2z + 48 \\
& *B^3*a^3c^5d^2e^2f^3z - 16B^3*a^2c^6d^3e^2f^2z - 3B^3*a^4*b^2*c^ \\
& ^2e^2f^5z - 120B^3*a^2*b^2*c^4d^3f^4z + 112B^3*a^3*b^2c^3d^2f^5* \\
& z + 112A^3*a^2*b^3c^3e^2f^5z + 12A^3*a^2*b^2c^4e^3f^4z - 120A^3* \\
& a^c^7d^5e^5fz - 52A^3*a*b^c^6e^6fz + 10A^3*a*b^6*c^e^f^6z - 2A*B^2 \\
& *b^8d^5e^f^5z - 2A^2B*a*b^7e^f^6z - 24A^2B*a^c^7d^5e^6z + 2A*B^2*a \\
& *b^7d^f^6z - 12A^2B*a*b^c^6e^7z - 2A^3*b^7*c^d^f^6z - 4A^3*b^c^7d \\
& *e^6z + 16B^3*a^5c^3e^2f^5z + 11B^3*b^6c^2d^3f^4z - 11A^3*b^4c^ \\
& ^4e^5f^2z - 8B^3*b^4c^4d^4f^3z - 4B^3*b^2c^6d^5f^2z + 4B^3*a^ \\
& 4c^4e^4f^3z + 4A^3*b^5c^3e^4f^3z - A^3*b^6c^2e^3f^4z + 136A^3 \\
& *a^3c^5e^3f^4z + 68A^3*a^2c^6e^5f^2z - 64A^3*b^3c^5d^3f^4z + \\
& 2B^3*b^3c^5d^2e^5z - B^3*b^2c^6d^3e^4z + 96A^3*a^3*b^3c^2f^7z \\
& + A*B^2*a^2*b^6e^f^6z + 32A^3*c^8d^4e^f^2z - 24A^3*c^8d^3e^3f^z + \\
& 10A^3*b^3c^5e^6fz + 2A^3*b^7*c^e^2f^5z + 128A^3*a^4c^4e^f^6z - \\
& 32A^3*b^c^7d^4f^3z - 4B^3*a^2c^6d^5e^6z - B^3*a^2*b^6d^f^6z - 128 \\
& *A^3*a^4*b^c^3f^7z - 24A^3*a^2*b^5c^f^7z - 16A^2B*c^8d^5f^2z - 4* \\
& A^2B*c^8d^3e^4z + 64A^2B*a^5c^3f^7z + 2A^2B*b^3c^5e^7z + 4A* \\
& B^2*a^2c^6e^7z - A^2B*a^2*b^6f^7z + 4A^3*c^8d^2e^5z - 3A^3*b^2c^ \\
& ^6e^7z + A^2B*b^8d^f^6z - A^3*b^8e^f^6z + 16A^3*a^c^7e^7z + 2A^3 \\
& *a*b^7f^7z + A^2B*b^8e^2f^5z + B^3*b^8d^2f^5z - 48A^2B^2*a*b^c^4 \\
& *d^5e^f^4 + 28A*B^3*a*b^2c^3d^5e^f^4 - 16A*B^3*a*b^c^4d^5e^2f^3 + 16A^3 \\
& *B*a^c^5d^5e^f^4 + 32A^3*B*a*b^c^4d^f^5 + 12A^2B^2*b^3c^3d^5e^f^4 + 5* \\
& A*B^3*b^2c^4d^2e^f^3 + 4A*B^3*b^3c^3d^5e^2f^3 + 24A^2B^2*a^c^5d^5e^ \\
& ^2f^3 + 24A^2B^2*a^2*b^c^3e^f^5 + 12A^2B^2*a*b^c^4e^3f^3 - 6A^2B^2 \\
& *a*b^3c^2e^f^5 + 4A*B^3*a^2*b^c^3e^2f^4 + 3A*B^3*a^2*b^2c^2e^f^5 - \\
& 18A^2B^2*a*b^2c^3d^f^5 - 4B^4*a^2*b^c^3d^5e^f^4 + 4B^4*a*b^c^4d^2e^ \\
& f^3 - 6A*B^3*b^4c^2d^5e^f^4 + 4A^3*B*b^c^5d^5e^2f^3 - 2A^3*B*b^2c^4d \\
& *e^f^4 - 8A*B^3*a^2c^4d^5e^f^4 - 8A*B^3*a^c^5d^2e^f^3 + 26A^3*B*a*b^2 \\
& *c^3e^f^5 + 8A^3*B*a*b^c^4e^2f^4 + 32A*B^3*a*b^c^4d^2f^4 - 28A*B^3* \\
& a^2*b^c^3d^f^5 + 6A*B^3*a*b^3c^2d^f^5 - 9A^2B^2*b^2c^4d^5e^2f^3 - 1 \\
& 8A^2B^2*a*b^2c^3e^2f^4 - 4A^3*B^c^6d^2e^f^3 - 3A^3*B*b^4c^2e^f^5 \\
& - 44A^3*B*a^2c^4e^f^5 - 16A^3*B*a^c^5e^3f^3 - 16A*B^3*a^3c^3e^f^5 \\
& - 10A^3*B*b^3c^3d^f^5 - 4A^3*B*b^c^5d^2f^4 - 4A*B^3*b^c^5d^3f^3 - \\
& 28A^3*B*a^2*b^c^3f^6 + 6A^3*B*a*b^3c^2f^6 - 4A^4*b^c^5d^5e^f^4 - 20* \\
& A^4*a*b^c^4e^f^5 + 3A^2B^2*b^4c^2e^2f^4 - 2A^2B^2*b^3c^3e^3f^3 + \\
& 12A^2B^2*a^2c^4e^2f^4 + 9A^2B^2*b^2c^4d^2f^4 - 3A^2B^2*a^2*b^2 \\
& *c^2f^6 - 2B^4*b^3c^3d^2e^f^3 + 4B^4*a^2c^4d^5e^2f^3 - 10B^4*a*b^2 \\
& *c^3d^2f^4 - 3B^4*a^2*b^2c^2d^f^5 + 3A^3*B*b^2c^4e^3f^3 - 2A^3*B* \\
& b^3c^3e^2f^4 - 10A*B^3*b^3c^3d^2f^4 - 4A*B^3*a^2c^4e^3f^3 + 3A^ \\
& 2B^2*b^4c^2d^f^5 + 36A^2B^2*a^2c^4d^f^5 - 24A^2B^2*a^c^5d^2f^4 + \\
& 4A^2B^2*c^6d^3f^3 + 16A^2B^2*a^3c^3f^6 + 4A^4*b^3c^3e^f^5 + 16* \\
& B^4*a^3c^3d^f^5 + 16A^4*a^c^5e^2f^4 + 8A^4*b^2c^4d^f^5 - 8A^4*a*b^ \\
& 2c^3f^6 - 24A^4*a^c^5d^f^5 + 3B^4*b^4c^2d^2f^4 - 3A^4*b^2c^4e^2*
\end{aligned}$$

$$\begin{aligned}
& f^4 + 4A^4c^6d^2f^4 + 36A^4a^2c^4f^6 + B^4b^2c^4d^3f^3, z, k) \cdot \\
& \text{root}(48416a^6b^2c^6d^4e^2f^4z^4 - 41544a^5b^4c^5d^4e^2f^4z^4 \\
& - 31872a^7b^2c^5d^3e^2f^5z^4 - 31872a^5b^2c^7d^5e^2f^3z^4 - 2 \\
& 9184a^6b^2c^6d^3e^4f^3z^4 + 28800a^5b^4c^5d^3e^4f^3z^4 + 2151 \\
& 0a^4b^6c^4d^4e^2f^4z^4 + 21408a^6b^4c^4d^3e^2f^5z^4 + 21408a \\
& ^4b^4c^6d^5e^2f^3z^4 - 18112a^7b^3c^4d^2e^3f^5z^4 - 18112a^4b \\
& ^3c^7d^5e^3f^2z^4 - 15600a^5b^5c^4d^3e^3f^4z^4 - 15600a^4b^5 \\
& ^5c^5d^4e^3f^3z^4 + 15296a^6b^3c^5d^3e^3f^4z^4 + 15296a^5b^3c^ \\
& ^6d^4e^3f^3z^4 + 14016a^7b^2c^5d^2e^4f^4z^4 + 14016a^5b^2c^7d \\
& ^4e^4f^2z^4 - 13920a^4b^6c^4d^3e^4f^3z^4 - 11648a^6b^3c^5d^2e \\
& ^5f^3z^4 - 11648a^5b^3c^6d^3e^5f^2z^4 + 10432a^6b^2c^6d^2e^6 \\
& ^5f^2z^4 + 9008a^6b^5c^3d^2e^3f^5z^4 + 9008a^3b^5c^6d^5e^3f^2z \\
& ^4 + 8544a^5b^5c^4d^2e^5f^3z^4 + 8544a^4b^5c^5d^3e^5f^2z^4 - \\
& 8496a^5b^4c^5d^2e^6f^2z^4 + 7488a^8b^2c^4d^2e^2f^6z^4 + 7488 \\
& ^4a^4b^2c^8d^6e^2f^2z^4 + 7380a^4b^7c^3d^3e^3f^4z^4 + 7380a^3b \\
& ^7c^4d^4e^3f^3z^4 - 6720a^3b^8c^3d^4e^2f^4z^4 - 5784a^5b^6c^ \\
& ^3d^3e^2f^5z^4 - 5784a^3b^6c^5d^5e^2f^3z^4 - 3440a^6b^4c^4d^ \\
& ^2e^4f^4z^4 - 3440a^4b^4c^6d^4e^4f^2z^4 + 3360a^3b^8c^3d^3e^4 \\
& ^3f^3z^4 + 3140a^4b^6c^4d^2e^6f^2z^4 - 2760a^4b^7c^3d^2e^5f^3z \\
& ^4 - 2760a^3b^7c^4d^3e^5f^2z^4 - 1764a^5b^7c^2d^2e^3f^5z^4 - \\
& 1764a^2b^7c^5d^5e^3f^2z^4 - 1640a^3b^9c^2d^3e^3f^4z^4 - 1640 \\
& ^2a^2b^9c^3d^4e^3f^3z^4 - 1604a^6b^6c^2d^2e^2f^6z^4 - 1604a^2b \\
& ^6c^6d^6e^2f^2z^4 - 1500a^5b^6c^3d^2e^4f^4z^4 - 1500a^3b^6c^ \\
& ^5d^4e^4f^2z^4 + 1140a^2b^10c^2d^4e^2f^4z^4 + 810a^4b^8c^2d^ \\
& ^2e^4f^4z^4 + 810a^2b^8c^4d^4e^4f^2z^4 - 544a^3b^8c^3d^2e^6f \\
& ^2z^4 + 416a^3b^9c^2d^2e^5f^3z^4 + 416a^2b^9c^3d^3e^5f^2z^4 \\
& - 384a^2b^10c^2d^3e^4f^3z^4 + 180a^4b^8c^2d^3e^2f^5z^4 + 180 \\
& ^2a^2b^8c^4d^5e^2f^3z^4 + 48a^7b^4c^3d^2e^2f^6z^4 + 48a^3b^4c \\
& ^7d^6e^2f^2z^4 + 36a^2b^10c^2d^2e^6f^2z^4 - 1024a^10b^3c^3d^e \\
& ^8z^4 - 1024a^3b^3c^10d^8e^8z^4 - 192a^8b^5c^d^e^8z^4 - 192a^b \\
& ^5c^8d^8e^8z^4 + 16128a^7b^3c^4d^3e^6z^4 + 16128a^4b^3c^7d^ \\
& ^6e^6z^4 - 11712a^6b^5c^3d^3e^6z^4 - 11712a^3b^5c^6d^6e^6z \\
& ^4 + 11520a^8b^3c^5d^2e^3f^5z^4 + 11520a^5b^3c^8d^5e^3f^2z^4 - \\
& 9984a^6b^3c^5d^4e^5z^4 - 9984a^5b^3c^6d^5e^4z^4 + 8640a^5b \\
& ^5c^4d^4e^5z^4 + 8640a^4b^5c^5d^5e^4z^4 - 7424a^7b^3c^6d^3 \\
& ^e^3f^4z^4 - 7424a^6b^3c^7d^4e^3f^3z^4 - 6912a^8b^3c^3d^2e^7z \\
& ^4 - 6912a^3b^3c^8d^7e^7z^4 + 4800a^7b^3c^4d^5e^5f^4z^4 + 480 \\
& 0a^4b^3c^7d^4e^5f^3z^4 + 4608a^7b^3c^6d^2e^5f^3z^4 + 4608a^6b^3c \\
& ^7d^3e^5f^2z^4 - 4560a^4b^7c^3d^4e^5z^4 - 4560a^3b^7c^4d^5e \\
& ^5f^4z^4 + 4176a^5b^7c^2d^3e^6z^4 + 4176a^2b^7c^5d^6e^6z^4 \\
& + 3264a^7b^5c^2d^2e^7z^4 + 3264a^2b^5c^7d^7e^7z^4 + 3008a \\
& ^8b^3c^3d^e^3f^6z^4 + 3008a^3b^3c^8d^6e^3f^6z^4 + 2880a^6b^3c^ \\
& ^5d^e^7f^2z^4 + 2880a^5b^3c^6d^2e^7f^6z^4 - 2240a^7b^4c^3d^e^4f \\
& ^5z^4 - 2240a^3b^4c^7d^5e^4f^5z^4 - 1488a^5b^5c^4d^e^7f^2z^4 - \\
& 1488a^4b^5c^5d^2e^7f^5z^4 + 1440a^3b^9c^2d^4e^5z^4 + 1440a^2b \\
& ^9c^3d^5e^4z^4 - 1328a^6b^5c^3d^e^5f^4z^4 - 1328a^3b^5c^6d^ \\
& ^4e^5f^4z^4 - 1152a^7b^2c^5d^e^6f^3z^4 - 1152a^5b^2c^7d^3e^6f^ \\
& ^3z^4 - 1120a^6b^4c^4d^e^6f^3z^4 - 1120a^4b^4c^6d^3e^6f^3z^4 + 912 \\
& ^6a^6b^6c^2d^e^4f^5z^4 + 912a^2b^6c^6d^5e^4f^5z^4 + 872a^5b^6c^ \\
& ^3d^e^6f^3z^4 + 872a^3b^6c^5d^3e^6f^3z^4 + 768a^8b^2c^4d^e^4f^5 \\
& ^5z^4 + 768a^4b^2c^8d^5e^4f^5z^4 - 672a^8b^4c^2d^e^2f^7z^4 - 672 \\
& ^2a^2b^4c^8d^7e^2f^7z^4 - 624a^7b^5c^2d^e^3f^6z^4 - 624a^2b^5c^7 \\
& ^d^6e^3f^6z^4 + 480a^5b^8c^d^2e^2f^6z^4 + 480a^8b^8c^5d^6e^2f^2z \\
& ^4 + 316a^4b^7c^3d^e^7f^2z^4 + 316a^3b^7c^4d^2e^7f^2z^4 - 204a \\
& ^4b^8c^2d^e^6f^3z^4 - 204a^2b^8c^4d^3e^6f^3z^4 + 168a^3b^10c^d \\
& ^3e^2f^5z^4 + 168a^b^10c^3d^5e^2f^3z^4 + 156a^2b^11c^d^3e^3f^ \\
& ^4z^4 + 156a^b^11c^2d^4e^3f^3z^4 + 128a^9b^2c^3d^e^2f^7z^4 + 12 \\
& 8a^3b^2c^9d^7e^2f^7z^4 - 124a^3b^10c^d^2e^4f^4z^4 - 124a^b^10c \\
& ^3d^4e^4f^2z^4 + 100a^4b^9c^d^2e^3f^5z^4 + 100a^b^9c^4d^5e^3
\end{aligned}$$

$$\begin{aligned}
& f^2z^4 + 36a^5b^7c^2d^5e^5f^4z^4 + 36a^2b^7c^5d^4e^5f^4z^4 - 24a^3b^9c^2d^5e^7f^2z^4 - 24a^2b^{11}c^2d^2e^5f^3z^4 - 24a^2b^9c^3d^2e^7f^2z^4 - 24a^2b^{11}c^2d^3e^5f^2z^4 - 9216a^8b^5c^5d^3e^5f^6z^4 - 9216a^5b^5c^8d^6e^5f^3z^4 - 5376a^8b^5c^5d^5e^5f^4z^4 - 5376a^5b^5c^8d^4e^5f^4z^4 + 5120a^9b^5c^4d^2e^5f^7z^4 + 5120a^7b^5c^6d^4e^5f^5z^4 + 5120a^6b^5c^7d^5e^5f^4z^4 + 5120a^4b^5c^9d^7e^5f^2z^4 - 4352a^9b^5c^4d^5e^3f^6z^4 - 4352a^4b^5c^9d^6e^3f^5z^4 - 1792a^7b^5c^6d^5e^7f^2z^4 - 1792a^6b^5c^7d^2e^7f^2z^4 - 1600a^6b^2c^6d^5e^8f^2z^4 + 912a^5b^4c^5d^5e^8f^2z^4 + 768a^9b^3c^2d^5e^8f^2z^4 + 768a^2b^3c^9d^8e^8f^2z^4 - 720a^4b^9c^4d^6e^5f^3z^4 - 720a^2b^9c^4d^6e^5f^3z^4 - 656a^6b^7c^6d^2e^5f^7z^4 - 656a^2b^7c^6d^7e^5f^2z^4 - 240a^2b^{11}c^2d^4e^5f^5z^4 - 240a^2b^{11}c^2d^5e^5f^4z^4 + 216a^7b^6c^5d^2e^2f^7z^4 + 216a^2b^6c^7d^7e^2f^2z^4 - 204a^4b^6c^4d^5e^8f^2z^4 - 144a^5b^8c^2d^4e^4f^5z^4 - 144a^2b^8c^5d^5e^4f^2z^4 - 84a^2b^{12}c^2d^4e^2f^4z^4 + 36a^4b^9c^5d^5e^5f^4z^4 + 36a^2b^9c^4d^4e^5f^4z^4 + 20a^6b^7c^5d^3e^3f^6z^4 + 20a^2b^7c^6d^6e^3f^2z^4 + 16a^3b^{10}c^5d^6e^6f^3z^4 + 16a^3b^8c^3d^5e^8f^2z^4 + 16a^2b^{12}c^3d^3e^4f^3z^4 + 16a^2b^{10}c^3d^3e^6f^2z^4 + 48b^{11}c^3d^6e^5f^3z^4 + 48b^9c^5d^7e^5f^2z^4 - 20b^8c^6d^7e^2f^2z^4 + 8b^{10}c^4d^5e^4f^2z^4 - 4b^{13}c^2d^4e^3f^3z^4 - 4b^{11}c^3d^4e^5f^2z^4 + 4b^9c^5d^6e^3f^2z^4 + 3072a^9c^5d^5e^4f^5z^4 + 3072a^5c^9d^5e^4f^2z^4 + 2560a^8c^6d^5e^6f^3z^4 + 2560a^6c^8d^3e^6f^2z^4 + 1536a^{10}c^4d^2e^2f^7z^4 + 1536a^4c^{10}d^7e^2f^2z^4 + 48a^5b^9d^2e^5f^7z^4 + 48a^3b^{11}d^3e^5f^6z^4 - 20a^6b^8d^2e^2f^7z^4 + 8a^4b^{10}d^4e^4f^5z^4 + 4a^5b^9d^3e^3f^6z^4 - 4a^3b^{11}d^2e^5f^4z^4 - 4a^2b^{13}d^3e^3f^4z^4 + 768a^9b^5c^4e^5f^5z^4 + 768a^8b^5c^5e^7f^3z^4 + 256a^{10}b^3c^3e^3f^7z^4 - 192a^6b^3c^5e^9f^2z^4 - 68a^7b^6c^4e^4f^6z^4 + 48a^8b^5c^3e^3f^7z^4 + 48a^5b^5c^4e^9f^2z^4 + 36a^6b^7c^5e^5f^5z^4 - 12a^9b^4c^2e^2f^8z^4 - 4a^4b^9c^5e^7f^3z^4 - 4a^4b^7c^3e^9f^2z^4 + 384a^5b^8c^2d^3f^7z^4 + 384a^2b^8c^5d^7f^3z^4 + 288a^3b^{10}c^4d^4f^6z^4 + 288a^2b^{10}c^3d^6f^4z^4 + 224a^7b^6c^2d^2f^8z^4 + 224a^2b^6c^7d^8f^2z^4 - 192a^{10}b^2c^2d^2f^9z^4 - 192a^2b^2c^{10}d^9f^2z^4 + 768a^5b^5c^8d^3e^7z^4 + 768a^4b^5c^9d^5e^5z^4 + 256a^3b^5c^{10}d^7e^3z^4 - 192a^5b^3c^6d^5e^9z^4 - 68a^2b^6c^7d^6e^4z^4 + 48a^4b^5c^5d^5e^9z^4 + 48a^2b^5c^8d^7e^3z^4 + 36a^2b^7c^6d^5e^5z^4 - 12a^2b^4c^9d^8e^2z^4 - 4a^3b^7c^4d^5e^9z^4 - 4a^2b^9c^4d^3e^7z^4 + 16b^{13}c^2d^5e^5f^4z^4 + 16b^7c^7d^8e^5f^2z^4 + 768a^7c^7d^5e^8f^2z^4 + 16a^7b^7d^5e^8f^2z^4 + 16a^2b^{13}d^4e^5f^5z^4 + 256a^7b^5c^6e^9f^2z^4 + 80a^2b^{12}c^5d^5f^5z^4 + 48a^9b^4c^4d^4f^9z^4 + 48a^2b^4c^9d^9f^2z^4 + 256a^6b^5c^7d^5e^9z^4 - 42b^{10}c^4d^6e^2f^2z^4 - 20b^{12}c^2d^5e^2f^3z^4 + 6b^{12}c^2d^4e^4f^2z^4 + 4b^{11}c^3d^5e^3f^2z^4 - 24960a^7c^7d^4e^2f^4z^4 + 18944a^8c^6d^3e^2f^5z^4 + 18944a^6c^8d^5e^2f^3z^4 + 14336a^7c^7d^3e^4f^3z^4 - 9984a^8c^6d^2e^4f^4z^4 - 9984a^6c^8d^4e^4f^2z^4 - 7936a^9c^5d^2e^2f^6z^4 - 7936a^5c^9d^6e^2f^2z^4 - 4352a^7c^7d^2e^6f^2z^4 - 42a^4b^{10}d^2e^2f^6z^4 - 20a^2b^{12}d^3e^2f^5z^4 + 6a^2b^{12}d^2e^4f^4z^4 + 4a^3b^{11}d^2e^3f^5z^4 - 480a^8b^2c^4e^6f^4z^4 + 440a^7b^4c^3e^6f^4z^4 - 320a^8b^3c^3e^5f^5z^4 - 320a^7b^3c^4e^7f^3z^4 + 240a^8b^4c^2e^4f^6z^4 + 240a^6b^4c^4e^8f^2z^4 - 192a^9b^3c^2e^3f^7z^4 - 192a^9b^2c^3e^4f^6z^4 - 192a^7b^2c^5e^8f^2z^4 - 90a^6b^6c^2e^6f^4z^4 - 68a^5b^6c^3e^8f^2z^4 + 48a^{10}b^2c^2e^2f^8z^4 - 48a^7b^5c^2e^5f^5z^4 - 48a^6b^5c^3e^7f^3z^4 + 36a^5b^7c^2e^7f^3z^4 + 6a^4b^8c^2e^8f^2z^4 - 33920a^6b^2c^6d^5f^5z^4 + 27936a^5b^4c^5d^5f^5z^4 + 26112a^7b^2c^5d^4f^6z^4 + 26112a^5b^2c^7d^6f^4z^4 - 20352a^6b^4c^4d^4f^6z^4 - 20352a^4b^4c^6d^6f^4z^4 - 13080a^4b^6c^4d^5f^5z^4 - 11520a^8b^2c^4d^3f^7z^4 - 11520a^4b^2c^8d^7f^3z^4 + 8736a^5b^6c^3d^4f^6z^4 + 8736a^3b^6c^5d^6f^4z^4 + 7488a^7b^4c^3d^3f^7z^4 + 7488a^3b^4c^7d^7f^3z^4 + 3840a^3b^8c^3d^5f^5z^4 + 2560a^9b^2c^3d^2f^8z^4 + 2560a^3b^2c^9d^8f^2z^4 - 24
\end{aligned}$$

$$\begin{aligned}
& 16a^6b^6c^2d^3f^7z^4 - 2416a^2b^6c^6d^7f^3z^4 - 2160a^4b^8c^2d^4f^6z^4 - 2160a^2b^8c^4d^6f^4z^4 - 1152a^8b^4c^2d^2f^8z^4 \\
& - 1152a^2b^4c^8d^8f^2z^4 - 720a^2b^{10}c^2d^5f^5z^4 - 480a^4b^2c^8d^4e^6z^4 + 440a^3b^4c^7d^4e^6z^4 - 320a^4b^3c^7d^3e^7z^4 \\
& - 320a^3b^3c^8d^5e^5z^4 + 240a^4b^4c^6d^2e^8z^4 + 240a^2b^4c^8d^6e^4z^4 - 192a^5b^2c^7d^2e^8z^4 - 192a^3b^2c^9d^6e^4z^4 \\
& - 192a^2b^3c^9d^7e^3z^4 - 90a^2b^6c^6d^4e^6z^4 - 68a^3b^6c^5d^2e^8z^4 - 48a^3b^5c^6d^3e^7z^4 - 48a^2b^5c^7d^5e^5z^4 + \\
& 48a^2b^2c^{10}d^8e^2z^4 + 36a^2b^7c^5d^3e^7z^4 + 6a^2b^8c^4d^2e^8z^4 - 4b^6c^8d^9f^7z^4 + 256a^{11}c^3d^f^9z^4 + 256a^3c^{11}d^9f^7z^4 \\
& - 4a^8b^6d^f^9z^4 - 384a^9c^5e^6f^4z^4 - 256a^{10}c^4e^4f^6z^4 - 256a^8c^6e^8f^2z^4 - 64a^{11}c^3e^2f^8z^4 - 24b^{10}c^4d^7f^3z^4 \\
& - 16b^{12}c^2d^6f^4z^4 - 16b^8c^6d^8f^2z^4 + 17920a^7c^7d^5f^5z^4 - 14336a^8c^6d^4f^6z^4 - 14336a^6c^8d^6f^4z^4 + 7168a^9c^5d^3f^7z^4 \\
& + 7168a^5c^9d^7f^3z^4 - 2048a^{10}c^4d^2f^8z^4 - 2048a^4c^{10}d^8f^2z^4 + 6b^8c^6d^6e^4z^4 + 6a^6b^8e^4f^6z^4 - 4b^9c^5d^5e^5z^4 \\
& - 4b^7c^7d^7e^3z^4 - 4a^7b^7e^3f^7z^4 - 4a^5b^9e^5f^5z^4 - 384a^5c^9d^4e^6z^4 - 256a^6c^8d^2e^8z^4 - 256a^4c^{10}d^6e^4z^4 \\
& - 64a^3c^{11}d^8e^2z^4 - 24a^4b^{10}d^3f^7z^4 - 16a^6b^8d^2f^8z^4 - 16a^2b^{12}d^4f^6z^4 + 48a^6b^2c^6e^{10}z^4 - 12a^5b^4c^5e^{10}z^4 \\
& - 4b^{14}d^5f^5z^4 - 64a^7c^7e^{10}z^4 + b^{14}d^4e^2f^4z^4 + b^{10}c^4d^4e^6z^4 + b^6c^8d^8e^2z^4 + a^8b^6e^2f^8z^4 + a^4b^{10}e^6f^4z^4 \\
& + a^4b^6c^4e^{10}z^4 - 4820A^2B^2a^4b^6c^5d^2e^2f^4z^2 + 2976A^2B^2a^3b^6c^6d^3e^2f^3z^2 - 2328A^2B^2a^3b^6c^6d^2e^4f^2z^2 \\
& + 1848A^2B^2a^2b^4c^4d^3e^4f^4z^2 - 1768A^2B^2a^3b^4c^3d^2e^5f^5z^2 + 1528A^2B^2a^4b^2c^4d^2e^5f^5z^2 - 1136A^2B^2a^3b^2c^5d^3e^4f^4z^2 \\
& - 974A^2B^2a^4b^3c^3d^2e^2f^5z^2 + 692A^2B^2a^2b^6c^7d^4e^2f^2z^2 + 588A^2B^2a^6b^6c^3d^2e^3f^3z^2 - 580A^2B^2a^3b^3c^4d^4e^4f^3z^2 \\
& + 488A^2B^2a^3b^4c^3d^2e^3f^4z^2 - 444A^2B^2a^2b^2c^6d^2e^5f^5z^2 - 412A^2B^2a^6b^5c^4d^2e^4f^2z^2 + 366A^2B^2a^2b^6c^2d^2e^5f^5z^2 \\
& - 352A^2B^2a^2b^2c^6d^4e^5f^3z^2 + 326A^2B^2a^2b^4c^4d^4e^5f^2z^2 + 324A^2B^2a^6b^5c^4d^3e^2f^3z^2 - 302A^2B^2a^6b^3c^6d^4e^2f^2z^2 \\
& - 296A^2B^2a^6b^7c^2d^2e^2f^4z^2 + 122A^2B^2a^4b^2c^4d^4e^3f^4z^2 - 122A^2B^2a^2b^6c^2d^2e^3f^4z^2 - 84A^2B^2a^3b^2c^5d^2e^5f^2z^2 + 72A^2B^2a^6b^4c^5d^3e^3f^2z^2 \\
& - 64A^2B^2a^2b^5c^3d^2e^4f^3z^2 + 60A^2B^2a^3b^5c^2d^2e^2f^5z^2 + 1312A^2B^2a^5b^6c^4d^2e^2f^5z^2 + 1040A^2B^2a^4b^6c^5d^4e^4f^3z^2 \\
& - 500A^2B^2a^6b^6c^3d^3e^4f^4z^2 - 376A^2B^2a^6b^2c^7d^5e^4f^2z^2 + 276A^2B^2a^4b^4c^2d^2e^6f^6z^2 - 262A^2B^2a^2b^3c^5d^2e^6f^6z^2 \\
& + 238A^2B^2a^6b^2c^7d^4e^3f^4z^2 + 232A^2B^2a^5b^2c^3d^2e^6f^6z^2 - 176A^2B^2a^2b^6c^7d^3e^4f^4z^2 - 120A^2B^2a^6b^6c^3d^2e^5f^2z^2 - 108A^2B^2a^6b^4c^5d^4e^5f^3z^2 \\
& + 68A^2B^2a^6b^7c^2d^2e^4f^3z^2 + 68A^2B^2a^6b^4c^5d^2e^5f^3z^2 + 46A^2B^2a^2b^7c^3d^2e^2f^5z^2 - 36A^2B^2a^6b^3c^6d^3e^4f^4z^2 \\
& - 1932A^2B^2a^2b^3c^5d^3e^2f^3z^2 - 1818A^2B^2a^2b^4c^4d^2e^3f^3z^2 + 1620A^2B^2a^3b^3c^4d^2e^2f^4z^2 + 1560A^2B^2a^2b^3c^5d^2e^4f^2z^2 \\
& + 1244A^2B^2a^3b^2c^5d^2e^3f^3z^2 + 820A^2B^2a^2b^2c^6d^3e^3f^2z^2 + 480A^2B^2a^2b^5c^3d^2e^2f^4z^2 + 352A^2B^2a^3b^6c^6d^2e^6f^6z^2 \\
& - 108A^2B^2a^3b^6c^6d^2e^6f^6z^2 + 82A^2B^2a^6b^5c^4d^4e^6f^6z^2 - 64A^2B^2a^6b^8c^8d^5e^2f^4z^2 + 16A^2B^2a^6b^8c^8d^2e^5f^5z^2 - 4A^2B^2a^6b^8c^8d^2e^3f^4z^2 \\
& + 16B^2a^6b^8c^8d^6e^6f^6z^2 + 56A^2B^2a^6b^2c^8d^6e^6f^6z^2 - 8A^2B^2a^6b^9c^8d^4e^4f^3z^2 - 8A^2B^2a^6b^7c^3d^2e^6f^6z^2 - 800A^2B^2a^6c^4d^4e^6f^6z^2 \\
& + 10A^2B^2a^2b^8d^4e^6f^6z^2 - 6A^2B^2a^6b^9d^2e^2f^5z^2 - 12A^2B^2a^5b^4c^6e^7f^7z^2 + 912A^2B^2a^6b^6c^3d^2f^7z^2 + 192A^2B^2a^4b^5c^6d^4f^7z^2 \\
& + 192A^2B^2a^6b^6c^8d^6f^2z^2 - 20A^2B^2a^6b^4c^5d^2e^7z^2 + 4A^2B^2a^6b^6c^8d^4e^4z^2 + 2144B^2a^4b^6c^5d^3e^4f^4z^2 - 1120B^2a^3b^6c^6d^4e^4f^3z^2 \\
& - 688B^2a^5b^6c^4d^2e^5f^5z^2 - 256B^2a^3b^6c^6d^2e^5f^5z^2 + 152B^2a^6b^3c^6d^5e^5f^2z^2 + 120B^2a^5b^3c^2d^2e^6f^6z^2 - 116B^2a^5b^6c^4d^4e^3f^4z^2 \\
& + 110B^2a^6b^7c^2d^3e^4f^4z^2 - 80B^2a^2b^6c^7d^5e^4f^2z^2 - 72B^2a^6b^5c^4d^4e^4f^3z^2 - 48B^2a^4b^6c^5d^2e^5f^2z^2 - 46B^2a^6b^3c^6d^4e^3f^4z^2 - 44B^2a^6b^4c^5d^3e^6f^6z^2
\end{aligned}$$

$$\begin{aligned}
& ^4fz^2 - 34B^2a^2b^5c^4d^2e^5fz^2 + 20B^2a^2b^6c^7d^4e^3fz^2 \\
& - 10B^2a^3b^6c^4d^2e^5fz^2 - 10B^2a^2b^7c^4d^2e^5fz^2 - 10B^2a^2b^2c^7d^5e^2fz^2 - 7B^2a^2b^4c^4d^2e^6fz^2 - 6B^2a^3b^2c^5 \\
& *d^2e^6fz^2 + 4B^2a^2b^8c^4d^2e^2f^4z^2 - 2B^2a^2b^7c^4d^2e^3f^4z^2 \\
& + 3196A^2a^4b^3c^3d^2e^5f^6z^2 - 3184A^2a^4b^3c^5d^2e^5f^5z^2 + 1 \\
& 568A^2a^3b^3c^6d^3e^5f^4z^2 + 1504A^2a^3b^3c^6d^2e^5f^2z^2 - 656A^2 \\
& a^4b^3c^3d^2e^5f^6z^2 - 400A^2a^2b^6c^3d^2e^4f^3z^2 + 314A^2a^2b^5 \\
& c^4d^2e^5f^2z^2 - 264A^2a^3b^5c^2d^2e^5f^6z^2 + 240A^2a^2b^2c^6d^2 \\
& e^6fz^2 - 224A^2a^2b^7c^4d^4e^5f^3z^2 + 216A^2a^2b^5c^4d^3e^5f^4 \\
& z^2 - 192A^2a^2b^7c^4d^2e^5f^5z^2 + 178A^2a^2b^7c^2d^2e^3f^4z^2 - \\
& 154A^2a^2b^7c^2d^2e^5f^5z^2 + 128A^2a^2b^3c^6d^4e^5f^3z^2 + 106A^2 \\
& a^2b^3c^6d^2e^5f^5z^2 - 12A^2a^2b^2c^7d^3e^4f^4z^2 - 58A^2a^2b^8c^2d^2 \\
& e^3f^3z^2 + 40A^2a^2b^7c^3d^2e^4f^2z^2 - 28A^2a^2b^7c^3d^3e^2f^3 \\
& z^2 - 24A^2a^2b^5c^5d^4e^2f^2z^2 - 20A^2a^2b^6c^4d^3e^3f^2z^2 + \\
& 2768A^2a^2b^4c^6d^2e^3f^3z^2 - 1712A^2a^2b^3c^7d^3e^3f^2z^2 - 156A^2 \\
& a^2b^4c^4e^5f^3z^2 + 146A^2a^4b^3c^3e^4f^4z^2 - 106A^2a^5b^2c^3e^3f^5 \\
& z^2 + 90A^2a^5b^3c^2e^2f^6z^2 + 38A^2a^3b^3c^4e^6f^2z^2 - 36A^2a^3b^5c^2 \\
& e^4f^4z^2 + 16A^2a^3b^4c^3e^5f^3z^2 - 9A^2a^4b^4c^2e^3f^5z^2 - 8A^2a^2b^5c^3 \\
& e^6f^2z^2 + 2A^2a^2b^6c^2e^5f^3z^2 + 920A^2a^4b^3c^3d^2e^2f^6z^2 - 480A^2a^2b^5c^3 \\
& d^3e^2f^5z^2 - 336A^2a^2b^3c^5d^4e^4f^4z^2 - 272A^2a^3b^3c^4d^3 \\
& e^5f^5z^2 + 240A^2a^3b^5c^2d^2e^2f^6z^2 - 32A^2a^3c^9d^6e^5fz^2 - 792 \\
& B^2a^2b^3c^5d^3e^3f^2z^2 + 714B^2a^2b^4c^4d^3e^2f^3z^2 - 57 \\
& 2B^2a^3b^2c^5d^3e^2f^3z^2 - 475B^2a^2b^2c^6d^4e^2f^2z^2 + 2 \\
& 65B^2a^4b^2c^4d^2e^2f^4z^2 + 260B^2a^3b^3c^4d^2e^3f^3z^2 - \\
& 212B^2a^3b^4c^3d^2e^2f^4z^2 + 180B^2a^3b^2c^5d^2e^4f^2z^2 - \\
& 158B^2a^2b^4c^4d^2e^4f^2z^2 + 47B^2a^2b^6c^2d^2e^2f^4z^2 + \\
& 16B^2a^2b^5c^3d^2e^3f^3z^2 + 2752A^2a^3b^2c^5d^2e^2f^4z^2 \\
& - 2148A^2a^2b^4c^4d^2e^2f^4z^2 + 2064A^2a^2b^3c^5d^2e^3f^3z^2 \\
& - 424A^2a^2b^2c^6d^3e^2f^3z^2 - 198A^2a^2b^2c^6d^2e^4f^2z^2 \\
& - 272B^2a^6b^3c^3d^2e^5f^6z^2 - 24B^2a^4b^5c^4d^2e^5f^6z^2 + 1808A^2 \\
& a^5b^3c^4d^2e^5f^6z^2 - 244A^2a^2b^7c^8d^4e^3f^5z^2 + 208A^2a^2b^7c^8 \\
& d^5e^5f^2z^2 + 134A^2a^2b^7c^4d^2e^5f^6z^2 - 76A^2a^2b^4c^5d^2e^6f^5z^2 \\
& + 4A^2a^2b^8c^4d^2e^2f^5z^2 + 148A^2a^2b^4c^6d^5e^5f^2z^2 + 65A^2a^2b^6 \\
& c^4d^4e^5f^3z^2 + 46A^2a^2b^8c^2d^3e^5f^4z^2 - 38A^2a^2b^3c^7d^5e^2 \\
& f^5z^2 + 34A^2a^2b^9c^4d^2e^2f^4z^2 - 29A^2a^2b^4c^6d^4e^3f^5z^2 + 20A^2 \\
& a^2b^5c^5d^3e^4f^5z^2 + 12A^2a^2b^8c^2d^2e^5f^2z^2 - 7A^2a^2b^6c^4d^2 \\
& e^5f^5z^2 - 2880A^2a^4c^6d^3e^5f^4z^2 + 2784A^2a^5c^5d^2e^5f^5z^2 \\
& - 1112A^2a^5c^5d^2e^3f^4z^2 + 896A^2a^3c^7d^4e^5f^3z^2 + 848A^2 \\
& a^3c^7d^2e^5f^5z^2 - 560A^2a^4c^6d^2e^5f^2z^2 + 96A^2a^2c^8d^5 \\
& e^5f^2z^2 - 88A^2a^2c^8d^4e^3f^5z^2 - 100A^2a^6b^3c^3e^2f^6z^2 \\
& - 76A^2a^5b^3c^4e^4f^4z^2 + 48A^2a^6b^2c^2e^5f^7z^2 - 42A^2a^3b^2 \\
& c^5e^7f^5z^2 + 36A^2a^4b^3c^5e^6f^2z^2 - 24A^2a^4b^5c^2e^2f^6 \\
& z^2 + 10A^2a^3b^6c^3e^3f^5z^2 + 7A^2a^2b^4c^4e^7f^5z^2 + 2A^2a^2b^7 \\
& c^4e^4f^4z^2 - 2496A^2a^5b^3c^4d^2e^2f^6z^2 + 1872A^2a^4b^3c^5 \\
& d^3e^5f^5z^2 - 744A^2a^5b^3c^2d^2e^7f^7z^2 - 720A^2a^2b^3c^7d^5f^3z^2 \\
& + 504A^2a^3b^3c^6d^5f^3z^2 + 256A^2a^3b^3c^6d^4f^4z^2 + 168A^2a^2b^7 \\
& c^2d^3e^5f^5z^2 - 144A^2a^2b^7c^4d^2e^6f^6z^2 + 144A^2a^2b^5c^4d^4 \\
& e^4f^4z^2 + 66A^2a^2b^2c^6d^2e^7f^7z^2 - 36A^2a^2b^2c^7d^3e^5f^5z^2 + \\
& 20A^2a^2b^3c^6d^2e^6f^6z^2 + 12A^2a^2b^3c^7d^2e^6f^6z^2 + 1208B^2a^3 \\
& b^3c^6d^3e^3f^2z^2 - 848B^2a^3b^3c^4d^3e^5f^4z^2 + 672B^2a^2b^3 \\
& c^5d^4e^5f^3z^2 - 632B^2a^4b^3c^5d^2e^3f^3z^2 + 432B^2a^4b^3c^3 \\
& d^2e^5f^5z^2 + 276B^2a^2b^2c^6d^3e^4f^4z^2 - 196B^2a^2b^6c^3d^3 \\
& e^2f^3z^2 - 168B^2a^2b^5c^3d^3e^5f^4z^2 + 154B^2a^2b^3c^5d^2e^5 \\
& f^5z^2 + 148B^2a^2b^5c^4d^3e^3f^2z^2 + 96B^2a^2b^4c^5d^4e^2f^2 \\
& z^2 - 72B^2a^3b^5c^2d^2e^5f^5z^2 + 70B^2a^5b^2c^3d^2e^2f^5z^2 \\
& - 60B^2a^4b^3c^3d^2e^3f^4z^2 + 52B^2a^2b^6c^3d^2e^4f^2z^2 + 36 \\
& B^2a^4b^2c^4d^2e^4f^3z^2 - 32B^2a^2b^7c^2d^2e^3f^3z^2 + 24B^2a^3 \\
& b^5c^2d^2e^3f^4z^2 + 15B^2a^4b^4c^2d^2e^2f^5z^2 - 8B^2a^3b^
\end{aligned}$$

$$\begin{aligned}
& 4*c^3*d*e^4*f^3*z^2 + 8*B^2*a^2*b^5*c^3*d*e^5*f^2*z^2 - 2*B^2*a^3*b^3*c^4*d \\
& *e^5*f^2*z^2 - 2*B^2*a^2*b^6*c^2*d*e^4*f^3*z^2 - 3176*A^2*a^3*b^6*c^4*d^2*e^3 \\
& *f^3*z^2 - 2252*A^2*a^4*b^2*c^4*d*e^2*f^5*z^2 + 1952*A^2*a^3*b^4*c^3*d*e^2* \\
& f^5*z^2 - 1496*A^2*a^3*b^3*c^4*d*e^3*f^4*z^2 + 1378*A^2*a^2*b^4*c^4*d*e^4*f \\
& ^3*z^2 + 1184*A^2*a^3*b^3*c^4*d^2*e*f^5*z^2 - 1166*A^2*a^2*b^3*c^5*d*e^5*f^ \\
& 2*z^2 - 1164*A^2*a^3*b^2*c^5*d*e^4*f^3*z^2 - 1152*A^2*a^2*b^3*c^5*d^3*e*f^4 \\
& *z^2 + 578*A^2*a*b^6*c^3*d^2*e^2*f^4*z^2 - 548*A^2*a*b^5*c^4*d^2*e^3*f^3*z^ \\
& 2 + 440*A^2*a*b^2*c^7*d^4*e^2*f^2*z^2 - 412*A^2*a^2*b^6*c^2*d*e^2*f^5*z^2 - \\
& 360*A^2*a*b^3*c^6*d^3*e^3*f^2*z^2 + 312*A^2*a*b^4*c^5*d^3*e^2*f^3*z^2 + 24 \\
& 8*A^2*a^2*b^6*c^7*d^3*e^3*f^2*z^2 - 224*A^2*a^2*b^5*c^3*d*e^3*f^4*z^2 + 216*A \\
& ^2*a^2*b^5*c^3*d^2*e*f^5*z^2 + 52*A^2*a*b^4*c^5*d^2*e^4*f^2*z^2 - 16*B^2*b^ \\
& 3*c^7*d^6*e*f*z^2 - 14*B^2*b^9*c*d^3*e*f^4*z^2 + 32*B^2*a^4*c^6*d*e^6*f*z^2 \\
& - 20*A^2*b^9*c*d*e^3*f^4*z^2 + 18*A^2*b^9*c*d^2*e*f^5*z^2 + 8*A^2*b^6*c^4* \\
& d*e^6*f*z^2 - 360*A^2*a^3*c^7*d*e^6*f*z^2 + 136*A^2*a*c^9*d^5*e^2*f*z^2 + 2 \\
& *B^2*a^3*b^7*d*e*f^6*z^2 + 2*B^2*a*b^9*d^2*e*f^5*z^2 + 12*B^2*a^4*b^6*c^5*e^7 \\
& *f*z^2 - 204*A^2*a^3*b^6*c^6*e^7*f*z^2 - 128*A^2*a^6*b^6*c^3*e*f^7*z^2 - 48*A^2 \\
& *a*b^5*c^4*e^7*f*z^2 - 36*B^2*a^5*b^4*c*d*f^7*z^2 - 24*A^2*a^4*b^5*c*e*f^7* \\
& z^2 - 16*B^2*a*b^8*c*d^3*f^5*z^2 - 164*A^2*a^3*b^6*c*d*f^7*z^2 - 16*A^2*a*b \\
& ^8*c*d^2*f^6*z^2 + 4*B^2*a^3*b^6*c^6*d*e^7*z^2 - 4*B^2*a*b^8*c^8*d^5*e^3*z^2 + \\
& 48*A^2*a*b^8*c^8*d^3*e^5*z^2 + 36*A^2*a^2*b^6*c^7*d*e^7*z^2 - 6*A^2*a*b^3*c^6*d \\
& *e^7*z^2 + 136*A*B*a^6*c^4*e^3*f^5*z^2 - 96*A*B*b^5*c^5*d^5*f^3*z^2 + 80*A* \\
& B*a^5*c^5*e^5*f^3*z^2 - 72*A*B*b^3*c^7*d^6*f^2*z^2 - 24*A*B*b^7*c^3*d^4*f^4 \\
& *z^2 + 14*A*B*b^3*c^7*d^4*e^4*z^2 - 14*A*B*b^2*c^8*d^5*e^3*z^2 - 2*A*B*b^5* \\
& c^5*d^2*e^6*z^2 - 2*A*B*b^4*c^6*d^3*e^5*z^2 + 2*A*B*a^3*b^7*e^2*f^6*z^2 - A \\
& *B*a^2*b^8*e^3*f^5*z^2 + 16*A*B*a^2*c^8*d^3*e^5*z^2 - 2*A*B*a^2*b^3*c^5*e^8 \\
& *z^2 + 22*B^2*b^8*c^2*d^3*e^2*f^3*z^2 - 12*B^2*b^7*c^3*d^3*e^3*f^2*z^2 + 12 \\
& *B^2*b^6*c^4*d^4*e^2*f^2*z^2 - 6*B^2*b^8*c^2*d^2*e^4*f^2*z^2 - 864*B^2*a^4* \\
& c^6*d^3*e^2*f^3*z^2 + 496*B^2*a^3*c^7*d^4*e^2*f^2*z^2 + 224*B^2*a^5*c^5*d^2 \\
& *e^2*f^4*z^2 + 136*B^2*a^4*c^6*d^2*e^4*f^2*z^2 - 53*A^2*b^8*c^2*d^2*e^2*f^4 \\
& *z^2 + 52*A^2*b^7*c^3*d^2*e^3*f^3*z^2 + 52*A^2*b^5*c^5*d^3*e^3*f^2*z^2 - 36 \\
& *A^2*b^6*c^4*d^3*e^2*f^3*z^2 - 12*A^2*b^4*c^6*d^4*e^2*f^2*z^2 - 9*A^2*b^6*c \\
& ^4*d^2*e^4*f^2*z^2 + 836*A^2*a^4*c^6*d^2*e^2*f^4*z^2 - 668*A^2*a^2*c^8*d^4* \\
& e^2*f^2*z^2 + 656*A^2*a^3*c^7*d^2*e^4*f^2*z^2 + 368*A^2*a^3*c^7*d^3*e^2*f^3 \\
& *z^2 - 45*B^2*a^6*b^2*c^2*e^2*f^6*z^2 - 18*B^2*a^5*b^2*c^3*e^4*f^4*z^2 - 9* \\
& B^2*a^4*b^2*c^4*e^6*f^2*z^2 - 6*B^2*a^5*b^3*c^2*e^3*f^5*z^2 + 3*B^2*a^4*b^4 \\
& *c^2*e^4*f^4*z^2 - 2*B^2*a^4*b^3*c^3*e^5*f^3*z^2 - 580*B^2*a^4*b^2*c^4*d^3* \\
& f^5*z^2 + 536*B^2*a^3*b^4*c^3*d^3*f^5*z^2 + 471*A^2*a^4*b^2*c^4*e^4*f^4*z^2 \\
& - 436*A^2*a^3*b^4*c^3*e^4*f^4*z^2 - 348*B^2*a^4*b^4*c^2*d^2*f^6*z^2 + 316* \\
& B^2*a^2*b^2*c^6*d^5*f^3*z^2 + 310*A^2*a^3*b^3*c^4*e^5*f^3*z^2 + 232*A^2*a^5 \\
& *b^2*c^3*e^2*f^6*z^2 - 229*A^2*a^2*b^4*c^4*e^6*f^2*z^2 - 216*A^2*a^4*b^4*c^ \\
& 2*e^2*f^6*z^2 + 204*A^2*a^4*b^3*c^3*e^3*f^5*z^2 + 200*B^2*a^5*b^2*c^3*d^2*f \\
& ^6*z^2 + 150*A^2*a^3*b^2*c^5*e^6*f^2*z^2 - 120*B^2*a^2*b^4*c^4*d^4*f^4*z^2 \\
& + 91*A^2*a^2*b^6*c^2*e^4*f^4*z^2 + 72*A^2*a^3*b^5*c^2*e^3*f^5*z^2 - 66*B^2* \\
& a^2*b^6*c^2*d^3*f^5*z^2 + 44*A^2*a^2*b^5*c^3*e^5*f^3*z^2 - 16*B^2*a^3*b^2*c \\
& ^5*d^4*f^4*z^2 + 1952*A^2*a^4*b^2*c^4*d^2*f^6*z^2 - 1792*A^2*a^3*b^2*c^5*d^ \\
& 3*f^5*z^2 - 1272*A^2*a^3*b^4*c^3*d^2*f^6*z^2 + 976*A^2*a^2*b^2*c^6*d^4*f^4* \\
& z^2 + 960*A^2*a^2*b^4*c^4*d^3*f^5*z^2 + 282*A^2*a^2*b^6*c^2*d^2*f^6*z^2 - 4 \\
& 5*B^2*a^2*b^2*c^6*d^2*e^6*z^2 - 48*A^2*b^6*c^9*d^6*e*f*z^2 - 14*A^2*a*b^9*d*e \\
& *f^6*z^2 - 7*A*B*b^10*d^2*e*f^5*z^2 + 2*A*B*b^10*d*e^3*f^4*z^2 - 64*A*B*a^7 \\
& *c^3*e*f^7*z^2 - 16*A*B*b^9*c*d^3*f^5*z^2 + 8*A*B*a^4*c^6*e^7*f*z^2 + 4*A*B \\
& *b^9*d^6*e^2*z^2 + 2*A*B*b^6*c^4*d*e^7*z^2 - 120*A*B*a^3*c^7*d*e^7*z^2 - \\
& 16*A*B*a^3*b^7*d*f^7*z^2 + 16*A*B*a*b^9*d^2*f^6*z^2 + 8*A*B*a*c^9*d^5*e^3*z \\
& ^2 + 12*A*B*a^3*b^6*c^6*e^8*z^2 - 48*B^2*b^5*c^5*d^5*e*f^2*z^2 + 15*B^2*b^4*c \\
& ^6*d^5*e^2*f*z^2 - 14*B^2*b^7*c^3*d^4*e*f^3*z^2 + 4*B^2*b^9*c*d^2*e^3*f^3*z \\
& ^2 + 4*B^2*b^7*c^3*d^2*e^5*f*z^2 + 4*B^2*b^5*c^5*d^4*e^3*f*z^2 - B^2*b^6*c^ \\
& 4*d^3*e^4*f*z^2 - 336*B^2*a^3*c^7*d^3*e^4*f*z^2 + 112*B^2*a^5*c^5*d*e^4*f^3 \\
& *z^2 - 112*A^2*b^3*c^7*d^5*e*f^2*z^2 + 80*B^2*a^6*c^4*d*e^2*f^5*z^2 - 48*A^ \\
& 2*b^5*c^5*d^4*e*f^3*z^2 + 36*A^2*b^8*c^2*d*e^4*f^3*z^2 + 36*A^2*b^3*c^7*d^4 \\
& *e^3*f*z^2 - 28*A^2*b^7*c^3*d*e^5*f^2*z^2 + 20*A^2*b^2*c^8*d^5*e^2*f*z^2 +
\end{aligned}$$

$$\begin{aligned}
& 16*B^2*a^2*c^8*d^5*e^2*f*z^2 - 14*A^2*b^7*c^3*d^3*e*f^4*z^2 - 14*A^2*b^4*c^6*d^3*e^4*f*z^2 - 10*A^2*b^5*c^5*d^2*e^5*f*z^2 - 1008*A^2*a^4*c^6*d*e^4*f^3 \\
& *z^2 - 760*A^2*a^5*c^5*d*e^2*f^5*z^2 + 272*A^2*a^2*c^8*d^3*e^4*f*z^2 + 48*B^2*a^5*b*c^4*e^5*f^3*z^2 + 36*B^2*a^6*b*c^3*e^3*f^5*z^2 + 12*B^2*a^5*b^4*c* \\
& e^2*f^6*z^2 - 624*A^2*a^4*b*c^5*e^5*f^3*z^2 - 548*A^2*a^5*b*c^4*e^3*f^5*z^2 + 182*A^2*a^2*b^3*c^5*e^7*f*z^2 - 180*B^2*a*b^4*c^5*d^5*f^3*z^2 + 132*B^2*a^6*b^2*c^2*d*f^7*z^2 + 108*B^2*a^3*b^6*c*d^2*f^6*z^2 + 96*A^2*a^5*b^3*c^2* \\
& e*f^7*z^2 + 68*A^2*a*b^6*c^3*e^6*f^2*z^2 + 58*A^2*a^3*b^6*c*e^2*f^6*z^2 - 56*B^2*a*b^2*c^7*d^6*f^2*z^2 - 38*A^2*a^2*b^7*c*e^3*f^5*z^2 - 36*A^2*a*b^7*c^2*e^5*f^3*z^2 + 20*B^2*a*b^6*c^3*d^4*f^4*z^2 - 736*A^2*a^5*b^2*c^3*d*f^7*z^2 + 624*A^2*a^4*b^4*c^2*d*f^7*z^2 - 416*A^2*a*b^2*c^7*d^5*f^3*z^2 - 276*A^2*a*b^4*c^5*d^4*f^4*z^2 - 196*A^2*a*b^6*c^3*d^3*f^5*z^2 + 8*B^2*a*b^4*c^5*d^2*e^6*z^2 + 6*B^2*a*b^2*c^7*d^4*e^4*z^2 + 2*B^2*a^2*b^3*c^5*d*e^7*z^2 + 2*B^2*a*b^3*c^6*d^3*e^5*z^2 - 18*A^2*a*b^2*c^7*d^2*e^6*z^2 - 16*A*B*b*c^9*d^7*f*z^2 - B^2*b^10*d^2*e^2*f^4*z^2 + 48*B^2*a^7*c^3*e^2*f^6*z^2 - 36*B^2*a^6*c^4*e^4*f^4*z^2 + 31*B^2*b^6*c^4*d^5*f^3*z^2 - 24*B^2*a^5*c^5*e^6*f^2*z^2 + 20*B^2*b^4*c^6*d^6*f^2*z^2 - 6*A^2*b^8*c^2*e^6*f^2*z^2 + 2*B^2*b^8*c^2*d^4*f^4*z^2 - 768*B^2*a^5*c^5*d^3*f^5*z^2 + 512*B^2*a^6*c^4*d^2*f^6*z^2 + 512*B^2*a^4*c^6*d^4*f^4*z^2 + 232*A^2*a^5*c^5*e^4*f^4*z^2 + 188*A^2*a^4*c^6*e^6*f^2*z^2 - 128*B^2*a^3*c^7*d^5*f^3*z^2 + 92*A^2*a^6*c^4*e^2*f^6*z^2 + 80*A^2*b^4*c^6*d^5*f^3*z^2 + 64*A^2*b^2*c^8*d^6*f^2*z^2 + 31*A^2*b^6*c^4*d^4*f^4*z^2 + 14*A^2*b^8*c^2*d^3*f^5*z^2 - 5*B^2*b^4*c^6*d^4*e^4*z^2 + 4*B^2*b^3*c^7*d^5*e^3*z^2 + 2*B^2*b^5*c^5*d^3*e^5*z^2 - B^2*b^6*c^4*d^2*e^6*z^2 - B^2*b^2*c^8*d^6*e^2*z^2 - B^2*a^4*b^6*e^2*f^6*z^2 - 1152*A^2*a^3*c^7*d^4*f^4*z^2 + 1008*A^2*a^4*c^6*d^3*f^5*z^2 + 624*A^2*a^2*c^8*d^5*f^3*z^2 - 288*A^2*a^5*c^5*d^2*f^6*z^2 + 56*B^2*a^3*c^7*d^2*e^6*z^2 - 10*B^2*a^2*b^8*d^2*f^6*z^2 - 9*A^2*b^2*c^8*d^4*e^4*z^2 - 5*A^2*a^2*b^8*e^2*f^6*z^2 - 4*B^2*a^2*c^8*d^4*e^4*z^2 + 3*A^2*b^4*c^6*d^2*e^6*z^2 - 2*A^2*b^3*c^7*d^3*e^5*z^2 - 36*A^2*a^2*c^8*d^2*e^6*z^2 - 48*A^2*a^6*b^2*c^2*f^8*z^2 - 45*A^2*a^2*b^2*c^6*e^8*z^2 + 4*A^2*b^10*d*e^2*f^5*z^2 + 4*B^2*b^2*c^8*d^7*f*z^2 + 4*A^2*b^9*c*e^5*f^3*z^2 + 4*A^2*b^7*c^3*e^7*f*z^2 - 128*B^2*a^7*c^3*d*f^7*z^2 - 160*A^2*a*c^9*d^6*f^2*z^2 - 112*A^2*a^6*c^4*d*f^7*z^2 + 12*A^2*b*c^9*d^5*e^3*z^2 + 4*A^2*a*b^9*e^3*f^5*z^2 + 3*B^2*a^4*b^6*d*f^7*z^2 + 2*A^2*a^3*b^7*e*f^7*z^2 - 24*A^2*a*c^9*d^4*e^4*z^2 + 14*A^2*a^2*b^8*d*f^7*z^2 + 12*A^2*a^5*b^4*c*f^8*z^2 + 12*A^2*a*b^4*c^5*e^8*z^2 + A*B*a^4*b^6*e*f^7*z^2 + B^2*a^2*b^8*d*e^2*f^5*z^2 + 16*A^2*c^10*d^7*f*z^2 + 3*B^2*b^10*d^3*f^5*z^2 - A^2*b^10*e^4*f^4*z^2 - 4*A^2*c^10*d^6*e^2*z^2 - A^2*b^10*d^2*f^6*z^2 + 64*A^2*a^7*c^3*f^8*z^2 - 4*B^2*a^4*c^6*e^8*z^2 - A^2*b^6*c^4*e^8*z^2 + 48*A^2*a^3*c^7*e^8*z^2 - A^2*a^4*b^6*f^8*z^2 + 720*A^2*B*a*b^2*c^5*d^2*e^2*f^3*z - 600*A^2*B*a^2*b^2*c^4*d*e^2*f^4*z + 576*A*B^2*a^2*b^2*c^4*d^2*e*f^4*z + 348*A*B^2*a*b^2*c^5*d^2*e^3*f^2*z - 336*A*B^2*a^2*b*c^5*d^2*e^2*f^3*z - 260*A*B^2*a*b^3*c^4*d^2*e^2*f^3*z - 240*A*B^2*a^2*b^2*c^4*d*e^3*f^3*z + 196*A*B^2*a^2*b^3*c^3*d*e^2*f^4*z + 172*A^2*B*a*b*c^6*d*e^5*f*z + 20*A*B^2*a*b^6*c*d*e*f^5*z - 912*A^2*B*a^2*b*c^5*d^2*e*f^4*z - 644*A^2*B*a*b*c^6*d^2*e^3*f^2*z - 432*A*B^2*a*b^2*c^5*d^3*e*f^3*z + 372*A^2*B*a^2*b*c^5*d*e^3*f^3*z - 330*A^2*B*a*b^2*c^5*d*e^4*f^2*z + 312*A*B^2*a*b*c^6*d^3*e^2*f^2*z - 208*A*B^2*a^3*b^2*c^3*d*e*f^5*z + 192*A^2*B*a^2*b^3*c^3*d*e*f^5*z + 172*A^2*B*a*b^3*c^4*d*e^3*f^3*z + 108*A*B^2*a^2*b*c^5*d*e^4*f^2*z + 104*A*B^2*a^3*b*c^4*d*e^2*f^4*z - 80*A^2*B*a*b^3*c^4*d^2*e*f^4*z + 68*A^2*B*a*b^4*c^3*d*e^2*f^4*z - 60*A*B^2*a*b^5*c^2*d*e*f^5*z + 20*B^3*a*b*c^6*d^3*e^3*f*z + 16*B^3*a^4*b*c^3*d*e*f^5*z - 16*B^3*a*b*c^6*d^4*e*f^2*z + 12*B^3*a^2*b*c^5*d*e^5*f*z + 320*A^3*a*b*c^6*d*e^4*f^2*z + 40*A^3*a*b^4*c^3*d*e*f^5*z - 48*A^2*B*b*c^7*d^4*e*f^2*z - 44*A^2*B*b^3*c^5*d*e^5*f*z - 20*A*B^2*b*c^7*d^4*e^2*f*z + 14*A*B^2*b^4*c^4*d*e^5*f*z + 12*A^2*B*b*c^7*d^3*e^3*f*z + 4*A*B^2*b^7*c*d*e^2*f^4*z + 160*A*B^2*a^4*c^4*d*e*f^5*z + 152*A^2*B*a*c^7*d^2*e^4*f*z - 40*A*B^2*a*c^7*d^3*e^3*f*z + 32*A
\end{aligned}$$

$$\begin{aligned}
& B^2 a^c^7 d^4 e f^2 z - 16 A B^2 a^2 c^6 d^5 e^5 f^2 z + 128 A^2 B a^4 b^c^3 e^6 f^6 z + 42 A^2 B a^2 b^2 c^5 e^6 f^6 z + 24 A^2 B a^2 b^5 c^e f^6 z - 12 A B^2 a^3 b^4 c^e f^6 z - 12 A B^2 a^2 b^c^5 e^6 f^6 z - 10 A^2 B a^2 b^6 c^e^2 f^5 z \\
& - 160 A B^2 a^2 b^c^6 d^4 f^3 z + 112 A B^2 a^4 b^c^3 d^4 f^6 z - 24 A B^2 a^2 b^5 c^d f^6 z - 84 B^3 a^2 b^2 c^5 d^3 e^2 f^2 z - 80 B^3 a^2 b^3 c^3 d^2 e f^4 z - 60 B^3 a^2 b^c^5 d^2 e^3 f^2 z - 20 B^3 a^3 b^2 c^3 d^e^2 f^4 z - 2 \\
& 0 B^3 a^2 b^3 c^4 d^2 e^3 f^2 z - 9 B^3 a^2 b^2 c^4 d^e^4 f^2 z - 8 B^3 a^2 b^4 c^3 d^2 e^2 f^3 z + 6 B^3 a^2 b^4 c^2 d^e^2 f^4 z - 4 B^3 a^2 b^3 c^3 d^e^3 f^3 z - 216 A^2 B b^4 c^4 d^2 e^2 f^3 z + 196 A^2 B b^3 c^5 d^2 e^3 f^2 z \\
& - 108 A B^2 b^3 c^5 d^3 e^2 f^2 z - 94 A B^2 b^4 c^4 d^2 e^3 f^2 z + 88 A^2 B b^2 c^6 d^3 e^2 f^2 z + 80 A B^2 b^5 c^3 d^2 e^2 f^3 z + 360 A^2 B a^2 c^6 d^2 e^2 f^3 z + 8 A B^2 a^2 c^6 d^2 e^3 f^2 z + 153 A^2 B a^2 b^2 c^4 e^4 f^3 z - 144 A^2 B a^2 b^3 c^3 e^3 f^4 z + 80 A^2 B a^3 b^2 c^3 e^2 f^5 z \\
& + 36 A B^2 a^3 b^2 c^3 e^3 f^4 z + 12 A^2 B a^2 b^4 c^2 e^2 f^5 z + 12 A B^2 a^3 b^3 c^2 e^2 f^5 z + 9 A B^2 a^2 b^2 c^4 e^5 f^2 z - 6 A B^2 a^2 b^4 c^2 e^3 f^4 z + 4 A B^2 a^2 b^3 c^3 e^4 f^3 z + 480 A^2 B a^2 b^2 c^4 d^2 f^5 z - 176 A B^2 a^2 b^3 c^3 d^2 f^5 z - 10 A^2 B a^2 b^6 c^d f^6 z + 16 A B^2 a^2 b^c^6 d^e^6 z + 80 B^3 a^2 b^3 c^4 d^3 e f^3 z - 48 B^3 a^3 b^c^4 d^2 e f^4 z + 48 B^3 a^2 b^c^5 d^3 e f^3 z + 44 B^3 a^3 b^c^4 d^e^3 f^3 z + 24 B^3 a^2 b^5 c^2 d^2 e f^4 z + 18 B^3 a^2 b^2 c^5 d^2 e^4 f^2 z + 696 A^3 a^2 b^c^5 d^e^2 f^4 z - 504 A^3 a^2 b^c^6 d^2 e^2 f^3 z - 192 A^3 a^2 b^2 c^5 d^e^3 f^3 z - 144 A^3 a^2 b^2 c^4 d^e f^5 z + 96 A^3 a^2 b^2 c^5 d^2 e f^4 z - 72 A^3 a^2 b^3 c^4 d^e^2 f^4 z - 208 A^2 B b^3 c^5 d^3 e f^3 z + 152 A B^2 b^4 c^4 d^3 e f^3 z + 80 A^2 B b^5 c^3 d^2 e f^4 z + 75 A^2 B b^4 c^4 d^e^4 f^2 z - 59 A^2 B b^2 c^6 d^2 e^4 f^2 z - 52 A^2 B b^5 c^3 d^e^3 f^3 z + 42 A B^2 b^3 c^5 d^2 e^4 f^2 z - 21 A B^2 b^6 c^2 d^2 e f^4 z - 16 A B^2 b^5 c^3 d^e^4 f^2 z + 16 A B^2 b^2 c^6 d^4 e f^2 z + 16 A B^2 b^2 c^6 d^3 e^3 f^2 z + 11 A^2 B b^6 c^2 d^e^2 f^4 z + 4 A B^2 b^6 c^2 d^e^3 f^3 z - 256 A^2 B a^c^7 d^3 e^2 f^2 z - 96 A B^2 a^3 c^5 d^2 e f^4 z - 36 A^2 B a^2 c^6 d^e^4 f^2 z - 32 A^2 B a^3 c^5 d^e^2 f^4 z - 32 A B^2 a^2 c^6 d^3 e f^3 z + 8 A B^2 a^3 c^5 d^e^3 f^3 z - 96 A^2 B a^3 b^3 c^2 e f^6 z + 68 A^2 B a^3 b^c^4 e^3 f^4 z - 60 A B^2 a^4 b^c^3 e^2 f^5 z - 60 A B^2 a^3 b^c^4 e^4 f^3 z + 48 A B^2 a^4 b^2 c^2 e f^6 z - 38 A^2 B a^2 b^3 c^4 e^5 f^2 z - 36 A^2 B a^2 b^c^5 e^5 f^2 z + 36 A^2 B a^2 b^5 c^2 e^3 f^4 z - 16 A^2 B a^2 b^4 c^3 e^4 f^3 z + 384 A B^2 a^2 b^c^5 d^3 f^4 z - 352 A B^2 a^3 b^c^4 d^2 f^5 z - 288 A^2 B a^2 b^2 c^5 d^3 f^4 z - 160 A^2 B a^3 b^2 c^3 d^4 f^6 z - 148 A^2 B a^2 b^4 c^3 d^2 f^5 z + 112 A B^2 a^2 b^3 c^4 d^3 f^4 z + 72 A^2 B a^2 b^4 c^2 d^4 f^6 z + 72 A B^2 a^2 b^5 c^2 d^2 f^5 z + 48 A B^2 a^3 b^3 c^2 d^4 f^6 z + 102 B^3 a^2 b^2 c^4 d^2 e^2 f^3 z - 32 B^3 b^5 c^3 d^3 e f^3 z - 8 B^3 b^3 c^5 d^3 e^3 f^2 z - 7 B^3 b^4 c^4 d^2 e^4 f^2 z + 5 B^3 b^2 c^6 d^4 e^2 f^2 z + 80 A^3 b^2 c^6 d^3 e f^3 z - 74 A^3 b^3 c^5 d^e^4 f^2 z - 64 A^3 b^4 c^4 d^2 e f^4 z + 60 A^3 b^4 c^4 d^e^3 f^3 z - 48 B^3 a^4 c^4 d^e^2 f^4 z - 24 B^3 a^3 c^5 d^e^4 f^2 z + 20 B^3 a^2 c^6 d^2 e^4 f^2 z - 16 A^3 b^5 c^3 d^e^2 f^4 z + 8 A^3 b^c^7 d^3 e^2 f^2 z + 480 A^3 a^2 c^6 d^2 e f^4 z - 392 A^3 a^2 c^6 d^e^3 f^3 z + 280 A^3 a^c^7 d^2 e^3 f^2 z - 4 B^3 a^4 b^c^3 e^3 f^4 z - 200 A^3 a^3 b^c^4 e^2 f^5 z - 144 A^3 a^2 b^c^5 e^4 f^3 z + 48 B^3 a^2 b^2 c^5 d^4 f^3 z + 42 A^3 a^2 b^2 c^5 e^5 f^2 z - 36 B^3 a^4 b^2 c^2 d^4 f^6 z - 32 A^3 a^3 b^2 c^3 e f^6 z - 24 A^3 a^2 b^4 c^2 e f^6 z - 24 A^3 a^2 b^5 c^2 e^2 f^5 z + 10 A^3 a^2 b^3 c^4 e^4 f^3 z - 4 B^3 a^2 b^4 c^3 d^3 f^4 z - 4 A^3 a^2 b^4 c^3 e^3 f^4 z - 480 A^3 a^2 b^c^5 d^2 f^5 z - 160 A^3 a^2 b^3 c^3 d^4 f^6 z + 128 A^3 a^2 b^3 c^4 d^2 f^5 z + 8 A^2 B b^5 c^3 e^5 f^2 z - 2 A^2 B b^6 c^2 e^4 f^3 z + 112 A^2 B b^4 c^4 d^3 f^4 z - 92 A^2 B a^4 c^4 e^2 f^5 z - 64 A^2 B a^3 c^5 e^4 f^3 z - 64 A B^2 b^5 c^3 d^3 f^4 z + 24 A B^2 a^4 c^4 e^3 f^4 z + 24 A B^2 a^3 c^5 e^5 f^2 z + 16 A^2 B b^2 c^6 d^4 f^3 z + 16 A B^2 b^3 c^5 d^4 f^3 z - A^2 B b^6 c^2 d^2 f^5 z + 448 A^2 B a^3 c^5 d^2 f^5 z - 352 A^2 B a^2 c^6 d^3 f^4 z - 5 A B^2 b^2 c^6 d^2 e^5 z - 48 A^2 B a^4 b^2 c^2 f^7 z - 2 B^3 b^7 c^d^2 e f^4 z + 34 A^3 b^2 c^6 d^e^5 f^2 z + 16 A^3 b^c^7 d^2 e^4 f^2 z + 2 A^3 b^6 c^2 d^e f^5 z - 416 A^3 a^3 c^5 d^e f^5 z - 224 A^3 a^c^7 d^3 e f^3 z + 12 B^3 a^3 b^4 c^d f^6 z - 10 B^3 a^2 b^6 c^d^2 f^5 z + 416 A^3 a^3 b^c^
\end{aligned}$$

$$\begin{aligned}
& 4*d*f^6*z + 224*A^3*a*b*c^6*d^3*f^4*z + 24*A^3*a*b^5*c^2*d*f^6*z - 4*B^3*a* \\
& b*c^6*d^2*e^5*z + 20*A^2*B*c^8*d^4*e^2*f*z - 7*A^2*B*b^4*c^4*e^6*f*z - 2*A^ \\
& 2*B*b^7*c*e^3*f^4*z - 64*A*B^2*a^5*c^3*e*f^6*z + 16*A*B^2*b*c^7*d^5*f^2*z - \\
& 8*A^2*B*a^2*c^6*e^6*f*z - 2*A*B^2*b^7*c*d^2*f^5*z - 272*A^2*B*a^4*c^4*d*f^ \\
& 6*z + 128*A^2*B*a*c^7*d^4*f^3*z + 9*A^2*B*b^2*c^6*d*e^6*z - 4*A*B^2*b^3*c^5 \\
& *d*e^6*z + 4*A*B^2*b*c^7*d^3*e^4*z + 8*A*B^2*a*c^7*d^2*e^5*z + 12*A^2*B*a^3 \\
& *b^4*c*f^7*z + 30*B^3*b^4*c^4*d^3*e^2*f^2*z + 8*B^3*b^5*c^3*d^2*e^3*f^2*z - \\
& 2*B^3*b^6*c^2*d^2*e^2*f^3*z + 152*A^3*b^3*c^5*d^2*e^2*f^3*z - 108*A^3*b^2* \\
& c^6*d^2*e^3*f^2*z + 48*B^3*a^3*c^5*d^2*e^2*f^3*z - 16*B^3*a^2*c^6*d^3*e^2*f \\
& ^2*z - 3*B^3*a^4*b^2*c^2*e^2*f^5*z - 120*B^3*a^2*b^2*c^4*d^3*f^4*z + 112*B^ \\
& 3*a^3*b^2*c^3*d^2*f^5*z + 112*A^3*a^2*b^3*c^3*e^2*f^5*z + 12*A^3*a^2*b^2*c^ \\
& 4*e^3*f^4*z - 120*A^3*a*c^7*d*e^5*f*z - 52*A^3*a*b*c^6*e^6*f*z + 10*A^3*a*b \\
& ^6*c*e*f^6*z - 2*A*B^2*b^8*d*e*f^5*z - 2*A^2*B*a*b^7*e*f^6*z - 24*A^2*B*a*c \\
& ^7*d*e^6*z + 2*A*B^2*a*b^7*d*f^6*z - 12*A^2*B*a*b*c^6*e^7*z - 2*A^3*b^7*c*d \\
& *f^6*z - 4*A^3*b*c^7*d*e^6*z + 16*B^3*a^5*c^3*e^2*f^5*z + 11*B^3*b^6*c^2*d^ \\
& 3*f^4*z - 11*A^3*b^4*c^4*e^5*f^2*z - 8*B^3*b^4*c^4*d^4*f^3*z - 4*B^3*b^2*c^ \\
& 6*d^5*f^2*z + 4*B^3*a^4*c^4*e^4*f^3*z + 4*A^3*b^5*c^3*e^4*f^3*z - A^3*b^6*c \\
& ^2*e^3*f^4*z + 136*A^3*a^3*c^5*e^3*f^4*z + 68*A^3*a^2*c^6*e^5*f^2*z - 64*A^ \\
& 3*b^3*c^5*d^3*f^4*z + 2*B^3*b^3*c^5*d^2*e^5*z - B^3*b^2*c^6*d^3*e^4*z + 96* \\
& A^3*a^3*b^3*c^2*f^7*z + A*B^2*a^2*b^6*e*f^6*z + 32*A^3*c^8*d^4*e*f^2*z - 24 \\
& *A^3*c^8*d^3*e^3*f*z + 10*A^3*b^3*c^5*e^6*f*z + 2*A^3*b^7*c*e^2*f^5*z + 128 \\
& *A^3*a^4*c^4*e*f^6*z - 32*A^3*b*c^7*d^4*f^3*z - 4*B^3*a^2*c^6*d*e^6*z - B^3 \\
& *a^2*b^6*d*f^6*z - 128*A^3*a^4*b*c^3*f^7*z - 24*A^3*a^2*b^5*c*f^7*z - 16*A^ \\
& 2*B*c^8*d^5*f^2*z - 4*A^2*B*c^8*d^3*e^4*z + 64*A^2*B*a^5*c^3*f^7*z + 2*A^2* \\
& B*b^3*c^5*e^7*z + 4*A*B^2*a^2*c^6*e^7*z - A^2*B*a^2*b^6*f^7*z + 4*A^3*c^8*d \\
& ^2*e^5*z - 3*A^3*b^2*c^6*e^7*z + A^2*B*b^8*d*f^6*z - A^3*b^8*e*f^6*z + 16*A \\
& ^3*a*c^7*e^7*z + 2*A^3*a*b^7*f^7*z + A^2*B*b^8*e^2*f^5*z + B^3*b^8*d^2*f^5* \\
& z - 48*A^2*B^2*a*b*c^4*d*e*f^4 + 28*A*B^3*a*b^2*c^3*d*e*f^4 - 16*A*B^3*a*b* \\
& c^4*d*e^2*f^3 + 16*A^3*B*a*c^5*d*e*f^4 + 32*A^3*B*a*b*c^4*d*f^5 + 12*A^2*B^ \\
& 2*b^3*c^3*d*e*f^4 + 5*A*B^3*b^2*c^4*d^2*e*f^3 + 4*A*B^3*b^3*c^3*d*e^2*f^3 + \\
& 24*A^2*B^2*a*c^5*d*e^2*f^3 + 24*A^2*B^2*a^2*b*c^3*e*f^5 + 12*A^2*B^2*a*b*c \\
& ^4*e^3*f^3 - 6*A^2*B^2*a*b^3*c^2*e*f^5 + 4*A*B^3*a^2*b*c^3*e^2*f^4 + 3*A*B^ \\
& 3*a^2*b^2*c^2*e*f^5 - 18*A^2*B^2*a*b^2*c^3*d*f^5 - 4*B^4*a^2*b*c^3*d*e*f^4 \\
& + 4*B^4*a*b*c^4*d^2*e*f^3 - 6*A*B^3*b^4*c^2*d*e*f^4 + 4*A^3*B*b*c^5*d*e^2*f \\
& ^3 - 2*A^3*B*b^2*c^4*d*e*f^4 - 8*A*B^3*a^2*c^4*d*e*f^4 - 8*A*B^3*a*c^5*d^2* \\
& e*f^3 + 26*A^3*B*a*b^2*c^3*e*f^5 + 8*A^3*B*a*b*c^4*e^2*f^4 + 32*A*B^3*a*b*c \\
& ^4*d^2*f^4 - 28*A*B^3*a^2*b*c^3*d*f^5 + 6*A*B^3*a*b^3*c^2*d*f^5 - 9*A^2*B^2 \\
& *b^2*c^4*d*e^2*f^3 - 18*A^2*B^2*a*b^2*c^3*e^2*f^4 - 4*A^3*B*c^6*d^2*e*f^3 - \\
& 3*A^3*B*b^4*c^2*e*f^5 - 44*A^3*B*a^2*c^4*e*f^5 - 16*A^3*B*a*c^5*e^3*f^3 - \\
& 16*A*B^3*a^3*c^3*e*f^5 - 10*A^3*B*b^3*c^3*d*f^5 - 4*A^3*B*b*c^5*d^2*f^4 - 4 \\
& *A*B^3*b*c^5*d^3*f^3 - 28*A^3*B*a^2*b*c^3*f^6 + 6*A^3*B*a*b^3*c^2*f^6 - 4*A \\
& ^4*b*c^5*d*e*f^4 - 20*A^4*a*b*c^4*e*f^5 + 3*A^2*B^2*b^4*c^2*e^2*f^4 - 2*A^2 \\
& *B^2*b^3*c^3*e^3*f^3 + 12*A^2*B^2*a^2*c^4*e^2*f^4 + 9*A^2*B^2*b^2*c^4*d^2*f \\
& ^4 - 3*A^2*B^2*a^2*b^2*c^2*f^6 - 2*B^4*b^3*c^3*d^2*e*f^3 + 4*B^4*a^2*c^4*d* \\
& e^2*f^3 - 10*B^4*a*b^2*c^3*d^2*f^4 - 3*B^4*a^2*b^2*c^2*d*f^5 + 3*A^3*B*b^2* \\
& c^4*e^3*f^3 - 2*A^3*B*b^3*c^3*e^2*f^4 - 10*A*B^3*b^3*c^3*d^2*f^4 - 4*A*B^3* \\
& a^2*c^4*e^3*f^3 + 3*A^2*B^2*b^4*c^2*d*f^5 + 36*A^2*B^2*a^2*c^4*d*f^5 - 24*A \\
& ^2*B^2*a*c^5*d^2*f^4 + 4*A^2*B^2*c^6*d^3*f^3 + 16*A^2*B^2*a^3*c^3*f^6 + 4*A \\
& ^4*b^3*c^3*e*f^5 + 16*B^4*a^3*c^3*d*f^5 + 16*A^4*a*c^5*e^2*f^4 + 8*A^4*b^2* \\
& c^4*d*f^5 - 8*A^4*a*b^2*c^3*f^6 - 24*A^4*a*c^5*d*f^5 + 3*B^4*b^4*c^2*d^2*f^ \\
& 4 - 3*A^4*b^2*c^4*e^2*f^4 + 4*A^4*c^6*d^2*f^4 + 36*A^4*a^2*c^4*f^6 + B^4*b^ \\
& 2*c^4*d^3*f^3, z, k)*((64*a^9*c^4*e*f^8 - 64*a^6*c^7*e^7*f^2 - 64*a^7*c^6*e \\
& ^5*f^4 + 64*a^8*c^5*e^3*f^6 + 4*b^5*c^8*d^7*f^2 + 4*b^7*c^6*d^6*f^3 - 4*b^9 \\
& *c^4*d^5*f^4 - 4*b^11*c^2*d^4*f^5 - 612*a^2*b^5*c^6*d^5*f^4 - 712*a^2*b^7*c \\
& ^4*d^4*f^5 - 132*a^2*b^9*c^2*d^3*f^6 + 1696*a^3*b^3*c^7*d^5*f^4 + 2736*a^3* \\
& b^5*c^5*d^4*f^5 + 896*a^3*b^7*c^3*d^3*f^6 - 5120*a^4*b^3*c^6*d^4*f^5 - 3140 \\
& *a^4*b^5*c^4*d^3*f^6 - 220*a^4*b^7*c^2*d^2*f^7 + 5664*a^5*b^3*c^5*d^3*f^6 + \\
& 1128*a^5*b^5*c^3*d^2*f^7 - 2560*a^6*b^3*c^4*d^2*f^7 + 4*a^3*b^6*c^4*e^7*f^ \\
& 2 - 6*a^3*b^7*c^3*e^6*f^3 + 4*a^3*b^8*c^2*e^5*f^4 - 36*a^4*b^4*c^5*e^7*f^2
\end{aligned}$$

$$\begin{aligned}
& + 57*a^4*b^5*c^4*e^6*f^3 - 37*a^4*b^6*c^3*e^5*f^4 + 7*a^4*b^7*c^2*e^4*f^5 + \\
& 96*a^5*b^2*c^6*e^7*f^2 - 168*a^5*b^3*c^5*e^6*f^3 + 100*a^5*b^4*c^4*e^5*f^4 \\
& - 3*a^5*b^5*c^3*e^4*f^5 - 10*a^5*b^6*c^2*e^3*f^6 - 48*a^6*b^2*c^5*e^5*f^4 \\
& - 56*a^6*b^3*c^4*e^4*f^5 + 36*a^6*b^4*c^3*e^3*f^6 - 13*a^6*b^5*c^2*e^2*f^7 \\
& - 64*a^7*b^2*c^4*e^3*f^6 + 56*a^7*b^3*c^3*e^2*f^7 - 1472*a^4*c^9*d^4*e^3*f^2 \\
& - 1088*a^5*c^8*d^2*e^5*f^2 + 3584*a^5*c^8*d^3*e^3*f^3 - 3200*a^6*c^7*d^2* \\
& e^3*f^4 - 2*b^7*c^6*d^5*e^2*f^2 - b^8*c^5*d^4*e^3*f^2 + 4*b^9*c^4*d^3*e^4*f^2 \\
& - 5*b^9*c^4*d^4*e^2*f^3 - 6*b^10*c^3*d^3*e^3*f^3 + 4*b^11*c^2*d^3*e^2*f^4 \\
& + 8*a*b^11*c*d^3*f^6 + 8*a^5*b^7*c*d*f^8 - 448*a^8*b*c^4*d*f^8 - 16*a^5*b \\
& *c^7*e^8*f - a^6*b^6*c*e*f^8 + 128*a^5*c^8*d*e^7*f + 128*a^8*c^5*d*e*f^7 - \\
& b^12*c*d^3*e*f^5 - 32*a*b^3*c^9*d^7*f^2 - 24*a*b^5*c^7*d^6*f^3 + 88*a*b^7*c^5 \\
& *d^5*f^4 + 88*a*b^9*c^3*d^4*f^5 + 64*a^2*b*c^10*d^7*f^2 + 128*a^3*b*c^9*d^6 \\
& *f^3 + 16*a^3*b^9*c*d^2*f^7 - 1600*a^4*b*c^8*d^5*f^4 + 3840*a^5*b*c^7*d^4 \\
& *f^5 - 4160*a^6*b*c^6*d^3*f^6 - 92*a^6*b^5*c^2*d*f^8 + 2176*a^7*b*c^5*d^2*f^7 \\
& + 352*a^7*b^3*c^3*d*f^8 - a^3*b^5*c^5*e^8*f - a^3*b^9*c*e^4*f^5 + 8*a^4*b^3 \\
& *c^6*e^8*f + a^4*b^8*c*e^3*f^6 + a^5*b^7*c*e^2*f^7 + 144*a^6*b*c^6*e^6*f^3 \\
& + 80*a^7*b*c^5*e^4*f^5 + 12*a^7*b^4*c^2*e*f^8 - 80*a^8*b*c^4*e^2*f^7 - 4 \\
& 8*a^8*b^2*c^3*e*f^8 + 128*a^3*c^10*d^5*e^3*f - 448*a^3*c^10*d^6*e*f^2 + 256 \\
& *a^4*c^9*d^3*e^5*f + 2176*a^4*c^9*d^5*e*f^3 - 4160*a^5*c^8*d^4*e*f^4 + 896* \\
& a^6*c^7*d*e^5*f^3 + 3840*a^6*c^7*d^3*e*f^5 + 896*a^7*c^6*d*e^3*f^5 - 1600*a^7 \\
& *c^6*d^2*e*f^6 - b^5*c^8*d^6*e^2*f + b^6*c^7*d^5*e^3*f - 5*b^6*c^7*d^6*e* \\
& f^2 + b^7*c^6*d^4*e^4*f - b^8*c^5*d^3*e^5*f + 5*b^8*c^5*d^5*e*f^3 + 9*b^10*c^3 \\
& *d^4*e*f^4 + 8*a*b^3*c^9*d^6*e^2*f + 12*a*b^4*c^8*d^6*e*f^2 - 27*a*b^5*c^7 \\
& *d^4*e^4*f + 18*a*b^6*c^6*d^3*e^5*f - 154*a*b^6*c^6*d^5*e*f^3 + a*b^7*c^5 \\
& *d^2*e^6*f - 200*a*b^8*c^4*d^4*e*f^4 - 2*a*b^10*c^2*d^3*e*f^5 + a*b^11*c*d^2 \\
& *e^2*f^5 - 16*a^2*b*c^10*d^6*e^2*f + a^2*b^6*c^5*d*e^7*f + a^2*b^10*c*d*e^3 \\
& *f^5 - 19*a^2*b^10*c*d^2*e*f^6 - 304*a^3*b*c^9*d^4*e^4*f + 10*a^3*b^9*c*d* \\
& e^2*f^6 - 304*a^4*b*c^8*d^2*e^6*f - 48*a^4*b^2*c^7*d*e^7*f - 160*a^5*b*c^7* \\
& d*e^6*f^2 + 214*a^5*b^6*c^2*d*e*f^7 - 2048*a^6*b*c^6*d*e^4*f^4 - 792*a^6*b^4 \\
& *c^3*d*e*f^7 - 1824*a^7*b*c^5*d*e^2*f^6 + 928*a^7*b^2*c^4*d*e*f^7 + 78*a*b^5 \\
& *c^7*d^5*e^2*f^2 + 10*a*b^6*c^6*d^4*e^3*f^2 - 68*a*b^7*c^5*d^3*e^4*f^2 + \\
& 129*a*b^7*c^5*d^4*e^2*f^3 - 4*a*b^8*c^4*d^2*e^5*f^2 + 96*a*b^8*c^4*d^3*e^3* \\
& f^3 + 6*a*b^9*c^3*d^2*e^4*f^3 - 52*a*b^9*c^3*d^3*e^2*f^4 - 4*a*b^10*c^2*d^2 \\
& *e^3*f^4 - 48*a^2*b^2*c^9*d^5*e^3*f + 144*a^2*b^2*c^9*d^6*e*f^2 + 168*a^2*b^3 \\
& *c^8*d^4*e^4*f - 80*a^2*b^4*c^7*d^3*e^5*f + 1128*a^2*b^4*c^7*d^5*e*f^3 - \\
& 27*a^2*b^5*c^6*d^2*e^6*f + 1481*a^2*b^6*c^5*d^4*e*f^4 - 4*a^2*b^7*c^4*d*e^6 \\
& *f^2 + 6*a^2*b^8*c^3*d*e^5*f^3 + 126*a^2*b^8*c^3*d^3*e*f^5 - 4*a^2*b^9*c^2* \\
& d*e^4*f^4 + 992*a^3*b*c^9*d^5*e^2*f^2 + 32*a^3*b^2*c^8*d^3*e^5*f - 2912*a^3 \\
& *b^2*c^8*d^5*e*f^3 + 168*a^3*b^3*c^7*d^2*e^6*f - 4812*a^3*b^4*c^6*d^4*e*f^4 \\
& + 22*a^3*b^5*c^5*d*e^6*f^2 - 68*a^3*b^6*c^4*d*e^5*f^3 - 860*a^3*b^6*c^4*d^3 \\
& *e*f^5 + 80*a^3*b^7*c^3*d*e^4*f^4 - 44*a^3*b^8*c^2*d*e^3*f^5 + 232*a^3*b^8 \\
& *c^2*d^2*e*f^6 + 1280*a^4*b*c^8*d^3*e^4*f^2 - 816*a^4*b*c^8*d^4*e^2*f^3 + 7 \\
& 088*a^4*b^2*c^7*d^4*e*f^4 + 16*a^4*b^3*c^6*d*e^6*f^2 + 312*a^4*b^4*c^5*d*e^5 \\
& *f^3 + 2864*a^4*b^4*c^5*d^3*e*f^5 - 576*a^4*b^5*c^4*d*e^4*f^4 + 393*a^4*b^6 \\
& *c^3*d*e^3*f^5 - 995*a^4*b^6*c^3*d^2*e*f^6 - 78*a^4*b^7*c^2*d*e^2*f^6 + 99 \\
& 2*a^5*b*c^7*d^2*e^4*f^3 - 3264*a^5*b*c^7*d^3*e^2*f^4 - 768*a^5*b^2*c^6*d*e^5 \\
& *f^3 - 5184*a^5*b^2*c^6*d^3*e*f^5 + 1792*a^5*b^3*c^5*d*e^4*f^4 - 1232*a^5*b^4 \\
& *c^4*d*e^3*f^5 + 1588*a^5*b^4*c^4*d^2*e*f^6 + 30*a^5*b^5*c^3*d*e^2*f^6 + \\
& 5008*a^6*b*c^6*d^2*e^2*f^5 + 976*a^6*b^2*c^5*d*e^3*f^5 - 16*a^6*b^2*c^5*d^2 \\
& *e*f^6 + 944*a^6*b^3*c^4*d*e^2*f^6 - 19*a^4*b^8*c*d*e*f^7 - 528*a^2*b^3*c^8 \\
& *d^5*e^2*f^2 - 124*a^2*b^4*c^7*d^4*e^3*f^2 + 432*a^2*b^5*c^6*d^3*e^4*f^2 - \\
& 843*a^2*b^5*c^6*d^4*e^2*f^3 + 83*a^2*b^6*c^5*d^2*e^5*f^2 - 722*a^2*b^6*c^5 \\
& *d^3*e^3*f^3 - 80*a^2*b^7*c^4*d^2*e^4*f^3 + 376*a^2*b^7*c^4*d^3*e^2*f^4 + 4 \\
& 3*a^2*b^9*c^2*d^2*e^2*f^5 + 768*a^3*b^2*c^8*d^4*e^3*f^2 - 1216*a^3*b^3*c^7*d^3 \\
& *e^4*f^2 + 1832*a^3*b^3*c^7*d^4*e^2*f^3 - 540*a^3*b^4*c^6*d^2*e^5*f^2 + \\
& 2928*a^3*b^4*c^6*d^3*e^3*f^3 + 414*a^3*b^5*c^5*d^2*e^4*f^3 - 1740*a^3*b^5*c^5 \\
& *d^3*e^2*f^4 + 344*a^3*b^6*c^4*d^2*e^3*f^4 - 634*a^3*b^7*c^3*d^2*e^2*f^5 \\
& + 1360*a^4*b^2*c^7*d^2*e^5*f^2 - 5664*a^4*b^2*c^7*d^3*e^3*f^3 - 1008*a^4*b^3 \\
& *c^6*d^2*e^4*f^3 + 4064*a^4*b^3*c^6*d^3*e^2*f^4 - 1928*a^4*b^4*c^5*d^2*e^3
\end{aligned}$$

$$\begin{aligned}
& *f^4 + 3065*a^4*b^5*c^4*d^2*e^2*f^5 + 4032*a^5*b^2*c^6*d^2*e^3*f^4 - 6376*a^5*b^3*c^5*d^2*e^2*f^5)/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + 16*a^4*c^4*e^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 2*a^3*b^5*e*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 - 2*b^5*c^3*d^3*e + 2*b^6*c^2*d^3*f + a^2*b^4*c^2*e^4 - 8*a^3*b^2*c^3*e^4 + 32*a^3*c^5*d^2*e^2 + a^2*b^6*e^2*f^2 + 96*a^4*c^4*d^2*f^2 + b^6*c^2*d^2*e^2 + 32*a^5*c^3*e^2*f^2 - 2*a*b^7*d*e*f^2 - 2*b^7*c*d^2*e*f + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 + 16*a*b^3*c^4*d^3*e - 2*a*b^5*c^2*d*e^3 - 32*a^2*b*c^5*d^3*e - 32*a^3*b*c^4*d*e^3 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 - 2*a^2*b^5*c*e^3*f - 32*a^4*b*c^3*e^3*f + 16*a^4*b^3*c*e*f^3 - 32*a^5*b*c^2*e*f^3 - 64*a^4*c^4*d*e^2*f - 6*a*b^4*c^3*d^2*e^2 + 16*a^2*b^3*c^3*d*e^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3 + 16*a^3*b^3*c^2*e^3*f - 6*a^3*b^4*c*e^2*f^2 - 48*a^2*b^3*c^3*d^2*e*f - 36*a^2*b^4*c^2*d*e^2*f + 96*a^3*b^2*c^3*d*e^2*f - 48*a^3*b^3*c^2*d*e*f^2 + 4*a*b^6*c*d*e^2*f + 18*a*b^5*c^2*d^2*e*f + 18*a^2*b^5*c*d*e*f^2 + 32*a^3*b*c^4*d^2*e*f + 32*a^4*b*c^3*d*e*f^2) + (x*(128*a^9*c^4*f^9 - 2*a^6*b^6*c*f^9 - 640*a^8*c^5*d*f^8 + 96*a^5*c^8*e^8*f + 6*b^12*c*d^3*f^6 + 24*a^7*b^4*c^2*f^9 - 96*a^8*b^2*c^3*f^9 + 128*a^2*c^11*d^7*f^2 - 640*a^3*c^10*d^6*f^3 + 1152*a^4*c^9*d^5*f^4 - 640*a^5*c^8*d^4*f^5 - 640*a^6*c^7*d^3*f^6 + 1152*a^7*c^6*d^2*f^7 + 288*a^6*c^7*e^6*f^3 + 416*a^7*c^6*e^4*f^5 + 352*a^8*c^5*e^2*f^7 + 8*b^4*c^9*d^7*f^2 + 22*b^6*c^7*d^6*f^3 + 26*b^8*c^5*d^5*f^4 + 18*b^10*c^3*d^4*f^5 + 672*a^2*b^2*c^9*d^6*f^3 + 1224*a^2*b^4*c^7*d^5*f^4 + 1202*a^2*b^6*c^5*d^4*f^5 + 564*a^2*b^8*c^3*d^3*f^6 - 2048*a^3*b^2*c^8*d^5*f^4 - 2744*a^3*b^4*c^6*d^4*f^5 - 1736*a^3*b^6*c^4*d^3*f^6 - 128*a^3*b^8*c^2*d^2*f^7 + 2656*a^4*b^2*c^7*d^4*f^5 + 2648*a^4*b^4*c^5*d^3*f^6 + 570*a^4*b^6*c^3*d^2*f^7 - 1344*a^5*b^2*c^6*d^3*f^6 - 904*a^5*b^4*c^4*d^2*f^7 - 160*a^6*b^2*c^5*d^2*f^7 + 8*a^2*b^7*c^4*e^7*f^2 - 12*a^2*b^8*c^3*e^6*f^3 + 8*a^2*b^9*c^2*e^5*f^4 - 90*a^3*b^5*c^5*e^7*f^2 + 132*a^3*b^6*c^4*e^6*f^3 - 76*a^3*b^7*c^3*e^5*f^4 + 6*a^3*b^8*c^2*e^4*f^5 + 336*a^4*b^3*c^6*e^7*f^2 - 462*a^4*b^4*c^5*e^6*f^3 + 164*a^4*b^5*c^4*e^5*f^4 + 106*a^4*b^6*c^3*e^4*f^5 - 56*a^4*b^7*c^2*e^3*f^6 + 432*a^5*b^2*c^6*e^6*f^3 + 288*a^5*b^3*c^5*e^5*f^4 - 598*a^5*b^4*c^4*e^4*f^5 + 102*a^5*b^5*c^3*e^3*f^6 + 90*a^5*b^6*c^2*e^2*f^7 + 720*a^6*b^2*c^5*e^4*f^5 + 336*a^6*b^3*c^4*e^3*f^6 - 314*a^6*b^4*c^3*e^2*f^7 + 240*a^7*b^2*c^4*e^2*f^7 + 64*a^3*c^10*d^5*e^2*f^2 - 768*a^4*c^9*d^3*e^4*f^2 + 416*a^4*c^9*d^4*e^2*f^3 + 1856*a^5*c^8*d^2*e^4*f^3 - 1664*a^5*c^8*d^3*e^2*f^4 + 2336*a^6*c^7*d^2*e^2*f^5 + 26*b^6*c^7*d^5*e^2*f^2 - 8*b^7*c^6*d^4*e^3*f^2 - 10*b^8*c^5*d^3*e^4*f^2 + 58*b^8*c^5*d^4*e^2*f^3 + 8*b^9*c^4*d^2*e^5*f^2 - 12*b^9*c^4*d^3*e^3*f^3 - 12*b^10*c^3*d^2*e^4*f^3 + 36*b^10*c^3*d^3*e^2*f^4 + 8*b^11*c^2*d^2*e^3*f^4 + 2*a^4*b^8*c*d*f^8 + 6*a^5*b^7*c*e*f^8 - 384*a^8*b*c^4*e*f^8 - 64*a*b^2*c^10*d^7*f^2 - 216*a*b^4*c^8*d^6*f^3 - 300*a*b^6*c^6*d^5*f^4 - 240*a*b^8*c^4*d^4*f^5 - 92*a*b^10*c^2*d^3*f^6 + 10*a^2*b^10*c*d^2*f^7 - 12*a^5*b^6*c^2*d*f^8 - 40*a^6*b^4*c^3*d*f^8 + 384*a^7*b^2*c^4*d*f^8 - 2*a^2*b^6*c^5*e^8*f - 2*a^2*b^10*c*e^4*f^5 + 22*a^3*b^4*c^6*e^8*f + 6*a^3*b^9*c*e^3*f^6 - 80*a^4*b^2*c^7*e^8*f - 8*a^4*b^8*c*e^2*f^7 - 416*a^5*b*c^7*e^7*f^2 - 960*a^6*b*c^6*e^5*f^4 - 72*a^6*b^5*c^2*e*f^8 - 928*a^7*b*c^5*e^3*f^6 + 288*a^7*b^3*c^3*e*f^8 - 32*a^2*c^11*d^6*e^2*f + 32*a^3*c^10*d^4*e^4*f + 160*a^4*c^9*d^2*e^6*f - 704*a^5*c^8*d*e^6*f^2 - 1536*a^6*c^7*d*e^4*f^4 - 1472*a^7*c^6*d*e^2*f^6 - 2*b^4*c^9*d^6*e^2*f + 6*b^5*c^8*d^5*e^3*f - 24*b^5*c^8*d^6*e*f^2 - 8*b^6*c^7*d^4*e^4*f + 6*b^7*c^6*d^3*e^5*f - 58*b^7*c^6*d^5*e*f^3 - 2*b^8*c^5*d^2*e^6*f - 60*b^9*c^4*d^4*e*f^4 - 26*b^11*c^2*d^3*e*f^5 - 2*b^12*c*d^2*e^2*f^5 + 16*a*b^2*c^10*d^6*e^2*f - 48*a*b^3*c^9*d^5*e^3*f + 192*a*b^3*c^9*d^6*e*f^2 + 66*a*b^4*c^8*d^4*e^4*f - 52*a*b^5*c^7*d^3*e^5*f + 576*a*b^5*c^7*d^5*e*f^3 + 14*a*b^6*c^6*d^2*e^6*f + 718*a*b^7*c^5*d^4*e*f^4 - 16*a*b^8*c^4*d*e^6*f^2 + 24*a*b^9*c^3*d*e^5*f^3 + 356*a*b^9*c^3*d^3*e*f^5 - 16*a*b^10*c^2*d*e^4*f^4 + 96*a^2*b*c^10*d^5*e^3*f - 384*a^2*b*c^10*d^6*e*f^2 - 42*a^2*b^5*c^6*d*e^7*f - 2*a^2*b^10*c*d*e^2*f^6 - 64*a^3*b*c^9*d^3*e^5*f + 1792*a^3*b*c^9*d^5*e*f^3 + 144*a^3*b^3*c^7*d*e^7*f - 3712*a^4*b*c^8*d^4*e*f^4 + 14*a^4*b^7*c^2*d*e*f^7 + 2368*a^5*b*c^7*d*e^5*f^3 + 4608*a^5*b*c^7*d^3*e*f^5 + 192*a^5*b^5*c^3*d*e*f^7 + 3808*a^6*b*c^6*d*e^3*f^5
\end{aligned}$$

$$\begin{aligned}
& - 3712*a^6*b*c^6*d^2*e*f^6 - 1184*a^6*b^3*c^4*d*e*f^7 - 204*a*b^4*c^8*d^5*e \\
& ^2*f^2 + 46*a*b^5*c^7*d^4*e^3*f^2 + 132*a*b^6*c^6*d^3*e^4*f^2 - 590*a*b^6*c \\
& ^6*d^4*e^2*f^3 - 90*a*b^7*c^5*d^2*e^5*f^2 + 64*a*b^7*c^5*d^3*e^3*f^3 + 196* \\
& a*b^8*c^4*d^2*e^4*f^3 - 408*a*b^8*c^4*d^3*e^2*f^4 - 188*a*b^9*c^3*d^2*e^3*f \\
& ^4 + 78*a*b^10*c^2*d^2*e^2*f^5 - 144*a^2*b^2*c^9*d^4*e^4*f + 128*a^2*b^3*c^ \\
& 8*d^3*e^5*f - 1824*a^2*b^3*c^8*d^5*e*f^3 - 6*a^2*b^4*c^7*d^2*e^6*f - 3096*a \\
& ^2*b^5*c^6*d^4*e*f^4 + 190*a^2*b^6*c^5*d*e^6*f^2 - 316*a^2*b^7*c^4*d*e^5*f^ \\
& 3 - 1908*a^2*b^7*c^4*d^3*e*f^5 + 228*a^2*b^8*c^3*d*e^4*f^4 - 58*a^2*b^9*c^2 \\
& *d*e^3*f^5 + 92*a^2*b^9*c^2*d^2*e*f^6 - 288*a^3*b*c^9*d^4*e^3*f^2 - 112*a^3 \\
& *b^2*c^8*d^2*e^6*f + 5664*a^3*b^3*c^7*d^4*e*f^4 - 796*a^3*b^4*c^6*d*e^6*f^2 \\
& + 1524*a^3*b^5*c^5*d*e^5*f^3 + 5120*a^3*b^5*c^5*d^3*e*f^5 - 1176*a^3*b^6*c \\
& ^4*d*e^4*f^4 + 240*a^3*b^7*c^3*d*e^3*f^5 - 116*a^3*b^7*c^3*d^2*e*f^6 + 68*a \\
& ^3*b^8*c^2*d*e^2*f^6 + 192*a^4*b*c^8*d^2*e^5*f^2 + 2240*a^4*b*c^8*d^3*e^3*f \\
& ^3 + 1344*a^4*b^2*c^7*d*e^6*f^2 - 3168*a^4*b^3*c^6*d*e^5*f^3 - 7232*a^4*b^3 \\
& *c^6*d^3*e*f^5 + 2464*a^4*b^4*c^5*d*e^4*f^4 + 78*a^4*b^5*c^4*d*e^3*f^5 - 11 \\
& 60*a^4*b^5*c^4*d^2*e*f^6 - 574*a^4*b^6*c^3*d*e^2*f^6 - 4928*a^5*b*c^7*d^2*e \\
& ^3*f^4 - 1152*a^5*b^2*c^6*d*e^4*f^4 - 2416*a^5*b^3*c^5*d*e^3*f^5 + 4096*a^5 \\
& *b^3*c^5*d^2*e*f^6 + 1748*a^5*b^4*c^4*d*e^2*f^6 - 1280*a^6*b^2*c^5*d*e^2*f^ \\
& 6 + 4*a*b^7*c^5*d*e^7*f + 4*a*b^11*c*d*e^3*f^5 - 10*a*b^11*c*d^2*e*f^6 - 4* \\
& a^3*b^9*c*d*e*f^7 - 160*a^4*b*c^8*d*e^7*f + 1792*a^7*b*c^5*d*e*f^7 + 384*a^ \\
& 2*b^2*c^9*d^5*e^2*f^2 + 16*a^2*b^3*c^8*d^4*e^3*f^2 - 624*a^2*b^4*c^7*d^3*e^ \\
& 4*f^2 + 1962*a^2*b^4*c^7*d^4*e^2*f^3 + 348*a^2*b^5*c^6*d^2*e^5*f^2 + 204*a^ \\
& 2*b^5*c^6*d^3*e^3*f^3 - 1214*a^2*b^6*c^5*d^2*e^4*f^3 + 1636*a^2*b^6*c^5*d^3 \\
& *e^2*f^4 + 1520*a^2*b^7*c^4*d^2*e^3*f^4 - 750*a^2*b^8*c^3*d^2*e^2*f^5 + 121 \\
& 6*a^3*b^2*c^8*d^3*e^4*f^2 - 2224*a^3*b^2*c^8*d^4*e^2*f^3 - 512*a^3*b^3*c^7* \\
& d^2*e^5*f^2 - 1632*a^3*b^3*c^7*d^3*e^3*f^3 + 3492*a^3*b^4*c^6*d^2*e^4*f^3 - \\
& 2824*a^3*b^4*c^6*d^3*e^2*f^4 - 5492*a^3*b^5*c^5*d^2*e^3*f^4 + 2868*a^3*b^6 \\
& *c^4*d^2*e^2*f^5 - 4480*a^4*b^2*c^7*d^2*e^4*f^3 + 2432*a^4*b^2*c^7*d^3*e^2* \\
& f^4 + 8864*a^4*b^3*c^6*d^2*e^3*f^4 - 4206*a^4*b^4*c^5*d^2*e^2*f^5 + 432*a^5 \\
& *b^2*c^6*d^2*e^2*f^5)/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + 16*a^4*c^4*e^4 + b^4 \\
& *c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 \\
& + 2*a^2*b^6*d*f^3 - 2*a^3*b^5*e*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 \\
& - 2*b^5*c^3*d^3*e + 2*b^6*c^2*d^3*f + a^2*b^4*c^2*e^4 - 8*a^3*b^2*c^3*e^4 + \\
& 32*a^3*c^5*d^2*e^2 + a^2*b^6*e^2*f^2 + 96*a^4*c^4*d^2*f^2 + b^6*c^2*d^2*e^ \\
& 2 + 32*a^5*c^3*e^2*f^2 - 2*a*b^7*d*e*f^2 - 2*b^7*c*d^2*e*f + 54*a^2*b^4*c^2 \\
& *d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 + 16*a*b^3*c^4*d^3*e - 2*a*b^5*c^2*d*e^3 \\
& - 32*a^2*b*c^5*d^3*e - 32*a^3*b*c^4*d*e^3 - 20*a*b^4*c^3*d^3*f - 12*a*b^6* \\
& c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 - 2*a^2*b^5*c*e^3*f - 32*a^4*b*c^3*e^3*f + 1 \\
& 6*a^4*b^3*c*e*f^3 - 32*a^5*b*c^2*e*f^3 - 64*a^4*c^4*d*e^2*f - 6*a*b^4*c^3*d \\
& ^2*e^2 + 16*a^2*b^3*c^3*d*e^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3 \\
& + 16*a^3*b^3*c^2*e^3*f - 6*a^3*b^4*c*e^2*f^2 - 48*a^2*b^3*c^3*d^2*e*f - 36 \\
& *a^2*b^4*c^2*d*e^2*f + 96*a^3*b^2*c^3*d*e^2*f - 48*a^3*b^3*c^2*d*e*f^2 + 4* \\
& a*b^6*c*d*e^2*f + 18*a*b^5*c^2*d^2*e*f + 18*a^2*b^5*c*d*e*f^2 + 32*a^3*b*c^ \\
& 4*d^2*e*f + 32*a^4*b*c^3*d*e*f^2) - (64*A*a^7*c^4*f^8 - A*a^4*b^6*c*f^8 + \\
& 32*A*a*c^10*d^6*f^2 - 352*A*a^6*c^5*d*f^7 - A*b^10*c*d^2*f^6 + 8*B*a^4*c^7* \\
& e^7*f - 64*B*a^7*c^4*e*f^7 + 12*A*a^5*b^4*c^2*f^8 - 48*A*a^6*b^2*c^3*f^8 - \\
& 224*A*a^2*c^9*d^5*f^3 + 640*A*a^3*c^8*d^4*f^4 - 960*A*a^4*c^7*d^3*f^5 + 800 \\
& *A*a^5*c^6*d^2*f^6 - 40*A*a^4*c^7*e^6*f^2 - 80*A*a^5*c^6*e^4*f^4 + 24*A*a^6 \\
& *c^5*e^2*f^6 - 8*A*b^2*c^9*d^6*f^2 - 16*A*b^4*c^7*d^5*f^3 - A*b^6*c^5*d^4*f \\
& ^4 + 6*A*b^8*c^3*d^3*f^5 + 16*B*a^5*c^6*e^5*f^3 - 56*B*a^6*c^5*e^3*f^5 + 4* \\
& B*b^3*c^8*d^6*f^2 + 12*B*b^5*c^6*d^5*f^3 + 4*B*b^7*c^4*d^4*f^4 - 4*B*b^9*c^ \\
& 2*d^3*f^5 + 120*A*a*b^2*c^8*d^5*f^3 + 60*A*a*b^4*c^6*d^4*f^4 - 36*A*a*b^6*c \\
& ^4*d^3*f^5 + 8*A*a*b^8*c^2*d^2*f^6 + 20*A*a^3*b^6*c^2*d*f^7 - 80*A*a^4*b^4* \\
& c^3*d*f^7 + 216*A*a^5*b^2*c^4*d*f^7 + 4*A*a*b^6*c^4*e^6*f^2 - 6*A*a*b^7*c^3 \\
& *e^5*f^3 + 4*A*a*b^8*c^2*e^4*f^4 + 9*A*a^2*b^3*c^6*e^7*f + 2*A*a^2*b^8*c*e^ \\
& 2*f^6 + 88*A*a^4*b*c^6*e^5*f^3 + 172*A*a^5*b*c^5*e^3*f^5 - 92*B*a*b^3*c^7*d \\
& ^5*f^3 - 72*B*a*b^5*c^5*d^4*f^4 + 20*B*a*b^7*c^3*d^3*f^5 + 176*B*a^2*b*c^8* \\
& d^5*f^3 - 544*B*a^3*b*c^7*d^4*f^4 + 736*B*a^4*b*c^6*d^3*f^5 + 4*B*a^4*b^5*c \\
& ^2*d*f^7 - 464*B*a^5*b*c^5*d^2*f^6 - 44*B*a^5*b^3*c^3*d*f^7 - 2*B*a^3*b^2*c
\end{aligned}$$

$$\begin{aligned}
& ^6e^7f - B^3b^7c^2e^2f^6 - 28B^4b^6c^6e^6f^2 - 56B^5b^5c^5e^4f^4 - 12B^5b^4c^2e^2f^7 + 36B^6b^3c^4e^2f^6 + 48B^6b^2c^3e^2f^7 - 16A^2c^9d^3e^4f + 48A^4c^7d^4e^4f^3 - 168A^5c^6d^2e^2f^5 + 2Ab^2c^9d^5e^2f - 3Ab^3c^8d^4e^3f + 12Ab^3c^8d^5e^2f^2 - 4Ab^7c^4d^3e^5f^2 - 12Ab^7c^4d^3e^5f^4 + 6Ab^8c^3d^4e^4f^3 - 4Ab^9c^2d^2e^3f^4 + 8Ab^9c^2d^2e^3f^5 + 8B^2c^9d^4e^3f - 32B^2c^9d^5e^2f^2 + 16B^3c^8d^2e^5f + 64B^3c^8d^4e^2f^3 + 64B^4c^7d^3e^2f^4 + 96B^5c^6d^2e^3f^4 - 256B^5c^6d^2e^2f^5 - B^3c^8d^5e^2f + 2B^4c^7d^4e^3f - 8B^4c^7d^5e^2f^2 - B^6c^5d^2e^5f - 3B^6c^5d^4e^2f^3 + 12B^6c^3d^3e^2f^4 - 384A^2b^2c^7d^4f^4 - 32A^2b^4c^5d^3f^5 - 14A^2b^6c^3d^2f^6 + 560A^3b^2c^6d^3f^5 + 56A^3b^4c^4d^2f^6 - 456A^4b^2c^5d^2f^6 - 38A^2b^4c^5e^6f^2 + 58A^2b^5c^4e^5f^3 - 36A^2b^6c^3e^4f^4 + 5A^2b^7c^2e^3f^5 + 98A^3b^2c^6e^6f^2 - 158A^3b^3c^5e^5f^3 + 80A^3b^4c^4e^4f^4 + 22A^3b^5c^3e^3f^5 - 22A^3b^6c^2e^2f^6 + 20A^4b^2c^5e^4f^4 - 147A^4b^3c^4e^3f^5 + 80A^4b^4c^3e^2f^6 - 102A^5b^2c^4e^2f^6 + 360B^2b^3c^6d^4f^4 + 64B^2b^5c^4d^3f^5 - 504B^3b^3c^5d^3f^5 - 40B^3b^5c^3d^2f^6 + 276B^4b^3c^4d^2f^6 + 7B^3b^3c^5e^6f^2 - 8B^3b^4c^4e^5f^3 + 2B^3b^5c^3e^4f^4 + 2B^3b^6c^2e^3f^5 + 28B^4b^2c^5e^5f^3 + 6B^4b^3c^4e^4f^4 - 26B^4b^4c^3e^3f^5 + 11B^4b^5c^2e^2f^6 + 86B^5b^2c^4e^3f^5 - 37B^5b^3c^3e^2f^6 + 120A^2c^9d^4e^2f^2 + 48A^3c^8d^2e^4f^2 - 336A^3c^8d^3e^2f^3 + 368A^4c^7d^2e^2f^4 + 4Ab^4c^7d^4e^2f^2 - 5Ab^6c^5d^2e^4f^2 + 6Ab^6c^5d^3e^2f^3 + 16Ab^7c^4d^2e^3f^3 - 18Ab^8c^3d^2e^2f^4 - 32B^3c^8d^3e^3f^2 - 16B^4c^7d^2e^3f^3 - 3B^5c^6d^4e^2f^2 + 4B^6c^5d^3e^3f^2 + 4B^7c^4d^2e^4f^2 - 12B^7c^4d^3e^2f^3 - 6B^8c^3d^2e^3f^3 + 4B^9c^2d^2e^2f^4 - 2A^2b^8c^d^7 - Ab^5c^5e^7f - Ab^9c^e^3f^5 - 20A^3b^c^7e^7f - 16B^2b^c^9d^6f^2 + 112B^6b^c^4d^7f + B^4b^6c^e^7f - 8A^2c^10d^5e^2f - 8A^3c^8d^e^6f + Ab^6c^5d^e^6f + Ab^10c^d^e^2f^5 + 224B^6c^5d^e^6f - B^10c^d^2e^f^5 + 12A^2b^c^9d^4e^3f - 48A^2b^c^9d^5e^2f - 10A^2b^4c^6d^e^6f + 80A^5b^c^5d^e^6f + 4B^2b^c^9d^5e^2f + B^2b^5c^5d^e^6f + B^2b^9c^d^e^2f^5 + 4B^3b^c^7d^e^6f - 92A^2b^2c^7d^2e^4f^2 + 132A^2b^2c^7d^3e^2f^3 + 446A^2b^3c^6d^2e^3f^3 - 604A^2b^4c^5d^2e^2f^4 + 340A^3b^2c^6d^2e^2f^4 + 72B^2b^2c^7d^3e^3f^2 + 134B^2b^3c^6d^2e^4f^2 - 306B^2b^3c^6d^3e^2f^3 - 264B^2b^4c^5d^2e^3f^3 + 188B^2b^5c^4d^2e^2f^4 + 292B^3b^2c^6d^2e^3f^3 + 6B^3b^3c^5d^2e^2f^4 + 4A^2b^2c^8d^3e^4f + 2A^2b^3c^7d^2e^5f - 16A^2b^3c^7d^4e^2f^3 + 48A^2b^5c^5d^e^5f^2 + 72A^2b^5c^5d^3e^2f^4 - 84A^2b^6c^4d^e^4f^3 + 64A^2b^7c^3d^e^3f^4 - 88A^2b^7c^3d^2e^2f^5 - 18A^2b^8c^2d^e^2f^5 - 8A^2b^8c^8d^2e^5f + 64A^2b^8c^8d^4e^2f^3 + 26A^2b^2c^7d^e^6f + 16A^2b^7c^2d^e^6f + 192A^3b^c^7d^e^5f^2 + 96A^3b^c^7d^3e^2f^4 - 136A^3b^5c^3d^e^6f + 80A^4b^c^6d^e^3f^4 - 192A^4b^c^6d^2e^2f^5 + 268A^4b^3c^4d^e^6f - 10B^2b^2c^8d^4e^3f + 40B^2b^2c^8d^5e^2f^2 - 2B^2b^3c^7d^3e^4f + 8B^2b^4c^6d^2e^5f + 36B^2b^4c^6d^4e^2f^3 - 4B^2b^6c^4d^e^5f^2 - 88B^2b^6c^4d^3e^2f^4 + 6B^2b^7c^3d^e^4f^3 - 4B^2b^8c^2d^e^3f^4 + 16B^2b^8c^2d^2e^2f^5 + 8B^2b^2c^8d^3e^4f - 5B^2b^3c^6d^e^6f - 8B^3b^6c^2d^e^2f^6 + 40B^4b^c^6d^e^4f^3 + 104B^4b^4c^3d^e^6f + 148B^5b^c^5d^e^2f^5 - 344B^5b^2c^4d^e^6f - 46A^2b^2c^8d^4e^2f^2 - 4A^2b^3c^7d^3e^3f^2 + 40A^2b^4c^6d^2e^4f^2 - 36A^2b^4c^6d^3e^2f^3 - 158A^2b^5c^5d^2e^3f^3 + 196A^2b^6c^4d^2e^2f^4 + 16A^2b^6c^8d^3e^3f^2 - 176A^2b^3c^6d^e^5f^2 - 120A^2b^3c^6d^3e^2f^4 + 380A^2b^4c^5d^e^4f^3 - 324A^2b^5c^4d^e^3f^4 + 272A^2b^5c^4d^2e^2f^5 + 80A^2b^6c^3d^e^2f^5 - 280A^3b^c^7d^2e^3f^3 - 572A^3b^2c^6d^e^4f^3 + 508A^3b^3c^5d^e^3f^4 - 144A^3b^3c^5d^e^3f^4 - 144A^3b^3c^5d^e^3f^4
\end{aligned}$$

$$\begin{aligned}
& b^3c^5d^2ef^5 - 4Aa^3b^4c^4d^2ef^5 - 326Aa^4b^2c^5d^2ef^5 \\
& + 31Bab^3c^7d^4e^2f^2 - 32Bab^4c^6d^3e^3f^2 - 40Bab^5c^5 \\
& d^2e^4f^2 + 102Bab^5c^5d^3e^2f^3 + 72Bab^6c^4d^2e^3f^3 - 5 \\
& 6Bab^7c^3d^2e^2f^4 - 76Bab^2b^8c^4d^4e^2f^2 - 20Bab^2b^2c^7d \\
& ^2e^5f - 112Bab^2b^2c^7d^4e^2f^3 + 24Bab^2b^4c^5d^2e^5f^2 + 192B \\
& ab^2b^4c^5d^3e^2f^4 - 42Bab^2b^5c^4d^2e^4f^3 + 32Bab^2b^6c^3d^2e^ \\
& 3f^4 - 38Bab^2b^6c^3d^2e^2f^5 - 9Bab^2b^7c^2d^2e^2f^5 - 152Bab^3 \\
& b^7c^7d^2e^4f^2 + 360Bab^3b^7c^7d^3e^2f^3 - 32Bab^3b^2c^6d^2e^5f^ \\
& 2 - 144Bab^3b^2c^6d^3e^2f^4 + 62Bab^3b^3c^5d^2e^4f^3 - 40Bab^3b^4 \\
& c^4d^2e^3f^4 - 152Bab^3b^4c^4d^2e^2f^5 + 14Bab^3b^5c^3d^2e^2f^5 - \\
& 472Bab^4b^7c^6d^2e^2f^4 - 120Bab^4b^2c^5d^2e^3f^4 + 512Bab^4b^2 \\
& c^5d^2e^2f^5 - 13Bab^4b^3c^4d^2e^2f^5)/(16a^2c^6d^4 + a^4b^4f^4 + \\
& 16a^4c^4e^4 + b^4c^4d^4 + 16a^6c^2f^4 + b^8d^2f^2 - 8ab^2c^5 \\
& d^4 - 8a^5b^2c^2f^4 + 2a^2b^6d^2f^3 - 2a^3b^5e^2f^3 - 64a^3c^5d^3 \\
& f - 64a^5c^3d^2f^3 - 2b^5c^3d^3e + 2b^6c^2d^3f + a^2b^4c^2e^4 \\
& - 8a^3b^2c^3e^4 + 32a^3c^5d^2e^2 + a^2b^6e^2f^2 + 96a^4c^4d^2 \\
& f^2 + b^6c^2d^2e^2 + 32a^5c^3e^2f^2 - 2ab^7d^2e^2f^2 - 2b^7c^2d^2 \\
& e^2f + 54a^2b^4c^2d^2f^2 - 112a^3b^2c^3d^2f^2 + 16ab^3c^4d^3 \\
& e - 2ab^5c^2d^2e^3 - 32a^2b^7c^5d^3e - 32a^3b^7c^4d^2e^3 - 20ab^4 \\
& c^3d^3f - 12ab^6c^2d^2f^2 - 20a^3b^4c^2d^2f^3 - 2a^2b^5c^2e^3f - 3 \\
& 2a^4b^3c^3e^3f + 16a^4b^3c^2e^2f^3 - 32a^5b^2c^2e^2f^3 - 64a^4c^4d^2 \\
& e^2f - 6ab^4c^3d^2e^2 + 16a^2b^3c^3d^2e^3 + 64a^2b^2c^4d^3f + \\
& 64a^4b^2c^2d^2f^3 + 16a^3b^3c^2e^3f - 6a^3b^4c^2e^2f^2 - 48a^2 \\
& b^3c^3d^2e^2f - 36a^2b^4c^2d^2e^2f + 96a^3b^2c^3d^2e^2f - 48a^3 \\
& b^3c^2d^2e^2f^2 + 4ab^6c^2d^2e^2f + 18ab^5c^2d^2e^2f + 18a^2b^5c^2 \\
& d^2e^2f^2 + 32a^3b^7c^4d^2e^2f + 32a^4b^7c^3d^2e^2f^2) + (x*(64Bab^7c^4f \\
& ^8 + 4Aa^3b^7c^4f^8 - 256Aa^6b^7c^4f^8 - Bab^4b^6c^4f^8 + 48Aa^3c \\
& ^8e^7f + 256Aa^6c^5e^7f - 320Bab^6c^5d^7f + 3Bab^10c^d^2f^6 - \\
& 48Aa^4b^5c^2f^8 + 192Aa^5b^3c^3f^8 + 12Bab^5b^4c^2f^8 - 48B \\
& ab^6b^2c^3f^8 + 256Aa^4c^7e^5f^3 + 464Aa^5c^6e^3f^5 - 16Ab^3 \\
& c^8d^5f^3 - 48Ab^5c^6d^4f^4 - 36Ab^7c^4d^3f^5 - 4Ab^9c^2d^ \\
& 2f^6 - 64Bab^2c^9d^5f^3 + 320Bab^3c^8d^4f^4 - 640Bab^4c^7d^3f^ \\
& 5 + 640Bab^5c^6d^2f^6 - 16Bab^4c^7e^6f^2 - 64Bab^5c^6e^4f^4 + 1 \\
& 6Bab^6c^5e^2f^6 + 4Bab^4c^7d^5f^3 + 23Bab^6c^5d^4f^4 + 22Bab^8 \\
& c^3d^3f^5 + 320Aab^3c^7d^4f^4 + 352Aab^5c^5d^3f^5 + 76Aab^7 \\
& c^3d^2f^6 - 512Aab^2b^8c^4d^4f^4 - 60Aab^2b^7c^2d^2f^7 + 1408Aa \\
& a^3b^7c^7d^3f^5 + 352Aa^3b^5c^3d^2f^7 - 1792Aa^4b^7c^6d^2f^6 - 97 \\
& 6Aa^4b^3c^4d^2f^7 - 6Aab^5c^5e^6f^2 + 4Aab^6c^4e^5f^3 + 4A \\
& ab^7c^3e^4f^4 - 6Aab^8c^2e^3f^5 - 20Aa^2b^2c^7e^7f - 144A \\
& a^3b^7c^7e^6f^2 + 68Aa^3b^6c^2e^2f^7 - 640Aa^4b^7c^6e^4f^4 - 240 \\
& Aa^4b^4c^3e^2f^7 - 848Aa^5b^5c^5e^2f^6 + 192Aa^5b^2c^4e^2f^7 - \\
& 132Bab^4c^6d^4f^4 - 196Bab^6c^4d^3f^5 - 40Bab^8c^2d^2f^6 \\
& - 20Bab^3b^6c^2d^2f^7 + 52Bab^4b^4c^3d^2f^7 + 64Bab^5b^2c^4d^2f^7 \\
& + 2Bab^2b^3c^6e^7f + Bab^2b^8c^2e^2f^6 + 16Bab^4b^7c^6e^5f^3 + 12 \\
& 0Bab^5b^7c^5e^3f^5 + 64Aa^2c^9d^2e^5f + 512Aa^2c^9d^4e^2f^3 - \\
& 384Aa^3c^8d^2e^5f^2 - 1408Aa^3c^8d^3e^2f^4 - 1280Aa^4c^7d^2e^3f \\
& ^4 + 1792Aa^4c^7d^2e^2f^5 - 4Ab^2c^9d^4e^3f + 16Ab^2c^9d^5e^ \\
& f^2 + 8Ab^3c^8d^3e^4f - 2Ab^4c^7d^2e^5f + 80Ab^4c^7d^4e^2f^3 \\
& + 6Ab^6c^5d^2e^5f^2 + 76Ab^6c^5d^3e^2f^4 - 4Ab^7c^4d^2e^4f^3 \\
& - 4Ab^8c^3d^2e^3f^4 - 2Ab^8c^3d^2e^2f^5 + 6Ab^9c^2d^2e^2f^5 - 3 \\
& 2Bab^2c^9d^3e^4f - 96Bab^4c^7d^2e^4f^3 - 192Bab^5c^6d^2e^2f^5 + \\
& 2Bab^3c^8d^4e^3f - 8Bab^3c^8d^5e^2f^2 - 6Bab^4c^7d^3e^4f + 4B \\
& ab^5c^6d^2e^5f - 48Bab^5c^6d^4e^2f^3 - 60Bab^7c^4d^3e^2f^4 - 8B \\
& ab^9c^2d^2e^2f^5 + 4Aab^9c^2d^2f^7 - 2Ab^10c^2d^2e^2f^6 - 1184Aa^2b^3 \\
& c^6d^3f^5 - 544Aa^2b^5c^4d^2f^6 + 1664Aa^3b^3c^5d^2f^6 + 60 \\
& Aa^2b^3c^6e^6f^2 - 30Aa^2b^4c^5e^5f^3 - 64Aa^2b^5c^4e^4f^4 \\
& + 72Aa^2b^6c^3e^3f^5 - 12Aa^2b^7c^2e^2f^6 - 8Aa^3b^2c^6e^ \\
& 5f^3 + 352Aa^3b^3c^5e^4f^4 - 268Aa^3b^4c^4e^3f^5 - 52Aa^3b^ \\
& 5c^3e^2f^6 + 188Aa^4b^2c^5e^3f^5 + 484Aa^4b^3c^4e^2f^6 + 80*
\end{aligned}$$

$$\begin{aligned}
& B^2a^2b^2c^7d^4f^4 + 520B^2a^2b^4c^5d^3f^5 + 210B^2a^2b^6c^3d^2f^6 - 192B^2a^3b^2c^6d^3f^5 - 456B^2a^3b^4c^4d^2f^6 + 96B^2a^4b^2c^5d^2f^6 - 7B^2a^2b^4c^5e^6f^2 + 8B^2a^2b^5c^4e^5f^3 - 2B^2a^2b^6c^3e^4f^4 - 2B^2a^2b^7c^2e^3f^5 + 32B^2a^3b^2c^6e^6f^2 - 36B^2a^3b^3c^5e^5f^3 - 4B^2a^3b^4c^4e^4f^4 + 28B^2a^3b^5c^3e^3f^5 - 12B^2a^3b^6c^2e^2f^6 + 64B^2a^4b^2c^5e^4f^4 - 110B^2a^4b^3c^4e^3f^5 + 47B^2a^4b^4c^3e^2f^6 - 64B^2a^5b^2c^4e^2f^6 - 384A^2a^2c^9d^3e^3f^2 + 1184A^2a^3c^8d^2e^3f^3 - 28A^2b^3c^8d^4e^2f^2 - 12A^2b^4c^7d^3e^3f^2 + 20A^2b^5c^6d^2e^4f^2 - 36A^2b^5c^6d^3e^2f^3 - 44A^2b^6c^5d^2e^3f^3 + 32A^2b^7c^4d^2e^2f^4 + 144B^2a^2c^9d^4e^2f^2 + 192B^2a^3c^8d^2e^4f^2 - 448B^2a^3c^8d^3e^2f^3 + 480B^2a^4c^7d^2e^2f^4 + 23B^2b^4c^7d^4e^2f^2 - 4B^2b^5c^6d^3e^3f^2 - 13B^2b^6c^5d^2e^4f^2 + 48B^2b^6c^5d^3e^2f^3 + 12B^2b^7c^4d^2e^3f^3 + 2B^2b^8c^3d^2e^2f^4 + 64A^2a^2b^2c^9d^5f^3 + 1088A^2a^5b^2c^5d^2f^7 + 2A^2a^2b^4c^6e^7f + 2A^2a^2b^9c^2e^2f^6 - 6A^2a^2b^8c^2e^7f + 2B^2a^2b^8c^2d^2f^7 - 8B^2a^3b^2c^7e^7f + 16A^2a^2c^10d^4e^3f - 64A^2a^2c^10d^5e^2f^2 - 1088A^2a^5c^6d^2e^6f - 2A^2b^5c^6d^2e^6f - 32B^2a^3c^8d^2e^6f - 32A^2a^2b^2c^9d^3e^4f + 24A^2a^2b^3c^7d^2e^6f + 24A^2a^2b^8c^2d^2e^6f - 64A^2a^2b^2c^8d^2e^6f - 8B^2a^2b^2c^9d^4e^3f + 32B^2a^2b^2c^9d^5e^2f^2 - 6B^2a^2b^4c^6d^2e^6f + 288B^2a^5b^2c^5d^2e^6f - 1496A^2a^2b^2c^7d^2e^3f^3 + 1496A^2a^2b^3c^6d^2e^2f^4 - 272B^2a^2b^2c^7d^2e^4f^2 + 640B^2a^2b^2c^7d^3e^2f^3 + 636B^2a^2b^3c^6d^2e^3f^3 - 286B^2a^2b^4c^5d^2e^2f^4 + 192B^2a^3b^2c^6d^2e^2f^4 + 112A^2a^2b^2c^9d^4e^2f^2 - 8A^2a^2b^2c^8d^2e^5f - 448A^2a^2b^2c^8d^4e^2f^3 - 88A^2a^2b^4c^6d^2e^5f^2 - 576A^2a^2b^4c^6d^3e^2f^4 + 84A^2a^2b^5c^5d^2e^4f^3 + 32A^2a^2b^6c^4d^2e^3f^4 + 12A^2a^2b^6c^4d^2e^2f^5 - 80A^2a^2b^7c^3d^2e^2f^5 - 108A^2a^2b^6c^3d^2e^2f^6 + 736A^2a^3b^2c^7d^2e^4f^3 + 192A^2a^3b^4c^4d^2e^2f^6 + 2048A^2a^4b^2c^6d^2e^2f^5 + 208A^2a^4b^2c^5d^2e^2f^6 + 32B^2a^2b^2c^8d^3e^4f - 20B^2a^2b^3c^7d^2e^5f + 288B^2a^2b^3c^7d^4e^2f^3 + 20B^2a^2b^5c^5d^2e^5f^2 + 480B^2a^2b^5c^5d^3e^2f^4 - 20B^2a^2b^6c^4d^2e^4f^3 + 72B^2a^2b^7c^3d^2e^2f^5 + 10B^2a^2b^8c^2d^2e^2f^5 + 16B^2a^2b^2c^8d^2e^5f - 384B^2a^2b^2c^8d^4e^2f^3 + 32B^2a^2b^2c^7d^2e^6f + 44B^2a^2b^7c^2d^2e^2f^6 + 192B^2a^3b^2c^7d^2e^5f^2 + 960B^2a^3b^2c^7d^3e^2f^4 - 160B^2a^3b^5c^3d^2e^2f^6 + 544B^2a^4b^2c^6d^2e^3f^4 - 896B^2a^4b^2c^6d^2e^2f^5 + 120B^2a^4b^3c^4d^2e^2f^6 - 4B^2a^2b^9c^2d^2e^2f^6 + 144A^2a^2b^2c^8d^3e^3f^2 - 144A^2a^2b^3c^7d^2e^4f^2 + 112A^2a^2b^3c^7d^3e^2f^3 + 476A^2a^2b^4c^6d^2e^3f^3 - 412A^2a^2b^5c^5d^2e^2f^4 + 256A^2a^2b^2c^8d^2e^4f^2 + 128A^2a^2b^2c^8d^3e^2f^3 + 352A^2a^2b^2c^7d^2e^5f^2 + 1440A^2a^2b^2c^7d^3e^2f^4 - 456A^2a^2b^3c^6d^2e^4f^3 - 116A^2a^2b^4c^5d^2e^3f^4 + 224A^2a^2b^4c^5d^2e^2f^5 + 452A^2a^2b^5c^4d^2e^2f^5 - 1440A^2a^3b^2c^7d^2e^2f^4 + 528A^2a^3b^2c^6d^2e^3f^4 - 1408A^2a^3b^2c^6d^2e^2f^5 - 1424A^2a^3b^3c^5d^2e^2f^5 - 128B^2a^2b^2c^8d^4e^2f^2 + 8B^2a^2b^3c^7d^3e^3f^2 + 108B^2a^2b^4c^6d^2e^4f^2 - 324B^2a^2b^4c^6d^3e^2f^3 - 164B^2a^2b^5c^5d^2e^3f^3 + 44B^2a^2b^6c^4d^2e^2f^4 + 32B^2a^2b^2c^8d^3e^3f^2 - 128B^2a^2b^3c^6d^2e^5f^2 - 1200B^2a^2b^3c^6d^3e^2f^4 + 142B^2a^2b^4c^5d^2e^4f^3 + 20B^2a^2b^5c^4d^2e^3f^4 - 304B^2a^2b^5c^4d^2e^2f^5 - 112B^2a^2b^6c^3d^2e^2f^5 - 688B^2a^3b^2c^7d^2e^3f^3 - 224B^2a^3b^2c^6d^2e^4f^3 - 216B^2a^3b^3c^5d^2e^3f^4 + 800B^2a^3b^3c^5d^2e^2f^5 + 460B^2a^3b^4c^4d^2e^2f^5 - 640B^2a^4b^2c^5d^2e^2f^5)) / ((16a^2c^6d^4 + a^4b^4f^4 + 16a^4c^4e^4 + b^4c^4d^4 + 16a^6c^2f^4 + b^8d^2f^2 - 8a^2b^2c^5d^4 - 8a^5b^2c^2f^4 + 2a^2b^6d^2f^3 - 2a^3b^5e^2f^3 - 64a^3c^5d^3f - 64a^5c^3d^2f^3 - 2b^5c^3d^3e + 2b^6c^2d^3f + a^2b^4c^2e^4 - 8a^3b^2c^3e^4 + 32a^3c^5d^2e^2 + a^2b^6e^2f^2 + 96a^4c^4d^2f^2 + b^6c^2d^2e^2 + 32a^5c^3e^2f^2 - 2a^2b^7d^2e^2f - 2b^7c^2d^2e^2f + 54a^2b^4c^2d^2f^2 - 112a^3b^2c^3d^2f^2 + 16a^2b^3c^4d^3e - 2a^2b^5c^2d^2e^3 - 32a^2b^2c^5d^3e - 32a^3b^2c^4d^2e^3 - 20a^2b^4c^3d^3f - 12a^2b^6c^2d^2f^2 - 20a^3b^4c^2d^2f^3 - 2a^2b^5c^2e^3f - 32a^4b^2c^3e^3f + 16a^4b^3c^2e^3f - 32a^5b^2c^2e^2f^3 - 64a^4c^4d^2e^2f - 6a^2b^4c^3d^2e^2 + 16a^
\end{aligned}$$

$$\begin{aligned}
& ^2b^3c^3de^3 + 64a^2b^2c^4d^3f + 64a^4b^2c^2d^2f^3 + 16a^3b^3 \\
& *c^2e^3f - 6a^3b^4c^2e^2f^2 - 48a^2b^3c^3d^2e^2f - 36a^2b^4c^2* \\
& d^2e^2f + 96a^3b^2c^3d^2e^2f - 48a^3b^3c^2d^2e^2f^2 + 4a^2b^6c^2d^2e^2 \\
& *f + 18a^2b^5c^2d^2e^2f + 18a^2b^5c^2d^2e^2f^2 + 32a^3b^2c^4d^2e^2f + 3 \\
& 2a^4b^2c^3d^2e^2f^2) - (56A^2a^3b^3c^3f^7 - 13A^2a^2b^5c^2f^7 - \\
& 24A^2a^2c^7e^5f^2 - 16A^2b^3c^6d^3f^4 - A^2b^5c^4d^2f^5 + 2A \\
& ^2b^4c^5e^5f^2 - A^2b^5c^4e^4f^3 - A^2b^6c^3e^3f^4 + 2A^2b^7* \\
& c^2e^2f^5 - 12B^2a^4c^5e^3f^4 - 8B^2b^3c^6d^4f^3 - 9B^2b^5c^ \\
& 4d^3f^4 + 64A^2B^2a^5c^4f^7 + A^2a^2b^7c^2f^7 - A^2b^8c^2e^2f^6 - 80A^2 \\
& *a^4b^2c^4f^7 - 16A^2b^2c^8d^4f^3 + 28A^2a^4c^5e^2f^6 - 2A^2b^7c^ \\
& 2d^2f^6 - A^2b^3c^6e^6f - 16B^2a^5c^4e^2f^6 - 4A^2c^9d^3e^3f + \\
& 12A^2c^9d^4e^2f^2 + 48A^2a^2b^2c^7d^3f^4 + 22A^2a^2b^5c^3d^2f^6 + 48 \\
& *A^2a^3b^2c^5d^2f^6 + 12A^2a^2b^6c^2e^2f^6 + 16B^2a^2b^2c^7d^4f^3 + 64 \\
& *B^2a^4b^2c^4d^2f^6 - 16A^2a^2c^8d^3e^2f^3 + 80A^2a^3c^6d^2e^2f^5 + 4* \\
& A^2b^2c^8d^2e^4f + A^2b^2c^7d^2e^5f + 2A^2b^6c^3d^2e^2f^5 + 4B^2a \\
& ^2c^7d^2e^5f - 64B^2a^4c^5d^2e^2f^5 + A^2b^8c^2d^2f^6 + 8A^2a^2b^3c^5 \\
& *d^2f^5 - 64A^2a^2b^3c^4d^2f^6 - 2A^2a^2b^2c^6e^5f^2 - 10A^2a^2b^ \\
& 3c^5e^4f^3 + 20A^2a^2b^4c^4e^3f^4 - 25A^2a^2b^5c^3e^2f^5 + 56A^ \\
& 2a^2b^2c^6e^4f^3 - 44A^2a^2b^4c^3e^2f^6 - 76A^2a^3b^2c^5e^2f^5 + \\
& 40A^2a^3b^2c^4e^2f^6 + 56B^2a^2b^3c^5d^3f^4 - 2B^2a^2b^5c^3d^2* \\
& f^5 - 96B^2a^2b^2c^6d^3f^4 + 11B^2a^2b^5c^2d^2f^6 + 16B^2a^3b^2c^ \\
& 5d^2f^5 - 40B^2a^3b^3c^3d^2f^6 + 16B^2a^4b^2c^4e^2f^5 + 3B^2a^4 \\
& *b^2c^3e^2f^6 + 24A^2a^2c^8d^2e^3f^2 + 92A^2a^2c^7d^2e^3f^3 - 104* \\
& A^2a^2c^7d^2e^2f^4 - 4A^2b^2c^8d^3e^2f^2 + 20A^2b^2c^7d^3e^2f^3 \\
& - A^2b^4c^5d^2e^3f^3 - 8A^2b^4c^5d^2e^2f^4 + 32B^2a^2c^7d^3e^2f^ \\
& 3 - 8B^2a^3c^6d^2e^3f^3 + 48B^2a^3c^6d^2e^2f^4 - B^2b^2c^7d^3e^ \\
& 3f + 3B^2b^2c^7d^4e^2f^2 + B^2b^3c^6d^2e^4f + 8B^2b^4c^5d^3e \\
& *f^3 - 3B^2b^6c^3d^2e^2f^4 - A^2b^6c^2f^7 - 32A^2B^2a^2c^8d^4f^3 - \\
& 32A^2B^2a^4c^5d^2f^6 - 8A^2B^2a^2c^7e^6f + 4A^2a^2b^2c^7e^6f - B^2a^2b \\
& ^7c^2d^2f^6 - 8A^2a^2c^8d^2e^5f - 65A^2a^2b^2c^5e^3f^4 + 88A^2a^2* \\
& b^3c^4e^2f^5 + 8B^2a^2b^3c^4d^2f^5 + 2B^2a^3b^2c^4e^3f^4 - 3 \\
& *B^2a^3b^3c^3e^2f^5 - 13A^2b^2c^7d^2e^3f^2 + 16A^2b^3c^6d^2* \\
& e^2f^3 - 28B^2a^2c^7d^2e^3f^2 + B^2b^3c^6d^3e^2f^2 - 5B^2b^4* \\
& c^5d^2e^3f^2 + 7B^2b^5c^4d^2e^2f^3 + 12A^2B^2a^3b^4c^2f^7 - 48A \\
& *B^2a^4b^2c^3f^7 + 160A^2B^2a^2c^7d^3f^4 - 160A^2B^2a^3c^6d^2f^5 - 16 \\
& *A^2B^2a^3c^6e^4f^3 + 48A^2B^2a^4c^5e^2f^5 + 24A^2B^2b^2c^7d^4f^3 + 24 \\
& *A^2B^2b^4c^5d^3f^4 - A^2B^2b^6c^3d^2f^5 + 20B^2a^2b^2c^6d^2e^3f^2 - \\
& 49B^2a^2b^3c^5d^2e^2f^3 + 96B^2a^2b^2c^6d^2e^2f^3 + 19B^2a^2b \\
& ^2c^5d^2e^3f^3 - 102B^2a^2b^2c^5d^2e^2f^4 + 3B^2a^2b^3c^4d^2e^2* \\
& f^4 - 120A^2B^2a^2b^2c^6d^3f^4 + 4A^2B^2a^2b^4c^4d^2f^5 + 24A^2B^2a^2b^4* \\
& c^3d^2f^6 - 8A^2B^2a^3b^2c^4d^2f^6 - 7A^2B^2a^2b^3c^5e^5f^2 + 8A^2B^2a^2b^4 \\
& *c^4e^4f^3 - 2A^2B^2a^2b^5c^3e^3f^4 - 2A^2B^2a^2b^6c^2e^2f^5 + 28A^2B^2a \\
& ^2b^2c^6e^5f^2 - 11A^2B^2a^2b^5c^2e^2f^6 + 72A^2B^2a^3b^2c^5e^3f^4 + 34 \\
& *A^2B^2a^3b^3c^3e^2f^6 + 16A^2B^2a^2c^8d^3e^2f^2 + 80A^2B^2a^2c^7d^2e^4f^ \\
& 2 + 16A^2B^2a^3c^6d^2e^2f^4 - 4A^2B^2b^2c^7d^2e^4f - 22A^2B^2b^3c^6d^3 \\
& *e^2f^3 + 3A^2B^2b^4c^5d^2e^4f^2 - 6A^2B^2b^5c^4d^2e^3f^3 + 15A^2B^2b^5c^4 \\
& *d^2e^2f^4 + 4A^2B^2b^6c^3d^2e^2f^4 + 12A^2a^2b^2c^7d^2e^4f^2 - 28A^2a^2* \\
& b^4c^4d^2e^2f^5 - 2B^2a^2b^2c^6d^2e^5f + 2B^2a^2b^6c^2d^2e^2f^5 + A^2B^2a \\
& ^2b^7c^2e^2f^6 + 24A^2B^2a^2b^2c^5d^2f^5 - 28A^2B^2a^2b^2c^5e^4f^3 - 9* \\
& A^2B^2a^2b^3c^4e^3f^4 + 29A^2B^2a^2b^4c^3e^2f^5 - 100A^2B^2a^3b^2c^4* \\
& e^2f^5 - 208A^2B^2a^2c^7d^2e^2f^3 + 13A^2B^2b^3c^6d^2e^3f^2 - 23A^2B^2 \\
& *b^4c^5d^2e^2f^3 - 52A^2a^2b^2c^7d^2e^2f^3 - 34A^2a^2b^2c^6d^2e^3* \\
& f^3 + 48A^2a^2b^2c^6d^2e^2f^4 + 40A^2a^2b^3c^5d^2e^2f^4 - 108A^2a^2* \\
& b^2c^6d^2e^2f^4 + 36A^2a^2b^2c^5d^2e^2f^5 - 8B^2a^2b^2c^7d^3e^2f^2 - \\
& 24B^2a^2b^2c^6d^3e^2f^3 + 7B^2a^2b^3c^5d^2e^4f^2 - 8B^2a^2b^4c^4d \\
& *e^3f^3 + 32B^2a^2b^4c^4d^2e^2f^4 + 2B^2a^2b^5c^3d^2e^2f^4 - 16B^2* \\
& a^2b^2c^6d^2e^4f^2 - 20B^2a^2b^4c^3d^2e^2f^5 + 8B^2a^3b^2c^5d^2e^2f^ \\
& 4 + 40B^2a^3b^2c^4d^2e^2f^5 - 10A^2B^2a^2b^6c^2d^2f^6 + 2A^2B^2a^2b^2c^6e \\
& ^6f - 20A^2B^2a^4b^2c^4e^2f^6 + 4A^2B^2b^2c^8d^3e^3f - 12A^2B^2b^2c^8d^4e^2
\end{aligned}$$

$$\begin{aligned}
& f^2 - 2* A * B * b^7 * c^2 * d * e * f^5 + 132 * A * B * a * b^2 * c^6 * d^2 * e^2 * f^3 + 96 * A * B * a^2 * b^2 * c^5 * d * e^2 * f^4 + 24 * A * B * a * b * c^7 * d^3 * e * f^3 + 20 * A * B * a * b^5 * c^3 * d * e * f^5 - 24 * A * B * a^3 * b * c^5 * d * e * f^5 - 24 * A * B * a * b * c^7 * d^2 * e^3 * f^2 - 44 * A * B * a * b^2 * c^6 * d * e^4 * f^2 + 84 * A * B * a * b^3 * c^5 * d * e^3 * f^3 - 122 * A * B * a * b^3 * c^5 * d^2 * e * f^4 - 54 * A * B * a * b^4 * c^4 * d * e^2 * f^4 - 180 * A * B * a^2 * b * c^6 * d * e^3 * f^3 + 288 * A * B * a^2 * b * c^6 * d^2 * e * f^4 - 18 * A * B * a^2 * b^3 * c^4 * d * e * f^5 + 4 * A * B * a * b * c^7 * d * e^5 * f) / (16 * a^2 * c^6 * d^4 + a^4 * b^4 * f^4 + 16 * a^4 * c^4 * e^4 + b^4 * c^4 * d^4 + 16 * a^6 * c^2 * f^4 + b^8 * d^2 * f^2 - 8 * a * b^2 * c^5 * d^4 - 8 * a^5 * b^2 * c * f^4 + 2 * a^2 * b^6 * d * f^3 - 2 * a^3 * b^5 * e * f^3 - 64 * a^3 * c^5 * d^3 * f - 64 * a^5 * c^3 * d * f^3 - 2 * b^5 * c^3 * d^3 * e + 2 * b^6 * c^2 * d^3 * f + a^2 * b^4 * c^2 * e^4 - 8 * a^3 * b^2 * c^3 * e^4 + 32 * a^3 * c^5 * d^2 * e^2 + a^2 * b^6 * e^2 * f^2 + 96 * a^4 * c^4 * d^2 * f^2 + b^6 * c^2 * d^2 * e^2 + 32 * a^5 * c^3 * e^2 * f^2 - 2 * a * b^7 * d * e * f^2 - 2 * b^7 * c * d^2 * e * f + 54 * a^2 * b^4 * c^2 * d^2 * f^2 - 112 * a^3 * b^2 * c^3 * d^2 * f^2 + 16 * a * b^3 * c^4 * d^3 * e - 2 * a * b^5 * c^2 * d * e^3 - 32 * a^2 * b * c^5 * d^3 * e - 32 * a^3 * b * c^4 * d * e^3 - 20 * a * b^4 * c^3 * d^3 * f - 12 * a * b^6 * c * d^2 * f^2 - 20 * a^3 * b^4 * c * d * f^3 - 2 * a^2 * b^5 * c * e^3 * f - 32 * a^4 * b * c^3 * e^3 * f + 16 * a^4 * b^3 * c * e * f^3 - 32 * a^5 * b * c^2 * e * f^3 - 64 * a^4 * c^4 * d * e^2 * f - 6 * a * b^4 * c^3 * d^2 * e^2 + 16 * a^2 * b^3 * c^3 * d * e^3 + 64 * a^2 * b^2 * c^4 * d^3 * f + 64 * a^4 * b^2 * c^2 * d * f^3 + 16 * a^3 * b^3 * c^2 * e^3 * f - 6 * a^3 * b^4 * c * e^2 * f^2 - 48 * a^2 * b^3 * c^3 * d^2 * e * f - 36 * a^2 * b^4 * c^2 * d * e^2 * f + 96 * a^3 * b^2 * c^3 * d * e^2 * f - 48 * a^3 * b^3 * c^2 * d * e * f^2 + 4 * a * b^6 * c * d * e^2 * f + 18 * a * b^5 * c^2 * d^2 * e * f + 18 * a^2 * b^5 * c * d * e * f^2 + 32 * a^3 * b * c^4 * d^2 * e * f + 32 * a^4 * b * c^3 * d * e * f^2) + (x * (104 * A^2 * a^4 * c^5 * f^7 - 32 * B^2 * a^5 * c^4 * f^7 + 8 * A^2 * c^9 * d^4 * f^3 + A^2 * b^8 * c * f^7 + 50 * A^2 * a^2 * b^4 * c^3 * f^7 - 96 * A^2 * a^3 * b^2 * c^4 * f^7 - 12 * B^2 * a^3 * b^4 * c^2 * f^7 + 42 * B^2 * a^4 * b^2 * c^3 * f^7 + 208 * A^2 * a^2 * c^7 * d^2 * f^5 + 36 * A^2 * a^2 * c^7 * e^4 * f^3 + 72 * A^2 * a^3 * c^6 * e^2 * f^5 + 8 * A^2 * b^2 * c^7 * d^3 * f^4 + 18 * A^2 * b^4 * c^5 * d^2 * f^5 - 32 * B^2 * a^2 * c^7 * d^3 * f^4 + 32 * B^2 * a^3 * c^6 * d^2 * f^5 - 2 * A^2 * b^3 * c^6 * e^5 * f^2 + A^2 * b^4 * c^5 * e^4 * f^3 + A^2 * b^6 * c^3 * e^2 * f^5 + 24 * B^2 * a^3 * c^6 * e^4 * f^3 + 56 * B^2 * a^4 * c^5 * e^2 * f^5 + 2 * B^2 * b^2 * c^7 * d^4 * f^3 - 6 * B^2 * b^4 * c^5 * d^3 * f^4 + 9 * B^2 * b^6 * c^3 * d^2 * f^5 - 16 * A^2 * c^9 * d^3 * e^2 * f^2 - 12 * A^2 * a * b^6 * c^2 * f^7 + B^2 * a^2 * b^6 * c * f^7 - 64 * A^2 * a * c^8 * d^3 * f^4 - 256 * A^2 * a^3 * c^6 * d * f^6 + 2 * A^2 * b^6 * c^3 * d * f^6 + 32 * B^2 * a^4 * c^5 * d * f^6 + A^2 * b^2 * c^7 * e^6 * f - 2 * A^2 * b^7 * c^2 * e * f^6 + 4 * B^2 * a^2 * c^7 * e^6 * f + 4 * A^2 * c^9 * d^2 * e^4 * f - 36 * A^2 * a * b^4 * c^4 * d * f^6 - 4 * A^2 * a * b * c^7 * e^5 * f^2 + 22 * A^2 * a * b^5 * c^3 * e * f^6 - 16 * A^2 * a^3 * b * c^5 * e * f^6 - 2 * B^2 * a * b^6 * c^2 * d * f^6 - 40 * B^2 * a^4 * b * c^4 * e * f^6 + 8 * A^2 * a * c^8 * d * e^4 * f^2 + 16 * A^2 * b * c^8 * d^3 * e * f^3 + 6 * A^2 * b^5 * c^4 * d * e * f^5 - 2 * A * B * a * b^7 * c * f^7 - 144 * A^2 * a * b^2 * c^6 * d^2 * f^5 + 168 * A^2 * a^2 * b^2 * c^5 * d * f^6 + 2 * A^2 * a * b^2 * c^6 * e^4 * f^3 + 10 * A^2 * a * b^3 * c^5 * e^3 * f^4 - 18 * A^2 * a * b^4 * c^4 * e^2 * f^5 - 80 * A^2 * a^2 * b * c^6 * e^3 * f^4 - 56 * A^2 * a^2 * b^3 * c^4 * e * f^6 + 24 * B^2 * a * b^2 * c^6 * d^3 * f^4 - 64 * B^2 * a * b^4 * c^4 * d^2 * f^5 + 26 * B^2 * a^2 * b^4 * c^3 * d * f^6 - 88 * B^2 * a^3 * b^2 * c^4 * d * f^6 - 12 * B^2 * a^2 * b * c^6 * e^5 * f^2 - 40 * B^2 * a^3 * b * c^5 * e^3 * f^4 + 6 * B^2 * a^3 * b^3 * c^3 * e * f^6 + 8 * A^2 * a * c^8 * d^2 * e^2 * f^3 - 128 * A^2 * a^2 * c^7 * d * e^2 * f^4 + 8 * A^2 * b * c^8 * d^2 * e^3 * f^2 + 4 * A^2 * b^2 * c^7 * d * e^4 * f^2 + 10 * A^2 * b^3 * c^6 * d * e^3 * f^3 - 18 * A^2 * b^4 * c^5 * d * e^2 * f^4 - 32 * B^2 * a^2 * c^7 * d * e^4 * f^2 - 80 * B^2 * a^3 * c^6 * d * e^2 * f^4 + B^2 * b^2 * c^7 * d^2 * e^4 * f + 10 * B^2 * b^3 * c^6 * d^3 * e * f^3 - 12 * B^2 * b^5 * c^4 * d^2 * e * f^4 + 72 * A * B * a^4 * b * c^4 * f^7 - 8 * A * B * b * c^8 * d^4 * f^3 - 176 * A * B * a^4 * c^5 * e * f^6 + 2 * A * B * b^7 * c^2 * d * f^6 - 4 * A^2 * b * c^8 * d * e^5 * f + 54 * A^2 * a^2 * b^2 * c^5 * e^2 * f^5 + 84 * B^2 * a^2 * b^2 * c^5 * d^2 * f^5 + 9 * B^2 * a^2 * b^2 * c^5 * e^4 * f^3 + 2 * B^2 * a^3 * b^2 * c^4 * e^2 * f^5 - 26 * A^2 * b^2 * c^7 * d^2 * e^2 * f^3 + 88 * B^2 * a^2 * c^7 * d^2 * e^2 * f^3 - 4 * B^2 * b^2 * c^7 * d^3 * e^2 * f^2 - 4 * B^2 * b^3 * c^6 * d^2 * e^3 * f^2 + 8 * B^2 * b^4 * c^5 * d^2 * e^2 * f^3 + 24 * A * B * a^2 * b^5 * c^2 * f^7 - 84 * A * B * a^3 * b^3 * c^3 * f^7 + 8 * A * B * a^2 * c^7 * e^5 * f^2 - 32 * A * B * a^3 * c^6 * e^3 * f^4 + 4 * A * B * b^3 * c^6 * d^3 * f^4 - 20 * A * B * b^5 * c^4 * d^2 * f^5 - 4 * B^2 * a * b * c^7 * d * e^5 * f - 78 * B^2 * a * b^2 * c^6 * d^2 * e^2 * f^3 - 48 * B^2 * a^2 * b^2 * c^5 * d * e^2 * f^4 + 148 * A * B * a * b^3 * c^5 * d^2 * f^5 - 192 * A * B * a^2 * b * c^6 * d^2 * f^5 - 4 * A * B * a^2 * b^3 * c^4 * d * f^6 + 10 * A * B * a * b^2 * c^6 * e^5 * f^2 - 2 * A * B * a * b^3 * c^5 * e^4 * f^3 - 12 * A * B * a * b^4 * c^4 * e^3 * f^4 + 8 * A * B * a * b^5 * c^3 * e^2 * f^5 - 52 * A * B * a^2 * b * c^6 * e^4 * f^3 - 44 * A * B * a^2 * b^4 * c^3 * e * f^6 - 48 * A * B * a^3 * b * c^5 * e^2 * f^5 + 204 * A * B * a^3 * b^2 * c^4 * e * f^6 - 48 * A * B * a * c^8 * d^2 * e^3 * f^2 - 48 * A * B * a^2 * c^7 * d * e^3 * f^3 - 16 * A * B * a^2 * c^7 * d^2 * e * f^4 + 16 * A * B * b * c^8 * d^3 * e^2 * f^2 - 28 * A * B * b^2 * c^7 * d^3 * e * f^3 - 6 * A * B * b^3 * c^6 * d * e^4 * f^2 + 8 * A * B * b^4 * c^5 * d^2 * e * f^4 + 12 * A * B * b^5 * c^4 * d * e^2 * f^4 - 40 * A^2 * a * b * c^7 * d * e^3 * f^3 + 80 * A^2 * a * b * c^7 * d^2 * e * f^4 - 24 * A^2 * a * b^3 * c^5 * d * e * f^5 + 48 * A^2 * a^2 * b * c^6 * d * e * f^5 - 24 *
\end{aligned}$$

$$\begin{aligned}
& B^2 a^2 b^2 c^7 d^3 e^2 f^3 - 8 B^2 a^2 b^5 c^3 d^2 e^2 f^5 + 56 B^2 a^3 b^2 c^5 d^2 e^2 f^5 \\
& - 4 A^2 B^2 a^2 b^2 c^7 e^6 f + 8 A^2 B^2 a^2 c^8 d^2 e^5 f + 96 A^2 B^2 a^2 b^2 c^5 e^3 f^4 - \\
& 36 A^2 B^2 a^2 b^3 c^4 e^2 f^5 + 4 A^2 B^2 b^2 c^7 d^2 e^3 f^2 + 8 A^2 B^2 b^3 c^6 d^2 e^2 f^3 + 84 A^2 a^2 b^2 c^6 d^2 e^2 f^4 \\
& + 24 B^2 a^2 b^2 c^7 d^2 e^3 f^2 + 14 B^2 a^2 b^2 c^6 d^2 e^4 f^2 - 16 B^2 a^2 b^3 c^5 d^2 e^3 f^3 + 98 B^2 a^2 b^3 c^5 d^2 e^2 f^4 \\
& + 12 B^2 a^2 b^4 c^4 d^2 e^2 f^4 + 64 B^2 a^2 b^2 c^6 d^2 e^3 f^3 - 120 B^2 a^2 b^2 c^6 d^2 e^2 f^4 + 30 B^2 a^2 b^3 c^4 d^2 e^2 f^5 \\
& + 16 A^2 B^2 a^2 b^2 c^7 d^3 f^4 - 12 A^2 B^2 a^2 b^5 c^3 d^2 f^6 + 112 A^2 B^2 a^3 b^2 c^5 d^2 f^6 + 2 A^2 B^2 a^2 b^6 c^2 e^2 f^6 + 48 A^2 \\
& B^2 a^2 c^8 d^3 e^2 f^3 + 144 A^2 B^2 a^3 c^6 d^2 e^2 f^5 - 4 A^2 B^2 b^2 c^8 d^2 e^4 f + 2 A^2 B^2 b^2 c^7 d^2 e^5 f - 10 A^2 B^2 b^6 c^3 d^2 e^2 f^5 \\
& + 100 A^2 B^2 a^2 b^4 c^4 d^2 e^2 f^5 + 64 A^2 B^2 a^2 b^2 c^7 d^2 e^2 f^3 + 12 A^2 B^2 a^2 b^2 c^6 d^2 e^3 f^3 - 108 A^2 B^2 a^2 b^2 c^6 d^2 e^2 f^4 \\
& - 100 A^2 B^2 a^2 b^3 c^5 d^2 e^2 f^4 + 288 A^2 B^2 a^2 b^2 c^6 d^2 e^2 f^4 - 324 A^2 B^2 a^2 b^2 c^5 d^2 e^2 f^5) / (16 a^2 c^6 d^4 + a^4 b^4 f^4 + 16 a^4 c^4 e^4 + \\
& b^4 c^4 d^4 + 16 a^6 c^2 f^4 + b^8 d^2 f^2 - 8 a^2 b^2 c^5 d^4 - 8 a^5 b^2 c^2 f^4 + 2 a^2 b^6 d^2 f^3 - 2 a^3 b^5 e^2 f^3 - 64 a^3 c^5 d^3 f \\
& - 64 a^5 c^3 d^2 f^3 - 2 b^5 c^3 d^3 e + 2 b^6 c^2 d^3 f + a^2 b^4 c^2 e^4 - 8 a^3 b^2 c^3 e^4 + 32 a^3 c^5 d^2 e^2 + a^2 b^6 e^2 f^2 + 96 a^4 c^4 d^2 f^2 \\
& + b^6 c^2 d^2 e^2 + 32 a^5 c^3 e^2 f^2 - 2 a^2 b^7 d^2 e^2 f - 2 b^7 c^2 d^2 e^2 f + 54 a^2 b^4 c^2 d^2 f^2 - 112 a^3 b^2 c^3 d^2 f^2 + 16 a^2 b^3 c^4 d^3 e \\
& - 2 a^2 b^5 c^2 d^2 e^3 - 32 a^2 b^2 c^5 d^3 e - 32 a^3 b^2 c^4 d^2 e^3 - 20 a^2 b^4 c^3 d^3 f - 12 a^2 b^6 c^2 d^2 f^2 - 20 a^3 b^4 c^2 d^2 f^3 \\
& - 2 a^2 b^5 c^2 e^3 f - 32 a^4 b^2 c^3 e^3 f + 16 a^4 b^3 c^2 e^2 f^3 - 32 a^5 b^2 c^2 e^2 f^3 - 64 a^4 c^4 d^2 e^2 f - 6 a^2 b^4 c^3 d^2 e^2 \\
& + 16 a^2 b^3 c^3 d^2 e^3 + 64 a^2 b^2 c^4 d^3 f + 64 a^4 b^2 c^2 d^2 f^3 + 16 a^3 b^3 c^2 e^3 f - 6 a^3 b^4 c^2 e^2 f^2 - 48 a^2 b^3 c^3 d^2 e^2 f \\
& - 36 a^2 b^4 c^2 d^2 e^2 f + 96 a^3 b^2 c^3 d^2 e^2 f - 48 a^3 b^3 c^2 d^2 e^2 f^2 + 4 a^2 b^6 c^2 d^2 e^2 f + 18 a^2 b^5 c^2 d^2 e^2 f + 18 a^2 b^5 c^2 d^2 e^2 f \\
& + 32 a^3 b^2 c^4 d^2 e^2 f + 32 a^4 b^2 c^3 d^2 e^2 f) - (3 A^3 b^2 c^5 e^2 f^4 - 4 A^3 c^7 d^2 f^4 - 12 A^3 a^2 c^5 f^6 - B^3 b^3 c^4 d^2 f^4 + 16 A^3 a^2 c^6 d^2 f^5 - \\
& 16 A^2 B^2 a^3 c^4 f^6 + 2 A^3 a^2 b^2 c^4 f^6 - 16 A^3 a^2 c^6 e^2 f^4 - 6 A^3 b^2 c^5 d^2 f^5 - 3 A^3 b^3 c^4 e^2 f^5 + 4 B^3 a^2 b^2 c^5 d^2 f^4 + 3 B^3 a^2 b^3 c^3 d^2 f^5 \\
& - 12 B^3 a^2 b^2 c^4 d^2 f^5 + 8 B^3 a^2 c^5 d^2 e^2 f^4 + 3 A^2 B^2 a^2 b^2 c^3 f^6 - 12 A^2 B^2 a^2 c^5 e^2 f^4 + A^2 B^2 b^2 c^5 d^2 f^4 - 3 A^2 B^2 b^3 c^4 e^2 f^4 \\
& + 16 A^3 a^2 b^2 c^5 e^2 f^5 + 4 A^3 b^2 c^6 d^2 e^2 f^4 - 3 A^2 B^2 a^2 b^3 c^3 f^6 + 16 A^2 B^2 a^2 b^2 c^4 f^6 - 8 A^2 B^2 a^2 c^6 d^2 f^4 + 24 A^2 B^2 a^2 c^5 d^2 f^5 \\
& - 3 A^2 B^2 b^4 c^3 d^2 f^5 + 4 A^2 B^2 b^2 c^6 d^2 f^4 - 8 A^2 B^2 a^2 c^5 e^2 f^5 + 9 A^2 B^2 b^3 c^4 d^2 f^5 + 3 A^2 B^2 b^4 c^3 e^2 f^5 + 4 A^2 B^2 a^2 b^2 c^4 d^2 f^5 - \\
& 3 A^2 B^2 a^2 b^3 c^3 e^2 f^5 + 16 A^2 B^2 a^2 b^2 c^4 e^2 f^5 + 16 A^2 B^2 a^2 b^2 c^5 e^2 f^4 - 14 A^2 B^2 a^2 b^2 c^4 e^2 f^5 + 4 A^2 B^2 b^3 c^4 d^2 e^2 f^4 - 8 A^2 B^2 b^2 c^5 d^2 e^2 f^4 \\
& - 2 B^3 a^2 b^2 c^4 d^2 e^2 f^4 + 2 A^2 B^2 a^2 b^2 c^4 e^2 f^4 - 28 A^2 B^2 a^2 b^2 c^5 d^2 f^5 + 16 A^2 B^2 a^2 c^6 d^2 e^2 f^4 - 12 A^2 B^2 a^2 b^2 c^5 d^2 e^2 f^4) / (16 a^2 c^6 d^4 + a^4 b^4 f^4 + 16 a^4 c^4 e^4 + b^4 c^4 d^4 + 16 a^6 c^2 f^4 + b^8 d^2 f^2 - 8 a^2 b^2 c^5 d^4 - 8 a^5 b^2 c^2 f^4 + 2 a^2 b^6 d^2 f^3 - 2 a^3 b^5 e^2 f^3 - 64 a^3 c^5 d^3 f - 64 a^5 c^3 d^2 f^3 - 2 b^5 c^3 d^3 e + 2 b^6 c^2 d^3 f + a^2 b^4 c^2 e^4 - 8 a^3 b^2 c^3 e^4 + 32 a^3 c^5 d^2 e^2 + a^2 b^6 e^2 f^2 + 96 a^4 c^4 d^2 f^2 + b^6 c^2 d^2 e^2 + 32 a^5 c^3 e^2 f^2 - 2 a^2 b^7 d^2 e^2 f - 2 b^7 c^2 d^2 e^2 f + 54 a^2 b^4 c^2 d^2 f^2 - 112 a^3 b^2 c^3 d^2 f^2 + 16 a^2 b^3 c^4 d^3 e - 2 a^2 b^5 c^2 d^2 e^3 - 32 a^2 b^2 c^5 d^3 e - 32 a^3 b^2 c^4 d^2 e^3 - 20 a^2 b^4 c^3 d^3 f - 12 a^2 b^6 c^2 d^2 f^2 - 20 a^3 b^4 c^2 d^2 f^3 - 2 a^2 b^5 c^2 e^3 f - 32 a^4 b^2 c^3 e^3 f + 16 a^4 b^3 c^2 e^2 f^3 - 32 a^5 b^2 c^2 e^2 f^3 - 64 a^4 c^4 d^2 e^2 f - 6 a^2 b^4 c^3 d^2 e^2 + 16 a^2 b^3 c^3 d^2 e^3 + 64 a^2 b^2 c^4 d^3 f + 64 a^4 b^2 c^2 d^2 f^3 + 16 a^3 b^3 c^2 e^3 f - 6 a^3 b^4 c^2 e^2 f^2 - 48 a^2 b^3 c^3 d^2 e^2 f - 36 a^2 b^4 c^2 d^2 e^2 f + 96 a^3 b^2 c^3 d^2 e^2 f - 48 a^3 b^3 c^2 d^2 e^2 f + 4 a^2 b^6 c^2 d^2 e^2 f + 18 a^2 b^5 c^2 d^2 e^2 f + 18 a^2 b^5 c^2 d^2 e^2 f + 32 a^3 b^2 c^4 d^2 e^2 f + 32 a^4 b^2 c^3 d^2 e^2 f) * root(48416 a^6 b^2 c^6 d^4 e^2 f^4 z^4 - 41544 a^5 b^4 c^5 d^4 e^2 f^4 z^4 - 31872 a^7 b^2 c^5 d^3 e^2 f^5 z^4 - 31872 a^5 b^2 c^7 d^5 e^2 f^3 z^4 - 29184 a^6 b^2 c^6 d^3 e^4 f^3 z^4 + 28800 a^5 b^4 c^5 d^3 e^4 f^3 z^4 + 21510 a^4 b^6 c^4 d^4 e^2 f^4 z^4 + 21408 a^6 b^4 c^4 d^3 e^2 f^5 z^4 + 21408 a^4 b^4 c^6 d^5 e^2 f^3 z^4 - 18112 a^7 b^3 c^4 d^2 e^3 f^5 z^4 - 18112 a^
\end{aligned}$$

$$\begin{aligned}
& 4*b^3*c^7*d^5*e^3*f^2*z^4 - 15600*a^5*b^5*c^4*d^3*e^3*f^4*z^4 - 15600*a^4*b^5*c^5*d^4*e^3*f^3*z^4 + 15296*a^6*b^3*c^5*d^3*e^3*f^4*z^4 + 15296*a^5*b^3*c^6*d^4*e^3*f^3*z^4 + 14016*a^7*b^2*c^5*d^2*e^4*f^4*z^4 + 14016*a^5*b^2*c^7*d^4*e^4*f^2*z^4 - 13920*a^4*b^6*c^4*d^3*e^4*f^3*z^4 - 11648*a^6*b^3*c^5*d^2*e^5*f^3*z^4 - 11648*a^5*b^3*c^6*d^3*e^5*f^2*z^4 + 10432*a^6*b^2*c^6*d^2*e^6*f^2*z^4 + 9008*a^6*b^5*c^3*d^2*e^3*f^5*z^4 + 9008*a^3*b^5*c^6*d^5*e^3*f^2*z^4 + 8544*a^5*b^5*c^4*d^2*e^5*f^3*z^4 + 8544*a^4*b^5*c^5*d^3*e^5*f^2*z^4 - 8496*a^5*b^4*c^5*d^2*e^6*f^2*z^4 + 7488*a^8*b^2*c^4*d^2*e^2*f^6*z^4 + 7488*a^4*b^2*c^8*d^6*e^2*f^2*z^4 + 7380*a^4*b^7*c^3*d^3*e^3*f^4*z^4 + 7380*a^3*b^7*c^4*d^4*e^3*f^3*z^4 - 6720*a^3*b^8*c^3*d^4*e^2*f^4*z^4 - 5784*a^5*b^6*c^3*d^3*e^2*f^5*z^4 - 5784*a^3*b^6*c^5*d^5*e^2*f^3*z^4 - 3440*a^6*b^4*c^4*d^2*e^4*f^4*z^4 - 3440*a^4*b^4*c^6*d^4*e^4*f^2*z^4 + 3360*a^3*b^8*c^3*d^3*e^4*f^3*z^4 + 3140*a^4*b^6*c^4*d^2*e^6*f^2*z^4 - 2760*a^4*b^7*c^3*d^2*e^5*f^3*z^4 - 2760*a^3*b^7*c^4*d^3*e^5*f^2*z^4 - 1764*a^5*b^7*c^2*d^2*e^3*f^5*z^4 - 1764*a^2*b^7*c^5*d^5*e^3*f^2*z^4 - 1640*a^3*b^9*c^2*d^3*e^3*f^4*z^4 - 1640*a^2*b^9*c^3*d^4*e^3*f^3*z^4 - 1604*a^6*b^6*c^2*d^2*e^2*f^6*z^4 - 1604*a^2*b^6*c^6*d^6*e^2*f^2*z^4 - 1500*a^5*b^6*c^3*d^2*e^4*f^4*z^4 - 1500*a^3*b^6*c^5*d^4*e^4*f^2*z^4 + 1140*a^2*b^10*c^2*d^4*e^2*f^4*z^4 + 810*a^4*b^8*c^2*d^2*e^4*f^4*z^4 + 810*a^2*b^8*c^4*d^4*e^4*f^2*z^4 - 544*a^3*b^8*c^3*d^2*e^6*f^2*z^4 + 416*a^3*b^9*c^2*d^2*e^5*f^3*z^4 + 416*a^2*b^9*c^3*d^3*e^5*f^2*z^4 - 384*a^2*b^10*c^2*d^3*e^4*f^3*z^4 + 180*a^4*b^8*c^2*d^3*e^2*f^5*z^4 + 180*a^2*b^8*c^4*d^5*e^2*f^3*z^4 + 48*a^7*b^4*c^3*d^2*e^2*f^6*z^4 + 48*a^3*b^4*c^7*d^6*e^2*f^2*z^4 + 36*a^2*b^10*c^2*d^2*e^6*f^2*z^4 - 1024*a^10*b*c^3*d*e*f^8*z^4 - 1024*a^3*b*c^10*d^8*e*f*z^4 - 192*a^8*b^5*c*d*e*f^8*z^4 - 192*a*b^5*c^8*d^8*e*f*z^4 + 16128*a^7*b^3*c^4*d^3*e*f^6*z^4 + 16128*a^4*b^3*c^7*d^6*e*f^3*z^4 - 11712*a^6*b^5*c^3*d^3*e*f^6*z^4 - 11712*a^3*b^5*c^6*d^6*e*f^3*z^4 + 11520*a^8*b*c^5*d^2*e^3*f^5*z^4 + 11520*a^5*b*c^8*d^5*e^3*f^2*z^4 - 9984*a^6*b^3*c^5*d^4*e*f^5*z^4 - 9984*a^5*b^3*c^6*d^5*e*f^4*z^4 + 8640*a^5*b^5*c^4*d^4*e*f^5*z^4 + 8640*a^4*b^5*c^5*d^5*e*f^4*z^4 - 7424*a^7*b*c^6*d^3*e^3*f^4*z^4 - 7424*a^6*b*c^7*d^4*e^3*f^3*z^4 - 6912*a^8*b^3*c^3*d^2*e*f^7*z^4 - 6912*a^3*b^3*c^8*d^7*e*f^2*z^4 + 4800*a^7*b^3*c^4*d*e^5*f^4*z^4 + 4800*a^4*b^3*c^7*d^4*e^5*f*z^4 + 4608*a^7*b*c^6*d^2*e^5*f^3*z^4 + 4608*a^6*b*c^7*d^3*e^5*f^2*z^4 - 4560*a^4*b^7*c^3*d^4*e*f^5*z^4 - 4560*a^3*b^7*c^4*d^5*e*f^4*z^4 + 4176*a^5*b^7*c^2*d^3*e*f^6*z^4 + 4176*a^2*b^7*c^5*d^6*e*f^3*z^4 + 3264*a^7*b^5*c^2*d^2*e*f^7*z^4 + 3264*a^2*b^5*c^7*d^7*e*f^2*z^4 + 3008*a^8*b^3*c^3*d*e^3*f^6*z^4 + 3008*a^3*b^3*c^8*d^6*e^3*f*z^4 + 2880*a^6*b^3*c^5*d*e^7*f^2*z^4 + 2880*a^5*b^3*c^6*d^2*e^7*f*z^4 - 2240*a^7*b^4*c^3*d*e^4*f^5*z^4 - 2240*a^3*b^4*c^7*d^5*e^4*f*z^4 - 1488*a^5*b^5*c^4*d*e^7*f^2*z^4 - 1488*a^4*b^5*c^5*d^2*e^7*f*z^4 + 1440*a^3*b^9*c^2*d^4*e*f^5*z^4 + 1440*a^2*b^9*c^3*d^5*e*f^4*z^4 - 1328*a^6*b^5*c^3*d*e^5*f^4*z^4 - 1328*a^3*b^5*c^6*d^4*e^5*f*z^4 - 1152*a^7*b^2*c^5*d*e^6*f^3*z^4 - 1152*a^5*b^2*c^7*d^3*e^6*f*z^4 - 1120*a^6*b^4*c^4*d*e^6*f^3*z^4 - 1120*a^4*b^4*c^6*d^3*e^6*f*z^4 + 912*a^6*b^6*c^2*d*e^4*f^5*z^4 + 912*a^2*b^6*c^6*d^5*e^4*f*z^4 + 872*a^5*b^6*c^3*d*e^6*f^3*z^4 + 872*a^3*b^6*c^5*d^3*e^6*f*z^4 + 768*a^8*b^2*c^4*d*e^4*f^5*z^4 + 768*a^4*b^2*c^8*d^5*e^4*f*z^4 - 672*a^8*b^4*c^2*d*e^2*f^7*z^4 - 672*a^2*b^4*c^8*d^7*e^2*f*z^4 - 624*a^7*b^5*c^2*d*e^3*f^6*z^4 - 624*a^2*b^5*c^7*d^6*e^3*f*z^4 + 480*a^5*b^8*c^d^2*e^2*f^6*z^4 + 480*a*b^8*c^5*d^6*e^2*f^2*z^4 + 316*a^4*b^7*c^3*d*e^7*f^2*z^4 + 316*a^3*b^7*c^4*d^2*e^7*f*z^4 - 204*a^4*b^8*c^2*d*e^6*f^3*z^4 - 204*a^2*b^8*c^4*d^3*e^6*f*z^4 + 168*a^3*b^10*c*d^3*e^2*f^5*z^4 + 168*a*b^10*c^3*d^5*e^2*f^3*z^4 + 156*a^2*b^11*c*d^3*e^3*f^4*z^4 + 156*a*b^11*c^2*d^4*e^3*f^3*z^4 + 128*a^9*b^2*c^3*d*e^2*f^7*z^4 + 128*a^3*b^2*c^9*d^7*e^2*f*z^4 - 124*a^3*b^10*c*d^2*e^4*f^4*z^4 - 124*a*b^10*c^3*d^4*e^4*f^2*z^4 + 100*a^4*b^9*c*d^2*e^3*f^5*z^4 + 100*a*b^9*c^4*d^5*e^3*f^2*z^4 + 36*a^5*b^7*c^2*d*e^5*f^4*z^4 + 36*a^2*b^7*c^5*d^4*e^5*f*z^4 - 24*a^3*b^9*c^2*d*e^7*f^2*z^4 - 24*a^2*b^11*c*d^2*e^5*f^3*z^4 - 24*a^2*b^9*c^3*d^2*e^7*f*z^4 - 24*a*b^11*c^2*d^3*e^5*f^2*z^4 - 9216*a^8*b*c^5*d^3*e*f^6*z^4 - 9216*a^5*b*c^8*d^6*e*f^3*z^4 - 5376*a^8*b*c^5*d*e^5*f^4*z^4 - 5376*a^5*b*c^8*d^4*e^5*f*z^4 + 5120*a^9*b*c^4*d^2*e*f^7*z^4 + 5120*a^7*b*c^6*d^4*e*f^5*z^4 + 5120*a^6*b*c^7*d^5*e*f^4*z^4 + 5120*a^4*b*c^9*d^7*e*f^2*z^4 - 43
\end{aligned}$$

$$\begin{aligned}
& 52*a^9*b*c^4*d*e^3*f^6*z^4 - 4352*a^4*b*c^9*d^6*e^3*f*z^4 - 1792*a^7*b*c^6* \\
& d*e^7*f^2*z^4 - 1792*a^6*b*c^7*d^2*e^7*f*z^4 - 1600*a^6*b^2*c^6*d*e^8*f*z^4 \\
& + 912*a^5*b^4*c^5*d*e^8*f*z^4 + 768*a^9*b^3*c^2*d*e*f^8*z^4 + 768*a^2*b^3* \\
& c^9*d^8*e*f*z^4 - 720*a^4*b^9*c*d^3*e*f^6*z^4 - 720*a*b^9*c^4*d^6*e*f^3*z^4 \\
& - 656*a^6*b^7*c*d^2*e*f^7*z^4 - 656*a*b^7*c^6*d^7*e*f^2*z^4 - 240*a^2*b^11 \\
& *c*d^4*e*f^5*z^4 - 240*a*b^11*c^2*d^5*e*f^4*z^4 + 216*a^7*b^6*c*d*e^2*f^7*z \\
& ^4 + 216*a*b^6*c^7*d^7*e^2*f*z^4 - 204*a^4*b^6*c^4*d*e^8*f*z^4 - 144*a^5*b^ \\
& 8*c*d*e^4*f^5*z^4 - 144*a*b^8*c^5*d^5*e^4*f*z^4 - 84*a*b^12*c*d^4*e^2*f^4*z \\
& ^4 + 36*a^4*b^9*c*d*e^5*f^4*z^4 + 36*a*b^9*c^4*d^4*e^5*f*z^4 + 20*a^6*b^7*c \\
& *d*e^3*f^6*z^4 + 20*a*b^7*c^6*d^6*e^3*f*z^4 + 16*a^3*b^10*c*d*e^6*f^3*z^4 + \\
& 16*a^3*b^8*c^3*d*e^8*f*z^4 + 16*a*b^12*c*d^3*e^4*f^3*z^4 + 16*a*b^10*c^3*d \\
& ^3*e^6*f*z^4 + 48*b^11*c^3*d^6*e*f^3*z^4 + 48*b^9*c^5*d^7*e*f^2*z^4 - 20*b^ \\
& 8*c^6*d^7*e^2*f*z^4 + 8*b^10*c^4*d^5*e^4*f*z^4 - 4*b^13*c*d^4*e^3*f^3*z^4 - \\
& 4*b^11*c^3*d^4*e^5*f*z^4 + 4*b^9*c^5*d^6*e^3*f*z^4 + 3072*a^9*c^5*d*e^4*f^ \\
& 5*z^4 + 3072*a^5*c^9*d^5*e^4*f*z^4 + 2560*a^8*c^6*d*e^6*f^3*z^4 + 2560*a^6* \\
& c^8*d^3*e^6*f*z^4 + 1536*a^10*c^4*d*e^2*f^7*z^4 + 1536*a^4*c^10*d^7*e^2*f*z \\
& ^4 + 48*a^5*b^9*d^2*e*f^7*z^4 + 48*a^3*b^11*d^3*e*f^6*z^4 - 20*a^6*b^8*d*e^ \\
& 2*f^7*z^4 + 8*a^4*b^10*d*e^4*f^5*z^4 + 4*a^5*b^9*d*e^3*f^6*z^4 - 4*a^3*b^11 \\
& *d*e^5*f^4*z^4 - 4*a*b^13*d^3*e^3*f^4*z^4 + 768*a^9*b*c^4*e^5*f^5*z^4 + 768 \\
& *a^8*b*c^5*e^7*f^3*z^4 + 256*a^10*b*c^3*e^3*f^7*z^4 - 192*a^6*b^3*c^5*e^9*f \\
& *z^4 - 68*a^7*b^6*c*e^4*f^6*z^4 + 48*a^8*b^5*c*e^3*f^7*z^4 + 48*a^5*b^5*c^4 \\
& *e^9*f*z^4 + 36*a^6*b^7*c*e^5*f^5*z^4 - 12*a^9*b^4*c*e^2*f^8*z^4 - 4*a^4*b^ \\
& 9*c*e^7*f^3*z^4 - 4*a^4*b^7*c^3*e^9*f*z^4 + 384*a^5*b^8*c*d^3*f^7*z^4 + 384 \\
& *a*b^8*c^5*d^7*f^3*z^4 + 288*a^3*b^10*c*d^4*f^6*z^4 + 288*a*b^10*c^3*d^6*f^ \\
& 4*z^4 + 224*a^7*b^6*c*d^2*f^8*z^4 + 224*a*b^6*c^7*d^8*f^2*z^4 - 192*a^10*b^ \\
& 2*c^2*d*f^9*z^4 - 192*a^2*b^2*c^10*d^9*f*z^4 + 768*a^5*b*c^8*d^3*e^7*z^4 + \\
& 768*a^4*b*c^9*d^5*e^5*z^4 + 256*a^3*b*c^10*d^7*e^3*z^4 - 192*a^5*b^3*c^6*d* \\
& e^9*z^4 - 68*a*b^6*c^7*d^6*e^4*z^4 + 48*a^4*b^5*c^5*d*e^9*z^4 + 48*a*b^5*c^ \\
& 8*d^7*e^3*z^4 + 36*a*b^7*c^6*d^5*e^5*z^4 - 12*a*b^4*c^9*d^8*e^2*z^4 - 4*a^3 \\
& *b^7*c^4*d*e^9*z^4 - 4*a*b^9*c^4*d^3*e^7*z^4 + 16*b^13*c*d^5*e*f^4*z^4 + 16 \\
& *b^7*c^7*d^8*e*f*z^4 + 768*a^7*c^7*d*e^8*f*z^4 + 16*a^7*b^7*d*e*f^8*z^4 + 1 \\
& 6*a*b^13*d^4*e*f^5*z^4 + 256*a^7*b*c^6*e^9*f*z^4 + 80*a*b^12*c*d^5*f^5*z^4 \\
& + 48*a^9*b^4*c*d*f^9*z^4 + 48*a*b^4*c^9*d^9*f*z^4 + 256*a^6*b*c^7*d*e^9*z^4 \\
& - 42*b^10*c^4*d^6*e^2*f^2*z^4 - 20*b^12*c^2*d^5*e^2*f^3*z^4 + 6*b^12*c^2*d \\
& ^4*e^4*f^2*z^4 + 4*b^11*c^3*d^5*e^3*f^2*z^4 - 24960*a^7*c^7*d^4*e^2*f^4*z^4 \\
& + 18944*a^8*c^6*d^3*e^2*f^5*z^4 + 18944*a^6*c^8*d^5*e^2*f^3*z^4 + 14336*a^ \\
& 7*c^7*d^3*e^4*f^3*z^4 - 9984*a^8*c^6*d^2*e^4*f^4*z^4 - 9984*a^6*c^8*d^4*e^4 \\
& *f^2*z^4 - 7936*a^9*c^5*d^2*e^2*f^6*z^4 - 7936*a^5*c^9*d^6*e^2*f^2*z^4 - 43 \\
& 52*a^7*c^7*d^2*e^6*f^2*z^4 - 42*a^4*b^10*d^2*e^2*f^6*z^4 - 20*a^2*b^12*d^3* \\
& e^2*f^5*z^4 + 6*a^2*b^12*d^2*e^4*f^4*z^4 + 4*a^3*b^11*d^2*e^3*f^5*z^4 - 480 \\
& *a^8*b^2*c^4*e^6*f^4*z^4 + 440*a^7*b^4*c^3*e^6*f^4*z^4 - 320*a^8*b^3*c^3*e^ \\
& 5*f^5*z^4 - 320*a^7*b^3*c^4*e^7*f^3*z^4 + 240*a^8*b^4*c^2*e^4*f^6*z^4 + 240 \\
& *a^6*b^4*c^4*e^8*f^2*z^4 - 192*a^9*b^3*c^2*e^3*f^7*z^4 - 192*a^9*b^2*c^3*e^ \\
& 4*f^6*z^4 - 192*a^7*b^2*c^5*e^8*f^2*z^4 - 90*a^6*b^6*c^2*e^6*f^4*z^4 - 68*a \\
& ^5*b^6*c^3*e^8*f^2*z^4 + 48*a^10*b^2*c^2*e^2*f^8*z^4 - 48*a^7*b^5*c^2*e^5*f \\
& ^5*z^4 - 48*a^6*b^5*c^3*e^7*f^3*z^4 + 36*a^5*b^7*c^2*e^7*f^3*z^4 + 6*a^4*b^ \\
& 8*c^2*e^8*f^2*z^4 - 33920*a^6*b^2*c^6*d^5*f^5*z^4 + 27936*a^5*b^4*c^5*d^5*f \\
& ^5*z^4 + 26112*a^7*b^2*c^5*d^4*f^6*z^4 + 26112*a^5*b^2*c^7*d^6*f^4*z^4 - 20 \\
& 352*a^6*b^4*c^4*d^4*f^6*z^4 - 20352*a^4*b^4*c^6*d^6*f^4*z^4 - 13080*a^4*b^6 \\
& *c^4*d^5*f^5*z^4 - 11520*a^8*b^2*c^4*d^3*f^7*z^4 - 11520*a^4*b^2*c^8*d^7*f^ \\
& 3*z^4 + 8736*a^5*b^6*c^3*d^4*f^6*z^4 + 8736*a^3*b^6*c^5*d^6*f^4*z^4 + 7488* \\
& a^7*b^4*c^3*d^3*f^7*z^4 + 7488*a^3*b^4*c^7*d^7*f^3*z^4 + 3840*a^3*b^8*c^3*d \\
& ^5*f^5*z^4 + 2560*a^9*b^2*c^3*d^2*f^8*z^4 + 2560*a^3*b^2*c^9*d^8*f^2*z^4 - \\
& 2416*a^6*b^6*c^2*d^3*f^7*z^4 - 2416*a^2*b^6*c^6*d^7*f^3*z^4 - 2160*a^4*b^8* \\
& c^2*d^4*f^6*z^4 - 2160*a^2*b^8*c^4*d^6*f^4*z^4 - 1152*a^8*b^4*c^2*d^2*f^8*z \\
& ^4 - 1152*a^2*b^4*c^8*d^8*f^2*z^4 - 720*a^2*b^10*c^2*d^5*f^5*z^4 - 480*a^4* \\
& b^2*c^8*d^4*e^6*z^4 + 440*a^3*b^4*c^7*d^4*e^6*z^4 - 320*a^4*b^3*c^7*d^3*e^7 \\
& *z^4 - 320*a^3*b^3*c^8*d^5*e^5*z^4 + 240*a^4*b^4*c^6*d^2*e^8*z^4 + 240*a^2* \\
& b^4*c^8*d^6*e^4*z^4 - 192*a^5*b^2*c^7*d^2*e^8*z^4 - 192*a^3*b^2*c^9*d^6*e^4
\end{aligned}$$

$$\begin{aligned}
& *z^4 - 192*a^2*b^3*c^9*d^7*e^3*z^4 - 90*a^2*b^6*c^6*d^4*e^6*z^4 - 68*a^3*b^6*c^5*d^2*e^8*z^4 - 48*a^3*b^5*c^6*d^3*e^7*z^4 - 48*a^2*b^5*c^7*d^5*e^5*z^4 \\
& + 48*a^2*b^2*c^10*d^8*e^2*z^4 + 36*a^2*b^7*c^5*d^3*e^7*z^4 + 6*a^2*b^8*c^4*d^2*e^8*z^4 - 4*b^6*c^8*d^9*f*z^4 + 256*a^11*c^3*d*f^9*z^4 + 256*a^3*c^11*d^9*f*z^4 \\
& - 4*a^8*b^6*d*f^9*z^4 - 384*a^9*c^5*e^6*f^4*z^4 - 256*a^10*c^4*e^4*f^6*z^4 - 256*a^8*c^6*e^8*f^2*z^4 - 64*a^11*c^3*e^2*f^8*z^4 - 24*b^10*c^4*d^7*f^3*z^4 \\
& - 16*b^12*c^2*d^6*f^4*z^4 - 16*b^8*c^6*d^8*f^2*z^4 + 17920*a^7*c^7*d^5*f^5*z^4 - 14336*a^8*c^6*d^4*f^6*z^4 - 14336*a^6*c^8*d^6*f^4*z^4 + 7168*a^9*c^5*d^3*f^7*z^4 \\
& + 7168*a^5*c^9*d^7*f^3*z^4 - 2048*a^10*c^4*d^2*f^8*z^4 - 2048*a^4*c^10*d^8*f^2*z^4 + 6*b^8*c^6*d^6*e^4*z^4 + 6*a^6*b^8*e^4*f^6*z^4 - 4*b^9*c^5*d^5*e^5*z^4 \\
& - 4*b^7*c^7*d^7*e^3*z^4 - 4*a^7*b^7*e^3*f^7*z^4 - 4*a^5*b^9*e^5*f^5*z^4 - 384*a^5*c^9*d^4*e^6*z^4 - 256*a^6*c^8*d^2*e^8*z^4 - 256*a^4*c^10*d^6*e^4*z^4 \\
& - 64*a^3*c^11*d^8*e^2*z^4 - 24*a^4*b^10*d^3*f^7*z^4 - 16*a^6*b^8*d^2*f^8*z^4 - 16*a^2*b^12*d^4*f^6*z^4 + 48*a^6*b^2*c^6*e^10*z^4 \\
& - 12*a^5*b^4*c^5*e^10*z^4 - 4*b^14*d^5*f^5*z^4 - 64*a^7*c^7*e^10*z^4 + b^14*d^4*e^2*f^4*z^4 + b^10*c^4*d^4*e^6*z^4 + b^6*c^8*d^8*e^2*z^4 + a^8*b^6*e^2*f^8*z^4 \\
& + a^4*b^10*e^6*f^4*z^4 + a^4*b^6*c^4*e^10*z^4 - 4820*A*B*a^4*b*c^5*d^2*e^2*f^4*z^2 + 2976*A*B*a^3*b*c^6*d^3*e^2*f^3*z^2 - 2328*A*B*a^3*b*c^6*d^2*e^4*f^2*z^2 \\
& + 1848*A*B*a^2*b^4*c^4*d^3*e*f^4*z^2 - 1768*A*B*a^3*b^4*c^3*d^2*e*f^5*z^2 + 1528*A*B*a^4*b^2*c^4*d^2*e*f^5*z^2 - 1136*A*B*a^3*b^2*c^5*d^3*e*f^4*z^2 \\
& - 974*A*B*a^4*b^3*c^3*d*e^2*f^5*z^2 + 692*A*B*a^2*b*c^7*d^4*e^2*f^2*z^2 + 588*A*B*a*b^6*c^3*d^2*e^3*f^3*z^2 - 580*A*B*a^3*b^3*c^4*d*e^4*f^3*z^2 \\
& + 488*A*B*a^3*b^4*c^3*d*e^3*f^4*z^2 - 444*A*B*a^2*b^2*c^6*d^2*e^5*f*z^2 - 412*A*B*a*b^5*c^4*d^2*e^4*f^2*z^2 + 366*A*B*a^2*b^6*c^2*d^2*e*f^5*z^2 \\
& - 352*A*B*a^2*b^2*c^6*d^4*e*f^3*z^2 + 326*A*B*a^2*b^4*c^4*d*e^5*f^2*z^2 + 324*A*B*a*b^5*c^4*d^3*e^2*f^3*z^2 - 302*A*B*a*b^3*c^6*d^4*e^2*f^2*z^2 \\
& - 296*A*B*a*b^7*c^2*d^2*e^2*f^4*z^2 + 122*A*B*a^4*b^2*c^4*d*e^3*f^4*z^2 - 122*A*B*a^2*b^6*c^2*d*e^3*f^4*z^2 - 84*A*B*a^3*b^2*c^5*d*e^5*f^2*z^2 + 72*A*B*a*b^4*c^5*d^3*e^3*f^2*z^2 \\
& - 64*A*B*a^2*b^5*c^3*d*e^4*f^3*z^2 + 60*A*B*a^3*b^5*c^2*d*e^2*f^5*z^2 + 1312*A*B*a^5*b*c^4*d*e^2*f^5*z^2 + 1040*A*B*a^4*b*c^5*d*e^4*f^3*z^2 - 500*A*B*a*b^6*c^3*d^3*e*f^4*z^2 \\
& - 376*A*B*a*b^2*c^7*d^5*e*f^2*z^2 + 276*A*B*a^4*b^4*c^2*d*e*f^6*z^2 - 262*A*B*a^2*b^3*c^5*d*e^6*f*z^2 + 238*A*B*a*b^2*c^7*d^4*e^3*f*z^2 + 232*A*B*a^5*b^2*c^3*d*e*f^6*z^2 \\
& - 176*A*B*a^2*b*c^7*d^3*e^4*f*z^2 - 120*A*B*a*b^6*c^3*d*e^5*f^2*z^2 - 108*A*B*a*b^4*c^5*d^4*e*f^3*z^2 + 68*A*B*a*b^7*c^2*d*e^4*f^3*z^2 + 68*A*B*a*b^4*c^5*d^2*e^5*f*z^2 \\
& + 46*A*B*a^2*b^7*c*d*e^2*f^5*z^2 - 36*A*B*a*b^3*c^6*d^3*e^4*f*z^2 - 1932*A*B*a^2*b^3*c^5*d^3*e^2*f^3*z^2 - 1818*A*B*a^2*b^4*c^4*d^2*e^3*f^3*z^2 \\
& + 1620*A*B*a^3*b^3*c^4*d^2*e^2*f^4*z^2 + 1560*A*B*a^2*b^3*c^5*d^2*e^4*f^2*z^2 + 1244*A*B*a^3*b^2*c^5*d^2*e^3*f^3*z^2 + 820*A*B*a^2*b^2*c^6*d^3*e^3*f^2*z^2 \\
& + 480*A*B*a^2*b^5*c^3*d^2*e^2*f^4*z^2 + 352*A*B*a^3*b*c^6*d*e^6*f*z^2 - 108*A*B*a^3*b^6*c*d*e*f^6*z^2 + 82*A*B*a*b^5*c^4*d*e^6*f*z^2 - 64*A*B*a*b*c^8*d^5*e^2*f*z^2 \\
& + 16*A*B*a*b^8*c*d^2*e*f^5*z^2 - 4*A*B*a*b^8*c*d*e^3*f^4*z^2 + 16*B^2*a*b*c^8*d^6*e*f*z^2 + 56*A*B*b^2*c^8*d^6*e*f*z^2 - 8*A*B*b^9*c*d*e^4*f^3*z^2 - 8*A*B*b^7*c^3*d*e^6*f*z^2 \\
& - 800*A*B*a^6*c^4*d*e*f^6*z^2 + 10*A*B*a^2*b^8*d*e*f^6*z^2 - 6*A*B*a*b^9*d*e^2*f^5*z^2 - 12*A*B*a^5*b^4*c*e*f^7*z^2 + 912*A*B*a^6*b*c^3*d*f^7*z^2 + 192*A*B*a^4*b^5*c*d*f^7*z^2 \\
& + 192*A*B*a*b*c^8*d^6*f^2*z^2 - 20*A*B*a*b^4*c^5*d*e^7*z^2 + 4*A*B*a*b*c^8*d^4*e^4*z^2 + 2144*B^2*a^4*b*c^5*d^3*e*f^4*z^2 - 1120*B^2*a^3*b*c^6*d^4*e*f^3*z^2 \\
& - 688*B^2*a^5*b*c^4*d^2*e*f^5*z^2 - 256*B^2*a^3*b*c^6*d^2*e^5*f*z^2 + 152*B^2*a*b^3*c^6*d^5*e*f^2*z^2 + 120*B^2*a^5*b^3*c^2*d*e*f^6*z^2 - 116*B^2*a^5*b*c^4*d*e^3*f^4*z^2 \\
& + 110*B^2*a*b^7*c^2*d^3*e*f^4*z^2 - 80*B^2*a^2*b*c^7*d^5*e*f^2*z^2 - 72*B^2*a*b^5*c^4*d^4*e*f^3*z^2 - 48*B^2*a^4*b*c^5*d*e^5*f^2*z^2 - 46*B^2*a*b^3*c^6*d^4*e^3*f*z^2 \\
& - 44*B^2*a*b^4*c^5*d^3*e^4*f*z^2 - 34*B^2*a*b^5*c^4*d^2*e^5*f*z^2 + 20*B^2*a^2*b*c^7*d^4*e^3*f*z^2 - 10*B^2*a^3*b^6*c*d*e^2*f^5*z^2 - 10*B^2*a^2*b^7*c*d^2*e*f^5*z^2 - 10*B^2*a*b^2*c^7*d^5*e^2*f*z^2 \\
& - 7*B^2*a^2*b^4*c^4*d*e^6*f*z^2 - 6*B^2*a^3*b^2*c^5*d*e^6*f*z^2 + 4*B^2*a*b^8*c*d^2*e^2*f^4*z^2 - 2*B^2*a^2*b^7*c*d*e^3*f^4*z^2 + 3196*A^2*a^4*b*c^5*d*e^3*f^4*z^2 - 3184*A^2*a^4*b*c^5*d^2*e*f^5*z^2 + 1568*A^2*a^3*b*c^6*d^3*e*f^4*z^2 + 1504*A^2*a^3*b*c^6*d*e^5*f^2*z^2 - 656*
\end{aligned}$$

$$\begin{aligned}
& A^2a^4b^3c^3d^2e^6f^2z^2 - 400A^2a^2b^6c^3d^2e^4f^3z^2 + 314A^2a^2b^5c^4d^2e^5f^2z^2 - 264A^2a^3b^5c^2d^2e^6f^2z^2 + 240A^2a^2b^2c^6d^2e^6f^2z^2 - 224A^2a^2b^2c^7d^4e^6f^3z^2 + 216A^2a^2b^5c^4d^3e^6f^4z^2 - 192A^2a^2b^2c^7d^2e^5f^3z^2 + 178A^2a^2b^7c^2d^2e^3f^4z^2 - 154A^2a^2b^7c^2d^2e^5f^3z^2 + 128A^2a^2b^3c^6d^4e^6f^3z^2 + 106A^2a^2b^3c^6d^2e^5f^3z^2 - 12A^2a^2b^2c^7d^3e^4f^3z^2 - 58A^2a^2b^8c^2d^2e^3f^3z^2 + 40A^2a^2b^7c^3d^2e^4f^2z^2 - 28A^2a^2b^7c^3d^3e^2f^3z^2 - 24A^2a^2b^5c^5d^4e^2f^2z^2 - 20A^2a^2b^6c^4d^3e^3f^2z^2 + 2768A^2a^2b^4c^6d^2e^3f^3z^2 - 1712A^2a^2b^3c^7d^3e^3f^2z^2 - 156A^2a^2b^4c^4e^5f^3z^2 + 146A^2a^2b^4c^3e^4f^4z^2 - 106A^2a^2b^5c^3e^3f^5z^2 + 90A^2a^2b^5c^2e^2f^6z^2 + 38A^2a^2b^3c^4e^6f^2z^2 - 36A^2a^2b^3c^5e^4f^4z^2 + 16A^2a^2b^3c^4e^5f^3z^2 - 9A^2a^2b^4c^2e^3f^5z^2 - 8A^2a^2b^5c^3e^6f^2z^2 + 2A^2a^2b^6c^2e^5f^3z^2 + 920A^2a^2b^4c^3d^2e^2f^6z^2 - 480A^2a^2b^5c^3d^3e^2f^5z^2 - 336A^2a^2b^3c^5d^4e^4f^4z^2 - 272A^2a^2b^3c^4d^3e^5f^5z^2 + 240A^2a^2b^3c^5d^2e^2f^6z^2 - 32A^2a^2b^3c^9d^6e^6f^2z^2 - 792B^2a^2b^3c^5d^3e^3f^2z^2 + 714B^2a^2b^4c^4d^3e^2f^3z^2 - 572B^2a^2b^2c^5d^3e^2f^3z^2 - 475B^2a^2b^2c^6d^4e^2f^2z^2 + 265B^2a^2b^4c^4d^2e^2f^4z^2 + 260B^2a^2b^3c^4d^2e^3f^3z^2 - 212B^2a^2b^3c^3d^2e^2f^4z^2 + 180B^2a^2b^2c^5d^2e^4f^2z^2 - 158B^2a^2b^4c^4d^2e^4f^2z^2 + 47B^2a^2b^6c^2d^2e^2f^4z^2 + 16B^2a^2b^5c^3d^2e^3f^3z^2 + 2752A^2a^3b^2c^5d^2e^2f^4z^2 - 2148A^2a^2b^4c^4d^2e^2f^4z^2 + 2064A^2a^2b^3c^5d^2e^3f^3z^2 - 424A^2a^2b^2c^6d^3e^2f^3z^2 - 198A^2a^2b^2c^6d^2e^4f^2z^2 - 272B^2a^2b^6c^3d^2e^6f^2z^2 - 24B^2a^2b^5c^4d^2e^6f^2z^2 + 1808A^2a^5b^2c^4d^2e^6f^2z^2 - 244A^2a^2b^8c^4d^4e^3f^2z^2 + 208A^2a^2b^8c^5d^5e^2f^2z^2 + 134A^2a^2b^7c^4d^2e^6f^2z^2 - 76A^2a^2b^4c^5d^2e^6f^2z^2 + 4A^2a^2b^8c^4d^2e^2f^5z^2 + 148A^2a^2b^4c^6d^5e^2f^2z^2 + 65A^2a^2b^6c^4d^4e^3f^3z^2 + 46A^2a^2b^8c^2d^3e^4f^4z^2 - 38A^2a^2b^3c^7d^5e^2f^2z^2 + 34A^2a^2b^9c^2d^2e^2f^4z^2 - 29A^2a^2b^4c^6d^4e^3f^2z^2 + 20A^2a^2b^5c^5d^3e^4f^2z^2 + 12A^2a^2b^8c^2d^2e^5f^2z^2 - 7A^2a^2b^6c^4d^2e^5f^2z^2 - 2880A^2a^4c^6d^3e^4f^4z^2 + 2784A^2a^5c^5d^2e^5f^5z^2 - 1112A^2a^5c^5d^3e^3f^4z^2 + 896A^2a^3c^7d^4e^3f^3z^2 + 848A^2a^3c^7d^2e^5f^3z^2 - 560A^2a^4c^6d^2e^5f^2z^2 + 96A^2a^2c^8d^5e^2f^2z^2 - 88A^2a^2c^8d^4e^3f^2z^2 - 100A^2a^6b^2c^3e^2f^6z^2 - 76A^2a^5b^2c^4e^4f^4z^2 + 48A^2a^6b^2c^2e^7f^2z^2 - 42A^2a^3b^2c^5e^7f^2z^2 + 36A^2a^4b^2c^5e^6f^2z^2 - 24A^2a^4b^5c^2e^6f^2z^2 + 10A^2a^3b^6c^2e^3f^5z^2 + 7A^2a^2b^4c^4e^7f^2z^2 + 2A^2a^2b^7c^2e^4f^4z^2 - 2496A^2a^5b^2c^4d^2e^2f^6z^2 + 1872A^2a^4b^2c^5d^3e^3f^5z^2 - 744A^2a^5b^3c^2d^2e^7f^2z^2 - 720A^2a^2b^2c^7d^5e^3f^3z^2 + 504A^2a^2b^3c^6d^5e^3f^3z^2 + 256A^2a^3b^2c^6d^4e^4f^4z^2 + 168A^2a^2b^7c^2d^3e^5f^5z^2 - 144A^2a^2b^7c^2d^2e^6f^6z^2 + 144A^2a^2b^5c^4d^4e^4f^4z^2 + 66A^2a^2b^2c^6d^2e^7f^2z^2 - 36A^2a^2b^2c^7d^3e^5f^5z^2 + 20A^2a^2b^3c^6d^2e^6f^6z^2 + 12A^2a^2b^2c^7d^2e^6f^6z^2 + 1208B^2a^3b^2c^6d^3e^3f^2z^2 - 848B^2a^3b^3c^4d^3e^4f^4z^2 + 672B^2a^2b^3c^5d^4e^4e^3f^3z^2 - 632B^2a^4b^2c^5d^2e^3f^3z^2 + 432B^2a^4b^3c^3d^2e^4e^5f^5z^2 + 276B^2a^2b^2c^6d^3e^4f^4z^2 - 196B^2a^2b^6c^3d^3e^2f^3z^2 - 168B^2a^2b^5c^3d^3e^4f^4z^2 + 154B^2a^2b^3c^5d^2e^5f^5z^2 + 148B^2a^2b^5c^4d^3e^3f^2z^2 + 96B^2a^2b^4c^5d^4e^2f^2z^2 - 72B^2a^3b^5c^2d^2e^5f^5z^2 + 70B^2a^5b^2c^3d^2e^2f^5z^2 - 60B^2a^4b^3c^3d^2e^3f^4z^2 + 52B^2a^2b^6c^3d^2e^4f^2z^2 + 36B^2a^4b^2c^4d^2e^4f^3z^2 - 32B^2a^2b^7c^2d^2e^3f^3z^2 + 24B^2a^3b^5c^2d^2e^3f^4z^2 + 15B^2a^4b^4c^2d^2e^2f^5z^2 - 8B^2a^3b^4c^3d^2e^4f^3z^2 + 8B^2a^2b^5c^3d^2e^5f^2z^2 - 2B^2a^3b^3c^4d^2e^5f^2z^2 - 2B^2a^2b^6c^2d^2e^4f^3z^2 - 3176A^2a^3b^2c^6d^2e^3f^3z^2 - 2252A^2a^4b^2c^4d^2e^2f^5z^2 + 1952A^2a^3b^4c^3d^2e^2f^5z^2 - 1496A^2a^3b^3c^4d^2e^3f^4z^2 + 1378A^2a^2b^4c^4d^2e^4f^3z^2 + 1184A^2a^3b^3c^4d^2e^5f^5z^2 - 1166A^2a^2b^3c^5d^2e^5f^2z^2 - 1164A^2a^3b^2c^5d^2e^4f^3z^2 - 1152A^2a^2b^3c^5d^3e^4f^2z^2
\end{aligned}$$

$$\begin{aligned}
&^4z^2 + 578A^2a^2b^6c^3d^2e^2f^4z^2 - 548A^2a^2b^5c^4d^2e^3f^3z^2 + 440A^2a^2b^2c^7d^4e^2f^2z^2 - 412A^2a^2b^6c^2d^2e^2f^5z^2 \\
&- 360A^2a^2b^3c^6d^3e^3f^2z^2 + 312A^2a^2b^4c^5d^3e^2f^3z^2 + 248A^2a^2b^2c^7d^3e^3f^2z^2 - 224A^2a^2b^5c^3d^2e^3f^4z^2 + 216 \\
&A^2a^2b^5c^3d^2e^2f^5z^2 + 52A^2a^2b^4c^5d^2e^4f^2z^2 - 16B^2b^3c^7d^6e^2fz^2 - 14B^2b^9c^d^3e^2f^4z^2 + 32B^2a^4c^6d^2e^6fz^2 \\
&- 20A^2b^9c^d^2e^3f^4z^2 + 18A^2b^9c^d^2e^2f^5z^2 + 8A^2b^6c^4d^2e^6fz^2 - 360A^2a^3c^7d^2e^6fz^2 + 136A^2a^2c^9d^5e^2fz^2 + \\
&2B^2a^3b^7d^2e^6fz^2 + 2B^2a^2b^9d^2e^2f^5z^2 + 12B^2a^4b^3c^5e^7fz^2 - 204A^2a^3b^3c^6e^7fz^2 - 128A^2a^6b^3c^3e^2f^7z^2 - 48A^2a^2b^5c^4e^7fz^2 - 36B^2a^5b^4c^d^2f^7z^2 - 24A^2a^4b^5c^e^2f^7z^2 - 16B^2a^2b^8c^d^3f^5z^2 - 164A^2a^3b^6c^d^2f^7z^2 - 16A^2a^2b^8c^d^2f^6z^2 + 4B^2a^3b^3c^6d^2e^7z^2 - 4B^2a^2b^3c^8d^5e^3z^2 + 48A^2a^2b^3c^8d^3e^5z^2 + 36A^2a^2b^3c^7d^2e^7z^2 - 6A^2a^2b^3c^6d^2e^7z^2 + 136A^2a^2b^3c^6d^2e^7z^2 - 96A^2a^2b^3c^5d^5e^3z^2 + 80A^2a^2b^3c^5d^5e^3z^2 - 72A^2a^2b^3c^7d^6e^2f^2z^2 - 24A^2a^2b^3c^3d^4e^4f^4z^2 + 14A^2a^2b^3c^7d^4e^4z^2 - 14A^2a^2b^2c^8d^5e^3z^2 - 2A^2a^2b^5c^5d^2e^6z^2 - 2A^2a^2b^4c^6d^3e^5z^2 + 2A^2a^2b^3c^7e^2f^6z^2 - A^2a^2b^8e^3f^5z^2 + 16A^2a^2b^2c^8d^3e^5z^2 - 2A^2a^2b^3c^5e^8z^2 + 22B^2b^8c^2d^3e^2f^3z^2 - 12B^2b^7c^3d^3e^3f^2z^2 + 12B^2b^6c^4d^4e^2f^2z^2 - 6B^2b^8c^2d^2e^4f^2z^2 - 864B^2a^4c^6d^3e^2f^3z^2 + 496B^2a^3c^7d^4e^2f^2z^2 + 224B^2a^5c^5d^2e^2f^4z^2 + 136B^2a^4c^6d^2e^4f^2z^2 - 53A^2b^8c^2d^2e^2f^4z^2 + 52A^2b^7c^3d^2e^3f^3z^2 + 52A^2b^5c^5d^3e^3f^2z^2 - 36A^2b^6c^4d^3e^2f^3z^2 - 12A^2b^4c^6d^4e^2f^2z^2 - 9A^2b^6c^4d^2e^4f^2z^2 + 836A^2a^4c^6d^2e^2f^4z^2 - 668A^2a^2c^8d^4e^2f^2z^2 + 656A^2a^3c^7d^2e^4f^2z^2 + 368A^2a^3c^7d^3e^2f^3z^2 - 45B^2a^6b^2c^2e^2f^6z^2 - 18B^2a^5b^2c^3e^4f^4z^2 - 9B^2a^4b^2c^4e^6f^2z^2 - 6B^2a^5b^3c^2e^3f^5z^2 + 3B^2a^4b^4c^2e^4f^4z^2 - 2B^2a^4b^3c^3e^5f^3z^2 - 580B^2a^4b^2c^4d^3f^5z^2 + 536B^2a^3b^4c^3d^3f^5z^2 + 471A^2a^4b^2c^4e^4f^4z^2 - 436A^2a^3b^4c^3e^4f^4z^2 - 348B^2a^4b^4c^2d^2f^6z^2 + 316B^2a^2b^2c^6d^5f^3z^2 + 310A^2a^3b^3c^4e^5f^3z^2 + 232A^2a^5b^2c^3e^2f^6z^2 - 229A^2a^2b^4c^4e^6f^2z^2 - 216A^2a^4b^4c^2e^2f^6z^2 + 204A^2a^4b^3c^3e^3f^5z^2 + 200B^2a^5b^2c^3d^2f^6z^2 + 150A^2a^3b^2c^5e^6f^2z^2 - 120B^2a^2b^4c^4d^4f^4z^2 + 91A^2a^2b^6c^2e^4f^4z^2 + 72A^2a^3b^5c^2e^3f^5z^2 - 66B^2a^2b^6c^2d^3f^5z^2 + 44A^2a^2b^5c^3e^5f^3z^2 - 16B^2a^3b^2c^5d^4f^4z^2 + 1952A^2a^4b^2c^4d^2f^6z^2 - 1792A^2a^3b^2c^5d^3f^5z^2 - 1272A^2a^3b^4c^3d^2f^6z^2 + 976A^2a^2b^2c^6d^4f^4z^2 + 960A^2a^2b^4c^4d^3f^5z^2 + 282A^2a^2b^6c^2d^2f^6z^2 - 45B^2a^2b^2c^6d^2e^6z^2 - 48A^2b^3c^9d^6e^2fz^2 - 14A^2a^2b^9d^2e^6z^2 - 7A^2b^10d^2e^2f^5z^2 + 2A^2b^10d^2e^3f^4z^2 - 64A^2b^7c^3e^2f^7z^2 - 16A^2b^9c^d^3f^5z^2 + 8A^2b^4c^6e^7fz^2 + 4A^2b^9c^d^6e^2z^2 + 2A^2b^6c^4d^2e^7z^2 - 120A^2b^3c^7d^2e^7z^2 - 16A^2b^3b^7d^2f^7z^2 + 16A^2b^9d^2f^6z^2 + 8A^2b^3c^9d^5e^3z^2 + 12A^2b^3b^3c^6e^8z^2 - 48B^2b^5c^5d^5e^2f^2z^2 + 15B^2b^4c^6d^5e^2fz^2 - 14B^2b^7c^3d^4e^3z^2 + 4B^2b^9c^d^2e^3f^3z^2 + 4B^2b^7c^3d^2e^5fz^2 + 4B^2b^5c^5d^4e^3fz^2 - B^2b^6c^4d^3e^4fz^2 - 336B^2a^3c^7d^3e^4fz^2 + 112B^2a^5c^5d^4e^4f^3z^2 - 112A^2b^3c^7d^5e^2f^2z^2 + 80B^2a^6c^4d^2e^2f^5z^2 - 48A^2b^5c^5d^4e^2f^3z^2 + 36A^2b^8c^2d^2e^4f^3z^2 + 36A^2b^3c^7d^4e^3fz^2 - 28A^2b^7c^3d^2e^5f^2z^2 + 20A^2b^2c^8d^5e^2fz^2 + 16B^2a^2c^8d^5e^2fz^2 - 14A^2b^7c^3d^3e^2f^4z^2 - 14A^2b^4c^6d^3e^4fz^2 - 10A^2b^5c^5d^2e^5fz^2 - 1008A^2a^4c^6d^4e^4f^3z^2 - 760A^2a^5c^5d^2e^2f^5z^2 + 272A^2a^2c^8d^3e^4fz^2 + 48B^2a^5b^3c^4e^5f^3z^2 + 36B^2a^6b^3c^3e^3f^5z^2 + 12B^2a^5b^4c^2e^2f^6z^2 - 624A^2a^4b^3c^5e^5f^3z^2 - 548A^2a^5b^3c^4e^3f^5z^2 + 182A^2a^2b^3c^5e^7fz^2 - 180B^2a^2b^4c^5d^5f^3z^2 + 132B^
\end{aligned}$$

$$\begin{aligned}
& 2*a^6*b^2*c^2*d*f^7*z^2 + 108*B^2*a^3*b^6*c*d^2*f^6*z^2 + 96*A^2*a^5*b^3*c^2 \\
& *e*f^7*z^2 + 68*A^2*a*b^6*c^3*e^6*f^2*z^2 + 58*A^2*a^3*b^6*c*e^2*f^6*z^2 - \\
& 56*B^2*a*b^2*c^7*d^6*f^2*z^2 - 38*A^2*a^2*b^7*c*e^3*f^5*z^2 - 36*A^2*a*b^7 \\
& *c^2*e^5*f^3*z^2 + 20*B^2*a*b^6*c^3*d^4*f^4*z^2 - 736*A^2*a^5*b^2*c^3*d*f^7 \\
& *z^2 + 624*A^2*a^4*b^4*c^2*d*f^7*z^2 - 416*A^2*a*b^2*c^7*d^5*f^3*z^2 - 276* \\
& A^2*a*b^4*c^5*d^4*f^4*z^2 - 196*A^2*a*b^6*c^3*d^3*f^5*z^2 + 8*B^2*a*b^4*c^5 \\
& *d^2*e^6*z^2 + 6*B^2*a*b^2*c^7*d^4*e^4*z^2 + 2*B^2*a^2*b^3*c^5*d*e^7*z^2 + \\
& 2*B^2*a*b^3*c^6*d^3*e^5*z^2 - 18*A^2*a*b^2*c^7*d^2*e^6*z^2 - 16*A*B*b*c^9*d \\
& ^7*f*z^2 - B^2*b^10*d^2*e^2*f^4*z^2 + 48*B^2*a^7*c^3*e^2*f^6*z^2 - 36*B^2*a \\
& ^6*c^4*e^4*f^4*z^2 + 31*B^2*b^6*c^4*d^5*f^3*z^2 - 24*B^2*a^5*c^5*e^6*f^2*z^2 \\
& + 20*B^2*b^4*c^6*d^6*f^2*z^2 - 6*A^2*b^8*c^2*e^6*f^2*z^2 + 2*B^2*b^8*c^2* \\
& d^4*f^4*z^2 - 768*B^2*a^5*c^5*d^3*f^5*z^2 + 512*B^2*a^6*c^4*d^2*f^6*z^2 + 5 \\
& 12*B^2*a^4*c^6*d^4*f^4*z^2 + 232*A^2*a^5*c^5*e^4*f^4*z^2 + 188*A^2*a^4*c^6* \\
& e^6*f^2*z^2 - 128*B^2*a^3*c^7*d^5*f^3*z^2 + 92*A^2*a^6*c^4*e^2*f^6*z^2 + 80 \\
& *A^2*b^4*c^6*d^5*f^3*z^2 + 64*A^2*b^2*c^8*d^6*f^2*z^2 + 31*A^2*b^6*c^4*d^4* \\
& f^4*z^2 + 14*A^2*b^8*c^2*d^3*f^5*z^2 - 5*B^2*b^4*c^6*d^4*e^4*z^2 + 4*B^2*b^ \\
& 3*c^7*d^5*e^3*z^2 + 2*B^2*b^5*c^5*d^3*e^5*z^2 - B^2*b^6*c^4*d^2*e^6*z^2 - B \\
& ^2*b^2*c^8*d^6*e^2*z^2 - B^2*a^4*b^6*e^2*f^6*z^2 - 1152*A^2*a^3*c^7*d^4*f^4 \\
& *z^2 + 1008*A^2*a^4*c^6*d^3*f^5*z^2 + 624*A^2*a^2*c^8*d^5*f^3*z^2 - 288*A^2 \\
& *a^5*c^5*d^2*f^6*z^2 + 56*B^2*a^3*c^7*d^2*e^6*z^2 - 10*B^2*a^2*b^8*d^2*f^6* \\
& z^2 - 9*A^2*b^2*c^8*d^4*e^4*z^2 - 5*A^2*a^2*b^8*e^2*f^6*z^2 - 4*B^2*a^2*c^8 \\
& *d^4*e^4*z^2 + 3*A^2*b^4*c^6*d^2*e^6*z^2 - 2*A^2*b^3*c^7*d^3*e^5*z^2 - 36*A \\
& ^2*a^2*c^8*d^2*e^6*z^2 - 48*A^2*a^6*b^2*c^2*f^8*z^2 - 45*A^2*a^2*b^2*c^6*e^ \\
& 8*z^2 + 4*A^2*b^10*d*e^2*f^5*z^2 + 4*B^2*b^2*c^8*d^7*f*z^2 + 4*A^2*b^9*c*e^ \\
& 5*f^3*z^2 + 4*A^2*b^7*c^3*e^7*f*z^2 - 128*B^2*a^7*c^3*d*f^7*z^2 - 160*A^2*a \\
& *c^9*d^6*f^2*z^2 - 112*A^2*a^6*c^4*d*f^7*z^2 + 12*A^2*b*c^9*d^5*e^3*z^2 + 4 \\
& *A^2*a*b^9*e^3*f^5*z^2 + 3*B^2*a^4*b^6*d*f^7*z^2 + 2*A^2*a^3*b^7*e*f^7*z^2 \\
& - 24*A^2*a*c^9*d^4*e^4*z^2 + 14*A^2*a^2*b^8*d*f^7*z^2 + 12*A^2*a^5*b^4*c*f^ \\
& 8*z^2 + 12*A^2*a*b^4*c^5*e^8*z^2 + A*B*a^4*b^6*e*f^7*z^2 + B^2*a^2*b^8*d*e^ \\
& 2*f^5*z^2 + 16*A^2*c^10*d^7*f*z^2 + 3*B^2*b^10*d^3*f^5*z^2 - A^2*b^10*e^4*f \\
& ^4*z^2 - 4*A^2*c^10*d^6*e^2*z^2 - A^2*b^10*d^2*f^6*z^2 + 64*A^2*a^7*c^3*f^8 \\
& *z^2 - 4*B^2*a^4*c^6*e^8*z^2 - A^2*b^6*c^4*e^8*z^2 + 48*A^2*a^3*c^7*e^8*z^2 \\
& - A^2*a^4*b^6*f^8*z^2 + 720*A^2*B*a*b^2*c^5*d^2*e^2*f^3*z - 600*A^2*B*a^2* \\
& b^2*c^4*d*e^2*f^4*z + 576*A*B^2*a^2*b^2*c^4*d^2*e*f^4*z + 348*A*B^2*a*b^2*c \\
& ^5*d^2*e^3*f^2*z - 336*A*B^2*a^2*b*c^5*d^2*e^2*f^3*z - 260*A*B^2*a*b^3*c^4* \\
& d^2*e^2*f^3*z - 240*A*B^2*a^2*b^2*c^4*d*e^3*f^3*z + 196*A*B^2*a^2*b^3*c^3*d \\
& *e^2*f^4*z + 172*A^2*B*a*b*c^6*d*e^5*f*z + 20*A*B^2*a*b^6*c*d*e*f^5*z - 912 \\
& *A^2*B*a^2*b*c^5*d^2*e*f^4*z - 644*A^2*B*a*b*c^6*d^2*e^3*f^2*z - 432*A*B^2* \\
& a*b^2*c^5*d^3*e*f^3*z + 372*A^2*B*a^2*b*c^5*d*e^3*f^3*z - 330*A^2*B*a*b^2*c \\
& ^5*d*e^4*f^2*z + 312*A*B^2*a*b*c^6*d^3*e^2*f^2*z - 208*A*B^2*a^3*b^2*c^3*d* \\
& e*f^5*z + 192*A^2*B*a^2*b^3*c^3*d*e*f^5*z + 172*A^2*B*a*b^3*c^4*d*e^3*f^3*z \\
& + 108*A*B^2*a^2*b*c^5*d*e^4*f^2*z + 104*A*B^2*a^3*b*c^4*d*e^2*f^4*z - 80*A \\
& ^2*B*a*b^3*c^4*d^2*e*f^4*z + 68*A^2*B*a*b^4*c^3*d*e^2*f^4*z - 60*A*B^2*a*b^ \\
& 5*c^2*d*e^2*f^4*z + 58*A*B^2*a*b^3*c^4*d*e^4*f^2*z - 36*A*B^2*a*b^4*c^3*d^2 \\
& *e*f^4*z - 24*A*B^2*a^2*b^4*c^2*d*e*f^5*z + 24*A*B^2*a*b^4*c^3*d*e^3*f^3*z \\
& + 592*A^2*B*a*b*c^6*d^3*e*f^3*z + 240*A^2*B*a^3*b*c^4*d*e*f^5*z - 132*A*B^2 \\
& *a*b*c^6*d^2*e^4*f*z - 60*A*B^2*a*b^2*c^5*d*e^5*f*z - 48*A^2*B*a*b^5*c^2*d* \\
& e*f^5*z + 20*B^3*a*b*c^6*d^3*e^3*f*z + 16*B^3*a^4*b*c^3*d*e*f^5*z - 16*B^3* \\
& a*b*c^6*d^4*e*f^2*z + 12*B^3*a^2*b*c^5*d*e^5*f*z + 320*A^3*a*b*c^6*d*e^4*f^ \\
& 2*z + 40*A^3*a*b^4*c^3*d*e*f^5*z - 48*A^2*B*b*c^7*d^4*e*f^2*z - 44*A^2*B*b^ \\
& 3*c^5*d*e^5*f*z - 20*A*B^2*b*c^7*d^4*e^2*f*z + 14*A*B^2*b^4*c^4*d*e^5*f*z + \\
& 12*A^2*B*b*c^7*d^3*e^3*f*z + 4*A*B^2*b^7*c*d*e^2*f^4*z + 160*A*B^2*a^4*c^4 \\
& *d*e*f^5*z + 152*A^2*B*a*c^7*d^2*e^4*f*z - 40*A*B^2*a*c^7*d^3*e^3*f*z + 32* \\
& A*B^2*a*c^7*d^4*e*f^2*z - 16*A*B^2*a^2*c^6*d*e^5*f*z + 128*A^2*B*a^4*b*c^3* \\
& e*f^6*z + 42*A^2*B*a*b^2*c^5*e^6*f*z + 24*A^2*B*a^2*b^5*c*e*f^6*z - 12*A*B^ \\
& 2*a^3*b^4*c*e*f^6*z - 12*A*B^2*a^2*b*c^5*e^6*f*z - 10*A^2*B*a*b^6*c*e^2*f^5 \\
& *z - 160*A*B^2*a*b*c^6*d^4*f^3*z + 112*A*B^2*a^4*b*c^3*d*f^6*z - 24*A*B^2*a \\
& ^2*b^5*c*d*f^6*z - 84*B^3*a*b^2*c^5*d^3*e^2*f^2*z - 80*B^3*a^2*b^3*c^3*d^2* \\
& e*f^4*z - 60*B^3*a^2*b*c^5*d^2*e^3*f^2*z - 20*B^3*a^3*b^2*c^3*d*e^2*f^4*z -
\end{aligned}$$

$$\begin{aligned}
& 20*B^3*a*b^3*c^4*d^2*e^3*f^2*z - 9*B^3*a^2*b^2*c^4*d*e^4*f^2*z - 8*B^3*a*b^4*c^3*d^2*e^2*f^3*z + 6*B^3*a^2*b^4*c^2*d*e^2*f^4*z - 4*B^3*a^2*b^3*c^3*d*e^3*f^3*z - 216*A^2*B*b^4*c^4*d^2*e^2*f^3*z + 196*A^2*B*b^3*c^5*d^2*e^3*f^2*z - 108*A*B^2*b^3*c^5*d^3*e^2*f^2*z - 94*A*B^2*b^4*c^4*d^2*e^3*f^2*z + 88*A^2*B*b^2*c^6*d^3*e^2*f^2*z + 80*A*B^2*b^5*c^3*d^2*e^2*f^3*z + 360*A^2*B*a^2*c^6*d^2*e^2*f^3*z + 8*A*B^2*a^2*c^6*d^2*e^3*f^2*z + 153*A^2*B*a^2*b^2*c^4*e^4*f^3*z - 144*A^2*B*a^2*b^3*c^3*e^3*f^4*z + 80*A^2*B*a^3*b^2*c^3*e^2*f^5*z + 36*A*B^2*a^3*b^2*c^3*e^3*f^4*z + 12*A^2*B*a^2*b^4*c^2*e^2*f^5*z + 12*A*B^2*a^3*b^3*c^2*e^2*f^5*z + 9*A*B^2*a^2*b^2*c^4*e^5*f^2*z - 6*A*B^2*a^2*b^4*c^2*e^3*f^4*z + 4*A*B^2*a^2*b^3*c^3*e^4*f^3*z + 480*A^2*B*a^2*b^2*c^4*d^2*f^5*z - 176*A*B^2*a^2*b^3*c^3*d^2*f^5*z - 10*A^2*B*a*b^6*c*d*f^6*z + 16*A*B^2*a*b*c^6*d*e^6*z + 80*B^3*a*b^3*c^4*d^3*e*f^3*z - 48*B^3*a^3*b*c^4*d^2*e*f^4*z + 48*B^3*a^2*b*c^5*d^3*e*f^3*z + 44*B^3*a^3*b*c^4*d*e^3*f^3*z + 24*B^3*a*b^5*c^2*d^2*e*f^4*z + 18*B^3*a*b^2*c^5*d^2*e^4*f*z + 696*A^3*a^2*b*c^5*d*e^2*f^4*z - 504*A^3*a*b*c^6*d^2*e^2*f^3*z - 192*A^3*a*b^2*c^5*d*e^3*f^3*z - 144*A^3*a^2*b^2*c^4*d*e*f^5*z + 96*A^3*a*b^2*c^5*d^2*e*f^4*z - 72*A^3*a*b^3*c^4*d*e^2*f^4*z - 208*A^2*B*b^3*c^5*d^3*e*f^3*z + 152*A*B^2*b^4*c^4*d^3*e*f^3*z + 80*A^2*B*b^5*c^3*d^2*e*f^4*z + 75*A^2*B*b^4*c^4*d*e^4*f^2*z - 59*A^2*B*b^2*c^6*d^2*e^4*f*z - 52*A^2*B*b^5*c^3*d*e^3*f^3*z + 42*A*B^2*b^3*c^5*d^2*e^4*f*z - 21*A*B^2*b^6*c^2*d^2*e*f^4*z - 16*A*B^2*b^5*c^3*d*e^4*f^2*z + 16*A*B^2*b^2*c^6*d^4*e*f^2*z + 16*A*B^2*b^2*c^6*d^3*e^3*f*z + 11*A^2*B*b^6*c^2*d*e^2*f^4*z + 4*A*B^2*b^6*c^2*d*e^3*f^3*z - 256*A^2*B*a*c^7*d^3*e^2*f^2*z - 96*A*B^2*a^3*c^5*d^2*e*f^4*z - 36*A^2*B*a^2*c^6*d*e^4*f^2*z - 32*A^2*B*a^3*c^5*d*e^2*f^4*z - 32*A*B^2*a^2*c^6*d^3*e*f^3*z + 8*A*B^2*a^3*c^5*d*e^3*f^3*z - 96*A^2*B*a^3*b^3*c^2*e*f^6*z + 68*A^2*B*a^3*b*c^4*e^3*f^4*z - 60*A*B^2*a^4*b*c^3*e^2*f^5*z - 60*A*B^2*a^3*b*c^4*e^4*f^3*z + 48*A*B^2*a^4*b^2*c^2*e*f^6*z - 38*A^2*B*a*b^3*c^4*e^5*f^2*z - 36*A^2*B*a^2*b*c^5*e^5*f^2*z + 36*A^2*B*a*b^5*c^2*e^3*f^4*z - 16*A^2*B*a*b^4*c^3*e^4*f^3*z + 384*A*B^2*a^2*b*c^5*d^3*f^4*z - 352*A*B^2*a^3*b*c^4*d^2*f^5*z - 288*A^2*B*a*b^2*c^5*d^3*f^4*z - 160*A^2*B*a^3*b^2*c^3*d*f^6*z - 148*A^2*B*a*b^4*c^3*d^2*f^5*z + 112*A*B^2*a*b^3*c^4*d^3*f^4*z + 72*A^2*B*a^2*b^4*c^2*d*f^6*z + 72*A*B^2*a*b^5*c^2*d^2*f^5*z + 48*A*B^2*a^3*b^3*c^2*d*f^6*z + 102*B^3*a^2*b^2*c^4*d^2*e^2*f^3*z - 32*B^3*b^5*c^3*d^3*e*f^3*z - 8*B^3*b^3*c^5*d^3*e^3*f*z - 7*B^3*b^4*c^4*d^2*e^4*f*z + 5*B^3*b^2*c^6*d^4*e^2*f*z + 80*A^3*b^2*c^6*d^3*e*f^3*z - 74*A^3*b^3*c^5*d*e^4*f^2*z - 64*A^3*b^4*c^4*d^2*e*f^4*z + 60*A^3*b^4*c^4*d*e^3*f^3*z - 48*B^3*a^4*c^4*d*e^2*f^4*z - 24*B^3*a^3*c^5*d*e^4*f^2*z + 20*B^3*a^2*c^6*d^2*e^4*f*z - 16*A^3*b^5*c^3*d*e^2*f^4*z + 8*A^3*b*c^7*d^3*e^2*f^2*z + 480*A^3*a^2*c^6*d^2*e*f^4*z - 392*A^3*a^2*c^6*d*e^3*f^3*z + 280*A^3*a*c^7*d^2*e^3*f^2*z - 4*B^3*a^4*b*c^3*e^3*f^4*z - 200*A^3*a^3*b*c^4*e^2*f^5*z - 144*A^3*a^2*b*c^5*e^4*f^3*z + 48*B^3*a*b^2*c^5*d^4*f^3*z + 42*A^3*a*b^2*c^5*e^5*f^2*z - 36*B^3*a^4*b^2*c^2*d*f^6*z - 32*A^3*a^3*b^2*c^3*e*f^6*z - 24*A^3*a^2*b^4*c^2*e*f^6*z - 24*A^3*a*b^5*c^2*e^2*f^5*z + 10*A^3*a*b^3*c^4*e^4*f^3*z - 4*B^3*a*b^4*c^3*d^3*f^4*z - 4*A^3*a*b^4*c^3*e^3*f^4*z - 480*A^3*a^2*b*c^5*d^2*f^5*z - 160*A^3*a^2*b^3*c^3*d*f^6*z + 128*A^3*a*b^3*c^4*d^2*f^5*z + 8*A^2*B*b^5*c^3*e^5*f^2*z - 2*A^2*B*b^6*c^2*e^4*f^3*z + 112*A^2*B*b^4*c^4*d^3*f^4*z - 92*A^2*B*a^4*c^4*e^2*f^5*z - 64*A^2*B*a^3*c^5*e^4*f^3*z - 64*A*B^2*b^5*c^3*d^3*f^4*z + 24*A*B^2*a^4*c^4*e^3*f^4*z + 24*A*B^2*a^3*c^5*e^5*f^2*z + 16*A^2*B*b^2*c^6*d^4*f^3*z + 16*A*B^2*b^3*c^5*d^4*f^3*z - A^2*B*b^6*c^2*d^2*f^5*z + 448*A^2*B*a^3*c^5*d^2*f^5*z - 352*A^2*B*a^2*c^6*d^3*f^4*z - 5*A*B^2*b^2*c^6*d^2*e^5*z - 48*A^2*B*a^4*b^2*c^2*f^7*z - 2*B^3*b^7*c*d^2*e*f^4*z + 34*A^3*b^2*c^6*d*e^5*f*z + 16*A^3*b*c^7*d^2*e^4*f*z + 2*A^3*b^6*c^2*d*e*f^5*z - 416*A^3*a^3*c^5*d*e*f^5*z - 224*A^3*a*c^7*d^3*e*f^3*z + 12*B^3*a^3*b^4*c*d*f^6*z - 10*B^3*a*b^6*c*d^2*f^5*z + 416*A^3*a^3*b*c^4*d*f^6*z + 224*A^3*a*b*c^6*d^3*f^4*z + 24*A^3*a*b^5*c^2*d*f^6*z - 4*B^3*a*b*c^6*d^2*e^5*z + 20*A^2*B*c^8*d^4*e^2*f*z - 7*A^2*B*b^4*c^4*e^6*f*z - 2*A^2*B*b^7*c*e^3*f^4*z - 64*A*B^2*a^5*c^3*e*f^6*z + 16*A*B^2*b*c^7*d^5*f^2*z - 8*A^2*B*a^2*c^6*e^6*f*z - 2*A*B^2*b^7*c*d^2*f^5*z - 272*A^2*B*a^4*c^4*d*f^6*z + 128*A^2*B*a*c^7*d^4*f^3*z + 9*A^2*B*b^2*c^6*d*e^6*z - 4*A*B^2*b^3*c^5*d*e^6*z + 4*A*B^2*b*c^7*d^3*e^4*z + 8*A*B^2*a*c^7*d^2*e^5*z + 12*A^2*B*a
\end{aligned}$$

$$\begin{aligned}
& ^3b^4c^7f^2z + 30B^3b^4c^4d^3e^2f^2z + 8B^3b^5c^3d^2e^3f^2z \\
& - 2B^3b^6c^2d^2e^2f^3z + 152A^3b^3c^5d^2e^2f^3z - 108A^3b^2 \\
& c^6d^2e^3f^2z + 48B^3a^3c^5d^2e^2f^3z - 16B^3a^2c^6d^3e^2 \\
& f^2z - 3B^3a^4b^2c^2e^2f^5z - 120B^3a^2b^2c^4d^3f^4z + 112 \\
& B^3a^3b^2c^3d^2f^5z + 112A^3a^2b^3c^3e^2f^5z + 12A^3a^2b^2c^4 \\
& e^3f^4z - 120A^3a^2c^7d^5e^5f^2z - 52A^3a^2b^6c^6e^6f^2z + 10A^3a \\
& b^6c^6e^6f^2z - 2A^2B^2b^8d^5e^5f^2z - 2A^2B^2a^7b^7e^6f^2z - 24A^2B^2a \\
& c^7d^5e^6z + 2A^2B^2a^7b^7d^6f^6z - 12A^2B^2a^7b^7c^6e^7z - 2A^3b^7c \\
& d^6f^6z - 4A^3b^7c^7d^6e^6z + 16B^3a^5c^3e^2f^5z + 11B^3b^6c^2d^3 \\
& f^4z - 11A^3b^4c^4e^5f^2z - 8B^3b^4c^4d^4f^3z - 4B^3b^2c^6d^5 \\
& f^2z + 4B^3a^4c^4e^4f^3z + 4A^3b^5c^3e^4f^3z - A^3b^6c^2e^3f^4z + \\
& 136A^3a^3c^5e^3f^4z + 68A^3a^2c^6e^5f^2z - 64A^3b^3c^5d^3f^4z + \\
& 2B^3b^3c^5d^2e^5z - B^3b^2c^6d^3e^4z + 96A^3a^3b^3c^2f^7z + A^2B^2a^2 \\
& b^6e^6f^6z + 32A^3c^8d^4e^6f^2z - 24A^3c^8d^3e^3f^2z + 10A^3b^3c^5e^6 \\
& f^2z + 2A^3b^7c^6e^2f^5z + 128A^3a^4c^4e^6f^6z - 32A^3b^7c^7d^4f^3z - \\
& 4B^3a^2c^6d^5e^6z - B^3a^2b^6d^6f^6z - 128A^3a^4b^6c^3f^7z - 24A^3a^2b^5 \\
& c^4f^7z - 16A^2B^2c^8d^5f^2z - 4A^2B^2c^8d^3e^4z + 64A^2B^2a^5c^3f^7z + \\
& 2A^2B^2b^3c^5e^7z + 4A^2B^2a^2c^6e^7z - A^2B^2a^2b^6f^7z + 4A^3c^8d^2 \\
& e^5z - 3A^3b^2c^6e^7z + A^2B^2b^8d^6f^6z - A^3b^8e^6f^6z + 16A^3a^2c^7 \\
& e^7z + 2A^3a^2b^7f^7z + A^2B^2b^8e^2f^5z + B^3b^8d^2f^5z - 48A^2B^2a^2 \\
& b^6c^4d^5e^6f^4 + 28A^2B^3a^2b^2c^3d^5e^6f^4 - 16A^2B^3a^2b^6c^4d^5e^6 \\
& f^4 + 16A^3B^2a^2c^5d^5e^6f^4 + 32A^3B^2a^2b^6c^4d^5e^6f^4 + 12A^2B^2a^2 \\
& b^6c^4d^5e^6f^4 + 5A^2B^3b^2c^4d^2e^6f^3 + 4A^2B^3b^3c^3d^5e^2f^3 + \\
& 24A^2B^2a^2c^5d^5e^2f^3 + 24A^2B^2a^2b^6c^3e^6f^5 + 12A^2B^2a^2b^6 \\
& c^4e^3f^3 - 6A^2B^2a^2b^3c^2e^6f^5 + 4A^2B^3a^2b^6c^3e^2f^4 + 3A^2B^3a^2 \\
& b^2c^2e^6f^5 - 18A^2B^2a^2b^2c^3d^6f^5 - 4B^4a^2b^6c^3d^5e^6f^4 + 4B^4a^2 \\
& b^6c^4d^2e^6f^3 - 6A^2B^3b^4c^2d^5e^6f^4 + 4A^3B^2b^6c^5d^5e^2f^3 - 2A^3B^2 \\
& b^2c^4d^5e^6f^4 - 8A^2B^3a^2c^4d^5e^6f^4 - 8A^2B^3a^2c^5d^2e^6f^3 + 26A^3B^2a^2 \\
& b^2c^3e^6f^5 + 8A^3B^2a^2b^6c^4e^2f^4 + 32A^2B^3a^2b^6c^4d^2e^6f^4 - 28A^2B^3 \\
& a^2b^6c^3d^6f^5 + 6A^2B^3a^2b^3c^2d^6f^5 - 9A^2B^2b^2c^4d^5e^2f^3 - 18A^2B^2 \\
& a^2b^2c^3e^2f^4 - 4A^3B^2c^6d^2e^6f^3 - 3A^3B^2b^4c^2e^6f^5 - 44A^3B^2a^2c^4 \\
& e^6f^5 - 16A^3B^2a^2c^5e^3f^3 - 16A^2B^3a^3c^3e^6f^5 - 10A^3B^2b^3c^3d^6f^5 - \\
& 4A^3B^2b^6c^5d^2f^4 - 4A^2B^3b^6c^5d^3f^3 - 28A^3B^2a^2b^6c^3f^6 + 6A^3B^2a^2 \\
& b^6c^2f^6 - 4A^4b^6c^5d^5e^6f^4 - 20A^4a^2b^6c^4e^6f^5 + 3A^2B^2b^4c^2e^2f^4 - \\
& 2A^2B^2b^3c^3e^3f^3 + 12A^2B^2a^2c^4e^2f^4 + 9A^2B^2b^2c^4d^2e^2f^4 - 3A^2B^2 \\
& a^2b^2c^2f^6 - 2B^4b^3c^3d^2e^6f^3 + 4B^4a^2c^4d^5e^2f^3 - 10B^4a^2b^2c^3d^2 \\
& f^4 - 3B^4a^2b^2c^2d^6f^5 + 3A^3B^2b^2c^4e^3f^3 - 2A^3B^2b^3c^3e^2f^4 - 10A^2B^3 \\
& b^3c^3d^2f^4 - 4A^2B^3a^2c^4e^3f^3 + 3A^2B^2b^4c^2d^6f^5 + 36A^2B^2a^2c^4d^6f^5 - \\
& 24A^2B^2a^2c^5d^2f^4 + 4A^2B^2c^6d^3f^3 + 16A^2B^2a^3c^3f^6 + 4A^4b^3c^3e^6f^5 + \\
& 16B^4a^3c^3d^6f^5 + 16A^4a^2c^5e^2f^4 + 8A^4b^2c^4d^6f^5 - 8A^4a^2b^2c^3f^6 - \\
& 24A^4a^2c^5d^6f^5 + 3B^4b^4c^2d^2f^4 - 3A^4b^2c^4e^2f^4 + 4A^4c^6d^2f^4 + \\
& 36A^4a^2c^4f^6 + B^4b^2c^4d^3f^3, z, k), k, 1, 4) - ((A^2b^3f + 2A^2a^2c^2e + A^2b^3c^2d - 2 \\
& B^2a^2c^2d - A^2b^2c^2e - B^2a^2b^2f + 2B^2a^2c^2f - 3A^2a^2b^2c^2f + B^2a^2b^2c^2e) / \\
& (a^2b^2f^2 - 4a^3c^2f^2 - 4a^2c^3d^2 - 4a^2c^2e^2 + b^2c^2d^2 + b^4d^2f - a^2b^3e^2f - \\
& b^3c^2d^2e + a^2b^2c^2e^2 + 8a^2c^2d^2f + 4a^2b^2c^2d^2e - 6a^2b^2c^2d^2f + 4a^2b^2c^2d^2e \\
& - (x*(2A^2a^2c^2f - 2A^2c^3d + A^2b^2c^2e - 2B^2a^2c^2e + B^2b^2c^2d - A^2b^2c^2f + B^2a^2b^2c^2f)) / (a^2b^2f^2 - 4a^3c^2 \\
& f^2 - 4a^2c^3d^2 - 4a^2c^2e^2 + b^2c^2d^2 + b^4d^2f - a^2b^3e^2f - b^3c^2d^2e + a^2b^2c^2e^2 + 8a^2c^2d^2f + 4a^2b^2c^2d^2e - 6a^2b^2c^2d^2f + 4a^2 \\
& b^2c^2e^2f)) / (a + b*x + c*x^2)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((B*x+A)/(c*x**2+b*x+a)**2/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

$$3.17 \quad \int \frac{g+hx}{(a+bx+cx^2)(ad+bdx+cdx^2)^2} dx$$

Optimal. Leaf size=140

$$\frac{3(b+2cx)(2cg-bh)}{2d^2(b^2-4ac)^2(a+bx+cx^2)} - \frac{-2ah+x(2cg-bh)+bg}{2d^2(b^2-4ac)(a+bx+cx^2)^2} - \frac{6c(2cg-bh)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d^2(b^2-4ac)^{5/2}}$$

Rubi [A] time = 0.13, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {998, 638, 614, 618, 206}

$$\frac{3(b+2cx)(2cg-bh)}{2d^2(b^2-4ac)^2(a+bx+cx^2)} - \frac{-2ah+x(2cg-bh)+bg}{2d^2(b^2-4ac)(a+bx+cx^2)^2} - \frac{6c(2cg-bh)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d^2(b^2-4ac)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(g + h*x)/((a + b*x + c*x^2)*(a*d + b*d*x + c*d*x^2)^2), x]

[Out] -(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*(b^2 - 4*a*c)*d^2*(a + b*x + c*x^2)^2) + (3*(2*c*g - b*h)*(b + 2*c*x))/(2*(b^2 - 4*a*c)^2*d^2*(a + b*x + c*x^2)) - (6*c*(2*c*g - b*h)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(5/2)*d^2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 638

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 998

Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Dist[(c/f)^p, Int[(g + h*x)^m*(d + e*x + f*x^2)^(p + q), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q}, x] && EqQ[c*d - a*f, 0] && EqQ[b*d - a*e, 0] && (IntegerQ[p] || GtQ[c/

f, 0]) && (!IntegerQ[q] || LeafCount[d + e*x + f*x^2] <= LeafCount[a + b*x + c*x^2])

Rubi steps

$$\begin{aligned} \int \frac{g + hx}{(a + bx + cx^2)(ad + bdx + cd^2x^2)^2} dx &= \int \frac{g + hx}{(a + bx + cx^2)^3} dx \\ &= -\frac{bg - 2ah + (2cg - bh)x}{2(b^2 - 4ac)d^2(a + bx + cx^2)^2} - \frac{(3(2cg - bh)) \int \frac{1}{(a + bx + cx^2)^2} dx}{2(b^2 - 4ac)d^2} \\ &= -\frac{bg - 2ah + (2cg - bh)x}{2(b^2 - 4ac)d^2(a + bx + cx^2)^2} + \frac{3(2cg - bh)(b + 2cx)}{2(b^2 - 4ac)^2 d^2(a + bx + cx^2)} \\ &= -\frac{bg - 2ah + (2cg - bh)x}{2(b^2 - 4ac)d^2(a + bx + cx^2)^2} + \frac{3(2cg - bh)(b + 2cx)}{2(b^2 - 4ac)^2 d^2(a + bx + cx^2)} \\ &= -\frac{bg - 2ah + (2cg - bh)x}{2(b^2 - 4ac)d^2(a + bx + cx^2)^2} + \frac{3(2cg - bh)(b + 2cx)}{2(b^2 - 4ac)^2 d^2(a + bx + cx^2)} \end{aligned}$$

Mathematica [A] time = 0.14, size = 131, normalized size = 0.94

$$\frac{\frac{(b^2 - 4ac)(2ah - bg + bhx - 2cgx)}{(a + x(b + cx))^2} - \frac{12c(bh - 2cg) \tan^{-1}\left(\frac{b + 2cx}{\sqrt{4ac - b^2}}\right)}{\sqrt{4ac - b^2}} + \frac{3(b + 2cx)(2cg - bh)}{a + x(b + cx)}}{2d^2(b^2 - 4ac)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(g + h*x)/((a + b*x + c*x^2)*(a*d + b*d*x + c*d*x^2)^2), x]

[Out] (((b^2 - 4*a*c)*(-b*g) + 2*a*h - 2*c*g*x + b*h*x))/(a + x*(b + c*x))^2 + (3*(2*c*g - b*h)*(b + 2*c*x))/(a + x*(b + c*x)) - (12*c*(-2*c*g + b*h)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]/(2*(b^2 - 4*a*c)^2*d^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{g + hx}{(a + bx + cx^2)(ad + bdx + cd^2x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(g + h*x)/((a + b*x + c*x^2)*(a*d + b*d*x + c*d*x^2)^2), x]

[Out] IntegrateAlgebraic[(g + h*x)/((a + b*x + c*x^2)*(a*d + b*d*x + c*d*x^2)^2), x]

fricas [B] time = 0.64, size = 1150, normalized size = 8.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)/(c*x^2+b*x+a)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="fricas")

[Out] [1/2*(6*(2*(b^2*c^3 - 4*a*c^4)*g - (b^3*c^2 - 4*a*b*c^3)*h)*x^3 + 9*(2*(b^3*c^2 - 4*a*b*c^3)*g - (b^4*c - 4*a*b^2*c^2)*h)*x^2 - 6*(2*a^2*c^2*g - a^2*b*c*h + (2*c^4*g - b*c^3*h)*x^4 + 2*(2*b*c^3*g - b^2*c^2*h)*x^3 + (2*(b^2*c^2 + 2*a*c^3)*g - (b^3*c + 2*a*b*c^2)*h)*x^2 + 2*(2*a*b*c^2*g - a*b^2*c*h)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c))*(2*c*x + b))/(c*x^2 + b*x + a) - (b^5 - 14*a*b^3*c + 40*a^2*b*c^2)*g - (a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2)*h + 2*(2*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*g - (b^5 + a*b^3*c - 20*a^2*b*c^2)*h)*x)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*d^2), 1/2*(6*(2*(b^2*c^3 - 4*a*c^4)*g - (b^3*c^2 - 4*a*b*c^3)*h)*x^3 + 9*(2*(b^3*c^2 - 4*a*b*c^3)*g - (b^4*c - 4*a*b^2*c^2)*h)*x^2 - 12*(2*a^2*c^2*g - a^2*b*c*h + (2*c^4*g - b*c^3*h)*x^4 + 2*(2*b*c^3*g - b^2*c^2*h)*x^3 + (2*(b^2*c^2 + 2*a*c^3)*g - (b^3*c + 2*a*b*c^2)*h)*x^2 + 2*(2*a*b*c^2*g - a*b^2*c*h)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (b^5 - 14*a*b^3*c + 40*a^2*b*c^2)*g - (a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2)*h + 2*(2*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*g - (b^5 + a*b^3*c - 20*a^2*b*c^2)*h)*x)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*d^2)]

giac [A] time = 0.17, size = 219, normalized size = 1.56

$$\frac{6(2c^2g - bch) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(b^4d^2 - 8ab^2cd^2 + 16a^2c^2d^2)\sqrt{-b^2+4ac}} + \frac{12c^3gx^3 - 6bc^2hx^3 + 18bc^2gx^2 - 9b^2chx^2 + 4b^2cgx + 20ac^2gx - 2b^3hx - 10abchx - b^3g + 10abcg - ab^2h - 8a^2ch}{2(b^4d^2 - 8ab^2cd^2 + 16a^2c^2d^2)(cx^2 + bx + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)/(c*x^2+b*x+a)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="giac")

[Out] 6*(2*c^2*g - b*c*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4*d^2 - 8*a*b^2*c*d^2 + 16*a^2*c^2*d^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(12*c^3*g*x^3 - 6*b*c^2*h*x^3 + 18*b*c^2*g*x^2 - 9*b^2*c*h*x^2 + 4*b^2*c*g*x + 20*a*c^2*g*x - 2*b^3*h*x - 10*a*b*c*h*x - b^3*g + 10*a*b*c*g - a*b^2*h - 8*a^2*c*h)/((b^4*d^2 - 8*a*b^2*c*d^2 + 16*a^2*c^2*d^2)*(c*x^2 + b*x + a)^2)

maple [B] time = 0.01, size = 340, normalized size = 2.43

$$\frac{3bchx}{(4ac-b^2)^2(c^2+bx+a)d^2} - \frac{6bch \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^2d^2} + \frac{6c^2gx}{(4ac-b^2)^2(c^2+bx+a)d^2} + \frac{12c^2g \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^2d^2} - \frac{3b^2h}{2(4ac-b^2)(c^2+bx+a)d^2} + \frac{3bcg}{(4ac-b^2)(c^2+bx+a)d^2} - \frac{bhx}{2(4ac-b^2)(c^2+bx+a)d^2} + \frac{cgx}{(4ac-b^2)(c^2+bx+a)d^2} + \frac{ab}{(4ac-b^2)(c^2+bx+a)d^2} + \frac{bg}{2(4ac-b^2)(c^2+bx+a)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h*x+g)/(c*x^2+b*x+a)/(c*d*x^2+b*d*x+a*d)^2,x)

[Out] -1/2/d^2/(4*a*c-b^2)/(c*x^2+b*x+a)^2*x*b*h+1/d^2/(4*a*c-b^2)/(c*x^2+b*x+a)^2*x*c*g-1/d^2/(4*a*c-b^2)/(c*x^2+b*x+a)^2*a*h+1/2/d^2/(4*a*c-b^2)/(c*x^2+b*x+a)^2*b*g-3/d^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)*c*x*b*h+6/d^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)*c^2*x*g-3/2/d^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)*b^2*h+3/d^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)*b*c*g-6/d^2/(4*a*c-b^2)^(5/2)*c*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*h+12/d^2/(4*a*c-b^2)^(5/2)*c^2*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*g

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(c*x^2+b*x+a)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="maxima")
```

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 0.42, size = 395, normalized size = 2.82

$$6 \operatorname{catan} \left(\frac{d^2 \left(\frac{6c^2x(bh-2cg)}{d^2(4ac-b^2)^{5/2}} + \frac{3c(bh-2cg)(16a^2b^2d^2-8ab^3cd^2+b^5d^2)}{d^4(4ac-b^2)^{5/2}(16a^2c^2-8ab^2c+b^4)} \right) (16a^2c^2-8ab^2c+b^4)}{6c^2g-3bch} \right) (bh-2cg) - \frac{8cht^2+ha^2b^2-10cga+gb^3}{2(16a^2c^2-8ab^2c+b^4)} + \frac{x(b^2+5ac)(bh-2cg)}{16a^2c^2-8ab^2c+b^4} + \frac{3c^2x^3(bh-2cg)}{16a^2c^2-8ab^2c+b^4} + \frac{9bct^2(bh-2cg)}{2(16a^2c^2-8ab^2c+b^4)}{d^2(4ac-b^2)^{5/2} x^2(b^2d^2+2acd^2)+a^2d^2+c^2d^2x^4+2abd^2x+2bcd^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)/((a*d + b*d*x + c*d*x^2)^2*(a + b*x + c*x^2)),x)
```

[Out] $(6*c*\operatorname{atan}((d^2*((6*c^2*x*(b*h - 2*c*g))/(d^2*(4*a*c - b^2)^{(5/2)})) + (3*c*(b*h - 2*c*g)*(b^5*d^2 + 16*a^2*b*c^2*d^2 - 8*a*b^3*c*d^2))/(d^4*(4*a*c - b^2)^{(5/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)}))*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(6*c^2*g - 3*b*c*h)*(b*h - 2*c*g))/(d^2*(4*a*c - b^2)^{(5/2)} - ((b^3*g + a*b^2*h + 8*a^2*c*h - 10*a*b*c*g)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x*(5*a*c + b^2)*(b*h - 2*c*g))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (3*c^2*x^3*(b*h - 2*c*g))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (9*b*c*x^2*(b*h - 2*c*g))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(b^2*d^2 + 2*a*c*d^2) + a^2*d^2 + c^2*d^2*x^4 + 2*a*b*d^2*x + 2*b*c*d^2*x^3)$

sympy [B] time = 2.44, size = 709, normalized size = 5.06

$$\frac{\sqrt{\frac{4ac-b^2}{4ac-b^2}} \operatorname{atan} \left(\frac{d^2 \left(\frac{6c^2x(bh-2cg)}{d^2(4ac-b^2)^{5/2}} + \frac{3c(bh-2cg)(16a^2b^2d^2-8ab^3cd^2+b^5d^2)}{d^4(4ac-b^2)^{5/2}(16a^2c^2-8ab^2c+b^4)} \right) (16a^2c^2-8ab^2c+b^4)}{6c^2g-3bch} \right) (bh-2cg) - \frac{8cht^2+ha^2b^2-10cga+gb^3}{2(16a^2c^2-8ab^2c+b^4)} + \frac{x(b^2+5ac)(bh-2cg)}{16a^2c^2-8ab^2c+b^4} + \frac{3c^2x^3(bh-2cg)}{16a^2c^2-8ab^2c+b^4} + \frac{9bct^2(bh-2cg)}{2(16a^2c^2-8ab^2c+b^4)}}{d^2(4ac-b^2)^{5/2} x^2(b^2d^2+2acd^2)+a^2d^2+c^2d^2x^4+2abd^2x+2bcd^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(c*x**2+b*x+a)/(c*d*x**2+b*d*x+a*d)**2,x)
```

[Out] $3*c*\sqrt{-1/(4*a*c - b**2)**5}*(b*h - 2*c*g)*\log(x + (-192*a**3*c**4*\sqrt{-1/(4*a*c - b**2)**5}*(b*h - 2*c*g) + 144*a**2*b**2*c**3*\sqrt{-1/(4*a*c - b**2)**5}*(b*h - 2*c*g) - 36*a*b**4*c**2*\sqrt{-1/(4*a*c - b**2)**5}*(b*h - 2*c*g) + 3*b**6*c*\sqrt{-1/(4*a*c - b**2)**5}*(b*h - 2*c*g) + 3*b**2*c*h - 6*b*c**2*g)/(6*b*c**2*h - 12*c**3*g))/d**2 - 3*c*\sqrt{-1/(4*a*c - b**2)**5}*(b*h - 2*c*g)*\log(x + (192*a**3*c**4*\sqrt{-1/(4*a*c - b**2)**5}*(b*h - 2*c*g) - 144*a**2*b**2*c**3*\sqrt{-1/(4*a*c - b**2)**5}*(b*h - 2*c*g) + 36*a*b**4*c**2*\sqrt{-1/(4*a*c - b**2)**5}*(b*h - 2*c*g) - 3*b**6*c*\sqrt{-1/(4*a*c - b**2)**5}*(b*h - 2*c*g) + 3*b**2*c*h - 6*b*c**2*g)/(6*b*c**2*h - 12*c**3*g))/d**2 + (-8*a**2*c*h - a*b**2*h + 10*a*b*c*g - b**3*g + x**3*(-6*b*c**2*h + 12*c**3*g) + x**2*(-9*b**2*c*h + 18*b*c**2*g) + x*(-10*a*b*c*h + 20*a*c**2*g - 2*b**3*h + 4*b**2*c*g))/(32*a**4*c**2*d**2 - 16*a**3*b**2*c*d**2 + 2*a**2*b**4*d**2 + x**4*(32*a**2*c**4*d**2 - 16*a*b**2*c**3*d**2 + 2*b**4*c**2*d**2) + x**3*(64*a**2*b*c**3*d**2 - 32*a*b**3*c**2*d**2 + 4*b**5*c*d**2) + x**2*(64*a**3*c**3*d**2 - 12*a*b**4*c*d**2 + 2*b**6*d**2) + x*(64*a**3*b*c**2*d**2 - 32*a**2*b**3*c*d**2 + 4*a*b**5*d**2))$

$$3.18 \quad \int \frac{g+hx}{(a+bx+cx^2)^2(ad+bdx+cdx^2)} dx$$

Optimal. Leaf size=140

$$\frac{3(b+2cx)(2cg-bh)}{2d(b^2-4ac)^2(a+bx+cx^2)} - \frac{-2ah+x(2cg-bh)+bg}{2d(b^2-4ac)(a+bx+cx^2)^2} - \frac{6c(2cg-bh)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d(b^2-4ac)^{5/2}}$$

Rubi [A] time = 0.10, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {998, 638, 614, 618, 206}

$$\frac{3(b+2cx)(2cg-bh)}{2d(b^2-4ac)^2(a+bx+cx^2)} - \frac{-2ah+x(2cg-bh)+bg}{2d(b^2-4ac)(a+bx+cx^2)^2} - \frac{6c(2cg-bh)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d(b^2-4ac)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(g + h*x)/((a + b*x + c*x^2)^2*(a*d + b*d*x + c*d*x^2)), x]

[Out] $-(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*(b^2 - 4*a*c)*d*(a + b*x + c*x^2)^2) + (3*(2*c*g - b*h)*(b + 2*c*x))/(2*(b^2 - 4*a*c)^2*d*(a + b*x + c*x^2)) - (6*c*(2*c*g - b*h)*\text{ArcTanh}[(b + 2*c*x)/\text{Sqrt}[b^2 - 4*a*c]])/((b^2 - 4*a*c)^{(5/2)}*d)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 638

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 998

Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Dist[(c/f)^p, Int[(g + h*x)^m*(d + e*x + f*x^2)^(p + q), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q}, x] && EqQ[c*d - a*f, 0] && EqQ[b*d - a*e, 0] && (IntegerQ[p] || GtQ[c/

f, 0]) && (!IntegerQ[q] || LeafCount[d + e*x + f*x^2] <= LeafCount[a + b*x + c*x^2])

Rubi steps

$$\begin{aligned} \int \frac{g + hx}{(a + bx + cx^2)^2 (ad + bdx + cdx^2)} dx &= \frac{\int \frac{g+hx}{(a+bx+cx^2)^3} dx}{d} \\ &= -\frac{bg - 2ah + (2cg - bh)x}{2(b^2 - 4ac)d(a + bx + cx^2)^2} - \frac{(3(2cg - bh)) \int \frac{1}{(a+bx+cx^2)^2} dx}{2(b^2 - 4ac)d} \\ &= -\frac{bg - 2ah + (2cg - bh)x}{2(b^2 - 4ac)d(a + bx + cx^2)^2} + \frac{3(2cg - bh)(b + 2cx)}{2(b^2 - 4ac)^2 d(a + bx + cx^2)} + \\ &= -\frac{bg - 2ah + (2cg - bh)x}{2(b^2 - 4ac)d(a + bx + cx^2)^2} + \frac{3(2cg - bh)(b + 2cx)}{2(b^2 - 4ac)^2 d(a + bx + cx^2)} - \\ &= -\frac{bg - 2ah + (2cg - bh)x}{2(b^2 - 4ac)d(a + bx + cx^2)^2} + \frac{3(2cg - bh)(b + 2cx)}{2(b^2 - 4ac)^2 d(a + bx + cx^2)} - \end{aligned}$$

Mathematica [A] time = 0.03, size = 131, normalized size = 0.94

$$\frac{\frac{(b^2-4ac)(2ah-bg+bx-2cgx)}{(a+x(b+cx))^2} - \frac{12c(bh-2cg) \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{3(b+2cx)(2cg-bh)}{a+x(b+cx)}}{2d(b^2-4ac)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(g + h*x)/((a + b*x + c*x^2)^2*(a*d + b*d*x + c*d*x^2)), x]

[Out] (((b^2 - 4*a*c)*(-(b*g) + 2*a*h - 2*c*g*x + b*h*x))/(a + x*(b + c*x))^2 + (3*(2*c*g - b*h)*(b + 2*c*x))/(a + x*(b + c*x)) - (12*c*(-2*c*g + b*h)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2*(b^2 - 4*a*c)^2*d)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{g + hx}{(a + bx + cx^2)^2 (ad + bdx + cdx^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(g + h*x)/((a + b*x + c*x^2)^2*(a*d + b*d*x + c*d*x^2)), x]

[Out] IntegrateAlgebraic[(g + h*x)/((a + b*x + c*x^2)^2*(a*d + b*d*x + c*d*x^2)), x]

fricas [B] time = 0.61, size = 1130, normalized size = 8.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)/(c*x^2+b*x+a)^2/(c*d*x^2+b*d*x+a*d),x, algorithm="fricas")

[Out] [1/2*(6*(2*(b^2*c^3 - 4*a*c^4)*g - (b^3*c^2 - 4*a*b*c^3)*h)*x^3 + 9*(2*(b^3*c^2 - 4*a*b*c^3)*g - (b^4*c - 4*a*b^2*c^2)*h)*x^2 - 6*(2*a^2*c^2*g - a^2*b*c*h + (2*c^4*g - b*c^3*h)*x^4 + 2*(2*b*c^3*g - b^2*c^2*h)*x^3 + (2*(b^2*c^2 + 2*a*c^3)*g - (b^3*c + 2*a*b*c^2)*h)*x^2 + 2*(2*a*b*c^2*g - a*b^2*c*h)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c))*(2*c*x + b))/(c*x^2 + b*x + a) - (b^5 - 14*a*b^3*c + 40*a^2*b*c^2)*g - (a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2)*h + 2*(2*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*g - (b^5 + a*b^3*c - 20*a^2*b*c^2)*h)*x)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*d), 1/2*(6*(2*(b^2*c^3 - 4*a*c^4)*g - (b^3*c^2 - 4*a*b*c^3)*h)*x^3 + 9*(2*(b^3*c^2 - 4*a*b*c^3)*g - (b^4*c - 4*a*b^2*c^2)*h)*x^2 - 12*(2*a^2*c^2*g - a^2*b*c*h + (2*c^4*g - b*c^3*h)*x^4 + 2*(2*b*c^3*g - b^2*c^2*h)*x^3 + (2*(b^2*c^2 + 2*a*c^3)*g - (b^3*c + 2*a*b*c^2)*h)*x^2 + 2*(2*a*b*c^2*g - a*b^2*c*h)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (b^5 - 14*a*b^3*c + 40*a^2*b*c^2)*g - (a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2)*h + 2*(2*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*g - (b^5 + a*b^3*c - 20*a^2*b*c^2)*h)*x)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*d)]

giac [A] time = 0.17, size = 207, normalized size = 1.48

$$\frac{6(2c^2g - bch) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(b^4d - 8ab^2cd + 16a^2c^2d)\sqrt{-b^2+4ac}} + \frac{12c^3gx^3 - 6bc^2hx^3 + 18bc^2gx^2 - 9b^2chx^2 + 4b^2cgx + 20ac^2gx - 2b^3hx - 10abchx - b^3g + 10abcg - ab^2h - 8a^2ch}{2(b^4d - 8ab^2cd + 16a^2c^2d)(cx^2 + bx + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)/(c*x^2+b*x+a)^2/(c*d*x^2+b*d*x+a*d),x, algorithm="giac")

[Out] 6*(2*c^2*g - b*c*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4*d - 8*a*b^2*c*d + 16*a^2*c^2*d)*sqrt(-b^2 + 4*a*c)) + 1/2*(12*c^3*g*x^3 - 6*b*c^2*h*x^3 + 18*b*c^2*g*x^2 - 9*b^2*c*h*x^2 + 4*b^2*c*g*x + 20*a*c^2*g*x - 2*b^3*h*x - 10*a*b*c*h*x - b^3*g + 10*a*b*c*g - a*b^2*h - 8*a^2*c*h)/((b^4*d - 8*a*b^2*c*d + 16*a^2*c^2*d)*(c*x^2 + b*x + a)^2)

maple [B] time = 0.01, size = 340, normalized size = 2.43

$$\frac{3bchx}{(4ac - b^2)^2 (cx^2 + bx + a)d} - \frac{6bch \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac - b^2)^2 d} + \frac{6c^2gx}{(4ac - b^2)^2 (cx^2 + bx + a)d} + \frac{12c^2g \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac - b^2)^2 d} - \frac{3b^2h}{2(4ac - b^2)^2 (cx^2 + bx + a)d} + \frac{3bcg}{(4ac - b^2)^2 (cx^2 + bx + a)d} - \frac{bhx}{2(4ac - b^2)(cx^2 + bx + a)^2 d} + \frac{cgx}{(4ac - b^2)(cx^2 + bx + a)^2 d} - \frac{ah}{(4ac - b^2)(cx^2 + bx + a)^2 d} + \frac{bg}{2(4ac - b^2)(cx^2 + bx + a)^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h*x+g)/(c*x^2+b*x+a)^2/(c*d*x^2+b*d*x+a*d),x)

[Out] -1/2/d/(4*a*c-b^2)/(c*x^2+b*x+a)^2*x*b*h+1/d/(4*a*c-b^2)/(c*x^2+b*x+a)^2*x*c*g-1/d/(4*a*c-b^2)/(c*x^2+b*x+a)^2*a*h+1/2/d/(4*a*c-b^2)/(c*x^2+b*x+a)^2*b*g-3/d/(4*a*c-b^2)^2/(c*x^2+b*x+a)*c*x*b*h+6/d/(4*a*c-b^2)^2/(c*x^2+b*x+a)*c^2*x*g-3/2/d/(4*a*c-b^2)^2/(c*x^2+b*x+a)*b^2*h+3/d/(4*a*c-b^2)^2/(c*x^2+b*x+a)*b*c*g-6/d/(4*a*c-b^2)^(5/2)*c*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*h+12/d/(4*a*c-b^2)^(5/2)*c^2*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*g

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)/(c*x^2+b*x+a)^2/(c*d*x^2+b*d*x+a*d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 4.02, size = 375, normalized size = 2.68

$$6c \operatorname{atan} \left(\frac{d \left(\frac{6c^2 x (bh-2cg)}{d(4ac-b^2)^{5/2}} + \frac{3c(bh-2cg)(16d^2 b^2 c^2 - 8da^2 b^3 c + d^4 b^5)}{d^2(4ac-b^2)^{5/2}(16a^2 c^2 - 8ab^2 c + b^4)} \right) (16a^2 c^2 - 8ab^2 c + b^4)}{6c^2 g - 3bch} \right) (bh-2cg) - \frac{8chd^2 + hab^2 - 10cga + gb^3}{2(16a^2 c^2 - 8ab^2 c + b^4)} + \frac{x(b^2 + 5ac)(bh-2cg)}{16a^2 c^2 - 8ab^2 c + b^4} + \frac{3c^2 x^3 (bh-2cg)}{16a^2 c^2 - 8ab^2 c + b^4} + \frac{9bcx^2 (bh-2cg)}{2(16a^2 c^2 - 8ab^2 c + b^4)}{d(4ac-b^2)^{5/2} - \frac{a^2 d + x^2 (db^2 + 2acd) + c^2 dx^4 + 2bcdx^3 + 2abd x}{a^2 d + x^2 (db^2 + 2acd) + c^2 dx^4 + 2bcdx^3 + 2abd x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g + h*x)/((a*d + b*d*x + c*d*x^2)*(a + b*x + c*x^2)^2),x)

[Out] (6*c*atan((d*((6*c^2*x*(b*h - 2*c*g))/(d*(4*a*c - b^2)^(5/2)) + (3*c*(b*h - 2*c*g)*(b^5*d - 8*a*b^3*c*d + 16*a^2*b*c^2*d))/(d^2*(4*a*c - b^2)^(5/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(6*c^2*g - 3*b*c*h))*(b*h - 2*c*g))/(d*(4*a*c - b^2)^(5/2)) - ((b^3*g + a*b^2*h + 8*a^2*c*h - 10*a*b*c*g)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x*(5*a*c + b^2)*(b*h - 2*c*g))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (3*c^2*x^3*(b*h - 2*c*g))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (9*b*c*x^2*(b*h - 2*c*g))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(a^2*d + x^2*(b^2*d + 2*a*c*d) + c^2*d*x^4 + 2*b*c*d*x^3 + 2*a*b*d*x)

sympy [B] time = 2.26, size = 680, normalized size = 4.86

$$\frac{\sqrt{\frac{d}{(4ac-b^2)^{5/2}} \log\left(\frac{d(6c^2x(bh-2cg) + 3c(bh-2cg)(16d^2b^2c^2 - 8da^2b^3c + d^4b^5)}{d^2(4ac-b^2)^{5/2}(16a^2c^2 - 8ab^2c + b^4)}\right) + \frac{3c(bh-2cg)(b^5d - 8ab^3cd + 16a^2b^2c^2d)}{d^2(4ac-b^2)^{5/2}(b^4 + 16a^2c^2 - 8ab^2c)}}{6c^2g - 3bch} - \frac{(b^3g + a^2bh + 8a^2ch - 10abcg)}{2(b^4 + 16a^2c^2 - 8ab^2c)} + \frac{x(5ac + b^2)(bh - 2cg)}{b^4 + 16a^2c^2 - 8ab^2c} + \frac{3c^2x^3(bh - 2cg)}{b^4 + 16a^2c^2 - 8ab^2c} + \frac{9bcx^2(bh - 2cg)}{2(b^4 + 16a^2c^2 - 8ab^2c)}}{a^2d + x^2(b^2d + 2acd) + c^2dx^4 + 2bcdx^3 + 2abd x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)/(c*x**2+b*x+a)**2/(c*d*x**2+b*d*x+a*d),x)

[Out] 3*c*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g)*log(x + (-192*a**3*c**4*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 144*a**2*b**2*c**3*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) - 36*a*b**4*c**2*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 3*b**6*c*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 3*b**2*c*h - 6*b*c**2*g)/(6*b*c**2*h - 12*c**3*g))/d - 3*c*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g)*log(x + (192*a**3*c**4*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) - 144*a**2*b**2*c**3*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 36*a*b**4*c**2*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) - 3*b**6*c*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 3*b**2*c*h - 6*b*c**2*g)/(6*b*c**2*h - 12*c**3*g))/d + (-8*a**2*c*h - a*b**2*h + 10*a*b*c*g - b**3*g + x**3*(-6*b*c**2*h + 12*c**3*g) + x**2*(-9*b**2*c*h + 18*b*c**2*g) + x*(-10*a*b*c*h + 20*a*c**2*g - 2*b**3*h + 4*b**2*c*g))/(32*a**4*c**2*d - 16*a**3*b**2*c*d + 2*a**2*b**4*d + x**4*(32*a**2*c**4*d - 16*a*b**2*c**3*d + 2*b**4*c**2*d) + x**3*(64*a**2*b*c**3*d - 32*a*b**3*c**2*d + 4*b**5*c*d) + x**2*(64*a**3*c**3*d - 12*a*b**4*c*d + 2*b**6*d) + x*(64*a**3*b*c**2*d - 32*a**2*b**3*c*d + 4*a*b**5*d))

3.19 $\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2} dx$

Optimal. Leaf size=617

$$\frac{(2f(Af(cd - af) - Bd(ce - bf)) - (e - \sqrt{e^2 - 4df})(B(f(be - af) - c(e^2 - df)) + Af(ce - bf))) \tanh^{-1}\left(\frac{4}{2\sqrt{2}\sqrt{a+bx+cx^2}}\right) + \sqrt{2}f^2\sqrt{e^2 - 4df}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}}{\dots}$$

Rubi [A] time = 9.00, antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 32, number of rules / integrand size = 0.188, Rules used = {1019, 1076, 621, 206, 1032, 724}

$$\frac{(2f(Af(cd - af) - Bd(ce - bf)) - (e - \sqrt{e^2 - 4df})(B(f(be - af) - c(e^2 - df)) + Af(ce - bf))) \tanh^{-1}\left(\frac{4}{2\sqrt{2}\sqrt{a+bx+cx^2}}\right) + \sqrt{2}f^2\sqrt{e^2 - 4df}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}}{\sqrt{2}f^2\sqrt{e^2 - 4df}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2), x]
```

```
[Out] (B*Sqrt[a + b*x + c*x^2])/f - ((2*B*c*e - b*B*f - 2*A*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*f^2) + ((2*f*(A*f*(c*d - a*f) - B*d*(c*e - b*f)) - (e - Sqrt[e^2 - 4*d*f])*(B*f*(b*e - a*f) + A*f*(c*e - b*f) - B*c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - ((2*f*(A*f*(c*d - a*f) - B*d*(c*e - b*f)) - (e + Sqrt[e^2 - 4*d*f])*(B*f*(b*e - a*f) + A*f*(c*e - b*f) - B*c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1019

```
Int[((g_) + (h_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[(h*(a + b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[h*p*(b*d - a*e) + a*(h*e - 2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(h*e - 2*g*f)*(p + q + 1)]]
```

) * x + (h * p * (c * e - b * f) + c * (h * e - 2 * g * f) * (p + q + 1)) * x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4 * a * c, 0] && NeQ[e^2 - 4 * d * f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]

Rule 1032

Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]

Rule 1076

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

Rubi steps

$$\int \frac{(A + Bx)\sqrt{a + bx + cx^2}}{d + ex + fx^2} dx = \frac{B\sqrt{a + bx + cx^2}}{f} - \frac{\int \frac{\frac{1}{2}(bBd - 2aAf) - \frac{1}{2}(2Abf - B(2cd + be - 2af))x + \frac{1}{2}(2Bce - bBf - 2Acf)x^2}{\sqrt{a + bx + cx^2}(d + ex + fx^2)} dx}{f}$$

$$= \frac{B\sqrt{a + bx + cx^2}}{f} - \frac{\int \frac{\frac{1}{2}f(bBd - 2aAf) - \frac{1}{2}d(2Bce - bBf - 2Acf) + \left(-\frac{1}{2}e(2Bce - bBf - 2Acf) + \frac{1}{2}f(-2aA + bB)\right)x + \frac{1}{2}(2Bce - bBf - 2Acf)x^2}{\sqrt{a + bx + cx^2}(d + ex + fx^2)} dx}{f^2}$$

$$= \frac{B\sqrt{a + bx + cx^2}}{f} - \frac{(2Bce - bBf - 2Acf) \text{Subst}\left(\int \frac{1}{4c - x^2} dx, x, \frac{b + 2cx}{\sqrt{a + bx + cx^2}}\right)}{f^2} + \frac{\int \frac{2f(-2aA + bB)}{\sqrt{a + bx + cx^2}} dx}{f^2}$$

$$= \frac{B\sqrt{a + bx + cx^2}}{f} - \frac{(2Bce - bBf - 2Acf) \tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right)}{2\sqrt{c}f^2} - \frac{(2f(Af - bB)) \text{Subst}\left(\int \frac{1}{\sqrt{a + bx + cx^2}} dx, x, \frac{b + 2cx}{\sqrt{a + bx + cx^2}}\right)}{2\sqrt{c}f^2}$$

$$= \frac{B\sqrt{a + bx + cx^2}}{f} - \frac{(2Bce - bBf - 2Acf) \tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right)}{2\sqrt{c}f^2} + \frac{(2f(Af - bB)) \text{Subst}\left(\int \frac{1}{\sqrt{a + bx + cx^2}} dx, x, \frac{b + 2cx}{\sqrt{a + bx + cx^2}}\right)}{2\sqrt{c}f^2}$$

Mathematica [A] time = 2.09, size = 517, normalized size = 0.84

$$\frac{-\sqrt{c} \left(b \left(\sqrt{d^2 - 4ef} + c \right) - 2ef \right) \sqrt{(2d - b)(\sqrt{d^2 - 4ef} + c)} + c \left(\sqrt{d^2 - 4ef} - 2d + c \right) \tanh^{-1}\left(\frac{4c^2 \sqrt{d^2 - 4ef} - 2c \left(\sqrt{d^2 - 4ef} - c \right)}{2 \sqrt{c} \sqrt{(2d - b)(\sqrt{d^2 - 4ef} + c)} + c \left(\sqrt{d^2 - 4ef} - 2d + c \right)}\right) - \sqrt{c} \left(2Af + b \left(\sqrt{d^2 - 4ef} - c \right) \right) \sqrt{(2d + b)(\sqrt{d^2 - 4ef} - c)} + c \left(-\sqrt{d^2 - 4ef} - 2d + c \right) \tanh^{-1}\left(\frac{4c^2 \sqrt{d^2 - 4ef} - 2c \left(\sqrt{d^2 - 4ef} - c \right)}{2 \sqrt{c} \sqrt{(2d + b)(\sqrt{d^2 - 4ef} - c)} + c \left(-\sqrt{d^2 - 4ef} - 2d + c \right)}\right) + 4B \sqrt{d^2 - 4ef} \sqrt{a + b(x + cx^2)}}{4f^2 \sqrt{d^2 - 4ef}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*x)*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2), x]

[Out] ((-2*B*c*e + b*B*f + 2*A*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(2*Sqrt[c]*f^2) + (4*B*f*Sqrt[e^2 - 4*d*f]*Sqrt[a + x*(b + c*x)] - Sqrt[2]*(-2*A*f + B*(e + Sqrt[e^2 - 4*d*f]))*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2

```
] *Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))] *Sqrt[a + x*(b + c*x))] - Sqrt[2]*(2*A*f + B*(-e + Sqrt[e^2 - 4*d*f])) *Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))] *ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))] *Sqrt[a + x*(b + c*x)])]) / (4*f^2*Sqrt[e^2 - 4*d*f])
```

IntegrateAlgebraic [C] time = 1.21, size = 893, normalized size = 1.45

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic(((A + B*x)*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x)
[Out] (B*Sqrt[a + b*x + c*x^2])/f + ((2*B*c*e - b*B*f - 2*A*c*f)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(2*Sqrt[c]*f^2) + RootSum[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (- (b*B*c*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]) + a*B*c*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + b^2*B*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + A*b*c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*B*c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*b*B*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*A*c*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a^2*B*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*B*c^(3/2)*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*b*B*Sqrt[c]*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*A*c^(3/2)*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*a*A*Sqrt[c]*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - B*c*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + B*c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + A*c*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - A*b*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - a*B*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(2*b*Sqrt[c]*d - a*Sqrt[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2*f*#1^3) & ]/f^2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.05, size = 16209, normalized size = 26.27

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*x+A)*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)`

[Out] result too large to display

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details) Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + Bx) \sqrt{cx^2 + bx + a}}{fx^2 + ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2),x)`

[Out] `int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx) \sqrt{a + bx + cx^2}}{d + ex + fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x+A)*(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)`

[Out] `Integral((A + B*x)*sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)`

$$3.20 \quad \int \frac{(A+Bx)(a+bx+cx^2)^{3/2}}{d+ex+fx^2} dx$$

Optimal. Leaf size=1092

$$\frac{B(cx^2 + bx + a)^{3/2}}{3f} \frac{(2Acf(4ce - 5bf) - B(8(e^2 - df)c^2 - 2f(5be - 4af)c + b^2f^2) + 2cf(2Bce - bBf - 2Acf)x)}{8cf^3}$$

Rubi [A] time = 18.87, antiderivative size = 1092, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {1019, 1066, 1076, 621, 206, 1032, 724}

Antiderivative was successfully verified.

[In] Int[((A + B*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x + f*x^2), x]

[Out] -((2*A*c*f*(4*c*e - 5*b*f) - B*(b^2*f^2 - 2*c*f*(5*b*e - 4*a*f) + 8*c^2*(e^2 - d*f)) + 2*c*f*(2*B*c*e - b*B*f - 2*A*c*f)*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^3) + (B*(a + b*x + c*x^2)^(3/2))/(3*f) + ((2*A*c*f*(3*b^2*f^2 - 12*c*f*(b*e - a*f) + 8*c^2*(e^2 - d*f)) - B*(b^3*f^3 + 6*b*c*f^2*(b*e - 2*a*f) - 2*4*c^2*f*(b*e^2 - b*d*f - a*e*f) + 16*c^3*(e^3 - 2*d*e*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*f^4) - ((2*c*f*(B*d*(c*e - b*f)*(c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2) + A*f*(2*c*d*f*(b*e - a*f) - f^2*(b^2*d - a^2*f) - c^2*d*(e^2 - d*f))) - c*(e - Sqrt[e^2 - 4*d*f])*(A*f*(c*e - b*f)*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) + B*(c^2*(e^4 - 3*d*e^2*f + d^2*f^2) - f^2*(2*a*b*e*f - a^2*f^2 - b^2*(e^2 - d*f)) + 2*c*f*(a*f*(e^2 - d*f) - b*(e^3 - 2*d*e*f))))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*c*f^4*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) + ((2*f*(B*d*(c*e - b*f)*(c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2) + A*f*(2*c*d*f*(b*e - a*f) - f^2*(b^2*d - a^2*f) - c^2*d*(e^2 - d*f))) - (e + Sqrt[e^2 - 4*d*f])*(A*f*(c*e - b*f)*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) + B*(c^2*(e^4 - 3*d*e^2*f + d^2*f^2) - f^2*(2*a*b*e*f - a^2*f^2 - b^2*(e^2 - d*f)) + 2*c*f*(a*f*(e^2 - d*f) - b*(e^3 - 2*d*e*f))))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^4*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2

$*a*e - b*d - (2*c*d - b*e)*x)/\text{Sqrt}[a + b*x + c*x^2]], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[2*c*d - b*e, 0]$

Rule 1019

$\text{Int}[(g_.) + (h_.)*(x_.))*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(p_.)*((d_.) + (e_.)*(x_.) + (f_.)*(x_.)^2)^(q_.), x_Symbol] \rightarrow \text{Simp}[(h*(a + b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - \text{Dist}[1/(2*f*(p + q + 1)), \text{Int}[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*\text{Simp}[h*p*(b*d - a*e) + a*(h*e - 2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(h*e - 2*g*f)*(p + q + 1))*x + (h*p*(c*e - b*f) + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, h, q\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[e^2 - 4*d*f, 0] \&\& \text{GtQ}[p, 0] \&\& \text{NeQ}[p + q + 1, 0]$

Rule 1032

$\text{Int}[(g_.) + (h_.)*(x_.))/(((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)*\text{Sqrt}[(d_.) + (e_.)*(x_.) + (f_.)*(x_.)^2]), x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(2*c*g - h*(b - q))/q, \text{Int}[1/((b - q + 2*c*x)*\text{Sqrt}[d + e*x + f*x^2]), x], x] - \text{Dist}[(2*c*g - h*(b + q))/q, \text{Int}[1/((b + q + 2*c*x)*\text{Sqrt}[d + e*x + f*x^2]), x], x]] /; \text{FreeQ}[\{a, b, c, d, e, f, g, h\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[e^2 - 4*d*f, 0] \&\& \text{PosQ}[b^2 - 4*a*c]$

Rule 1066

$\text{Int}[(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(p_.)*((A_.) + (B_.)*(x_.) + (C_.)*(x_.)^2)*((d_.) + (e_.)*(x_.) + (f_.)*(x_.)^2)^(q_.), x_Symbol] \rightarrow \text{Simp}[(B*c*f*(2*p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - \text{Dist}[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), \text{Int}[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*\text{Simp}[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3)))]*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3)))]*x^2, x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, C, q\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[e^2 - 4*d*f, 0] \&\& \text{GtQ}[p, 0] \&\& \text{NeQ}[p + q + 1, 0] \&\& \text{NeQ}[2*p + 2*q + 3, 0] \&\& !\text{IGtQ}[p, 0] \&\& !\text{IGtQ}[q, 0]$

Rule 1076

$\text{Int}[(A_.) + (B_.)*(x_.) + (C_.)*(x_.)^2)/(((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)*\text{Sqrt}[(d_.) + (e_.)*(x_.) + (f_.)*(x_.)^2]), x_Symbol] \rightarrow \text{Dist}[C/c, \text{Int}[1/\text{Sqrt}[d + e*x + f*x^2], x], x] + \text{Dist}[1/c, \text{Int}[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*\text{Sqrt}[d + e*x + f*x^2]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, C\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[e^2 - 4*d*f, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx)(a+bx+cx^2)^{3/2}}{d+ex+fx^2} dx &= \frac{B(a+bx+cx^2)^{3/2}}{3f} - \int \frac{\sqrt{a+bx+cx^2} \left(\frac{3}{2}(bBd-2aAf) - \frac{3}{2}(2Abf-B(2cd+be-2af))x + \frac{3}{2}(2Bce-bBf) \right)}{d+ex+fx^2} \\
&= -\frac{(2Acf(4ce-5bf) - B(b^2f^2 - 2cf(5be-4af) + 8c^2(e^2-df)) + 2cf(2Bce)}{8cf^3} \\
&= -\frac{(2Acf(4ce-5bf) - B(b^2f^2 - 2cf(5be-4af) + 8c^2(e^2-df)) + 2cf(2Bce)}{8cf^3} \\
&= -\frac{(2Acf(4ce-5bf) - B(b^2f^2 - 2cf(5be-4af) + 8c^2(e^2-df)) + 2cf(2Bce)}{8cf^3} \\
&= -\frac{(2Acf(4ce-5bf) - B(b^2f^2 - 2cf(5be-4af) + 8c^2(e^2-df)) + 2cf(2Bce)}{8cf^3} \\
&= -\frac{(2Acf(4ce-5bf) - B(b^2f^2 - 2cf(5be-4af) + 8c^2(e^2-df)) + 2cf(2Bce)}{8cf^3}
\end{aligned}$$

Mathematica [A] time = 6.56, size = 1627, normalized size = 1.49

result too large to display

Antiderivative was successfully verified.

[In] Integrate[((A + B*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x + f*x^2), x]

[Out] ((B - (B*e - 2*A*f)/Sqrt[e^2 - 4*d*f])*(a + x*(b + c*x))^(3/2))/(6*f) + ((B + (B*e - 2*A*f)/Sqrt[e^2 - 4*d*f])*(a + x*(b + c*x))^(3/2))/(6*f) - ((B + (-B*e) + 2*A*f)/Sqrt[e^2 - 4*d*f])*(a + x*(b + c*x))^(3/2)*(((4*c*f*(-4*a*f + b*(e - Sqrt[e^2 - 4*d*f])) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*(-(b*f) + 2*c*(e - Sqrt[e^2 - 4*d*f])) - 4*c*f*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^2) - (((-2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) - 4*c*f*(3*a*f - b*(e - Sqrt[e^2 - 4*d*f])))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*f) - (2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*(4*(e - Sqrt[e^2 - 4*d*f])*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) - 4*c*f*(3*a*f - b*(e - Sqrt[e^2 - 4*d*f])))) + 4*f*(2*c*f*(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]))^2 - (e - Sqrt[e^2 - 4*d*f])*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*(b^2*f + 4*a*c*f - 2*b*c*(e - Sqrt[e^2 - 4*d*f])))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) - (-2*b*f + 2*c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]))/(f*(16*a*f^2 - 8*b*f*(e - Sqrt[e^2 - 4*d*f]) + 4*c*(e - Sqrt[e^2 - 4*d*f])^2))/(16*c*f^2))/(4*f*(a + b*x + c*x^2)^(3/2) - ((B - (-B*e) + 2*A*f)/Sqrt[e^2 - 4*d*f])*(a + x*(b + c*x))^(3/2)*(((4*c*f*(-4*a*f + b*(e + Sqrt[e^2 - 4*d*f])) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*(-(b*f) + 2*c*(e + Sqrt[e^2 - 4*d*f])) - 4*c*f*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^2) - (((-2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) - 4*c*f*(3*a*f - b*(e + Sqrt[e^2 - 4*d*f])))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*f) - (2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]

$$\begin{aligned} &]*(4*(e + \text{Sqrt}[e^2 - 4*d*f])*(b*f - c*(e + \text{Sqrt}[e^2 - 4*d*f]))*(b^2*f^2 - 4 \\ & *c^2*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]) - 4*c*f*(3*a*f - b*(e + \text{Sqrt}[e^2 - \\ & 4*d*f]))) + 4*f*(2*c*f*(4*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]))^2 - (e + \text{Sqrt}[e \\ & ^2 - 4*d*f])*(b*f - c*(e + \text{Sqrt}[e^2 - 4*d*f]))*(b^2*f + 4*a*c*f - 2*b*c*(e \\ & + \text{Sqrt}[e^2 - 4*d*f]))) * \text{ArcTanh}[(4*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]) - (-2*b*f \\ & + 2*c*(e + \text{Sqrt}[e^2 - 4*d*f]))*x)/(2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f \\ & + 2*a*f^2 + c*e*\text{Sqrt}[e^2 - 4*d*f] - b*f*\text{Sqrt}[e^2 - 4*d*f])* \text{Sqrt}[a + b*x + \\ & c*x^2]])/(f*(16*a*f^2 - 8*b*f*(e + \text{Sqrt}[e^2 - 4*d*f]) + 4*c*(e + \text{Sqrt}[e^2 \\ & - 4*d*f])^2))/(16*c*f^2))/(4*f*(a + b*x + c*x^2)^(3/2)) \end{aligned}$$

IntegrateAlgebraic [C] time = 7.50, size = 2787, normalized size = 2.55

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic(((A + B*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x + f*x^2), x]
[Out] (Sqrt[a + b*x + c*x^2]*(24*B*c^2*e^2 - 24*B*c^2*d*f - 30*b*B*c*e*f - 24*A*c
^2*e*f + 3*b^2*B*f^2 + 30*A*b*c*f^2 + 32*a*B*c*f^2 - 12*B*c^2*e*f*x + 14*b*
B*c*f^2*x + 12*A*c^2*f^2*x + 8*B*c^2*f^2*x^2))/(24*c*f^3) + ((16*B*c^3*e^3
- 32*B*c^3*d*e*f - 24*b*B*c^2*e^2*f - 16*A*c^3*e^2*f + 24*b*B*c^2*d*f^2 + 1
6*A*c^3*d*f^2 + 6*b^2*B*c*e*f^2 + 24*A*b*c^2*e*f^2 + 24*a*B*c^2*e*f^2 + b^3
*B*f^3 - 6*A*b^2*c*f^3 - 12*a*b*B*c*f^3 - 24*a*A*c^2*f^3)*Log[b*c + 2*c^2*x
- 2*c^(3/2)*Sqrt[a + b*x + c*x^2]]/(16*c^(3/2)*f^4) - RootSum[b^2*d - a*b
*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 -
2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b*B*c^2*d*e^3*Log[-(Sqrt[c]*x)
+ Sqrt[a + b*x + c*x^2] - #1] - a*B*c^2*e^4*Log[-(Sqrt[c]*x) + Sqrt[a + b*x
+ c*x^2] - #1] - 2*b*B*c^2*d^2*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2
] - #1] - 2*b^2*B*c*d*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]
- A*b*c^2*d*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 3*a*B*c^
2*d*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*a*b*B*c*e^3*f*
Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a*A*c^2*e^3*f*Log[-(Sqrt[c
]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*b^2*B*c*d^2*f^2*Log[-(Sqrt[c]*x) + S
qrt[a + b*x + c*x^2] - #1] + A*b*c^2*d^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*
x + c*x^2] - #1] - a*B*c^2*d^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2]
- #1] + b^3*B*d*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*A
*b^2*c*d*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*a*b*B*c*d
*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*a*A*c^2*d*e*f^2*L
og[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*b^2*B*e^2*f^2*Log[-(Sqrt[
c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*a*A*b*c*e^2*f^2*Log[-(Sqrt[c]*x) +
Sqrt[a + b*x + c*x^2] - #1] - 2*a^2*B*c*e^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a +
b*x + c*x^2] - #1] - A*b^3*d*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2]
- #1] - a*b^2*B*d*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*a^
2*B*c*d*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a*A*b^2*e*f^3*
Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*a^2*b*B*e*f^3*Log[-(Sqrt
[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*a^2*A*c*e*f^3*Log[-(Sqrt[c]*x) + S
qrt[a + b*x + c*x^2] - #1] - a^2*A*b*f^4*Log[-(Sqrt[c]*x) + Sqrt[a + b*x +
c*x^2] - #1] - a^3*B*f^4*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2
*B*c^(5/2)*d*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 4*B*c^
(5/2)*d^2*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 4*b*B*c^(
3/2)*d*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*A*c^(5/2
)*d*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 4*b*B*c^(3/2)
*d^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*A*c^(5/2)*d^
2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*b^2*B*Sqrt[c]*d
*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 4*A*b*c^(3/2)*d*
e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 4*a*B*c^(3/2)*d*
e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*A*b^2*Sqrt[c]*d*
f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 4*a*b*B*Sqrt[c]*d*f
^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 4*a*A*c^(3/2)*d*f^3*
Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*a^2*A*Sqrt[c]*f^4*Log
```

$$\begin{aligned} & [-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1 + B*c^2*e^4*\text{Log}[-(\text{Sqrt}[c]*x) \\ & + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1^2 - 3*B*c^2*d*e^2*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + \\ & b*x + c*x^2] - \#1]*\#1^2 - 2*b*B*c*e^3*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + \\ & b*x + c*x^2] - \#1]*\#1^2 - A*c^2*e^3*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x \\ & ^2] - \#1]*\#1^2 + B*c^2*d^2*f^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \# \\ & 1]*\#1^2 + 4*b*B*c*d*e*f^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1 \\ & ^2 + 2*A*c^2*d*e*f^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1^2 + \\ & b^2*B*e^2*f^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1^2 + 2*A*b*c \\ & *e^2*f^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1^2 + 2*a*B*c*e^2* \\ & f^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1^2 - b^2*B*d*f^3*\text{Log}[- \\ & (\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1^2 - 2*A*b*c*d*f^3*\text{Log}[-(\text{Sqrt}[c] \\ &]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1^2 - 2*a*B*c*d*f^3*\text{Log}[-(\text{Sqrt}[c]*x) + \\ & \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1^2 - A*b^2*e*f^3*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + \\ & b*x + c*x^2] - \#1]*\#1^2 - 2*a*b*B*e*f^3*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c \\ & *x^2] - \#1]*\#1^2 - 2*a*A*c*e*f^3*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \\ & \#1]*\#1^2 + 2*a*A*b*f^4*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1^2 \\ & + a^2*B*f^4*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1^2)/(2*b*\text{Sqrt} \\ & [c]*d - a*\text{Sqrt}[c]*e - 4*c*d*\#1 - b*e*\#1 + 2*a*f*\#1 + 3*\text{Sqrt}[c]*e*\#1^2 - 2*f \\ & *\#1^3) \&]/f^4 \end{aligned}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:Error: Bad Argument Type

maple [B] time = 0.03, size = 59465, normalized size = 54.46

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x+A)*(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)

[Out] result too large to display

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + Bx)(cx^2 + bx + a)^{3/2}}{fx^2 + ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x + f*x^2), x)

[Out] int(((A + B*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x + f*x^2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)*(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d), x)

[Out] Timed out

$$3.21 \quad \int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+ex+fx^2}} dx$$

Optimal. Leaf size=416

$$\frac{\left(-B\sqrt{b^2-4ac}-2Ac+bB\right)\tanh^{-1}\left(\frac{2x\left(ce-f\left(b-\sqrt{b^2-4ac}\right)\right)-e\left(b-\sqrt{b^2-4ac}\right)+4cd}{2\sqrt{2}\sqrt{d+ex+fx^2}\sqrt{b^2-4ac}\left(ce-bf\right)-2acf+b^2f-bce+2c^2d}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b^2-4ac}\left(ce-bf\right)-2acf+b^2f-bce+2c^2d} + \frac{\left(2Ac-B\left(\sqrt{b^2-4ac}+b\right)\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{-}}$$

Rubi [A] time = 2.70, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 32, number of rules / integrand size = 0.094, Rules used = {1032, 724, 206}

$$\frac{\left(-B\sqrt{b^2-4ac}-2Ac+bB\right)\tanh^{-1}\left(\frac{2x\left(ce-f\left(b-\sqrt{b^2-4ac}\right)\right)-e\left(b-\sqrt{b^2-4ac}\right)+4cd}{2\sqrt{2}\sqrt{d+ex+fx^2}\sqrt{b^2-4ac}\left(ce-bf\right)-2acf+b^2f-bce+2c^2d}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b^2-4ac}\left(ce-bf\right)-2acf+b^2f-bce+2c^2d} + \frac{\left(2Ac-B\left(\sqrt{b^2-4ac}+b\right)\right)\tanh^{-1}\left(\frac{2x\left(ce-f\left(\sqrt{b^2-4ac}+b\right)\right)-e\left(\sqrt{b^2-4ac}+b\right)+4cd}{2\sqrt{2}\sqrt{d+ex+fx^2}\sqrt{-b^2-4ac}\left(ce-bf\right)-2acf+b^2f-bce+2c^2d}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{-b^2-4ac}\left(ce-bf\right)-2acf+b^2f-bce+2c^2d}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]),x]

[Out] ((b*B - 2*A*c - B*Sqrt[b^2 - 4*a*c])*ArcTanh[(4*c*d - (b - Sqrt[b^2 - 4*a*c])*e + 2*(c*e - (b - Sqrt[b^2 - 4*a*c])*f)*x]/(2*Sqrt[2]*Sqrt[2*c^2*d - b*c*e + b^2*f - 2*a*c*f + Sqrt[b^2 - 4*a*c]*(c*e - b*f)]*Sqrt[d + e*x + f*x^2]))/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2*d - b*c*e + b^2*f - 2*a*c*f + Sqrt[b^2 - 4*a*c]*(c*e - b*f)]) + ((2*A*c - B*(b + Sqrt[b^2 - 4*a*c]))*ArcTanh[(4*c*d - (b + Sqrt[b^2 - 4*a*c])*e + 2*(c*e - (b + Sqrt[b^2 - 4*a*c])*f)*x]/(2*Sqrt[2]*Sqrt[2*c^2*d - b*c*e + b^2*f - 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*e - b*f)]*Sqrt[d + e*x + f*x^2]))/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2*d - b*c*e + b^2*f - 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*e - b*f)])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1032

Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]

Rubi steps

$$\int \frac{A + Bx}{(a + bx + cx^2)\sqrt{d + ex + fx^2}} dx = \frac{(2Ac - B(b - \sqrt{b^2 - 4ac})) \int \frac{1}{(b - \sqrt{b^2 - 4ac} + 2cx)\sqrt{d + ex + fx^2}} dx}{\sqrt{b^2 - 4ac}} - \frac{(2Ac - B(b + \sqrt{b^2 - 4ac})) \int \frac{1}{(b + \sqrt{b^2 - 4ac} + 2cx)\sqrt{d + ex + fx^2}} dx}{\sqrt{b^2 - 4ac}}$$

$$= \frac{(2(bB - 2Ac - B\sqrt{b^2 - 4ac})) \operatorname{Subst}\left(\int \frac{1}{16c^2d - 8c(b - \sqrt{b^2 - 4ac})e + 4(b - \sqrt{b^2 - 4ac})e^2 + 4c^2x^2} dx\right)}{\sqrt{b^2 - 4ac}}$$

$$= \frac{(bB - 2Ac - B\sqrt{b^2 - 4ac}) \tanh^{-1}\left(\frac{4cd - (b - \sqrt{b^2 - 4ac})e + 2(ce - (b - \sqrt{b^2 - 4ac})e^2)}{2\sqrt{2}\sqrt{2c^2d - bce + b^2f - 2acf + \sqrt{b^2 - 4ac}(ce - bf)}}\right)}{\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{2c^2d - bce + b^2f - 2acf + \sqrt{b^2 - 4ac}(ce - bf)}}$$

Mathematica [A] time = 4.18, size = 393, normalized size = 0.94

$$\frac{(B\sqrt{b^2 - 4ac} + 2Ac - bB) \tanh^{-1}\left(\frac{(\sqrt{b^2 - 4ac} - b)(e + 2fx) + 2c(2d + ex)}{2\sqrt{2}\sqrt{d + x(e + fx)}\sqrt{c(e\sqrt{b^2 - 4ac} - 2af - be) + bf(b - \sqrt{b^2 - 4ac}) + 2c^2d}}\right)}{\sqrt{c(e\sqrt{b^2 - 4ac} - 2af - be) + bf(b - \sqrt{b^2 - 4ac}) + 2c^2d}} - \frac{(B\sqrt{b^2 - 4ac} - 2Ac + bB) \tanh^{-1}\left(\frac{2c(2d + ex) - (\sqrt{b^2 - 4ac} + b)(e + 2fx)}{2\sqrt{2}\sqrt{d + x(e + fx)}\sqrt{-2c(e\sqrt{b^2 - 4ac} + 2af + be) + 2bf(\sqrt{b^2 - 4ac} + b) + 4c^2d}}\right)}{\sqrt{-c(e\sqrt{b^2 - 4ac} + 2af + be) + bf(\sqrt{b^2 - 4ac} + b) + 2c^2d}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]),x]

[Out] (-(((-(b*B) + 2*A*c + B*Sqrt[b^2 - 4*a*c])*ArcTanh[(2*c*(2*d + e*x) + (-b + Sqrt[b^2 - 4*a*c])*(e + 2*f*x))/(2*Sqrt[2]*Sqrt[2*c^2*d + b*(b - Sqrt[b^2 - 4*a*c])]*f + c*(-(b*e) + Sqrt[b^2 - 4*a*c]*e - 2*a*f)]*Sqrt[d + x*(e + f*x)])]/Sqrt[2*c^2*d + b*(b - Sqrt[b^2 - 4*a*c])]*f + c*(-(b*e) + Sqrt[b^2 - 4*a*c]*e - 2*a*f])) - ((b*B - 2*A*c + B*Sqrt[b^2 - 4*a*c])*ArcTanh[(2*c*(2*d + e*x) - (b + Sqrt[b^2 - 4*a*c])*(e + 2*f*x))/(2*Sqrt[4*c^2*d + 2*b*(b + Sqrt[b^2 - 4*a*c])]*f - 2*c*(b*e + Sqrt[b^2 - 4*a*c]*e + 2*a*f)]*Sqrt[d + x*(e + f*x)])]/Sqrt[2*c^2*d + b*(b + Sqrt[b^2 - 4*a*c])]*f - c*(b*e + Sqrt[b^2 - 4*a*c]*e + 2*a*f)))/(Sqrt[2]*Sqrt[b^2 - 4*a*c])

IntegrateAlgebraic [C] time = 0.53, size = 278, normalized size = 0.67

$$-\operatorname{RootSum}\left[\#1^3c - 2\#1^2b\sqrt{f} + 4\#1^2af + \#1^2be - 2\#1^2cd - 4\#1ac\sqrt{f} + 2\#1bd\sqrt{f} + ac^2 - bde + cd^2 \& \frac{\#1^2(-B)\log(-\#1 + \sqrt{d + ex + fx^2} - \sqrt{f}x) + 2\#1A\sqrt{f}\log(-\#1 + \sqrt{d + ex + fx^2} - \sqrt{f}x) - Ac\log(-\#1 + \sqrt{d + ex + fx^2} - \sqrt{f}x) + Bd\log(-\#1 + \sqrt{d + ex + fx^2} - \sqrt{f}x)}{2\#1^3c - 3\#1^2b\sqrt{f} + 4\#1af + \#1be - 2\#1cd - 2ac\sqrt{f} + bd\sqrt{f}}\right]$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]),x]

[Out] -RootSum[c*d^2 - b*d*e + a*e^2 + 2*b*d*Sqrt[f]*#1 - 4*a*e*Sqrt[f]*#1 - 2*c*d*#1^2 + b*e*#1^2 + 4*a*f*#1^2 - 2*b*Sqrt[f]*#1^3 + c*#1^4 & , (B*d*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1] - A*e*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1] + 2*A*Sqrt[f]*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1]*#1 - B*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1]*#1^2)/(b*d*Sqrt[f] - 2*a*e*Sqrt[f] - 2*c*d*#1 + b*e*#1 + 4*a*f*#1 - 3*b*Sqrt[f]*#1^2 + 2*c*#1^3) &]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x);OUTPUT:Unable to divide, perhaps due to rounding error
 %%%{poly1[%%{-4, [3,2,0]%%}+%%{16, [1,3,1]%%}, %%{4, [4,2,0]%%}+%%{-24, [2,3,1]%%}+%%{32, [0,4,2]%%}]: [1,0,%%{-1, [2,0,0]%%}+%%{4, [0,1,1]%%}]%%}, [2,1,0,0,0]%%}+%%{%%{[%%{-4, [5,0,0]%%}+%%{24, [3,1,1]%%}+%%{-32, [1,2,2]%%}, %%{4, [6,0,0]%%}+%%{-32, [4,1,1]%%}+%%{72, [2,2,2]%%}+%%{-32, [0,3,3]%%}]: [1,0,%%{-1, [2,0,0]%%}+%%{4, [0,1,1]%%}]%%}, [2,0,0,1,0]%%}+%%{%%{[%%{4, [4,1,0]%%}+%%{-20, [2,2,1]%%}+%%{16, [0,3,2]%%}, %%{-4, [5,1,0]%%}+%%{28, [3,2,1]%%}+%%{-48, [1,3,2]%%}]: [1,0,%%{-1, [2,0,0]%%}+%%{4, [0,1,1]%%}]%%}, [2,0,0,0,1]%%}+%%{%%{poly1[%%{8, [2,3,0]%%}+%%{-32, [0,4,1]%%}, %%{-8, [3,3,0]%%}+%%{32, [1,4,1]%%}]: [1,0,%%{-1, [2,0,0]%%}+%%{4, [0,1,1]%%}]%%}, [1,1,1,0,0]%%}+%%{%%{[%%{8, [4,1,0]%%}+%%{-40, [2,2,1]%%}+%%{32, [0,3,2]%%}, %%{-8, [5,1,0]%%}+%%{56, [3,2,1]%%}+%%{-96, [1,3,2]%%}]: [1,0,%%{-1, [2,0,0]%%}+%%{4, [0,1,1]%%}]%%}, [1,0,1,1,0]%%}+%%{%%{[%%{-8, [3,2,0]%%}+%%{32, [1,3,1]%%}, %%{8, [4,2,0]%%}+%%{-48, [2,3,1]%%}+%%{64, [0,4,2]%%}]: [1,0,%%{-1, [2,0,0]%%}+%%{4, [0,1,1]%%}]%%}, [1,0,1,0,1]%%}+%%{%%{8, [2,4,0]%%}+%%{-32, [0,5,1]%%}, [0,1,2,0,0]%%}+%%{%%{poly1[%%{-4, [3,2,0]%%}+%%{16, [1,3,1]%%}, %%{4, [4,2,0]%%}+%%{-24, [2,3,1]%%}+%%{32, [0,4,2]%%}]: [1,0,%%{-1, [2,0,0]%%}+%%{4, [0,1,1]%%}]%%}, [0,0,2,1,0]%%}+%%{%%{poly1[%%{4, [2,3,0]%%}+%%{-16, [0,4,1]%%}, %%{-4, [3,3,0]%%}+%%{16, [1,4,1]%%}]: [1,0,%%{-1, [2,0,0]%%}+%%{4, [0,1,1]%%}]%%}, [0,0,2,0,1]%%} / %%{%%{[%%{4, [3,2,0]%%}+%%{-16, [1,3,1]%%}, %%{-4, [4,2,0]%%}+%%{24, [2,3,1]%%}+%%{-32, [0,4,2]%%}]: [1,0,%%{-1, [2,0,0]%%}+%%{4, [0,1,1]%%}]%%}, [0,1,0,0,0]%%}+%%{%%{[%%{4, [5,0,0]%%}+%%{-24, [3,1,1]%%}+%%{32, [1,2,2]%%}, %%{-4, [6,0,0]%%}+%%{32, [4,1,1]%%}+%%{-72, [2,2,2]%%}+%%{32, [0,3,3]%%}]: [1,0,%%{-1, [2,0,0]%%}+%%{4, [0,1,1]%%}]%%}, [0,0,0,1,0]%%}+%%{%%{[%%{-4, [4,1,0]%%}+%%{20, [2,2,1]%%}+%%{-16, [0,3,2]%%}, %%{4, [5,1,0]%%}+%%{-28, [3,2,1]%%}+%%{48, [1,3,2]%%}]: [1,0,%%{-1, [2,0,0]%%}+%%{4, [0,1,1]%%}]%%}, [0,0,0,0,1]%%} Error: Bad Argument ValueEvaluation time: 0.92Unable to divide, perhaps due to rounding error
 %%%{%%{[%%{1, [3,0,0]%%}+%%{-4, [1,1,1]%%}, %%{1, [4,0,0]%%}+%%{-6, [2,1,1]%%}+%%{8, [0,2,2]%%}]: [1,0,%%{-1, [2,0,0]%%}+%%{4, [0,1,1]%%}]%%} / %%{4, [0,0,2]%%}, [2,1,0,0,0]%%}+%%{%%{1, [2,0,0]%%}+%%{-4, [0,1,1]%%} / 2, [2,0,0,1,0]%%}+%%{%%{[%%{-1, [2,0,0]%%}+%%{4, [0,1,1]%%}, %%{-1, [3,0,0]%%}+%%{4, [1,1,1]%%}]: [1,0,%%{-1, [2,0,0]%%}+%%{4, [0,1,1]%%}]%%} / %%{4, [0,0,1]%%}, [2,0,0,0,1]%%}+%%{%%{[%%{-1, [4,0,0]%%}+%%{5, [2,1,1]%%}+%%{-4, [0,2,2]%%}, %%{-1, [5,0,0]%%}+%%{7, [3,1,1]%%}+%%{-12, [1,2,2]%%}]: [1,0,%%{-1, [2,0,0]%%}+%%{4, [0,1,1]%%}]%%} / %%{2, [0,0,3]%%}, [1,1,1,0,0]%%}+%%{%%{[%%{-1, [2,0,0]%%}+%%{4, [0,1,1]%%}, %%{-1, [3,0,0]%%}+%%{4, [1,1,1]%%}]: [1,0,%%{-1, [2,0,0]%%}+%%{4, [0,1,1]%%}]%%} / %%{2, [0,0,1]%%}, [1,0,1,1,0]%%}+%%{%%{[%%{1, [3,0,0]%%}+%%{-4, [1,1,1]%%}, %%{1, [4,0,0]%%}+%%{-6, [2,1,1]%%}+%%{8, [0,2,2]%%}]: [1,0,%%{-1, [2,0,0]%%}+%%{4, [0,1,1]%%}]%%} / %%{2, [0,0,2]%%}, [1,0,1,0,1]%%}+%%{%%{[%%{1, [5,0,0]%%}+%%{-6, [3,1,1]%%}+%%{8, [1,2,2]%%}, %%{1, [6,0,0]%%}+%%{-8, [4,1,1]%%}+%%{18, [2,2,2]%%}+%%{-8, [0,3,3]%%}]: [1,0,%%{-1, [2,0,0]%%}+%%{4, [0,1,1]%%}]%%} / %%{4, [0,0,4]%%}, [0,1,2,0,0]%%}+%%{%%{[%%{1, [3,0,0]%%}+%%{-4, [1,1,1]%%}, %%{1, [4,0,0]%%}+%%{-6, [2,1,1]%%}+%%{8, [0,2,2]%%}]: [1,0,%%{-1, [2,0,0]%%}+%%{4, [0,1,1]%%}]%%} / %%{4, [0,0,2]%%}, [0,0,2,1,0]%%}+%%{%%{[%%{-1, [4,0,0]%%}+%%{5, [2,1,1]%%}+%%{-4, [0,2,2]%%}, %%{-1, [5,0,0]%%}+%%{7, [3,1,1]%%}+%%{-12, [1,2,2]%%}]: [1,0,%%{-1, [2,0,0]%%}+%%{4, [0,1,1]%%}]%%} / %%{4, [0,0,3]%%}, [0,0,2,0,1]%%} / %%{%%{[%%{-1, [3,

```

0,0]%%}+%%{4,[1,1,1]%%},%%{-1,[4,0,0]%%}+%%{6,[2,1,1]%%}+%%{-8,[0,2,2]%%}] : [1,0,%%{-1,[2,0,0]%%}+%%{4,[0,1,1]%%}]%%}/%%{4,[0,0,4]%%}, [0,1,0,0,0]%%}+%%{%%{-1,[2,0,0]%%}+%%{4,[0,1,1]%%}}/%%{2,[0,0,2]%%}, [0,0,0,1,0]%%}+%%{%%{%%{1,[2,0,0]%%}+%%{-4,[0,1,1]%%},%%{1,[3,0,0]%%}}+%%{-4,[1,1,1]%%}} : [1,0,%%{-1,[2,0,0]%%}+%%{4,[0,1,1]%%}]%%}/%%{4,[0,0,3]%%}, [0,0,0,0,1]%%} Error: Bad Argument Value
  
```

maple [B] time = 0.04, size = 2269, normalized size = 5.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d)^(1/2),x)

[Out]
$$\frac{2}{(-4ac+b^2)^{1/2}} \frac{\ln\left(\frac{-(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2}{(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2} \frac{f(-4ac+b^2)^{1/2}+bf-ce}{f(-4ac+b^2)^{1/2}+bf-ce}\right)}{(x+1/2(b+(-4ac+b^2)^{1/2})/c)+1/2(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2} \left(\frac{4(x+1/2(b+(-4ac+b^2)^{1/2})/c)^2f-4(f(-4ac+b^2)^{1/2}+bf-ce)/c(x+1/2(b+(-4ac+b^2)^{1/2})/c)}{(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2}\right)^{1/2} \frac{A-1/c}{(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2} \frac{\ln\left(\frac{-(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2}{(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2} \frac{f(-4ac+b^2)^{1/2}+bf-ce}{f(-4ac+b^2)^{1/2}+bf-ce}\right)}{(x+1/2(b+(-4ac+b^2)^{1/2})/c)+1/2(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2} \left(\frac{4(x+1/2(b+(-4ac+b^2)^{1/2})/c)^2f-4(f(-4ac+b^2)^{1/2}+bf-ce)/c(x+1/2(b+(-4ac+b^2)^{1/2})/c)}{(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2}\right)^{1/2} \frac{B-1/(-4ac+b^2)^{1/2}/c}{(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2} \frac{\ln\left(\frac{-(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2}{(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2} \frac{f(-4ac+b^2)^{1/2}+bf-ce}{f(-4ac+b^2)^{1/2}+bf-ce}\right)}{(x-1/2(-b+(-4ac+b^2)^{1/2})/c)+1/2(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2} \left(\frac{4(x-1/2(-b+(-4ac+b^2)^{1/2})/c)^2f-4(-f(-4ac+b^2)^{1/2}+bf-ce)/c(x-1/2(-b+(-4ac+b^2)^{1/2})/c)}{(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2}\right)^{1/2} \frac{A-1/c}{(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2} \frac{\ln\left(\frac{-(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2}{(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2} \frac{f(-4ac+b^2)^{1/2}+bf-ce}{f(-4ac+b^2)^{1/2}+bf-ce}\right)}{(x-1/2(-b+(-4ac+b^2)^{1/2})/c)+1/2(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2} \left(\frac{4(x-1/2(-b+(-4ac+b^2)^{1/2})/c)^2f-4(-f(-4ac+b^2)^{1/2}+bf-ce)/c(x-1/2(-b+(-4ac+b^2)^{1/2})/c)}{(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2}\right)^{1/2} \frac{B+1/(-4ac+b^2)^{1/2}/c}{(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2} \frac{\ln\left(\frac{-(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2}{(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2} \frac{f(-4ac+b^2)^{1/2}+bf-ce}{f(-4ac+b^2)^{1/2}+bf-ce}\right)}{(x-1/2(-b+(-4ac+b^2)^{1/2})/c)+1/2(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2} \left(\frac{4(x-1/2(-b+(-4ac+b^2)^{1/2})/c)^2f-4(-f(-4ac+b^2)^{1/2}+bf-ce)/c(x-1/2(-b+(-4ac+b^2)^{1/2})/c)}{(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2}\right)^{1/2} \frac{A-1/c}{(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2} \frac{\ln\left(\frac{-(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2}{(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2} \frac{f(-4ac+b^2)^{1/2}+bf-ce}{f(-4ac+b^2)^{1/2}+bf-ce}\right)}{(x-1/2(-b+(-4ac+b^2)^{1/2})/c)+1/2(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2} \left(\frac{4(x-1/2(-b+(-4ac+b^2)^{1/2})/c)^2f-4(-f(-4ac+b^2)^{1/2}+bf-ce)/c(x-1/2(-b+(-4ac+b^2)^{1/2})/c)}{(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2}\right)^{1/2} \frac{B-2/(-4ac+b^2)^{1/2}/c}{(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2} \frac{\ln\left(\frac{-(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2}{(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2} \frac{f(-4ac+b^2)^{1/2}+bf-ce}{f(-4ac+b^2)^{1/2}+bf-ce}\right)}{(x-1/2(-b+(-4ac+b^2)^{1/2})/c)+1/2(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2} \left(\frac{4(x-1/2(-b+(-4ac+b^2)^{1/2})/c)^2f-4(-f(-4ac+b^2)^{1/2}+bf-ce)/c(x-1/2(-b+(-4ac+b^2)^{1/2})/c)}{(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2}\right)^{1/2} \frac{A-1/c}{(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2} \frac{\ln\left(\frac{-(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2}{(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2} \frac{f(-4ac+b^2)^{1/2}+bf-ce}{f(-4ac+b^2)^{1/2}+bf-ce}\right)}{(x-1/2(-b+(-4ac+b^2)^{1/2})/c)+1/2(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2} \left(\frac{4(x-1/2(-b+(-4ac+b^2)^{1/2})/c)^2f-4(-f(-4ac+b^2)^{1/2}+bf-ce)/c(x-1/2(-b+(-4ac+b^2)^{1/2})/c)}{(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2}\right)^{1/2} \frac{B-2/(-4ac+b^2)^{1/2}/c}{(-(-4ac+b^2)^{1/2}bf+(-4ac+b^2)^{1/2}ce+2acfb-b^2f+bce-2c^2d)/c^2}$$

$*f - (-4*a*c + b^2)^{(1/2)} * c * e + 2*a*c*f - b^2*f + b*c*e - 2*c^2*d) / c^2)^{(1/2)} / (x - 1/2 * (-b + (-4*a*c + b^2)^{(1/2)}) / c) * b * B$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see 'assume?' for more details) Is 4*a*c-b^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + Bx}{(cx^2 + bx + a) \sqrt{fx^2 + ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)^(1/2)),x)

[Out] int((A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx}{(a + bx + cx^2) \sqrt{d + ex + fx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+e*x+d)**(1/2),x)

[Out] Integral((A + B*x)/((a + b*x + c*x**2)*sqrt(d + e*x + f*x**2)), x)

$$3.22 \quad \int \frac{A+Bx}{(a+cx^2)\sqrt{d+ex+fx^2}} dx$$

Optimal. Leaf size=780

$$\sqrt{A\left(-\sqrt{a^2f^2+ac(e^2-2df)+c^2d^2}-af+cd\right)+aBe}\sqrt{B\left(\sqrt{a^2f^2+ac(e^2-2df)+c^2d^2}-af+cd\right)-Ace}$$

$$\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{e}\sqrt{a^2f^2+ac(e^2-2df)+c^2d^2}$$

Rubi [A] time = 5.16, antiderivative size = 780, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 29, number of rules / integrand size = 0.103, Rules used = {1036, 1030, 208}

$$\frac{\sqrt{A\left(-\sqrt{a^2f^2+ac(e^2-2df)+c^2d^2}-af+cd\right)+aBe}\sqrt{B\left(\sqrt{a^2f^2+ac(e^2-2df)+c^2d^2}-af+cd\right)-Ace}}{\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{e}\sqrt{a^2f^2+ac(e^2-2df)+c^2d^2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x]

[Out] (Sqrt[a*B*e + A*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))]]*Sqrt[-(A*c*e) + B*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))]]*ArcTanh[(Sqrt[e]*(a*(A*c*e - B*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))]) - c*(a*B*e + A*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))]))*x)/(Sqrt[2]*Sqrt[a]*Sqrt[c]*Sqrt[a*B*e + A*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))]]*Sqrt[-(A*c*e) + B*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))]]*Sqrt[d + e*x + f*x^2])]/(Sqrt[2]*Sqrt[a]*Sqrt[c]*Sqrt[e]*Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)]) - (Sqrt[-(A*c*e) + B*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))]]*Sqrt[a*B*e + A*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))]]*ArcTanh[(Sqrt[e]*(a*(A*c*e - B*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))]) - c*(a*B*e + A*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))]))*x)/(Sqrt[2]*Sqrt[a]*Sqrt[c]*Sqrt[-(A*c*e) + B*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))]]*Sqrt[a*B*e + A*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))]]*Sqrt[d + e*x + f*x^2])]/(Sqrt[2]*Sqrt[a]*Sqrt[c]*Sqrt[e]*Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1030

Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2], x] /; FreeQ[{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]

Rule 1036

Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist[1/(2*q), Int[Simp[-(a*h*e) - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[-(a*h*e) - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && NegQ[-(a*c)]

Rubi steps

$$\int \frac{A + Bx}{(a + cx^2)\sqrt{d + ex + fx^2}} dx = \frac{\int \frac{-aBe - A\left(cd - af + \sqrt{c^2d^2 + a^2f^2 + ac(e^2 - 2df)}\right) + \left(-Ace + B\left(cd - af - \sqrt{c^2d^2 + a^2f^2 + ac(e^2 - 2df)}\right)\right)x}{(a + cx^2)\sqrt{d + ex + fx^2}} dx}{2\sqrt{c^2d^2 + a^2f^2 + ac(e^2 - 2df)}} = \frac{\left(a\left(Ace - B\left(cd - af - \sqrt{c^2d^2 + a^2f^2 + ac(e^2 - 2df)}\right)\right)\right)\left(aBe + A\left(cd - af + \sqrt{c^2d^2 + a^2f^2 + ac(e^2 - 2df)}\right)\right)}{2\sqrt{c^2d^2 + a^2f^2 + ac(e^2 - 2df)}\sqrt{-Ace + B\left(cd - af + \sqrt{c^2d^2 + a^2f^2 + ac(e^2 - 2df)}\right)}} = \frac{\sqrt{aBe + A\left(cd - af - \sqrt{c^2d^2 + a^2f^2 + ac(e^2 - 2df)}\right)}\sqrt{-Ace + B\left(cd - af + \sqrt{c^2d^2 + a^2f^2 + ac(e^2 - 2df)}\right)}}{2\sqrt{c^2d^2 + a^2f^2 + ac(e^2 - 2df)}}$$

Mathematica [A] time = 0.41, size = 254, normalized size = 0.33

$$\frac{(A\sqrt{c} - \sqrt{-a}B) \tanh^{-1}\left(\frac{\sqrt{c}(2d+ex) - \sqrt{-a}(e+2fx)}{2\sqrt{d+ex+fx^2}\sqrt{-\sqrt{-a}\sqrt{c}e-af+cd}}\right)}{\sqrt{-\sqrt{-a}\sqrt{c}e-af+cd}} - \frac{(\sqrt{-a}B + A\sqrt{c}) \tanh^{-1}\left(\frac{\sqrt{-a}(e+2fx) + \sqrt{c}(2d+ex)}{2\sqrt{d+ex+fx^2}\sqrt{-\sqrt{-a}\sqrt{c}e-af+cd}}\right)}{\sqrt{\sqrt{-a}\sqrt{c}e-af+cd}}}{2\sqrt{-a}\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]),x]

[Out] ((((-Sqrt[-a]*B) + A*Sqrt[c])*ArcTanh[(Sqrt[c]*(2*d + e*x) - Sqrt[-a]*(e + 2*f*x))/(2*Sqrt[c*d - Sqrt[-a]*Sqrt[c]*e - a*f]*Sqrt[d + x*(e + f*x)])])/Sqrt[c*d - Sqrt[-a]*Sqrt[c]*e - a*f] - ((Sqrt[-a]*B + A*Sqrt[c])*ArcTanh[(Sqrt[c]*(2*d + e*x) + Sqrt[-a]*(e + 2*f*x))/(2*Sqrt[c*d + Sqrt[-a]*Sqrt[c]*e - a*f]*Sqrt[d + x*(e + f*x)])])/Sqrt[c*d + Sqrt[-a]*Sqrt[c]*e - a*f))/(2*Sqrt[-a]*Sqrt[c])

IntegrateAlgebraic [C] time = 0.44, size = 218, normalized size = 0.28

$$\frac{1}{2}\text{RootSum}\left[\#1^4c + 4\#1^2af - 2\#1^2cd - 4\#1ae\sqrt{f} + ac^2 + cd^2\&, \frac{\#1^2(-B)\log(-\#1 + \sqrt{d + ex + fx^2} - \sqrt{fx}) + 2\#1A\sqrt{f}\log(-\#1 + \sqrt{d + ex + fx^2} - \sqrt{fx}) - Ae\log(-\#1 + \sqrt{d + ex + fx^2} - \sqrt{fx}) + Bd\log(-\#1 + \sqrt{d + ex + fx^2} - \sqrt{fx})}{\#1^3(-c) - 2\#1af + \#1cd + ae\sqrt{f}}\right]$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]),x]

[Out] RootSum[c*d^2 + a*e^2 - 4*a*e*Sqrt[f]*#1 - 2*c*d*#1^2 + 4*a*f*#1^2 + c*#1^4 & , (B*d*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1] - A*e*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1] + 2*A*Sqrt[f]*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1]*#1 - B*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1]*#1^2)/(a*e*Sqrt[f] + c*d*#1 - 2*a*f*#1 - c*#1^3) &]/2

fricas [B] time = 47.87, size = 6861, normalized size = 8.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")

[Out] -1/4*sqrt(-(2*A*B*a*c*e - (B^2*a*c - A^2*c^2)*d + (B^2*a^2 - A^2*a*c)*f + (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2)*sqrt(-(4*A^2*B^2*c^2*d

$$\begin{aligned}
&^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)*f)/ \\
&(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c*f^4 \\
&+ 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2)*f)) \\
&)/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2))*\log(-(2*(A*B^3*a*c \\
&+ A^3*B*c^2)*d*e + (B^4*a^2 - A^4*c^2)*e^2 - 2*(A*B^3*a^2 + A^3*B*a*c)*e*f \\
&- 2*(2*(A*B^3*a^2 + A^3*B*a*c)*f^2 - (2*(A*B^3*a*c + A^3*B*c^2)*d + (B^4*a^2 \\
&- A^4*c^2)*e)*f)*x + 2*(2*A^2*B*c^3*d^2 + 2*A^2*B*a^2*c*f^2 + (3*A*B^2*a \\
&*c^2 - A^3*c^3)*d*e + (B^3*a^2*c - A^2*B*a*c^2)*e^2 - (4*A^2*B*a*c^2*d + (3 \\
&*A*B^2*a^2*c - A^3*a*c^2)*e)*f - (B*a*c^4*d^3 - A*a*c^4*d^2*e + B*a^2*c^3*d \\
&*e^2 - A*a^2*c^3*e^3 - B*a^4*c*f^3 + (3*B*a^3*c^2*d - A*a^3*c^2*e)*f^2 - (3 \\
&*B*a^2*c^3*d^2 - 2*A*a^2*c^3*d*e + B*a^3*c^2*e^2)*f)*\sqrt{-(4*A^2*B^2*c^2*d^2 \\
&^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B^2 \\
&^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)*f)/ \\
&(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c*f^4 \\
&+ 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2)*f)) \\
&)*\sqrt{f*x^2 + e*x + d)*\sqrt{-(2*A*B*a*c*e - (B^2*a*c - A^2*c^2)*d + (B^2*a^2 \\
&- A^2*a*c)*f + (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2)*\sqrt{ \\
&t(-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + \\
&(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 \\
&- A^3*B*a*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2 \\
&*d*f^3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3 \\
&*d*e^2)*f)))/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2)) \\
&- (2*(B^2*a*c^3 + A^2*c^4)*d^3 + 2*(B^2*a^2*c^2 + A^2*a*c^3)*d*e^2 - 4*(B^2 \\
&a^2*c^2 + A^2*a*c^3)*d^2*f + 2*(B^2*a^3*c + A^2*a^2*c^2)*d*f^2 + ((B^2*a*c^3 \\
&+ A^2*c^4)*d^2*e + (B^2*a^2*c^2 + A^2*a*c^3)*e^3 - 2*(B^2*a^2*c^2 + A^2 \\
&a*c^3)*d*e*f + (B^2*a^3*c + A^2*a^2*c^2)*e*f^2)*x)*\sqrt{-(4*A^2*B^2*c^2*d^2 \\
&^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B^2 \\
&^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)*f)/ \\
&(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c*f^4 + \\
&2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2)*f)) \\
&/x) + 1/4*\sqrt{-(2*A*B*a*c*e - (B^2*a*c - A^2*c^2)*d + (B^2*a^2 - A^2*a*c)* \\
&f + (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2)*\sqrt{-(4*A^2*B^2*c^2 \\
&d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B^2 \\
&a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e) \\
&)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c \\
&*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2) \\
&)*f)))/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2))*\log(-(2*(A*B^3 \\
&a*c + A^3*B*c^2)*d*e + (B^4*a^2 - A^4*c^2)*e^2 - 2*(A*B^3*a^2 + A^3*B*a*c) \\
&)*e*f - 2*(2*(A*B^3*a^2 + A^3*B*a*c)*f^2 - (2*(A*B^3*a*c + A^3*B*c^2)*d + (\\
&B^4*a^2 - A^4*c^2)*e)*f)*x - 2*(2*A^2*B*c^3*d^2 + 2*A^2*B*a^2*c*f^2 + (3*A \\
&B^2*a*c^2 - A^3*c^3)*d*e + (B^3*a^2*c - A^2*B*a*c^2)*e^2 - (4*A^2*B*a*c^2*d \\
&+ (3*A*B^2*a^2*c - A^3*a*c^2)*e)*f - (B*a*c^4*d^3 - A*a*c^4*d^2*e + B*a^2*c^3 \\
&d*e^2 - A*a^2*c^3*e^3 - B*a^4*c*f^3 + (3*B*a^3*c^2*d - A*a^3*c^2*e)*f^2 \\
&- (3*B*a^2*c^3*d^2 - 2*A*a^2*c^3*d*e + B*a^3*c^2*e^2)*f)*\sqrt{-(4*A^2*B^2*c^2 \\
&d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B^2 \\
&a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e) \\
&)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c \\
&*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2) \\
&)*f)))*\sqrt{f*x^2 + e*x + d)*\sqrt{-(2*A*B*a*c*e - (B^2*a*c - A^2*c^2)*d + (\\
&B^2*a^2 - A^2*a*c)*f + (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2) \\
&)*\sqrt{-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)* \\
&d*e + (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3 \\
&a^2 - A^3*B*a*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4 \\
&c^2*d*f^3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 \\
&+ a^3*c^3*d*e^2)*f)))/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2) \\
&- (2*(B^2*a*c^3 + A^2*c^4)*d^3 + 2*(B^2*a^2*c^2 + A^2*a*c^3)*d*e^2 - \\
&4*(B^2*a^2*c^2 + A^2*a*c^3)*d^2*f + 2*(B^2*a^3*c + A^2*a^2*c^2)*d*f^2 + ((B^2 \\
&>a*c^3 + A^2*c^4)*d^2*e + (B^2*a^2*c^2 + A^2*a*c^3)*e^3 - 2*(B^2*a^2*c^2
\end{aligned}$$

$$\begin{aligned}
& + A^2*a*c^3)*d*e*f + (B^2*a^3*c + A^2*a^2*c^2)*e*f^2)*x)*\sqrt{-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2)*f)))/x} - 1/4*\sqrt{-(2*A*B*a*c*e - (B^2*a*c - A^2*c^2)*d + (B^2*a^2 - A^2*a*c)*f - (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2)*\sqrt{-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2)*f)))/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2))*\log(-(2*(A*B^3*a*c + A^3*B*c^2)*d*e + (B^4*a^2 - A^4*c^2)*e^2 - 2*(A*B^3*a^2 + A^3*B*a*c)*e*f - 2*(2*(A*B^3*a^2 + A^3*B*a*c)*f^2 - (2*(A*B^3*a*c + A^3*B*c^2)*d + (B^4*a^2 - A^4*c^2)*e)*f)*x + 2*(2*A^2*B*c^3*d^2 + 2*A^2*B*a^2*c*f^2 + (3*A*B^2*a*c^2 - A^3*c^3)*d*e + (B^3*a^2*c - A^2*B*a*c^2)*e^2 - (4*A^2*B*a*c^2*d + (3*A*B^2*a^2*c - A^3*a*c^2)*e)*f + (B*a*c^4*d^3 - A*a*c^4*d^2*e + B*a^2*c^3*d*e^2 - A*a^2*c^3*e^3 - B*a^4*c*f^3 + (3*B*a^3*c^2*d - A*a^3*c^2*e)*f^2 - (3*B*a^2*c^3*d^2 - 2*A*a^2*c^3*d*e + B*a^3*c^2*e^2)*f)*\sqrt{-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2)*f)))*\sqrt{f*x^2 + e*x + d)*\sqrt{-(2*A*B*a*c*e - (B^2*a*c - A^2*c^2)*d + (B^2*a^2 - A^2*a*c)*f - (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2)*\sqrt{-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2)*f)))/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2)) + (2*(B^2*a*c^3 + A^2*c^4)*d^3 + 2*(B^2*a^2*c^2 + A^2*a*c^3)*d*e^2 - 4*(B^2*a^2*c^2 + A^2*a*c^3)*d^2*f + 2*(B^2*a^3*c + A^2*a^2*c^2)*d*f^2 + ((B^2*a*c^3 + A^2*c^4)*d^2*e + (B^2*a^2*c^2 + A^2*a*c^3)*e^3 - 2*(B^2*a^2*c^2 + A^2*a*c^3)*d*e*f + (B^2*a^3*c + A^2*a^2*c^2)*e*f^2)*x)*\sqrt{-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2)*f)))/x} + 1/4*\sqrt{-(2*A*B*a*c*e - (B^2*a*c - A^2*c^2)*d + (B^2*a^2 - A^2*a*c)*f - (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2)*\sqrt{-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2)*f)))/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2))*\log(-(2*(A*B^3*a*c + A^3*B*c^2)*d*e + (B^4*a^2 - A^4*c^2)*e^2 - 2*(A*B^3*a^2 + A^3*B*a*c)*e*f - 2*(2*(A*B^3*a^2 + A^3*B*a*c)*f^2 - (2*(A*B^3*a*c + A^3*B*c^2)*d + (B^4*a^2 - A^4*c^2)*e)*f)*x - 2*(2*A^2*B*c^3*d^2 + 2*A^2*B*a^2*c*f^2 + (3*A*B^2*a*c^2 - A^3*c^3)*d*e + (B^3*a^2*c - A^2*B*a*c^2)*e^2 - (4*A^2*B*a*c^2*d + (3*A*B^2*a^2*c - A^3*a*c^2)*e)*f + (B*a*c^4*d^3 - A*a*c^4*d^2*e + B*a^2*c^3*d*e^2 - A*a^2*c^3*e^3 - B*a^4*c*f^3 + (3*B*a^3*c^2*d - A*a^3*c^2*e)*f^2 - (3*B*a^2*c^3*d^2 - 2*A*a^2*c^3*d*e + B*a^3*c^2*e^2)*f)*\sqrt{-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2)*f)))*\sqrt{f*x^2 + e*x + d)*\sqrt{-(2*A*B*a*c*e - (B^2*a*c - A^2*c^2)*d + (B^2*a^2 - A^2*a*c)*f - (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2)*\sqrt{-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2)*f)))/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2))
\end{aligned}$$

$$\begin{aligned} &^3B^2c^2)*d*e + (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2)*f)))/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2)) + (2*(B^2*a*c^3 + A^2*c^4)*d^3 + 2*(B^2*a^2*c^2 + A^2*a*c^3)*d*e^2 - 4*(B^2*a^2*c^2 + A^2*a*c^3)*d^2*f + 2*(B^2*a^3*c + A^2*a^2*c^2)*d*f^2 + ((B^2*a*c^3 + A^2*c^4)*d^2*e + (B^2*a^2*c^2 + A^2*a*c^3)*e^3 - 2*(B^2*a^2*c^2 + A^2*a*c^3)*d*e*f + (B^2*a^3*c + A^2*a^2*c^2)*e*f^2)*x)*sqrt(-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2)*f)))/x) \end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument ValueConj Error: Bad Argument Type

maple [A] time = 0.04, size = 784, normalized size = 1.01

$$A_1 \frac{\sqrt{\frac{Bx+A}{cx^2+a}} \sqrt{\frac{fx^2+ex+d}{c}}}{2\sqrt{ac} \sqrt{\frac{Bx+A}{cx^2+a}}} + A_2 \frac{\sqrt{\frac{Bx+A}{cx^2+a}} \sqrt{\frac{fx^2+ex+d}{c}}}{2\sqrt{ac} \sqrt{\frac{Bx+A}{cx^2+a}}} + A_3 \frac{\sqrt{\frac{Bx+A}{cx^2+a}} \sqrt{\frac{fx^2+ex+d}{c}}}{2\sqrt{ac} \sqrt{\frac{Bx+A}{cx^2+a}}} + A_4 \frac{\sqrt{\frac{Bx+A}{cx^2+a}} \sqrt{\frac{fx^2+ex+d}{c}}}{2\sqrt{ac} \sqrt{\frac{Bx+A}{cx^2+a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x+A)/(c*x^2+a)/(f*x^2+e*x+d)^(1/2),x)

[Out]
$$\begin{aligned} &-1/2/(-a*c)^{(1/2)}/(-(-e*(-a*c)^{(1/2)}+a*f-c*d)/c)^{(1/2)}*\ln((-2*(-e*(-a*c)^{(1/2)}+a*f-c*d)/c+(2*f*(-a*c)^{(1/2)}+c*e)/c*(x-(-a*c)^{(1/2)}/c)+2*(-(-e*(-a*c)^{(1/2)}+a*f-c*d)/c)^{(1/2)}*((x-(-a*c)^{(1/2)}/c)^2*f+(2*f*(-a*c)^{(1/2)}+c*e)/c*(x-(-a*c)^{(1/2)}/c)-(-e*(-a*c)^{(1/2)}+a*f-c*d)/c)^{(1/2)})/(x-(-a*c)^{(1/2)}/c))*A-1/2/c/(-(-e*(-a*c)^{(1/2)}+a*f-c*d)/c)^{(1/2)}*\ln((-2*(-e*(-a*c)^{(1/2)}+a*f-c*d)/c+(2*f*(-a*c)^{(1/2)}+c*e)/c*(x-(-a*c)^{(1/2)}/c)+2*(-(-e*(-a*c)^{(1/2)}+a*f-c*d)/c)^{(1/2)}*((x-(-a*c)^{(1/2)}/c)^2*f+(2*f*(-a*c)^{(1/2)}+c*e)/c*(x-(-a*c)^{(1/2)}/c)-(-e*(-a*c)^{(1/2)}+a*f-c*d)/c)^{(1/2)})/(x-(-a*c)^{(1/2)}/c))*B+1/2/(-a*c)^{(1/2)}/(-e*(-a*c)^{(1/2)}+a*f-c*d)/c)^{(1/2)}*\ln((-2*(e*(-a*c)^{(1/2)}+a*f-c*d)/c+1/c*(-2*f*(-a*c)^{(1/2)}+c*e)*(x+(-a*c)^{(1/2)}/c)+2*(-(e*(-a*c)^{(1/2)}+a*f-c*d)/c)^{(1/2)}*((x+(-a*c)^{(1/2)}/c)^2*f+1/c*(-2*f*(-a*c)^{(1/2)}+c*e)*(x+(-a*c)^{(1/2)}/c)-e*(-a*c)^{(1/2)}+a*f-c*d)/c)^{(1/2)})/(x+(-a*c)^{(1/2)}/c))*A-1/2/c/(-e*(-a*c)^{(1/2)}+a*f-c*d)/c)^{(1/2)}*\ln((-2*(e*(-a*c)^{(1/2)}+a*f-c*d)/c+1/c*(-2*f*(-a*c)^{(1/2)}+c*e)*(x+(-a*c)^{(1/2)}/c)+2*(-(e*(-a*c)^{(1/2)}+a*f-c*d)/c)^{(1/2)}*((x+(-a*c)^{(1/2)}/c)^2*f+1/c*(-2*f*(-a*c)^{(1/2)}+c*e)*(x+(-a*c)^{(1/2)}/c)-e*(-a*c)^{(1/2)}+a*f-c*d)/c)^{(1/2)})/(x+(-a*c)^{(1/2)}/c))*B \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx + A}{(cx^2 + a)\sqrt{fx^2 + ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")

[Out] integrate((B*x + A)/((c*x^2 + a)*sqrt(f*x^2 + e*x + d)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + Bx}{(cx^2 + a)\sqrt{fx^2 + ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x)/((a + c*x^2)*(d + e*x + f*x^2)^(1/2)), x)

[Out] int((A + B*x)/((a + c*x^2)*(d + e*x + f*x^2)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx}{(a + cx^2)\sqrt{d + ex + fx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x**2+a)/(f*x**2+e*x+d)**(1/2), x)

[Out] Integral((A + B*x)/((a + c*x**2)*sqrt(d + e*x + f*x**2)), x)

$$3.23 \quad \int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+fx^2}} dx$$

Optimal. Leaf size=302

$$\frac{\left(-B\sqrt{b^2-4ac}-2Ac+bB\right)\tanh^{-1}\left(\frac{2cd-fx(b-\sqrt{b^2-4ac})}{\sqrt{2}\sqrt{d+fx^2}\sqrt{bf(b-\sqrt{b^2-4ac})-2acf+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{bf(b-\sqrt{b^2-4ac})-2acf+2c^2d}} + \frac{\left(2Ac-B(\sqrt{b^2-4ac}+b)\right)\tanh^{-1}\left(\frac{2cd-fx(\sqrt{b^2-4ac}+b)}{\sqrt{2}\sqrt{d+fx^2}\sqrt{bf(\sqrt{b^2-4ac}+b)-2acf+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{bf(\sqrt{b^2-4ac}+b)-2acf+2c^2d}}$$

Rubi [A] time = 0.84, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1034, 725, 206}

$$\frac{\left(-B\sqrt{b^2-4ac}-2Ac+bB\right)\tanh^{-1}\left(\frac{2cd-fx(b-\sqrt{b^2-4ac})}{\sqrt{2}\sqrt{d+fx^2}\sqrt{bf(b-\sqrt{b^2-4ac})-2acf+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{bf(b-\sqrt{b^2-4ac})-2acf+2c^2d}} + \frac{\left(2Ac-B(\sqrt{b^2-4ac}+b)\right)\tanh^{-1}\left(\frac{2cd-fx(\sqrt{b^2-4ac}+b)}{\sqrt{2}\sqrt{d+fx^2}\sqrt{bf(\sqrt{b^2-4ac}+b)-2acf+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{bf(\sqrt{b^2-4ac}+b)-2acf+2c^2d}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]),x]

[Out] ((b*B - 2*A*c - B*Sqrt[b^2 - 4*a*c])*ArcTanh[(2*c*d - (b - Sqrt[b^2 - 4*a*c])*f*x)/(Sqrt[2]*Sqrt[2*c^2*d - 2*a*c*f + b*(b - Sqrt[b^2 - 4*a*c])*f])*Sqrt[d + f*x^2]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2*d - 2*a*c*f + b*(b - Sqrt[b^2 - 4*a*c])*f]) + ((2*A*c - B*(b + Sqrt[b^2 - 4*a*c]))*ArcTanh[(2*c*d - (b + Sqrt[b^2 - 4*a*c])*f*x)/(Sqrt[2]*Sqrt[2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f])*Sqrt[d + f*x^2]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 725

Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 1034

Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rubi steps

$$\int \frac{A + Bx}{(a + bx + cx^2)\sqrt{d + fx^2}} dx = \frac{(2Ac - B(b - \sqrt{b^2 - 4ac})) \int \frac{1}{(b - \sqrt{b^2 - 4ac} + 2cx)\sqrt{d + fx^2}} dx}{\sqrt{b^2 - 4ac}} - \frac{(2Ac - B(b + \sqrt{b^2 - 4ac})) \int \frac{1}{(b + \sqrt{b^2 - 4ac} + 2cx)\sqrt{d + fx^2}} dx}{\sqrt{b^2 - 4ac}}$$

$$= \frac{(2Ac - B(b - \sqrt{b^2 - 4ac})) \operatorname{Subst}\left(\int \frac{1}{4c^2d + (b - \sqrt{b^2 - 4ac})^2 f - x^2} dx, x, \frac{2cd - (b - \sqrt{b^2 - 4ac})x}{\sqrt{d + fx^2}}\right)}{\sqrt{b^2 - 4ac}} - \frac{(2Ac - B(b + \sqrt{b^2 - 4ac})) \operatorname{Subst}\left(\int \frac{1}{4c^2d + (b + \sqrt{b^2 - 4ac})^2 f - x^2} dx, x, \frac{2cd - (b + \sqrt{b^2 - 4ac})x}{\sqrt{d + fx^2}}\right)}{\sqrt{b^2 - 4ac}}$$

$$= \frac{(bB - 2Ac - B\sqrt{b^2 - 4ac}) \tanh^{-1}\left(\frac{2cd - (b - \sqrt{b^2 - 4ac})fx}{\sqrt{2}\sqrt{2c^2d - 2acf + b(b - \sqrt{b^2 - 4ac})f}\sqrt{d + fx^2}}\right)}{\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{2c^2d - 2acf + b(b - \sqrt{b^2 - 4ac})f}} - \frac{(bB - 2Ac + B\sqrt{b^2 - 4ac}) \tanh^{-1}\left(\frac{2cd - (b + \sqrt{b^2 - 4ac})fx}{\sqrt{2}\sqrt{2c^2d - 2acf + b(b + \sqrt{b^2 - 4ac})f}\sqrt{d + fx^2}}\right)}{\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{2c^2d - 2acf + b(b + \sqrt{b^2 - 4ac})f}}$$

Mathematica [A] time = 0.43, size = 283, normalized size = 0.94

$$\sqrt{2} \left[\frac{(B\sqrt{b^2 - 4ac} + 2Ac - bB) \tanh^{-1}\left(\frac{fx(\sqrt{b^2 - 4ac} - b) + 2cd}{\sqrt{d + fx^2}\sqrt{2bf(b - \sqrt{b^2 - 4ac}) - 4acf + 4c^2d}}\right)}{2\sqrt{bf(b - \sqrt{b^2 - 4ac}) - 2acf + 2c^2d}} - \frac{(B\sqrt{b^2 - 4ac} - 2Ac + bB) \tanh^{-1}\left(\frac{2cd - fx(\sqrt{b^2 - 4ac} + b)}{\sqrt{d + fx^2}\sqrt{2bf(\sqrt{b^2 - 4ac} + b) - 4acf + 4c^2d}}\right)}{2\sqrt{bf(\sqrt{b^2 - 4ac} + b) - 2acf + 2c^2d}} \right] \frac{1}{\sqrt{b^2 - 4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]),x]

[Out] (Sqrt[2]*(-1/2*((-b*B) + 2*A*c + B*Sqrt[b^2 - 4*a*c])*ArcTanh[(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*f*x)/(Sqrt[4*c^2*d - 4*a*c*f + 2*b*(b - Sqrt[b^2 - 4*a*c])*f]*Sqrt[d + f*x^2])])/(Sqrt[2*c^2*d - 2*a*c*f + b*(b - Sqrt[b^2 - 4*a*c])*f] - ((b*B - 2*A*c + B*Sqrt[b^2 - 4*a*c])*ArcTanh[(2*c*d - (b + Sqrt[b^2 - 4*a*c])*f*x)/(Sqrt[4*c^2*d - 4*a*c*f + 2*b*(b + Sqrt[b^2 - 4*a*c])*f]*Sqrt[d + f*x^2])])/(2*Sqrt[2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f]))/Sqrt[b^2 - 4*a*c]

IntegrateAlgebraic [C] time = 0.44, size = 195, normalized size = 0.65

$$-\operatorname{RootSum}\left[\#1^4c - 2\#1^3b\sqrt{f} + 4\#1^2af - 2\#1^2cd + 2\#1bd\sqrt{f} + cd^2 \&, \frac{\#1^2(-B)\log(-\#1 + \sqrt{d + fx^2} - \sqrt{f}x) + 2\#1A\sqrt{f}\log(-\#1 + \sqrt{d + fx^2} - \sqrt{f}x) + Bd\log(-\#1 + \sqrt{d + fx^2} - \sqrt{f}x)}{2\#1^3c - 3\#1^2b\sqrt{f} + 4\#1af - 2\#1cd + bd\sqrt{f}} \& \right]$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]),x]

[Out] -RootSum[c*d^2 + 2*b*d*Sqrt[f]*#1 - 2*c*d*#1^2 + 4*a*f*#1^2 - 2*b*Sqrt[f]*#1^3 + c*#1^4 &, (B*d*Log[-(Sqrt[f]*x) + Sqrt[d + f*x^2] - #1] + 2*A*Sqrt[f]*Log[-(Sqrt[f]*x) + Sqrt[d + f*x^2] - #1]*#1 - B*Log[-(Sqrt[f]*x) + Sqrt[d + f*x^2] - #1]*#1^2)/(b*d*Sqrt[f] - 2*c*d*#1 + 4*a*f*#1 - 3*b*Sqrt[f]*#1^2 + 2*c*#1^3) &]

fricas [B] time = 60.52, size = 8977, normalized size = 29.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d)^(1/2),x, algorithm="fricas")

[Out] 1/4*sqrt(2)*sqrt(((B^2*b^2 + 2*A^2*c^2 - 2*(B^2*a + A*B*b)*c)*d + (2*B^2*a^2 - 2*A*B*a*b + A^2*b^2 - 2*A^2*a*c)*f + ((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 -

$$\begin{aligned}
& 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2)*\text{sqrt}(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))/((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2))*\log((2*(B^4*a*b^2 - A*B^3*b^3 - 2*A^3*B*b*c^2 - (2*A*B^3*a*b - 3*A^2*B^2*b^2)*c)*d^2 + 2*(2*A*B^3*a^2*b - 3*A^2*B^2*a*b^2 + A^3*B*b^3 + (2*A^3*B*a*b - A^4*b^2)*c)*d*f + \text{sqrt}(2)*((B^3*b^4 - 8*A^2*B*a*c^3 + 2*(6*A*B^2*a*b + A^2*B*b^2)*c^2 - (4*B^3*a*b^2 + 3*A*B^2*b^3)*c)*d^2 + (3*A*B^2*a*b^3 - A^2*B*b^4 + 4*(4*A^2*B*a^2 - A^3*a*b)*c^2 - (12*A*B^2*a^2*b - A^3*b^3)*c)*d*f + (2*A^2*B*a^2*b^2 - A^3*a*b^3 - 4*(2*A^2*B*a^3 - A^3*a^2*b)*c)*f^2 - ((B*b^4*c^2 + 4*(2*B*a^2 + A*a*b)*c^4 - (6*B*a*b^2 + A*b^3)*c^3)*d^3 + (B*b^6 - 4*(6*B*a^3 + A*a^2*b)*c^3 + (22*B*a^2*b^2 + 5*A*a*b^3)*c^2 - (8*B*a*b^4 + A*b^5)*c)*d^2*f + (3*B*a^2*b^4 - A*a*b^5 + 4*(6*B*a^4 - A*a^3*b)*c^2 - (18*B*a^3*b^2 - 5*A*a^2*b^3)*c)*d*f^2 + (2*B*a^4*b^2 - A*a^3*b^3 - 4*(2*B*a^5 - A*a^4*b)*c)*f^3)*\text{sqrt}(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4))*\text{sqrt}(f*x^2 + d)*\text{sqrt}(((B^2*b^2 + 2*A^2*c^2 - 2*(B^2*a + A*B*b)*c)*d + (2*B^2*a^2 - 2*A*B*a*b + A^2*b^2 - 2*A^2*a*c)*f + ((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2))*\text{sqrt}(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))/((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2)) - 4*((B^4*a^2*b - A*B^3*a*b^2 - 2*A^3*B*a*c^2 - (2*A*B^3*a^2 - 3*A^2*B^2*a*b)*c)*d*f + (2*A*B^3*a^3 - 3*A^2*B^2*a^2*b + A^3*B*a*b^2 + (2*A^3*B*a^2 - A^4*a*b)*c)*f^2)*x + 2*((4*A^2*a*c^4 + (4*B^2*a^2 - 4*A*B*a*b - A^2*b^2)*c^3 - (B^2*a*b^2 - A*B*b^3)*c^2)*d^3 - (B^2*a*b^4 - A*B*b^5 + 8*A^2*a^2*c^3 + 2*(4*B^2*a^3 - 4*A*B*a^2*b - 3*A^2*a*b^2)*c^2 - (6*B^2*a^2*b^2 - 6*A*B*a*b^3 - A^2*b^4)*c)*d^2*f - (B^2*a^3*b^2 - A*B*a^2*b^3 - 4*A^2*a^3*c^2 - (4*B^2*a^4 - 4*A*B*a^3*b - A^2*a^2*b^2)*c)*d*f^2)*\text{sqrt}(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))/x - 1/4*\text{sqrt}(2)*\text{sqrt}(((B^2*b^2 + 2*A^2*c^2 - 2*(B^2*a + A*B*b)*c)*d + (2*B^2*a^2 - 2*A*B*a*b + A^2*b^2 - 2*A^2*a*c)*f + ((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2))*\text{sqrt}(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))/((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2))*\log((2*(B^4*a*b^2 - A*B^3*b^3 - 2*A^3*B*b*c^2 - (2*A*B^3*a*b - 3*A^2*B^2*b^2)*c)*d^2 + 2*(2*A*B^3*a^2*b - 3*A^2*B^2*a*b^2 + A^3*B*b^3 + (2*A^3*B*a*b - A^4*b^2)*c)*d*f - \text{sqrt}(2)*((B^3*b^4 - 8*A^2*B*a*c^3 + 2*(6*A*B^2*a*b + A^2*B*b^2)*c^2 - (4*B^3*a*b^2 + 3*A*B^2*b^3)*c)*d^2 + (3*A*B^2*a*b^3 - A^2*B*b^4 + 4*(4*A^2*B*a^2 - A^3*a*b)*c^2 - (12*A*B^2*a^2*b - A^3*b^3)*c)*d*f + (2*A^2*B*a^2*b^2 - A^3*a*b^3 - 4*(2*A^2*B*a^3 - A^3*a^2*b)*c)*f^2 - ((B*b^4*c^2 + 4*(2*B*a^2 + A*a*b)*c^4 - (6*B*a*b^2 + A*b^3)*c^3)*d^3 + (B*b^6 - 4*(6*B*a^3 + A*a^2*b)*c^3 + (22*B*a^2*b^2 + 5*A*a*b^3)*c^2 - (8*B*a*b^4 +
\end{aligned}$$

$$\begin{aligned}
& A*b^5)*c)*d^2*f + (3*B*a^2*b^4 - A*a*b^5 + 4*(6*B*a^4 - A*a^3*b)*c^2 - (18* \\
& B*a^3*b^2 - 5*A*a^2*b^3)*c)*d*f^2 + (2*B*a^4*b^2 - A*a^3*b^3 - 4*(2*B*a^5 - \\
& A*a^4*b)*c)*f^3)*\sqrt{((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2* \\
& A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 \\
& - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a* \\
& b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3) \\
&)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5* \\
& c)*f^4))*\sqrt{f*x^2 + d)*\sqrt{((B^2*b^2 + 2*A^2*c^2 - 2*(B^2*a + A*B*b)*c) \\
& *d + (2*B^2*a^2 - 2*A*B*a*b + A^2*b^2 - 2*A^2*a*c)*f + ((b^2*c^2 - 4*a*c^3) \\
& *d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2))*\sqrt{((\\
& B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - \\
& 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2) \\
& *f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3* \\
& f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - \\
& 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4))/((b^2*c^2 - 4*a \\
& *c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2)) - \\
& 4*((B^4*a^2*b - A*B^3*a*b^2 - 2*A^3*B*a*c^2 - (2*A*B^3*a^2 - 3*A^2*B^2*a*b) \\
&)*c)*d*f + (2*A*B^3*a^3 - 3*A^2*B^2*a^2*b + A^3*B*a*b^2 + (2*A^3*B*a^2 - A^ \\
& 4*a*b)*c)*f^2)*x + 2*((4*A^2*a*c^4 + (4*B^2*a^2 - 4*A*B*a*b - A^2*b^2)*c^3 \\
& - (B^2*a*b^2 - A*B*b^3)*c^2)*d^3 - (B^2*a*b^4 - A*B*b^5 + 8*A^2*a^2*c^3 + 2 \\
& *(4*B^2*a^3 - 4*A*B*a^2*b - 3*A^2*a*b^2)*c^2 - (6*B^2*a^2*b^2 - 6*A*B*a*b^3 \\
& - A^2*b^4)*c)*d^2*f - (B^2*a^3*b^2 - A*B*a^2*b^3 - 4*A^2*a^3*c^2 - (4*B^2* \\
& a^4 - 4*A*B*a^3*b - A^2*a^2*b^2)*c)*d*f^2)*\sqrt{((B^4*b^2 - 4*A*B^3*b*c + 4 \\
& *A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b) \\
&)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5) \\
&)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22 \\
& *a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)* \\
& d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4))/x) + 1/4*\sqrt{2)*\sqrt{((B^2*b^2 + 2*A^2* \\
& c^2 - 2*(B^2*a + A*B*b)*c)*d + (2*B^2*a^2 - 2*A*B*a*b + A^2*b^2 - 2*A^2*a*c) \\
&)*f - ((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b \\
& ^2 - 4*a^3*c)*f^2))*\sqrt{((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2 \\
& *A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^ \\
& 2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a \\
& *b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^ \\
& 3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5 \\
& *c)*f^4))/((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (\\
& a^2*b^2 - 4*a^3*c)*f^2))*\log((2*(B^4*a*b^2 - A*B^3*b^3 - 2*A^3*B*b*c^2 - (2 \\
& *A*B^3*a*b - 3*A^2*B^2*b^2)*c)*d^2 + 2*(2*A*B^3*a^2*b - 3*A^2*B^2*a*b^2 + A \\
& ^3*B*b^3 + (2*A^3*B*a*b - A^4*b^2)*c)*d*f + \sqrt{2)*((B^3*b^4 - 8*A^2*B*a*c \\
& ^3 + 2*(6*A*B^2*a*b + A^2*B*b^2)*c^2 - (4*B^3*a*b^2 + 3*A*B^2*b^3)*c)*d^2 + \\
& (3*A*B^2*a*b^3 - A^2*B*b^4 + 4*(4*A^2*B*a^2 - A^3*a*b)*c^2 - (12*A*B^2*a^2 \\
& *b - A^3*b^3)*c)*d*f + (2*A^2*B*a^2*b^2 - A^3*a*b^3 - 4*(2*A^2*B*a^3 - A^3* \\
& a^2*b)*c)*f^2 + ((B*b^4*c^2 + 4*(2*B*a^2 + A*a*b)*c^4 - (6*B*a*b^2 + A*b^3) \\
&)*c^3)*d^3 + (B*b^6 - 4*(6*B*a^3 + A*a^2*b)*c^3 + (22*B*a^2*b^2 + 5*A*a*b^3) \\
&)*c^2 - (8*B*a*b^4 + A*b^5)*c)*d^2*f + (3*B*a^2*b^4 - A*a*b^5 + 4*(6*B*a^4 - \\
& A*a^3*b)*c^2 - (18*B*a^3*b^2 - 5*A*a^2*b^3)*c)*d*f^2 + (2*B*a^4*b^2 - A*a^ \\
& 3*b^3 - 4*(2*B*a^5 - A*a^4*b)*c)*f^3)*\sqrt{((B^4*b^2 - 4*A*B^3*b*c + 4*A^2* \\
& B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)* \\
& d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 \\
& + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2* \\
& b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 \\
& + (a^4*b^2 - 4*a^5*c)*f^4))*\sqrt{f*x^2 + d)*\sqrt{((B^2*b^2 + 2*A^2*c^2 - \\
& 2*(B^2*a + A*B*b)*c)*d + (2*B^2*a^2 - 2*A*B*a*b + A^2*b^2 - 2*A^2*a*c)*f - \\
& ((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4 \\
& *a^3*c)*f^2))*\sqrt{((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3 \\
& *a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4* \\
& A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c \\
& ^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2 \\
& *f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^
\end{aligned}$$

$$\begin{aligned}
& 4)) / ((b^2c^2 - 4a^3c^3) * d^2 + (b^4 - 6a^2b^2c + 8a^2c^2) * d * f + (a^2b^2 - 4a^3c) * f^2) - 4 * ((B^4a^2b - AB^3a^2b^2 - 2A^3B^2a^2c^2 - (2AB^3a^2 - 3A^2B^2a^2b) * c) * d * f + (2AB^3a^3 - 3A^2B^2a^2b + A^3B^2a^2b^2 + (2A^3B^2a^2 - A^4a^2b) * c) * f^2) * x - 2 * ((4A^2a^2c^4 + (4B^2a^2 - 4AB^2a^2b - A^2b^2) * c^3 - (B^2a^2b^2 - AB^2b^3) * c^2) * d^3 - (B^2a^2b^4 - AB^2b^5 + 8A^2a^2c^3 + 2 * (4B^2a^3 - 4AB^2a^2b - 3A^2a^2b^2) * c^2 - (6B^2a^2b^2 - 6AB^2a^2b^3 - A^2b^4) * c) * d^2 * f - (B^2a^3b^2 - AB^2a^2b^3 - 4A^2a^3c^2 - (4B^2a^4 - 4AB^2a^3b - A^2a^2b^2) * c) * d * f^2) * sqrt(((B^4b^2 - 4AB^3b^2c + 4A^2B^2c^2) * d^2 + 2 * (2AB^3a^2b - A^2B^2b^2 - 2 * (2A^2B^2a - A^3B^2b) * c) * d * f + (4A^2B^2a^2 - 4A^3B^2a^2b + A^4b^2) * f^2) / ((b^2c^4 - 4a^3c^5) * d^4 + 2 * (b^4c^2 - 6a^2b^2c^3 + 8a^2c^4) * d^3 * f + (b^6 - 8a^2b^4c + 22a^2b^2c^2 - 24a^3c^3) * d^2 * f^2 + 2 * (a^2b^4 - 6a^3b^2c + 8a^4c^2) * d * f^3 + (a^4b^2 - 4a^5c) * f^4))) / x) - 1/4 * sqrt(2) * sqrt(((B^2b^2 + 2A^2c^2 - 2 * (B^2a + AB^2b) * c) * d + (2B^2a^2 - 2AB^2a^2b + A^2b^2 - 2A^2a^2c) * f - ((b^2c^2 - 4a^3c^3) * d^2 + (b^4 - 6a^2b^2c + 8a^2c^2) * d * f + (a^2b^2 - 4a^3c) * f^2) * sqrt(((B^4b^2 - 4AB^3b^2c + 4A^2B^2c^2) * d^2 + 2 * (2AB^3a^2b - A^2B^2b^2 - 2 * (2A^2B^2a - A^3B^2b) * c) * d * f + (4A^2B^2a^2 - 4A^3B^2a^2b + A^4b^2) * f^2) / ((b^2c^4 - 4a^3c^5) * d^4 + 2 * (b^4c^2 - 6a^2b^2c^3 + 8a^2c^4) * d^3 * f + (b^6 - 8a^2b^4c + 22a^2b^2c^2 - 24a^3c^3) * d^2 * f^2 + 2 * (a^2b^4 - 6a^3b^2c + 8a^4c^2) * d * f^3 + (a^4b^2 - 4a^5c) * f^4))) / ((b^2c^2 - 4a^3c^3) * d^2 + (b^4 - 6a^2b^2c + 8a^2c^2) * d * f + (a^2b^2 - 4a^3c) * f^2)) * log((2 * (B^4a^2b^2 - AB^3b^3 - 2A^3B^2b^2c^2 - (2AB^3a^2b - 3A^2B^2b^2) * c) * d^2 + 2 * (2AB^3a^2b - 3A^2B^2a^2b^2 + A^3B^2b^3 + (2A^3B^2a^2b - A^4b^2) * c) * d * f - sqrt(2) * ((B^3b^4 - 8A^2B^2a^2c^3 + 2 * (6AB^2a^2b + A^2B^2b^2) * c^2 - (4B^3a^2b^2 + 3AB^2b^3) * c) * d^2 + (3AB^2a^2b^3 - A^2B^2b^4 + 4 * (4A^2B^2a^2 - A^3a^2b) * c^2 - (12AB^2a^2b - A^3b^3) * c) * d * f + (2A^2B^2a^2b^2 - A^3a^2b^3 - 4 * (2A^2B^2a^3 - A^3a^2b) * c) * f^2 + ((B^2b^4c^2 + 4 * (2B^2a^2 + A^2a^2b) * c^4 - (6B^2a^2b^2 + A^2b^3) * c^3) * d^3 + (B^2b^6 - 4 * (6B^2a^3 + A^2a^2b) * c^3 + (22B^2a^2b^2 + 5A^2a^2b^3) * c^2 - (8B^2a^2b^4 + A^2b^5) * c) * d^2 * f + (3B^2a^2b^4 - A^2a^2b^5 + 4 * (6B^2a^4 - A^2a^3b) * c^2 - (18B^2a^3b^2 - 5A^2a^2b^3) * c) * d * f^2 + (2B^2a^4b^2 - A^2a^3b^3 - 4 * (2B^2a^5 - A^2a^4b) * c) * f^3) * sqrt(((B^4b^2 - 4AB^3b^2c + 4A^2B^2c^2) * d^2 + 2 * (2AB^3a^2b - A^2B^2b^2 - 2 * (2A^2B^2a - A^3B^2b) * c) * d * f + (4A^2B^2a^2 - 4A^3B^2a^2b + A^4b^2) * f^2) / ((b^2c^4 - 4a^3c^5) * d^4 + 2 * (b^4c^2 - 6a^2b^2c^3 + 8a^2c^4) * d^3 * f + (b^6 - 8a^2b^4c + 22a^2b^2c^2 - 24a^3c^3) * d^2 * f^2 + 2 * (a^2b^4 - 6a^3b^2c + 8a^4c^2) * d * f^3 + (a^4b^2 - 4a^5c) * f^4))) * sqrt(f * x^2 + d) * sqrt(((B^2b^2 + 2A^2c^2 - 2 * (B^2a + AB^2b) * c) * d + (2B^2a^2 - 2AB^2a^2b + A^2b^2 - 2A^2a^2c) * f - ((b^2c^2 - 4a^3c^3) * d^2 + (b^4 - 6a^2b^2c + 8a^2c^2) * d * f + (a^2b^2 - 4a^3c) * f^2) * sqrt(((B^4b^2 - 4AB^3b^2c + 4A^2B^2c^2) * d^2 + 2 * (2AB^3a^2b - A^2B^2b^2 - 2 * (2A^2B^2a - A^3B^2b) * c) * d * f + (4A^2B^2a^2 - 4A^3B^2a^2b + A^4b^2) * f^2) / ((b^2c^4 - 4a^3c^5) * d^4 + 2 * (b^4c^2 - 6a^2b^2c^3 + 8a^2c^4) * d^3 * f + (b^6 - 8a^2b^4c + 22a^2b^2c^2 - 24a^3c^3) * d^2 * f^2 + 2 * (a^2b^4 - 6a^3b^2c + 8a^4c^2) * d * f^3 + (a^4b^2 - 4a^5c) * f^4))) / ((b^2c^2 - 4a^3c^3) * d^2 + (b^4 - 6a^2b^2c + 8a^2c^2) * d * f + (a^2b^2 - 4a^3c) * f^2)) - 4 * ((B^4a^2b - AB^3a^2b^2 - 2A^3B^2a^2c^2 - (2AB^3a^2 - 3A^2B^2a^2b) * c) * d * f + (2AB^3a^3 - 3A^2B^2a^2b + A^3B^2a^2b^2 + (2A^3B^2a^2 - A^4a^2b) * c) * f^2) * x - 2 * ((4A^2a^2c^4 + (4B^2a^2 - 4AB^2a^2b - A^2b^2) * c^3 - (B^2a^2b^2 - AB^2b^3) * c^2) * d^3 - (B^2a^2b^4 - AB^2b^5 + 8A^2a^2c^3 + 2 * (4B^2a^3 - 4AB^2a^2b - 3A^2a^2b^2) * c^2 - (6B^2a^2b^2 - 6AB^2a^2b^3 - A^2b^4) * c) * d^2 * f - (B^2a^3b^2 - AB^2a^2b^3 - 4A^2a^3c^2 - (4B^2a^4 - 4AB^2a^3b - A^2a^2b^2) * c) * d * f^2) * sqrt(((B^4b^2 - 4AB^3b^2c + 4A^2B^2c^2) * d^2 + 2 * (2AB^3a^2b - A^2B^2b^2 - 2 * (2A^2B^2a - A^3B^2b) * c) * d * f + (4A^2B^2a^2 - 4A^3B^2a^2b + A^4b^2) * f^2) / ((b^2c^4 - 4a^3c^5) * d^4 + 2 * (b^4c^2 - 6a^2b^2c^3 + 8a^2c^4) * d^3 * f + (b^6 - 8a^2b^4c + 22a^2b^2c^2 - 24a^3c^3) * d^2 * f^2 + 2 * (a^2b^4 - 6a^3b^2c + 8a^4c^2) * d * f^3 + (a^4b^2 - 4a^5c) * f^4))) / x)
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x)::OUTPUT:Unable to divide, perhaps due to rounding error
 %%%{poly1[%%{-4, [3,2,0]%%}+%%{16, [1,3,1]%%},%%{4, [4,2,0]%%}+%%{-24, [2,3,1]%%}+%%{32, [0,4,2]%%}]: [1,0,%%{-1, [2,0,0]%%}+%%{4, [0,1,1]%%}%%}, [2,1,0,0]%%}+%%{%%{[-4, [5,0,0]%%}+%%{24, [3,1,1]%%}+%%{-32, [1,2,2]%%},%%{4, [6,0,0]%%}+%%{-32, [4,1,1]%%}+%%{72, [2,2,2]%%}+%%{-32, [0,3,3]%%}]: [1,0,%%{-1, [2,0,0]%%}+%%{4, [0,1,1]%%}%%}, [2,0,0,1]%%}+%%{%%{poly1[%%{8, [2,3,0]%%}+%%{-32, [0,4,1]%%},%%{-8, [3,3,0]%%}+%%{32, [1,4,1]%%}]: [1,0,%%{-1, [2,0,0]%%}+%%{4, [0,1,1]%%}%%}, [1,1,1,0]%%}+%%{%%{[-8, [4,1,0]%%}+%%{-40, [2,2,1]%%}+%%{32, [0,3,2]%%},%%{-8, [5,1,0]%%}+%%{56, [3,2,1]%%}+%%{-96, [1,3,2]%%}]: [1,0,%%{-1, [2,0,0]%%}+%%{4, [0,1,1]%%}%%}, [1,0,1,1]%%}+%%{%%{8, [2,4,0]%%}+%%{-32, [0,5,1]%%}, [0,1,2,0]%%}+%%{%%{poly1[%%{-4, [3,2,0]%%}+%%{16, [1,3,1]%%},%%{4, [4,2,0]%%}+%%{-24, [2,3,1]%%}+%%{32, [0,4,2]%%}]: [1,0,%%{-1, [2,0,0]%%}+%%{4, [0,1,1]%%}%%}, [0,0,2,1]%%} / %%{%%{[-4, [3,2,0]%%}+%%{16, [1,3,1]%%},%%{4, [4,2,0]%%}+%%{-24, [2,3,1]%%}+%%{32, [0,4,2]%%}]: [1,0,%%{-1, [2,0,0]%%}+%%{4, [0,1,1]%%}%%}, [0,1,0,0]%%}+%%{%%{[-4, [5,0,0]%%}+%%{24, [3,1,1]%%}+%%{-32, [1,2,2]%%},%%{4, [6,0,0]%%}+%%{-32, [4,1,1]%%}+%%{72, [2,2,2]%%}+%%{-32, [0,3,3]%%}]: [1,0,%%{-1, [2,0,0]%%}+%%{4, [0,1,1]%%}%%}, [0,0,0,1]%%} Error: Bad Argument ValueUnable to divide, perhaps due to rounding error
 %%%{%%{[%%{1, [3,0,0]%%}+%%{-4, [1,1,1]%%},%%{1, [4,0,0]%%}+%%{-6, [2,1,1]%%}+%%{8, [0,2,2]%%}]: [1,0,%%{-1, [2,0,0]%%}+%%{4, [0,1,1]%%}%%}/%%{4, [0,0,2]%%}, [2,1,0,0]%%}+%%{%%{1, [2,0,0]%%}+%%{-4, [0,1,1]%%}/2, [2,0,0,1]%%}+%%{%%{[-1, [4,0,0]%%}+%%{5, [2,1,1]%%}+%%{-4, [0,2,2]%%},%%{-1, [5,0,0]%%}+%%{7, [3,1,1]%%}+%%{-12, [1,2,2]%%}]: [1,0,%%{-1, [2,0,0]%%}+%%{4, [0,1,1]%%}%%}/%%{2, [0,0,3]%%}, [1,1,1,0]%%}+%%{%%{[-1, [2,0,0]%%}+%%{4, [0,1,1]%%},%%{-1, [3,0,0]%%}+%%{4, [1,1,1]%%}]: [1,0,%%{-1, [2,0,0]%%}+%%{4, [0,1,1]%%}%%}/%%{2, [0,0,1]%%}, [1,0,1,1]%%}+%%{%%{[%%{1, [5,0,0]%%}+%%{-6, [3,1,1]%%}+%%{8, [1,2,2]%%},%%{1, [6,0,0]%%}+%%{-8, [4,1,1]%%}+%%{18, [2,2,2]%%}+%%{-8, [0,3,3]%%}]: [1,0,%%{-1, [2,0,0]%%}+%%{4, [0,1,1]%%}%%}/%%{4, [0,0,4]%%}, [0,1,2,0]%%}+%%{%%{[%%{1, [3,0,0]%%}+%%{-4, [1,1,1]%%},%%{1, [4,0,0]%%}+%%{-6, [2,1,1]%%}+%%{8, [0,2,2]%%}]: [1,0,%%{-1, [2,0,0]%%}+%%{4, [0,1,1]%%}%%}/%%{4, [0,0,2]%%}, [0,0,2,1]%%} / %%{%%{[%%{1, [3,0,0]%%}+%%{-4, [1,1,1]%%},%%{1, [4,0,0]%%}+%%{-6, [2,1,1]%%}+%%{8, [0,2,2]%%}]: [1,0,%%{-1, [2,0,0]%%}+%%{4, [0,1,1]%%}%%}/%%{4, [0,0,4]%%}, [0,1,0,0]%%}+%%{%%{1, [2,0,0]%%}+%%{-4, [0,1,1]%%}/%%{2, [0,0,2]%%}, [0,0,0,1]%%} Error: Bad Argument Value

maple [B] time = 0.03, size = 1771, normalized size = 5.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d)^(1/2),x)

[Out]
$$\frac{2/(-4*a*c+b^2)^{(1/2)}/(-2*(-(-4*a*c+b^2)^{(1/2)}*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^{(1/2)}*\ln((-(-4*a*c+b^2)^{(1/2)}*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2-f*(b+(-4*a*c+b^2)^{(1/2)})/c*(x+1/2*(b+(-4*a*c+b^2)^{(1/2)})/c)+1/2*(-2*(-(-4*a*c+b^2)^{(1/2)}*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^{(1/2)}*(4*(x+1/2*(b+(-4*a*c+b^2)^{(1/2)})/c)^2*f-4*f*(b+(-4*a*c+b^2)^{(1/2)})/c*(x+1/2*(b+(-4*a*c+b^2)^{(1/2)})/c)-2*(-(-4*a*c+b^2)^{(1/2)}*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^{(1/2)})/(x+1/2*(b+(-4*a*c+b^2)^{(1/2)})/c)*A-1/c/(-2*(-(-4*a*c+b^2)^{(1/2)}*b*f+2*a*c*f-b^2*f-2*c^2*d)/c$$

$$\begin{aligned} & ^2)^{(1/2)} * \ln\left(\frac{-(-(-4ac+b^2)^{(1/2)} * bf + 2ac * f - b^2 * f - 2c^2 * d) / c^2 - f * (b + (-4ac+b^2)^{(1/2)}) / c * (x + 1/2 * (b + (-4ac+b^2)^{(1/2)}) / c) + 1/2 * (-2 * (-(-4ac+b^2)^{(1/2)} * bf + 2ac * f - b^2 * f - 2c^2 * d) / c^2)^{(1/2)} * (4 * (x + 1/2 * (b + (-4ac+b^2)^{(1/2)}) / c))^2 * f - 4 * f * (b + (-4ac+b^2)^{(1/2)}) / c * (x + 1/2 * (b + (-4ac+b^2)^{(1/2)}) / c) - 2 * (-(-4ac+b^2)^{(1/2)} * bf + 2ac * f - b^2 * f - 2c^2 * d) / c^2)^{(1/2)}}{(x + 1/2 * (b + (-4ac+b^2)^{(1/2)}) / c)}\right) * B - 1 / (-4ac+b^2)^{(1/2)} / c / (-2 * (-(-4ac+b^2)^{(1/2)} * bf + 2ac * f - b^2 * f - 2c^2 * d) / c^2)^{(1/2)} * \ln\left(\frac{-(-(-4ac+b^2)^{(1/2)} * bf + 2ac * f - b^2 * f - 2c^2 * d) / c^2 - f * (b + (-4ac+b^2)^{(1/2)}) / c * (x + 1/2 * (b + (-4ac+b^2)^{(1/2)}) / c) + 1/2 * (-2 * (-(-4ac+b^2)^{(1/2)} * bf + 2ac * f - b^2 * f - 2c^2 * d) / c^2)^{(1/2)} * (4 * (x + 1/2 * (b + (-4ac+b^2)^{(1/2)}) / c))^2 * f - 4 * f * (b + (-4ac+b^2)^{(1/2)}) / c * (x + 1/2 * (b + (-4ac+b^2)^{(1/2)}) / c) - 2 * (-(-4ac+b^2)^{(1/2)} * bf + 2ac * f - b^2 * f - 2c^2 * d) / c^2)^{(1/2)}}{(x + 1/2 * (b + (-4ac+b^2)^{(1/2)}) / c)}\right) * b * B - 2 / (-4ac+b^2)^{(1/2)} / (-2 * ((-4ac+b^2)^{(1/2)} * bf + 2ac * f - b^2 * f - 2c^2 * d) / c^2)^{(1/2)} * \ln\left(\frac{-((-4ac+b^2)^{(1/2)} * bf + 2ac * f - b^2 * f - 2c^2 * d) / c^2 - f * (b - (-4ac+b^2)^{(1/2)}) / c * (x - 1/2 * (-b + (-4ac+b^2)^{(1/2)}) / c) + 1/2 * (-2 * ((-4ac+b^2)^{(1/2)} * bf + 2ac * f - b^2 * f - 2c^2 * d) / c^2)^{(1/2)} * (4 * (x - 1/2 * (-b + (-4ac+b^2)^{(1/2)}) / c))^2 * f - 4 * f * (b - (-4ac+b^2)^{(1/2)}) / c * (x - 1/2 * (-b + (-4ac+b^2)^{(1/2)}) / c) - 2 * ((-4ac+b^2)^{(1/2)} * bf + 2ac * f - b^2 * f - 2c^2 * d) / c^2)^{(1/2)}}{(x - 1/2 * (-b + (-4ac+b^2)^{(1/2)}) / c)}\right) * A - 1 / c / (-2 * ((-4ac+b^2)^{(1/2)} * bf + 2ac * f - b^2 * f - 2c^2 * d) / c^2)^{(1/2)} * \ln\left(\frac{-((-4ac+b^2)^{(1/2)} * bf + 2ac * f - b^2 * f - 2c^2 * d) / c^2 - f * (b - (-4ac+b^2)^{(1/2)}) / c * (x - 1/2 * (-b + (-4ac+b^2)^{(1/2)}) / c) + 1/2 * (-2 * ((-4ac+b^2)^{(1/2)} * bf + 2ac * f - b^2 * f - 2c^2 * d) / c^2)^{(1/2)} * (4 * (x - 1/2 * (-b + (-4ac+b^2)^{(1/2)}) / c))^2 * f - 4 * f * (b - (-4ac+b^2)^{(1/2)}) / c * (x - 1/2 * (-b + (-4ac+b^2)^{(1/2)}) / c) - 2 * ((-4ac+b^2)^{(1/2)} * bf + 2ac * f - b^2 * f - 2c^2 * d) / c^2)^{(1/2)}}{(x - 1/2 * (-b + (-4ac+b^2)^{(1/2)}) / c)}\right) * B + 1 / (-4ac+b^2)^{(1/2)} / c / (-2 * ((-4ac+b^2)^{(1/2)} * bf + 2ac * f - b^2 * f - 2c^2 * d) / c^2)^{(1/2)} * \ln\left(\frac{-((-4ac+b^2)^{(1/2)} * bf + 2ac * f - b^2 * f - 2c^2 * d) / c^2 - f * (b - (-4ac+b^2)^{(1/2)}) / c * (x - 1/2 * (-b + (-4ac+b^2)^{(1/2)}) / c) + 1/2 * (-2 * ((-4ac+b^2)^{(1/2)} * bf + 2ac * f - b^2 * f - 2c^2 * d) / c^2)^{(1/2)} * (4 * (x - 1/2 * (-b + (-4ac+b^2)^{(1/2)}) / c))^2 * f - 4 * f * (b - (-4ac+b^2)^{(1/2)}) / c * (x - 1/2 * (-b + (-4ac+b^2)^{(1/2)}) / c) - 2 * ((-4ac+b^2)^{(1/2)} * bf + 2ac * f - b^2 * f - 2c^2 * d) / c^2)^{(1/2)}}{(x - 1/2 * (-b + (-4ac+b^2)^{(1/2)}) / c)}\right) * b * B \end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details) Is 4*a*c-b^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + Bx}{\sqrt{fx^2 + d} (cx^2 + bx + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x)/((d + f*x^2)^(1/2)*(a + b*x + c*x^2)),x)

[Out] int((A + B*x)/((d + f*x^2)^(1/2)*(a + b*x + c*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx}{\sqrt{d + fx^2} (a + bx + cx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+d)**(1/2),x)
```

```
[Out] Integral((A + B*x)/(sqrt(d + f*x**2)*(a + b*x + c*x**2)), x)
```

$$3.24 \quad \int \frac{A+Bx}{(a+cx^2)\sqrt{d+fx^2}} dx$$

Optimal. Leaf size=101

$$\frac{A \tan^{-1}\left(\frac{x\sqrt{cd-af}}{\sqrt{a}\sqrt{d+fx^2}}\right)}{\sqrt{a}\sqrt{cd-af}} - \frac{B \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+fx^2}}{\sqrt{cd-af}}\right)}{\sqrt{c}\sqrt{cd-af}}$$

Rubi [A] time = 0.13, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1010, 377, 205, 444, 63, 208}

$$\frac{A \tan^{-1}\left(\frac{x\sqrt{cd-af}}{\sqrt{a}\sqrt{d+fx^2}}\right)}{\sqrt{a}\sqrt{cd-af}} - \frac{B \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+fx^2}}{\sqrt{cd-af}}\right)}{\sqrt{c}\sqrt{cd-af}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x)/((a + c*x^2)*Sqrt[d + f*x^2]),x]

[Out] (A*ArcTan[(Sqrt[c*d - a*f]*x)/(Sqrt[a]*Sqrt[d + f*x^2])]/(Sqrt[a]*Sqrt[c*d - a*f]) - (B*ArcTanh[(Sqrt[c]*Sqrt[d + f*x^2])/Sqrt[c*d - a*f]])/(Sqrt[c]*Sqrt[c*d - a*f])

Rule 63

Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rule 1010

Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,

Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}, x]

Rubi steps

$$\begin{aligned} \int \frac{A + Bx}{(a + cx^2)\sqrt{d + fx^2}} dx &= A \int \frac{1}{(a + cx^2)\sqrt{d + fx^2}} dx + B \int \frac{x}{(a + cx^2)\sqrt{d + fx^2}} dx \\ &= A \operatorname{Subst}\left(\int \frac{1}{a - (-cd + af)x^2} dx, x, \frac{x}{\sqrt{d + fx^2}}\right) + \frac{1}{2}B \operatorname{Subst}\left(\int \frac{1}{(a + cx)\sqrt{d + fx}} dx, x, \frac{x}{\sqrt{d + fx^2}}\right) \\ &= \frac{A \tan^{-1}\left(\frac{\sqrt{cd-af}x}{\sqrt{a}\sqrt{d+fx^2}}\right)}{\sqrt{a}\sqrt{cd-af}} + \frac{B \operatorname{Subst}\left(\int \frac{1}{a - \frac{cd}{f} + \frac{cx^2}{f}} dx, x, \sqrt{d + fx^2}\right)}{f} \\ &= \frac{A \tan^{-1}\left(\frac{\sqrt{cd-af}x}{\sqrt{a}\sqrt{d+fx^2}}\right)}{\sqrt{a}\sqrt{cd-af}} - \frac{B \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+fx^2}}{\sqrt{cd-af}}\right)}{\sqrt{c}\sqrt{cd-af}} \end{aligned}$$

Mathematica [A] time = 0.17, size = 154, normalized size = 1.52

$$\frac{(A\sqrt{c} - \sqrt{-a}B) \tanh^{-1}\left(\frac{\sqrt{cd-af}x}{\sqrt{d+fx^2}\sqrt{cd-af}}\right) - (\sqrt{-a}B + A\sqrt{c}) \tanh^{-1}\left(\frac{\sqrt{-a}fx + \sqrt{cd}}{\sqrt{d+fx^2}\sqrt{cd-af}}\right)}{2\sqrt{-a}\sqrt{c}\sqrt{cd-af}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x)/((a + c*x^2)*Sqrt[d + f*x^2]), x]

[Out] ((-(Sqrt[-a]*B) + A*Sqrt[c])*ArcTanh[(Sqrt[c]*d - Sqrt[-a]*f*x)/(Sqrt[c*d - a*f]*Sqrt[d + f*x^2])] - (Sqrt[-a]*B + A*Sqrt[c])*ArcTanh[(Sqrt[c]*d + Sqrt[-a]*f*x)/(Sqrt[c*d - a*f]*Sqrt[d + f*x^2])])/(2*Sqrt[-a]*Sqrt[c]*Sqrt[c*d - a*f])

IntegrateAlgebraic [B] time = 2.27, size = 563, normalized size = 5.57

$$\frac{A \operatorname{tanh}^{-1}\left(\frac{\sqrt{cd-af}x}{\sqrt{a}\sqrt{d+fx^2}}\right) - \frac{B \operatorname{Subst}\left(\int \frac{1}{a - \frac{cd}{f} + \frac{cx^2}{f}} dx, x, \sqrt{d + fx^2}\right)}{f}}{2\sqrt{-a}\sqrt{c}\sqrt{cd-af}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B*x)/((a + c*x^2)*Sqrt[d + f*x^2]), x]

[Out] ((B*c*d*Sqrt[-(c*d)] + 2*a*f - 2*Sqrt[a]*Sqrt[f]*Sqrt[-(c*d) + a*f]) - a*B*f*Sqrt[-(c*d) + 2*a*f - 2*Sqrt[a]*Sqrt[f]*Sqrt[-(c*d) + a*f]) - Sqrt[a]*B*Sqrt[f]*Sqrt[-(c*d) + a*f]*Sqrt[-(c*d) + 2*a*f - 2*Sqrt[a]*Sqrt[f]*Sqrt[-(c*d) + a*f]])*ArcTan[(Sqrt[c]*(Sqrt[f]*x - Sqrt[d + f*x^2]))/Sqrt[-(c*d) + 2*a*f - 2*Sqrt[a]*Sqrt[f]*Sqrt[-(c*d) + a*f]]]/(c^(3/2)*d*(c*d - a*f)) + ((B*c*d*Sqrt[-(c*d) + 2*a*f + 2*Sqrt[a]*Sqrt[f]*Sqrt[-(c*d) + a*f]) - a*B*f*Sqrt[-(c*d) + 2*a*f + 2*Sqrt[a]*Sqrt[f]*Sqrt[-(c*d) + a*f]) + Sqrt[a]*B*Sqrt[f]*Sqrt[-(c*d) + a*f]*Sqrt[-(c*d) + 2*a*f + 2*Sqrt[a]*Sqrt[f]*Sqrt[-(c*d) + a*f]])*ArcTan[(Sqrt[c]*(Sqrt[f]*x - Sqrt[d + f*x^2]))/Sqrt[-(c*d) + 2*a*f + 2*Sqrt[a]*Sqrt[f]*Sqrt[-(c*d) + a*f]]]/(c^(3/2)*d*(c*d - a*f)) + (A*ArcTanh[(Sqrt[a]*Sqrt[f])/Sqrt[-(c*d) + a*f] + (c*Sqrt[f]*x^2)/(Sqrt[a]*Sqrt[-(c*d) + a*f]) - (c*x*Sqrt[d + f*x^2])/(Sqrt[a]*Sqrt[-(c*d) + a*f])])/(Sqrt[a]*Sqrt[-(c*d) + a*f])

fricas [B] time = 0.54, size = 1515, normalized size = 15.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+d)^(1/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/4*\sqrt{(B^2*a - A^2*c + 2*(a*c^2*d - a^2*c*f)*\sqrt{-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)}}/(a*c^2*d - a^2*c*f)*\log(((A*B^3*a + A^3*B*c) \\ & *f*x + (A^2*B*c^2*d - A^2*B*a*c*f + (B*a*c^3*d^2 - 2*B*a^2*c^2*d*f + B*a^3*c*f^2)*\sqrt{-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)}))\sqrt{f*x^2 + d} \\ & *\sqrt{(B^2*a - A^2*c + 2*(a*c^2*d - a^2*c*f)*\sqrt{-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)}}/(a*c^2*d - a^2*c*f) + \sqrt{-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)} \\ & *((B^2*a*c^2 + A^2*c^3)*d^2 - (B^2*a^2*c + A^2*a*c^2)*d*f)/x) + 1/4*\sqrt{(B^2*a - A^2*c + 2*(a*c^2*d - a^2*c*f)*\sqrt{-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)}} \\ & /((A*B^3*a + A^3*B*c)*f*x - (A^2*B*c^2*d - A^2*B*a*c*f + (B*a*c^3*d^2 - 2*B*a^2*c^2*d*f + B*a^3*c*f^2)*\sqrt{-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)}))\sqrt{f*x^2 + d} \\ & *\sqrt{(B^2*a - A^2*c + 2*(a*c^2*d - a^2*c*f)*\sqrt{-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)}}/(a*c^2*d - a^2*c*f) + \sqrt{-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)} \\ & *((B^2*a*c^2 + A^2*c^3)*d^2 - (B^2*a^2*c + A^2*a*c^2)*d*f)/x) - 1/4*\sqrt{(B^2*a - A^2*c - 2*(a*c^2*d - a^2*c*f)*\sqrt{-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)}} \\ & /((A*B^3*a + A^3*B*c)*f*x + (A^2*B*c^2*d - A^2*B*a*c*f - (B*a*c^3*d^2 - 2*B*a^2*c^2*d*f + B*a^3*c*f^2)*\sqrt{-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)}))\sqrt{f*x^2 + d} \\ & *\sqrt{(B^2*a - A^2*c - 2*(a*c^2*d - a^2*c*f)*\sqrt{-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)}}/(a*c^2*d - a^2*c*f) - \sqrt{-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)} \\ & *((B^2*a*c^2 + A^2*c^3)*d^2 - (B^2*a^2*c + A^2*a*c^2)*d*f)/x) + 1/4*\sqrt{(B^2*a - A^2*c - 2*(a*c^2*d - a^2*c*f)*\sqrt{-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)}} \\ & /((A*B^3*a + A^3*B*c)*f*x - (A^2*B*c^2*d - A^2*B*a*c*f - (B*a*c^3*d^2 - 2*B*a^2*c^2*d*f + B*a^3*c*f^2)*\sqrt{-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)}))\sqrt{f*x^2 + d} \\ & *\sqrt{(B^2*a - A^2*c - 2*(a*c^2*d - a^2*c*f)*\sqrt{-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)}}/(a*c^2*d - a^2*c*f) - \sqrt{-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)} \\ & *((B^2*a*c^2 + A^2*c^3)*d^2 - (B^2*a^2*c + A^2*a*c^2)*d*f)/x) \end{aligned}$$

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+d)^(1/2),x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.02, size = 608, normalized size = 6.02

$$A \ln \left(\frac{\sqrt{\frac{2\sqrt{ac} + \sqrt{ad}}{c}} \sqrt{\frac{2\sqrt{ac} + \sqrt{ad}}{c}} \sqrt{\frac{2\sqrt{ac} + \sqrt{ad}}{c}} \sqrt{\frac{2\sqrt{ac} + \sqrt{ad}}{c}}}{2\sqrt{ac} \sqrt{\frac{2\sqrt{ac} + \sqrt{ad}}{c}}} \right) + A \ln \left(\frac{\sqrt{\frac{2\sqrt{ac} + \sqrt{ad}}{c}} \sqrt{\frac{2\sqrt{ac} + \sqrt{ad}}{c}} \sqrt{\frac{2\sqrt{ac} + \sqrt{ad}}{c}} \sqrt{\frac{2\sqrt{ac} + \sqrt{ad}}{c}}}{2\sqrt{ac} \sqrt{\frac{2\sqrt{ac} + \sqrt{ad}}{c}}} \right) - B \ln \left(\frac{\sqrt{\frac{2\sqrt{ac} + \sqrt{ad}}{c}} \sqrt{\frac{2\sqrt{ac} + \sqrt{ad}}{c}} \sqrt{\frac{2\sqrt{ac} + \sqrt{ad}}{c}} \sqrt{\frac{2\sqrt{ac} + \sqrt{ad}}{c}}}{2\sqrt{ac} \sqrt{\frac{2\sqrt{ac} + \sqrt{ad}}{c}}} \right) - B \ln \left(\frac{\sqrt{\frac{2\sqrt{ac} + \sqrt{ad}}{c}} \sqrt{\frac{2\sqrt{ac} + \sqrt{ad}}{c}} \sqrt{\frac{2\sqrt{ac} + \sqrt{ad}}{c}} \sqrt{\frac{2\sqrt{ac} + \sqrt{ad}}{c}}}{2\sqrt{ac} \sqrt{\frac{2\sqrt{ac} + \sqrt{ad}}{c}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x+A)/(c*x^2+a)/(f*x^2+d)^(1/2),x)

[Out]
$$\begin{aligned} & -1/2/(-a*c)^{(1/2)}/(-a*f-c*d)/c)^{(1/2)}*\ln((-2*(a*f-c*d)/c+2*f*(-a*c)^{(1/2)}/ \\ & c*(x-(-a*c)^{(1/2)}/c)+2*(-a*f-c*d)/c)^{(1/2)}*((x-(-a*c)^{(1/2)}/c)^2*f+2*f*(-a \\ & *c)^{(1/2)}/c*(x-(-a*c)^{(1/2)}/c)-(a*f-c*d)/c)^{(1/2)})/(x-(-a*c)^{(1/2)}/c))*A-1/ \\ & 2/c/(-a*f-c*d)/c)^{(1/2)}*\ln((-2*(a*f-c*d)/c+2*f*(-a*c)^{(1/2)}/c*(x-(-a*c)^{(1 \\ & /2)}/c)+2*(-a*f-c*d)/c)^{(1/2)}*((x-(-a*c)^{(1/2)}/c)^2*f+2*f*(-a*c)^{(1/2)}/c*(x \\ & -(-a*c)^{(1/2)}/c)-(a*f-c*d)/c)^{(1/2)})/(x-(-a*c)^{(1/2)}/c))*B+1/2/(-a*c)^{(1/2) \\ & /(-a*f-c*d)/c)^{(1/2)}*\ln((-2*(a*f-c*d)/c-2*f*(-a*c)^{(1/2)}/c*(x+(-a*c)^{(1/2) \\ & /c)+2*(-a*f-c*d)/c)^{(1/2)}*((x+(-a*c)^{(1/2)}/c)^2*f-2*f*(-a*c)^{(1/2)}/c*(x+(- \end{aligned}$$

$$\frac{a\sqrt{c} - (af - cd)\sqrt{c}}{(x + (-a\sqrt{c})/c)^{1/2}} \cdot \frac{A - 1/2c}{(-af - cd)\sqrt{c}} \cdot \ln\left(\frac{-2(af - cd)\sqrt{c} - 2f(-a\sqrt{c})/c}{(x + (-a\sqrt{c})/c)^{1/2} + 2(-af - cd)\sqrt{c}}\right) + \frac{2(-af - cd)\sqrt{c}}{(x + (-a\sqrt{c})/c)^{1/2}} \cdot \frac{f - 2f(-a\sqrt{c})/c}{(x + (-a\sqrt{c})/c)^{1/2}} \cdot \frac{B}{(af - cd)\sqrt{c}}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx + A}{(cx^2 + a)\sqrt{fx^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+d)^(1/2),x, algorithm="maxima")

[Out] integrate((B*x + A)/((c*x^2 + a)*sqrt(f*x^2 + d)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\left\{ \begin{array}{ll} \frac{B \operatorname{atan}\left(\frac{c\sqrt{fx^2+d}}{\sqrt{acf-c^2d}}\right)}{\sqrt{acf-c^2d}} + \frac{A \operatorname{atan}\left(\frac{x\sqrt{cd-af}}{\sqrt{a}\sqrt{fx^2+d}}\right)}{\sqrt{-a}(af-cd)} & \text{if } 0 < cd - af \\ \frac{A \ln\left(\frac{\sqrt{a}(fx^2+d)+x\sqrt{af-cd}}{\sqrt{a}(fx^2+d)-x\sqrt{af-cd}}\right)}{2\sqrt{a}(af-cd)} + \frac{B \operatorname{atan}\left(\frac{c\sqrt{fx^2+d}}{\sqrt{acf-c^2d}}\right)}{\sqrt{acf-c^2d}} & \text{if } cd - af < 0 \\ \int \frac{A+Bx}{(cx^2+a)\sqrt{fx^2+d}} dx & \text{if } cd - af \notin \mathbb{R} \vee af = cd \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x)/((a + c*x^2)*(d + f*x^2)^(1/2)),x)

[Out] piecewise(0 < - a*f + c*d, (B*atan((c*(d + f*x^2)^(1/2))/(- c^2*d + a*c*f)^(1/2)))/(- c^2*d + a*c*f)^(1/2) + (A*atan((x*(- a*f + c*d)^(1/2))/(a^(1/2)*(d + f*x^2)^(1/2))))/(-a*(a*f - c*d))^(1/2), - a*f + c*d < 0, (A*log(((a*(d + f*x^2)^(1/2) + x*(a*f - c*d)^(1/2))/((a*(d + f*x^2)^(1/2) - x*(a*f - c*d)^(1/2)))))/(2*(a*(a*f - c*d))^(1/2)) + (B*atan((c*(d + f*x^2)^(1/2))/(- c^2*d + a*c*f)^(1/2)))/(- c^2*d + a*c*f)^(1/2), ~in(- a*f + c*d, 'real') | a*f == c*d, int((A + B*x)/((a + c*x^2)*(d + f*x^2)^(1/2)), x))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx}{(a + cx^2)\sqrt{d + fx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(c*x**2+a)/(f*x**2+d)**(1/2),x)

[Out] Integral((A + B*x)/((a + c*x**2)*sqrt(d + f*x**2)), x)

$$3.25 \quad \int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x-2x^2}} dx$$

Optimal. Leaf size=139

$$\frac{1}{2}\sqrt{\sqrt{10}-\frac{13}{5}} \tan^{-1}\left(\frac{(1+4\sqrt{10})x+3(4-\sqrt{10})}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right) + \frac{1}{2}\sqrt{\frac{13}{5}+\sqrt{10}} \tanh^{-1}\left(\frac{(1-4\sqrt{10})x+3(4+\sqrt{10})}{2\sqrt{\sqrt{10}-1}\sqrt{-2x^2+3x+1}}\right)$$

Rubi [A] time = 0.22, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1032, 724, 204, 206}

$$\frac{1}{2}\sqrt{\sqrt{10}-\frac{13}{5}} \tan^{-1}\left(\frac{(1+4\sqrt{10})x+3(4-\sqrt{10})}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right) + \frac{1}{2}\sqrt{\frac{13}{5}+\sqrt{10}} \tanh^{-1}\left(\frac{(1-4\sqrt{10})x+3(4+\sqrt{10})}{2\sqrt{\sqrt{10}-1}\sqrt{-2x^2+3x+1}}\right)$$

Antiderivative was successfully verified.

[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x - 2*x^2]),x]

[Out] (Sqrt[-13/5 + Sqrt[10]]*ArcTan[(3*(4 - Sqrt[10]) + (1 + 4*Sqrt[10])*x)/(2*Sqrt[1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/2 + (Sqrt[13/5 + Sqrt[10]]*ArcTanh[(3*(4 + Sqrt[10]) + (1 - 4*Sqrt[10])*x)/(2*Sqrt[-1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/2

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1032

Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]

Rubi steps

$$\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x-2x^2}} dx = \frac{1}{5}(5-4\sqrt{10}) \int \frac{1}{(4-2\sqrt{10}-6x)\sqrt{1+3x-2x^2}} dx + \frac{1}{5}(5+4\sqrt{10}) \int \frac{1}{(4+2\sqrt{10}-6x)\sqrt{1+3x-2x^2}} dx$$

$$= -\left(\frac{1}{5}(2(5-4\sqrt{10}))\right) \text{Subst}\left(\int \frac{1}{144+72(4-2\sqrt{10})-8(4-2\sqrt{10})^2-x}\right)$$

$$= \frac{1}{10}\sqrt{-65+25\sqrt{10}} \tan^{-1}\left(\frac{3(4-\sqrt{10})+(1+4\sqrt{10})x}{2\sqrt{1+\sqrt{10}}\sqrt{1+3x-2x^2}}\right) + \frac{1}{10}\sqrt{65+25\sqrt{10}} \tan^{-1}\left(\frac{3(4+\sqrt{10})+(1+4\sqrt{10})x}{2\sqrt{1+\sqrt{10}}\sqrt{1+3x-2x^2}}\right)$$

Mathematica [A] time = 0.30, size = 140, normalized size = 1.01

$$\frac{(4\sqrt{10}-5) \tan^{-1}\left(\frac{4\sqrt{10}x+x-3\sqrt{10}+12}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right) + 3\sqrt{5(7+2\sqrt{10})} \tanh^{-1}\left(\frac{-4\sqrt{10}x+x+3(4+\sqrt{10})}{2\sqrt{\sqrt{10}-1}\sqrt{-2x^2+3x+1}}\right)}{10\sqrt{1+\sqrt{10}}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x - 2*x^2]), x]

[Out] ((-5 + 4*Sqrt[10])*ArcTan[(12 - 3*Sqrt[10] + x + 4*Sqrt[10]*x)/(2*Sqrt[1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])] + 3*Sqrt[5*(7 + 2*Sqrt[10])]*ArcTanh[(3*(4 + Sqrt[10]) + x - 4*Sqrt[10]*x)/(2*Sqrt[-1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/(10*Sqrt[1 + Sqrt[10]])

IntegrateAlgebraic [C] time = 0.34, size = 149, normalized size = 1.07

$$\frac{1}{2}\text{RootSum}\left[2\#1^4 - 8\#1^3 + 8\#1^2 + 20\#1 + 5\&, \frac{2\#1^2 \log(\#1(-x) + \sqrt{-2x^2 + 3x + 1} - 1) - 2\#1^2 \log(x) - 2\#1 \log(\#1(-x) + \sqrt{-2x^2 + 3x + 1} - 1) + 7 \log(\#1(-x) + \sqrt{-2x^2 + 3x + 1} - 1) + 2\#1 \log(x) - 7 \log(x)}{2\#1^3 - 6\#1^2 + 4\#1 + 5}\right]$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x - 2*x^2]), x]

[Out] -1/2*RootSum[5 + 20*#1 + 8*#1^2 - 8*#1^3 + 2*#1^4 &, (-7*Log[x] + 7*Log[-1 + Sqrt[1 + 3*x - 2*x^2] - x*#1] + 2*Log[x]*#1 - 2*Log[-1 + Sqrt[1 + 3*x - 2*x^2] - x*#1]*#1 - 2*Log[x]*#1^2 + 2*Log[-1 + Sqrt[1 + 3*x - 2*x^2] - x*#1]*#1^2)/(5 + 4*#1 - 6*#1^2 + 2*#1^3) &]

fricas [B] time = 0.44, size = 322, normalized size = 2.32

$$\frac{1}{2}\sqrt{5}\sqrt{5\sqrt{2}-13}\arctan\left(\frac{\sqrt{2}\sqrt{5}\sqrt{5\sqrt{2}-13}\sqrt{\frac{5\sqrt{2}\sqrt{5}\sqrt{5\sqrt{2}-13}\sqrt{5\sqrt{2}\sqrt{5}\sqrt{5\sqrt{2}-13}}}{18x}} + 2(\sqrt{2}\sqrt{5}\sqrt{5\sqrt{2}-13}\sqrt{5\sqrt{2}\sqrt{5}\sqrt{5\sqrt{2}-13}} + \sqrt{2}\sqrt{5}\sqrt{5\sqrt{2}-13}\sqrt{5\sqrt{2}\sqrt{5}\sqrt{5\sqrt{2}-13}})}{18x}\right) + \frac{1}{10}\sqrt{5}\sqrt{5\sqrt{2}-13}\log\left(\frac{4\sqrt{5}\sqrt{2}\sqrt{5}\sqrt{5\sqrt{2}-13}\sqrt{5\sqrt{2}\sqrt{5}\sqrt{5\sqrt{2}-13}} + 13\sqrt{5}\sqrt{2}\sqrt{5}\sqrt{5\sqrt{2}-13}\sqrt{5\sqrt{2}\sqrt{5}\sqrt{5\sqrt{2}-13}} + 18\sqrt{2}\sqrt{5}\sqrt{5}\sqrt{5\sqrt{2}-13}}{x}\right) + \frac{1}{10}\sqrt{5}\sqrt{5\sqrt{2}-13}\log\left(\frac{4\sqrt{5}\sqrt{2}\sqrt{5}\sqrt{5\sqrt{2}-13}\sqrt{5\sqrt{2}\sqrt{5}\sqrt{5\sqrt{2}-13}} + 13\sqrt{5}\sqrt{2}\sqrt{5}\sqrt{5\sqrt{2}-13}\sqrt{5\sqrt{2}\sqrt{5}\sqrt{5\sqrt{2}-13}} + 18\sqrt{2}\sqrt{5}\sqrt{5}\sqrt{5\sqrt{2}-13}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(1/2), x, algorithm="fricas")

[Out] 2/5*sqrt(5)*sqrt(5*sqrt(5)*sqrt(2) - 13)*arctan(1/18*(sqrt(2)*(2*sqrt(5)*x - sqrt(2)*x)*sqrt(5*sqrt(5)*sqrt(2) - 13)*sqrt((sqrt(5)*sqrt(2)*(3*x^2 + 2*x) + 6*x^2 - 2*(sqrt(5)*sqrt(2)*x + 2*x + 2)*sqrt(-2*x^2 + 3*x + 1) + 10*x + 4)/x^2) + 2*(sqrt(2)*(4*x - 1) + sqrt(5)*(x + 2) - sqrt(-2*x^2 + 3*x + 1)*(2*sqrt(5) - sqrt(2)))*sqrt(5*sqrt(5)*sqrt(2) - 13))/x) - 1/10*sqrt(5)*sqrt(5*sqrt(5)*sqrt(2) + 13)*log((9*sqrt(5)*sqrt(2)*x + (4*sqrt(5)*x - 7*sqrt(2)*x)*sqrt(5*sqrt(5)*sqrt(2) + 13) - 18*x + 18*sqrt(-2*x^2 + 3*x + 1) - 18)/x) + 1/10*sqrt(5)*sqrt(5*sqrt(5)*sqrt(2) + 13)*log((9*sqrt(5)*sqrt(2)*x - (4*sqrt(5)*x - 7*sqrt(2)*x)*sqrt(5*sqrt(5)*sqrt(2) + 13) - 18*x + 18*sqrt(-2*x^2 + 3*x + 1) - 18)/x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(1/2),x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.07, size = 324, normalized size = 2.33

$$\frac{2\sqrt{10} \operatorname{arctanh}\left(\frac{-1+\sqrt{10}+\sqrt{\frac{1}{2}\left(\frac{4\sqrt{10}}{2}\right)\left(-\frac{1}{2}+\frac{2\sqrt{10}}{2}\right)}}{\sqrt{-1+\sqrt{10}}\sqrt{-18\left(-\frac{1}{2}+\frac{2\sqrt{10}}{2}\right)+9\left(\frac{1}{2}\left(\frac{4\sqrt{10}}{2}\right)\left(-\frac{1}{2}+\frac{2\sqrt{10}}{2}\right)\right)+1+\sqrt{10}}}\right)}{5\sqrt{-1+\sqrt{10}}} + \frac{\operatorname{arctanh}\left(\frac{-1+\sqrt{10}+\sqrt{\frac{1}{2}\left(\frac{4\sqrt{10}}{2}\right)\left(-\frac{1}{2}+\frac{2\sqrt{10}}{2}\right)}}{\sqrt{-1+\sqrt{10}}\sqrt{-18\left(-\frac{1}{2}+\frac{2\sqrt{10}}{2}\right)+9\left(\frac{1}{2}\left(\frac{4\sqrt{10}}{2}\right)\left(-\frac{1}{2}+\frac{2\sqrt{10}}{2}\right)\right)+1+\sqrt{10}}}\right)}{2\sqrt{-1+\sqrt{10}}} + \frac{2\sqrt{10} \arctan\left(\frac{-1-\sqrt{10}+\sqrt{\frac{1}{2}\left(\frac{4\sqrt{10}}{2}\right)\left(-\frac{1}{2}+\frac{2\sqrt{10}}{2}\right)}}{\sqrt{1+\sqrt{10}}\sqrt{-18\left(-\frac{1}{2}+\frac{2\sqrt{10}}{2}\right)+9\left(\frac{1}{2}\left(\frac{4\sqrt{10}}{2}\right)\left(-\frac{1}{2}+\frac{2\sqrt{10}}{2}\right)\right)+1-\sqrt{10}}}\right)}{5\sqrt{1+\sqrt{10}}} - \frac{\arctan\left(\frac{-1-\sqrt{10}+\sqrt{\frac{1}{2}\left(\frac{4\sqrt{10}}{2}\right)\left(-\frac{1}{2}+\frac{2\sqrt{10}}{2}\right)}}{\sqrt{1+\sqrt{10}}\sqrt{-18\left(-\frac{1}{2}+\frac{2\sqrt{10}}{2}\right)+9\left(\frac{1}{2}\left(\frac{4\sqrt{10}}{2}\right)\left(-\frac{1}{2}+\frac{2\sqrt{10}}{2}\right)\right)+1-\sqrt{10}}}\right)}{2\sqrt{1+\sqrt{10}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(1/2),x)

[Out] $\frac{2}{5} \cdot 10^{(1/2)} / (1+10^{(1/2)})^{(1/2)} \cdot \arctan(9/2 \cdot (-2/9 - 2/9 \cdot 10^{(1/2)} + (1/3 + 4/3 \cdot 10^{(1/2)})(1/2)) \cdot (x - 2/3 + 1/3 \cdot 10^{(1/2)}) / (1+10^{(1/2)})^{(1/2)} / (-18 \cdot (x - 2/3 + 1/3 \cdot 10^{(1/2)})^2 + 9 \cdot (1/3 + 4/3 \cdot 10^{(1/2)}) \cdot (x - 2/3 + 1/3 \cdot 10^{(1/2)}) - 1 - 10^{(1/2)})^{(1/2)} - 1/2 / (1+10^{(1/2)})^{(1/2)} \cdot \arctan(9/2 \cdot (-2/9 - 2/9 \cdot 10^{(1/2)} + (1/3 + 4/3 \cdot 10^{(1/2)})(1/2)) \cdot (x - 2/3 + 1/3 \cdot 10^{(1/2)}) / (1+10^{(1/2)})^{(1/2)} / (-18 \cdot (x - 2/3 + 1/3 \cdot 10^{(1/2)})^2 + 9 \cdot (1/3 + 4/3 \cdot 10^{(1/2)}) \cdot (x - 2/3 + 1/3 \cdot 10^{(1/2)}) - 1 - 10^{(1/2)})^{(1/2)} + 2/5 \cdot 10^{(1/2)} / (-1+10^{(1/2)})^{(1/2)} \cdot \operatorname{arctanh}(9/2 \cdot (-2/9 + 2/9 \cdot 10^{(1/2)} + (1/3 - 4/3 \cdot 10^{(1/2)})(1/2)) \cdot (x - 2/3 - 1/3 \cdot 10^{(1/2)}) / (-1+10^{(1/2)})^{(1/2)} / (-18 \cdot (x - 2/3 - 1/3 \cdot 10^{(1/2)})^2 + 9 \cdot (1/3 - 4/3 \cdot 10^{(1/2)}) \cdot (x - 2/3 - 1/3 \cdot 10^{(1/2)}) - 1 + 10^{(1/2)})^{(1/2)} + 1/2 / (-1+10^{(1/2)})^{(1/2)} \cdot \operatorname{arctanh}(9/2 \cdot (-2/9 + 2/9 \cdot 10^{(1/2)} + (1/3 - 4/3 \cdot 10^{(1/2)})(1/2)) \cdot (x - 2/3 - 1/3 \cdot 10^{(1/2)}) / (-1+10^{(1/2)})^{(1/2)} / (-18 \cdot (x - 2/3 - 1/3 \cdot 10^{(1/2)})^2 + 9 \cdot (1/3 - 4/3 \cdot 10^{(1/2)}) \cdot (x - 2/3 - 1/3 \cdot 10^{(1/2)}) - 1 + 10^{(1/2)})^{(1/2)}$

maxima [B] time = 1.07, size = 361, normalized size = 2.60

$$\frac{1}{20} \sqrt{10} \left(\frac{\sqrt{10} \arcsin\left(\frac{8\sqrt{17}\sqrt{10}}{17\sqrt{10+2\sqrt{10}}-4} + \frac{2\sqrt{17}}{17\sqrt{10+2\sqrt{10}}-4} + \frac{24\sqrt{17}}{17\sqrt{10+2\sqrt{10}}-4}\right)}{\sqrt{10+1}} - \frac{\sqrt{10} \log\left(-\frac{1}{2}\sqrt{10} + \frac{2\sqrt{2+2\sqrt{10}}\sqrt{\sqrt{10}-1}}{3\sqrt{10+2\sqrt{10}}-4} + \frac{2\sqrt{10}}{9\sqrt{10+2\sqrt{10}}-4} + \frac{2}{9\sqrt{10+2\sqrt{10}}-4} + \frac{1}{3}\right)}{\sqrt{10-1}} - \frac{8 \arcsin\left(\frac{8\sqrt{17}\sqrt{10}}{17\sqrt{10+2\sqrt{10}}-4} + \frac{2\sqrt{17}}{17\sqrt{10+2\sqrt{10}}-4} + \frac{24\sqrt{17}}{17\sqrt{10+2\sqrt{10}}-4}\right)}{\sqrt{10+1}} - \frac{8 \log\left(-\frac{1}{2}\sqrt{10} + \frac{2\sqrt{2+2\sqrt{10}}\sqrt{\sqrt{10}-1}}{3\sqrt{10+2\sqrt{10}}-4} + \frac{2\sqrt{10}}{9\sqrt{10+2\sqrt{10}}-4} + \frac{2}{9\sqrt{10+2\sqrt{10}}-4} + \frac{1}{3}\right)}{\sqrt{10-1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(1/2),x, algorithm="maxima")

[Out] $-1/20 \cdot \sqrt{10} \cdot (\sqrt{10} \cdot \arcsin(8/17 \cdot \sqrt{17} \cdot \sqrt{10} \cdot x / \operatorname{abs}(6 \cdot x + 2 \cdot \sqrt{10}) - 4) + 2/17 \cdot \sqrt{17} \cdot x / \operatorname{abs}(6 \cdot x + 2 \cdot \sqrt{10}) - 4) - 6/17 \cdot \sqrt{17} \cdot \sqrt{10} / \operatorname{abs}(6 \cdot x + 2 \cdot \sqrt{10}) - 4) + 24/17 \cdot \sqrt{17} / \operatorname{abs}(6 \cdot x + 2 \cdot \sqrt{10}) - 4) / \sqrt{10} - \sqrt{10} \cdot \log(-2/9 \cdot \sqrt{10} + 2/3 \cdot \sqrt{-2 \cdot x^2 + 3 \cdot x + 1}) \cdot \sqrt{10} / \operatorname{abs}(6 \cdot x - 2 \cdot \sqrt{10}) - 4) + 2/9 \cdot \sqrt{10} / \operatorname{abs}(6 \cdot x - 2 \cdot \sqrt{10}) - 4) - 2/9 / \operatorname{abs}(6 \cdot x - 2 \cdot \sqrt{10}) - 4) + 1/18 / \sqrt{10} - 1) - 8 \cdot \arcsin(8/17 \cdot \sqrt{17} \cdot \sqrt{10} \cdot x / \operatorname{abs}(6 \cdot x + 2 \cdot \sqrt{10}) - 4) + 2/17 \cdot \sqrt{17} \cdot x / \operatorname{abs}(6 \cdot x + 2 \cdot \sqrt{10}) - 4) - 6/17 \cdot \sqrt{17} \cdot \sqrt{10} / \operatorname{abs}(6 \cdot x + 2 \cdot \sqrt{10}) - 4) + 24/17 \cdot \sqrt{17} / \operatorname{abs}(6 \cdot x + 2 \cdot \sqrt{10}) - 4) / \sqrt{10} - 8 \cdot \log(-2/9 \cdot \sqrt{10} + 2/3 \cdot \sqrt{-2 \cdot x^2 + 3 \cdot x + 1}) \cdot \sqrt{10} / \operatorname{abs}(6 \cdot x - 2 \cdot \sqrt{10}) - 4) + 2/9 \cdot \sqrt{10} / \operatorname{abs}(6 \cdot x - 2 \cdot \sqrt{10}) - 4) - 2/9 / \operatorname{abs}(6 \cdot x - 2 \cdot \sqrt{10}) - 4) + 1/18 / \sqrt{10} - 1)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+2}{\sqrt{-2x^2+3x+1}(-3x^2+4x+2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+2)/((3*x-2*x^2+1)^(1/2)*(4*x-3*x^2+2)),x)

[Out] `int((x + 2)/((3*x - 2*x^2 + 1)^(1/2)*(4*x - 3*x^2 + 2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{3x^2\sqrt{-2x^2+3x+1}-4x\sqrt{-2x^2+3x+1}-2\sqrt{-2x^2+3x+1}} dx - \int \frac{2}{3x^2\sqrt{-2x^2+3x+1}-4x\sqrt{-2x^2+3x+1}-2\sqrt{-2x^2+3x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2+x)/(-3*x**2+4*x+2)/(-2*x**2+3*x+1)**(1/2), x)`

[Out] `-Integral(x/(3*x**2*sqrt(-2*x**2 + 3*x + 1) - 4*x*sqrt(-2*x**2 + 3*x + 1) - 2*sqrt(-2*x**2 + 3*x + 1)), x) - Integral(2/(3*x**2*sqrt(-2*x**2 + 3*x + 1) - 4*x*sqrt(-2*x**2 + 3*x + 1) - 2*sqrt(-2*x**2 + 3*x + 1)), x)`

$$3.26 \quad \int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{3/2}} dx$$

Optimal. Leaf size=166

$$-\frac{2(14x+15)}{17\sqrt{-2x^2+3x+1}} - \frac{9}{2}\sqrt{\frac{1}{5}}(\sqrt{10}-3) \tan^{-1}\left(\frac{(1+4\sqrt{10})x+3(4-\sqrt{10})}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right) + \frac{9}{2}\sqrt{\frac{1}{5}}(3+\sqrt{10}) \tanh^{-1}\left(\frac{(1-4\sqrt{10})x+3(4+\sqrt{10})}{2\sqrt{1-\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)$$

Rubi [A] time = 0.22, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1016, 12, 1032, 724, 204, 206}

$$-\frac{2(14x+15)}{17\sqrt{-2x^2+3x+1}} - \frac{9}{2}\sqrt{\frac{1}{5}}(\sqrt{10}-3) \tan^{-1}\left(\frac{(1+4\sqrt{10})x+3(4-\sqrt{10})}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right) + \frac{9}{2}\sqrt{\frac{1}{5}}(3+\sqrt{10}) \tanh^{-1}\left(\frac{(1-4\sqrt{10})x+3(4+\sqrt{10})}{2\sqrt{1-\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)$$

Antiderivative was successfully verified.

[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(3/2)), x]

[Out] (-2*(15 + 14*x))/(17*sqrt[1 + 3*x - 2*x^2]) - (9*sqrt[(-3 + sqrt[10])/5])*ArcTan[(3*(4 - sqrt[10]) + (1 + 4*sqrt[10])*x)/(2*sqrt[1 + sqrt[10]]*sqrt[1 + 3*x - 2*x^2])]/2 + (9*sqrt[(3 + sqrt[10])/5])*ArcTanh[(3*(4 + sqrt[10]) + (1 - 4*sqrt[10])*x)/(2*sqrt[-1 + sqrt[10]]*sqrt[1 + 3*x - 2*x^2])]/2

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1016

Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*c*d - 2*a*c*e + a*b*f))*x)/(b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-h*c*e

```

))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
(-h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1])

```

Rule 1032

```

Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]

```

Rubi steps

$$\begin{aligned}
 \int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{3/2}} dx &= -\frac{2(15+14x)}{17\sqrt{1+3x-2x^2}} + \frac{2}{17} \int \frac{153x}{2(2+4x-3x^2)\sqrt{1+3x-2x^2}} dx \\
 &= -\frac{2(15+14x)}{17\sqrt{1+3x-2x^2}} + 9 \int \frac{x}{(2+4x-3x^2)\sqrt{1+3x-2x^2}} dx \\
 &= -\frac{2(15+14x)}{17\sqrt{1+3x-2x^2}} + \frac{1}{5} (9(5-\sqrt{10})) \int \frac{1}{(4-2\sqrt{10}-6x)\sqrt{1+3x-2x^2}} dx \\
 &= -\frac{2(15+14x)}{17\sqrt{1+3x-2x^2}} - \frac{1}{5} (18(5-\sqrt{10})) \text{Subst} \left[\int \frac{1}{144+72(4-2\sqrt{10})x} dx \right] \\
 &= -\frac{2(15+14x)}{17\sqrt{1+3x-2x^2}} - \frac{9}{2} \sqrt{\frac{1}{5}} (-3+\sqrt{10}) \tan^{-1} \left[\frac{3(4-\sqrt{10})+(1+4\sqrt{10})x}{2\sqrt{1+\sqrt{10}}\sqrt{1+3x-2x^2}} \right]
 \end{aligned}$$

Mathematica [A] time = 0.38, size = 167, normalized size = 1.01

$$\frac{1}{170} \left(153\sqrt{5(3+\sqrt{10})} \tanh^{-1} \left(\frac{-4\sqrt{10}x+x+3(4+\sqrt{10})}{2\sqrt{10}-1\sqrt{-2x^2+3x+1}} \right) - \frac{153\sqrt{5(\sqrt{10}-3)}\sqrt{-2x^2+3x+1} \tan^{-1} \left(\frac{4\sqrt{10}x+x-3\sqrt{10}+12}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}} \right) + 280x+300}{\sqrt{-2x^2+3x+1}} \right)$$

Antiderivative was successfully verified.

```

[In] Integrate[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(3/2)), x]
[Out] (-((300 + 280*x + 153*Sqrt[5*(-3 + Sqrt[10])])*Sqrt[1 + 3*x - 2*x^2]*ArcTan[
(12 - 3*Sqrt[10] + x + 4*Sqrt[10]*x)/(2*Sqrt[1 + Sqrt[10]]*Sqrt[1 + 3*x - 2
*x^2])])/Sqrt[1 + 3*x - 2*x^2]) + 153*Sqrt[5*(3 + Sqrt[10])]*ArcTanh[(3*(4
+ Sqrt[10]) + x - 4*Sqrt[10]*x)/(2*Sqrt[-1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2
])])/170

```

IntegrateAlgebraic [C] time = 0.39, size = 149, normalized size = 0.90

$$\frac{9}{2} \text{RootSum} \left[2\#1^4 - 8\#1^3 + 8\#1^2 + 20\#1 + 5\&, \frac{-3 \log(\#1(-x) + \sqrt{-2x^2+3x+1}-1) + 2\#1 \log(\#1(-x) + \sqrt{-2x^2+3x+1}-1) - 2\#1 \log(x) + 3 \log(x)}{2\#1^3 - 6\#1^2 + 4\#1 + 5} \& \right] + \frac{2\sqrt{-2x^2+3x+1}(14x+15)}{17(2x^2-3x-1)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(3/2)),x]
```

```
[Out] (2*(15 + 14*x)*Sqrt[1 + 3*x - 2*x^2])/((17*(-1 - 3*x + 2*x^2)) + (9*RootSum[5 + 20*#1 + 8*#1^2 - 8*#1^3 + 2*#1^4 & , (3*Log[x] - 3*Log[-1 + Sqrt[1 + 3*x - 2*x^2] - x*#1] - 2*Log[x]*#1 + 2*Log[-1 + Sqrt[1 + 3*x - 2*x^2] - x*#1]*#1)/(5 + 4*#1 - 6*#1^2 + 2*#1^3) & ])/2
```

fricas [B] time = 0.44, size = 344, normalized size = 2.07

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/170*(612*sqrt(5)*(2*x^2 - 3*x - 1)*sqrt(sqrt(10) - 3)*arctan(1/10*(sqrt(10)*sqrt(5)*sqrt(2)*x*sqrt(sqrt(10) - 3)*sqrt((6*x^2 + sqrt(10)*(3*x^2 + 2*x) - 2*sqrt(-2*x^2 + 3*x + 1)*(sqrt(10)*x + 2*x + 2) + 10*x + 4)/x^2) + 2*(sqrt(10)*sqrt(5)*(x + 1) - sqrt(10)*sqrt(5)*sqrt(-2*x^2 + 3*x + 1) + 5*sqrt(5)*x)*sqrt(sqrt(10) - 3))/x + 153*sqrt(5)*(2*x^2 - 3*x - 1)*sqrt(sqrt(10) + 3)*log(9*(5*sqrt(10)*x + (3*sqrt(10)*sqrt(5)*x - 10*sqrt(5)*x)*sqrt(sqrt(10) + 3) - 10*x + 10*sqrt(-2*x^2 + 3*x + 1) - 10)/x) - 153*sqrt(5)*(2*x^2 - 3*x - 1)*sqrt(sqrt(10) + 3)*log(9*(5*sqrt(10)*x - (3*sqrt(10)*sqrt(5)*x - 10*sqrt(5)*x)*sqrt(sqrt(10) + 3) - 10*x + 10*sqrt(-2*x^2 + 3*x + 1) - 10)/x) + 600*x^2 - 20*sqrt(-2*x^2 + 3*x + 1)*(14*x + 15) - 900*x - 300)/(2*x^2 - 3*x - 1)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.03, size = 760, normalized size = 4.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(3/2),x)
```

```
[Out] 26/255*10^(1/2)/(-1/9-1/9*10^(1/2))/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(1/2)+32/765/(-1/9-1/9*10^(1/2))/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(1/2)*10^(1/2)*x-62/153/(-1/9-1/9*10^(1/2))/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(1/2)*x+7/51/(-1/9-1/9*10^(1/2))/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(1/2)+2/5*10^(1/2)/(-1/9-1/9*10^(1/2))/((1+10^(1/2))^(1/2)*arctan(9/2*(-2/9-2/9*10^(1/2)+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))))/(1+10^(1/2))^(1/2)/(-18*(x-2/3+1/3*10^(1/2))^2+9*(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1-10^(1/2))^(1/2)-1/2/(-1/9-1/9*10^(1/2))/((1+10^(1/2))^(1/2)*arctan(9/2*(-2/9-2/9*10^(1/2)+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))))/(1+10^(1/2))^(1/2)/(-18*(x-2/3+1/3*10^(1/2))^2+9*(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1-10^(1/2))^(1/2)-26/255*10^(1/2)/(-1/9+1/9*10^(1/2))/(-2*(x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))^(1/2)-32/765/(-1/9+1/9*10^(1/2))/(-2*(x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))^(1/2)*10^(1/2)*x
```

$$-62/153/(-1/9+1/9*10^{(1/2)})/(-2*(x-2/3-1/3*10^{(1/2)})^2+(1/3-4/3*10^{(1/2)})*(x-2/3-1/3*10^{(1/2)})-1/9+1/9*10^{(1/2)})^{(1/2)}*x+7/51/(-1/9+1/9*10^{(1/2)})/(-2*(x-2/3-1/3*10^{(1/2)})^2+(1/3-4/3*10^{(1/2)})*(x-2/3-1/3*10^{(1/2)})-1/9+1/9*10^{(1/2)})^{(1/2)}+2/5*10^{(1/2)}/(-1/9+1/9*10^{(1/2)})/(-1+10^{(1/2)})^{(1/2)}*\operatorname{arctanh}(9/2*(-2/9+2/9*10^{(1/2)}+(1/3-4/3*10^{(1/2)})*(x-2/3-1/3*10^{(1/2)})))/(-1+10^{(1/2)})^{(1/2)}/(-18*(x-2/3-1/3*10^{(1/2)})^2+9*(1/3-4/3*10^{(1/2)})*(x-2/3-1/3*10^{(1/2)})-1+10^{(1/2)})^{(1/2)}+1/2/(-1/9+1/9*10^{(1/2)})/(-1+10^{(1/2)})^{(1/2)}*\operatorname{arctanh}(9/2*(-2/9+2/9*10^{(1/2)}+(1/3-4/3*10^{(1/2)})*(x-2/3-1/3*10^{(1/2)})))/(-1+10^{(1/2)})^{(1/2)}/(-18*(x-2/3-1/3*10^{(1/2)})^2+9*(1/3-4/3*10^{(1/2)})*(x-2/3-1/3*10^{(1/2)})-1+10^{(1/2)})^{(1/2)}$$

maxima [B] time = 1.11, size = 678, normalized size = 4.08



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(3/2),x, algorithm="maxima")

[Out]
$$\frac{1}{340}\sqrt{10}*(124\sqrt{10}x/(\sqrt{10}\sqrt{-2x^2+3x+1})+\sqrt{-2x^2+3x+1})-124\sqrt{10}x/(\sqrt{10}\sqrt{-2x^2+3x+1})-\sqrt{-2x^2+3x+1}+153\sqrt{10}\arcsin(8/17\sqrt{17}\sqrt{10}x/|6x+2\sqrt{10}-4|)+2/17\sqrt{17}x/|6x+2\sqrt{10}-4|-6/17\sqrt{17}\sqrt{10}/|6x+2\sqrt{10}-4|+24/17\sqrt{17}/|6x+2\sqrt{10}-4|)/(\sqrt{10}\sqrt{\sqrt{10}+1}+\sqrt{\sqrt{10}+1})-128x/(\sqrt{10}\sqrt{-2x^2+3x+1}+\sqrt{-2x^2+3x+1})-128x/(\sqrt{10}\sqrt{-2x^2+3x+1})-\sqrt{-2x^2+3x+1}-1224\arcsin(8/17\sqrt{17}\sqrt{10}x/|6x+2\sqrt{10}-4|)+2/17\sqrt{17}x/|6x+2\sqrt{10}-4|-6/17\sqrt{17}\sqrt{10}/|6x+2\sqrt{10}-4|+24/17\sqrt{17}/|6x+2\sqrt{10}-4|)/(\sqrt{10}\sqrt{\sqrt{10}+1}+\sqrt{\sqrt{10}+1})+153\sqrt{10}\log(-2/9\sqrt{10}+2/3\sqrt{-2x^2+3x+1}\sqrt{\sqrt{10}-1})/|6x-2\sqrt{10}-4|+2/9\sqrt{10}/|6x-2\sqrt{10}-4|-2/9/|6x-2\sqrt{10}-4|+1/18)/(\sqrt{10}-1)^{3/2}-42\sqrt{10}/(\sqrt{10}\sqrt{-2x^2+3x+1}+\sqrt{-2x^2+3x+1})+42\sqrt{10}/(\sqrt{10}\sqrt{-2x^2+3x+1})-\sqrt{-2x^2+3x+1}+1224\log(-2/9\sqrt{10}+2/3\sqrt{-2x^2+3x+1}\sqrt{\sqrt{10}-1})/|6x-2\sqrt{10}-4|+2/9\sqrt{10}/|6x-2\sqrt{10}-4|-2/9/|6x-2\sqrt{10}-4|+1/18)/(\sqrt{10}-1)^{3/2}-312/(\sqrt{10}\sqrt{-2x^2+3x+1})+\sqrt{-2x^2+3x+1}-312/(\sqrt{10}\sqrt{-2x^2+3x+1})-\sqrt{-2x^2+3x+1}$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+2}{(-2x^2+3x+1)^{3/2}(-3x^2+4x+2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+2)/((3*x-2*x^2+1)^(3/2)*(4*x-3*x^2+2)),x)

[Out] int((x+2)/((3*x-2*x^2+1)^(3/2)*(4*x-3*x^2+2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-6x^4\sqrt{-2x^2+3x+1}+17x^3\sqrt{-2x^2+3x+1}-5x^2\sqrt{-2x^2+3x+1}-10x\sqrt{-2x^2+3x+1}-2\sqrt{-2x^2+3x+1}} dx - \int \frac{2}{-6x^4\sqrt{-2x^2+3x+1}+17x^3\sqrt{-2x^2+3x+1}-5x^2\sqrt{-2x^2+3x+1}-10x\sqrt{-2x^2+3x+1}-2\sqrt{-2x^2+3x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(-3*x**2+4*x+2)/(-2*x**2+3*x+1)**(3/2),x)

[Out] -Integral(x/(-6*x**4*sqrt(-2*x**2+3*x+1)+17*x**3*sqrt(-2*x**2+3*x+1)-5*x**2*sqrt(-2*x**2+3*x+1)-10*x*sqrt(-2*x**2+3*x+1)-2*sqrt

$t(-2x^2 + 3x + 1), x) - \text{Integral}(2/(-6x^4\sqrt{-2x^2 + 3x + 1}) + 17x^3\sqrt{-2x^2 + 3x + 1} - 5x^2\sqrt{-2x^2 + 3x + 1} - 10x\sqrt{-2x^2 + 3x + 1} - 2\sqrt{-2x^2 + 3x + 1}), x)$

$$3.27 \quad \int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{5/2}} dx$$

Optimal. Leaf size=193

$$-\frac{2(14x+15)}{51(-2x^2+3x+1)^{3/2}} - \frac{2(4814x+291)}{867\sqrt{-2x^2+3x+1}} + \frac{9}{2}\sqrt{\frac{1}{5}(17\sqrt{10}-53)} \tan^{-1}\left(\frac{(1+4\sqrt{10})x+3(4-\sqrt{10})}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right) + \frac{9}{2}\sqrt{\frac{1}{5}(53+17\sqrt{10})} \tanh^{-1}\left(\frac{(1-4\sqrt{10})x+3(4+\sqrt{10})}{2\sqrt{\sqrt{10}-1}\sqrt{-2x^2+3x+1}}\right)$$

Rubi [A] time = 0.27, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1016, 1060, 1032, 724, 204, 206}

$$-\frac{2(14x+15)}{51(-2x^2+3x+1)^{3/2}} - \frac{2(4814x+291)}{867\sqrt{-2x^2+3x+1}} + \frac{9}{2}\sqrt{\frac{1}{5}(17\sqrt{10}-53)} \tan^{-1}\left(\frac{(1+4\sqrt{10})x+3(4-\sqrt{10})}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right) + \frac{9}{2}\sqrt{\frac{1}{5}(53+17\sqrt{10})} \tanh^{-1}\left(\frac{(1-4\sqrt{10})x+3(4+\sqrt{10})}{2\sqrt{\sqrt{10}-1}\sqrt{-2x^2+3x+1}}\right)$$

Antiderivative was successfully verified.

[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(5/2)), x]

[Out] (-2*(15 + 14*x))/(51*(1 + 3*x - 2*x^2)^(3/2)) - (2*(291 + 4814*x))/(867*sqrt[1 + 3*x - 2*x^2]) + (9*sqrt[(-53 + 17*sqrt[10])/5]*ArcTan[(3*(4 - sqrt[10]) + (1 + 4*sqrt[10])*x)/(2*sqrt[1 + sqrt[10]]*sqrt[1 + 3*x - 2*x^2])])/2 + (9*sqrt[(53 + 17*sqrt[10])/5]*ArcTanh[(3*(4 + sqrt[10]) + (1 - 4*sqrt[10])*x)/(2*sqrt[-1 + sqrt[10]]*sqrt[1 + 3*x - 2*x^2])])/2

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1016

Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*c*d - 2*a*c*e + a*b*f))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a

```
(-(h*c*e)))*(b*f*(p + 1) - c*e*(2*p + q + 4))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f
)* (p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))* (p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))* (p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
+ q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{5/2}} dx &= -\frac{2(15+14x)}{51(1+3x-2x^2)^{3/2}} + \frac{2}{51} \int \frac{-56 + \frac{235x}{2} + 84x^2}{(2+4x-3x^2)(1+3x-2x^2)^{3/2}} dx \\
&= -\frac{2(15+14x)}{51(1+3x-2x^2)^{3/2}} - \frac{2(291+4814x)}{867\sqrt{1+3x-2x^2}} + \frac{4}{867} \int \frac{\frac{7803}{2} + \dots}{(2+4x-3x^2)} dx \\
&= -\frac{2(15+14x)}{51(1+3x-2x^2)^{3/2}} - \frac{2(291+4814x)}{867\sqrt{1+3x-2x^2}} + \frac{1}{5} (27(5-2\sqrt{10})) \int \dots \\
&= -\frac{2(15+14x)}{51(1+3x-2x^2)^{3/2}} - \frac{2(291+4814x)}{867\sqrt{1+3x-2x^2}} - \frac{1}{5} (54(5-2\sqrt{10})) \text{Sul} \\
&= -\frac{2(15+14x)}{51(1+3x-2x^2)^{3/2}} - \frac{2(291+4814x)}{867\sqrt{1+3x-2x^2}} + \frac{9}{2} \sqrt{\frac{1}{5}(-53+17\sqrt{10})}
\end{aligned}$$

Mathematica [A] time = 0.60, size = 185, normalized size = 0.96

$$\frac{3}{10} \sqrt{1 + \sqrt{10}} (7\sqrt{10} - 25) \tan^{-1} \left(\frac{3(\sqrt{10} - 4) - (1 + 4\sqrt{10})x}{2\sqrt{1 + \sqrt{10}} \sqrt{-2x^2 + 3x + 1}} \right) - \frac{3}{10} \sqrt{\sqrt{10} - 1} (25 + 7\sqrt{10}) \tanh^{-1} \left(\frac{(4\sqrt{10} - 1)x - 3(4 + \sqrt{10})}{2\sqrt{\sqrt{10} - 1} \sqrt{-2x^2 + 3x + 1}} \right) - \frac{2(-9628x^3 + 13860x^2 + 5925x + 546)}{867(-2x^2 + 3x + 1)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(5/2)), x]

[Out] (-2*(546 + 5925*x + 13860*x^2 - 9628*x^3))/(867*(1 + 3*x - 2*x^2)^(3/2)) + (3*Sqrt[1 + Sqrt[10]]*(-25 + 7*Sqrt[10])*ArcTan[(3*(-4 + Sqrt[10]) - (1 + 4*Sqrt[10])*x)/(2*Sqrt[1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/10 - (3*Sqrt[-1 + Sqrt[10]]*(25 + 7*Sqrt[10])*ArcTanh[(-3*(4 + Sqrt[10]) + (-1 + 4*Sqrt[10])*x)/(2*Sqrt[-1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/10

IntegrateAlgebraic [C] time = 0.54, size = 195, normalized size = 1.01

$$\frac{2\sqrt{-2x^2 + 3x + 1} (9628x^3 - 13860x^2 - 5925x - 546)}{867(2x^2 - 3x - 1)^2} - \frac{9}{2} \text{RootSum} \left[2\#1^4 - 8\#1^3 + 8\#1^2 + 20\#1 + 5\&, \frac{2\#1^2 \log(\#1(-x) + \sqrt{-2x^2 + 3x + 1} - 1) - 2\#1^2 \log(x) - 6\#1 \log(\#1(-x) + \sqrt{-2x^2 + 3x + 1} - 1) + 13 \log(\#1(-x) + \sqrt{-2x^2 + 3x + 1} - 1) + 6\#1 \log(x) - 13 \log(x)}{2\#1^3 - 6\#1^2 + 4\#1 + 5} \right]$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(5/2)), x]

[Out] (2*Sqrt[1 + 3*x - 2*x^2]*(-546 - 5925*x - 13860*x^2 + 9628*x^3))/(867*(-1 - 3*x + 2*x^2)^2) - (9*RootSum[5 + 20*#1 + 8*#1^2 - 8*#1^3 + 2*#1^4 &, (-13*Log[x] + 13*Log[-1 + Sqrt[1 + 3*x - 2*x^2] - x*#1] + 6*Log[x]*#1 - 6*Log[-1 + Sqrt[1 + 3*x - 2*x^2] - x*#1]*#1 - 2*Log[x]*#1^2 + 2*Log[-1 + Sqrt[1 + 3*x - 2*x^2] - x*#1]*#1^2)/(5 + 4*#1 - 6*#1^2 + 2*#1^3) &])/2

fricas [B] time = 0.44, size = 439, normalized size = 2.27

$$\frac{2\sqrt{-2x^2 + 3x + 1} (9628x^3 - 13860x^2 - 5925x - 546)}{867(2x^2 - 3x - 1)^2} - \frac{9}{2} \text{RootSum} \left[2\#1^4 - 8\#1^3 + 8\#1^2 + 20\#1 + 5\&, \frac{2\#1^2 \log(\#1(-x) + \sqrt{-2x^2 + 3x + 1} - 1) - 2\#1^2 \log(x) - 6\#1 \log(\#1(-x) + \sqrt{-2x^2 + 3x + 1} - 1) + 13 \log(\#1(-x) + \sqrt{-2x^2 + 3x + 1} - 1) + 6\#1 \log(x) - 13 \log(x)}{2\#1^3 - 6\#1^2 + 4\#1 + 5} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(5/2), x, algorithm="fricas")

[Out] -1/8670*(43680*x^4 - 131040*x^3 - 31212*sqrt(5)*(4*x^4 - 12*x^3 + 5*x^2 + 6*x + 1)*sqrt(17*sqrt(10) - 53)*arctan(1/90*(sqrt(2)*(sqrt(10)*sqrt(5)*x + 10*sqrt(5)*x)*sqrt(17*sqrt(10) - 53)*sqrt((6*x^2 + sqrt(10)*(3*x^2 + 2*x) - 2*sqrt(-2*x^2 + 3*x + 1)*(sqrt(10)*x + 2*x + 2) + 10*x + 4)/x^2) + 2*(sqrt(10)*sqrt(5)*(6*x + 1) - sqrt(-2*x^2 + 3*x + 1)*(sqrt(10)*sqrt(5) + 10*sqrt(5))) + 5*sqrt(5)*(3*x + 2))*sqrt(17*sqrt(10) - 53))/x - 7803*sqrt(5)*(4*x^4 - 12*x^3 + 5*x^2 + 6*x + 1)*sqrt(17*sqrt(10) + 53)*log(9*(45*sqrt(10)*x + (13*sqrt(10)*sqrt(5)*x - 40*sqrt(5)*x)*sqrt(17*sqrt(10) + 53) - 90*x + 90*sqrt(-2*x^2 + 3*x + 1) - 90)/x) + 7803*sqrt(5)*(4*x^4 - 12*x^3 + 5*x^2 + 6*x + 1)*sqrt(17*sqrt(10) + 53)*log(9*(45*sqrt(10)*x - (13*sqrt(10)*sqrt(5)*x - 40*sqrt(5)*x)*sqrt(17*sqrt(10) + 53) - 90*x + 90*sqrt(-2*x^2 + 3*x + 1) - 90)/x) + 54600*x^2 - 20*(9628*x^3 - 13860*x^2 - 5925*x - 546)*sqrt(-2*x^2 + 3*x + 1) + 65520*x + 10920)/(4*x^4 - 12*x^3 + 5*x^2 + 6*x + 1)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(5/2), x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.02, size = 1560, normalized size = 8.08

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(5/2), x)$

[Out]
$$\frac{248}{2601} \frac{(-1/9 - 1/9 \cdot 10^{1/2})}{(-2 \cdot (x - 2/3 + 1/3 \cdot 10^{1/2})^2 + (1/3 + 4/3 \cdot 10^{1/2}) \cdot (x - 2/3 + 1/3 \cdot 10^{1/2}) - 1/9 - 1/9 \cdot 10^{1/2})^{1/2}} + \frac{248}{2601} \frac{(-1/9 + 1/9 \cdot 10^{1/2})}{(-2 \cdot (x - 2/3 - 1/3 \cdot 10^{1/2})^2 + (1/3 - 4/3 \cdot 10^{1/2}) \cdot (x - 2/3 - 1/3 \cdot 10^{1/2}) - 1/9 + 1/9 \cdot 10^{1/2})^{1/2}} + \frac{7}{153} \frac{(-1/9 + 1/9 \cdot 10^{1/2})}{(-2 \cdot (x - 2/3 - 1/3 \cdot 10^{1/2})^2 + (1/3 - 4/3 \cdot 10^{1/2}) \cdot (x - 2/3 - 1/3 \cdot 10^{1/2}) - 1/9 + 1/9 \cdot 10^{1/2})^{3/2}} + \frac{7}{153} \frac{(-1/9 - 1/9 \cdot 10^{1/2})}{(-2 \cdot (x - 2/3 + 1/3 \cdot 10^{1/2})^2 + (1/3 + 4/3 \cdot 10^{1/2}) \cdot (x - 2/3 + 1/3 \cdot 10^{1/2}) - 1/9 - 1/9 \cdot 10^{1/2})^{3/2}} + \frac{7}{51} \frac{(-1/9 + 1/9 \cdot 10^{1/2})^2}{(-2 \cdot (x - 2/3 - 1/3 \cdot 10^{1/2})^2 + (1/3 - 4/3 \cdot 10^{1/2}) \cdot (x - 2/3 - 1/3 \cdot 10^{1/2}) - 1/9 + 1/9 \cdot 10^{1/2})^{1/2}} + \frac{7}{51} \frac{(-1/9 - 1/9 \cdot 10^{1/2})^2}{(-2 \cdot (x - 2/3 + 1/3 \cdot 10^{1/2})^2 + (1/3 + 4/3 \cdot 10^{1/2}) \cdot (x - 2/3 + 1/3 \cdot 10^{1/2}) - 1/9 - 1/9 \cdot 10^{1/2})^{1/2}} - \frac{32}{765} \frac{(-1/9 + 1/9 \cdot 10^{1/2})^2}{(-2 \cdot (x - 2/3 - 1/3 \cdot 10^{1/2})^2 + (1/3 - 4/3 \cdot 10^{1/2}) \cdot (x - 2/3 - 1/3 \cdot 10^{1/2}) - 1/9 + 1/9 \cdot 10^{1/2})^{1/2}} - \frac{32}{2295} \frac{(-1/9 + 1/9 \cdot 10^{1/2}) \cdot 10^{1/2}}{(-2 \cdot (x - 2/3 - 1/3 \cdot 10^{1/2})^2 + (1/3 - 4/3 \cdot 10^{1/2}) \cdot (x - 2/3 - 1/3 \cdot 10^{1/2}) - 1/9 + 1/9 \cdot 10^{1/2})^{3/2}} \cdot x + \frac{2}{5} \cdot 10^{1/2} \frac{(-1/9 - 1/9 \cdot 10^{1/2})^2}{(-1 + 10^{1/2})^{1/2}} \cdot \arctan\left(\frac{9}{2} \cdot (-2/9 - 2/9 \cdot 10^{1/2}) + (1/3 + 4/3 \cdot 10^{1/2}) \cdot (x - 2/3 + 1/3 \cdot 10^{1/2})\right) \frac{1}{(1 + 10^{1/2})^{1/2}} - \frac{18 \cdot (x - 2/3 + 1/3 \cdot 10^{1/2})^2 + 9 \cdot (1/3 + 4/3 \cdot 10^{1/2}) \cdot (x - 2/3 + 1/3 \cdot 10^{1/2}) - 1 - 10^{1/2}}{(-1 + 10^{1/2})^{1/2}} + \frac{32}{2295} \frac{(-1/9 - 1/9 \cdot 10^{1/2}) \cdot 10^{1/2}}{(-2 \cdot (x - 2/3 + 1/3 \cdot 10^{1/2})^2 + (1/3 + 4/3 \cdot 10^{1/2}) \cdot (x - 2/3 + 1/3 \cdot 10^{1/2}) - 1/9 - 1/9 \cdot 10^{1/2})^{3/2}} \cdot x + \frac{32}{765} \frac{(-1/9 - 1/9 \cdot 10^{1/2})^2}{(-2 \cdot (x - 2/3 + 1/3 \cdot 10^{1/2})^2 + (1/3 + 4/3 \cdot 10^{1/2}) \cdot (x - 2/3 + 1/3 \cdot 10^{1/2}) - 1/9 - 1/9 \cdot 10^{1/2})^{1/2}} \cdot 10^{1/2} \cdot x + \frac{2}{5} \cdot 10^{1/2} \frac{(-1/9 + 1/9 \cdot 10^{1/2})^2}{(-1 + 10^{1/2})^{1/2}} \cdot \operatorname{arctanh}\left(\frac{9}{2} \cdot (-2/9 + 2/9 \cdot 10^{1/2}) + (1/3 - 4/3 \cdot 10^{1/2}) \cdot (x - 2/3 - 1/3 \cdot 10^{1/2})\right) \frac{1}{(-1 + 10^{1/2})^{1/2}} - \frac{18 \cdot (x - 2/3 - 1/3 \cdot 10^{1/2})^2 + 9 \cdot (1/3 - 4/3 \cdot 10^{1/2}) \cdot (x - 2/3 - 1/3 \cdot 10^{1/2}) - 1 + 10^{1/2}}{(-1 + 10^{1/2})^{1/2}} + \frac{512}{39015} \frac{(-1/9 - 1/9 \cdot 10^{1/2})}{(-2 \cdot (x - 2/3 + 1/3 \cdot 10^{1/2})^2 + (1/3 + 4/3 \cdot 10^{1/2}) \cdot (x - 2/3 + 1/3 \cdot 10^{1/2}) - 1/9 - 1/9 \cdot 10^{1/2})^{1/2}} \cdot 10^{1/2} \cdot x - \frac{512}{39015} \frac{(-1/9 + 1/9 \cdot 10^{1/2})}{(-2 \cdot (x - 2/3 - 1/3 \cdot 10^{1/2})^2 + (1/3 - 4/3 \cdot 10^{1/2}) \cdot (x - 2/3 - 1/3 \cdot 10^{1/2}) - 1/9 + 1/9 \cdot 10^{1/2})^{1/2}} \cdot 10^{1/2} \cdot x - \frac{62}{153} \frac{(-1/9 - 1/9 \cdot 10^{1/2})^2}{(-2 \cdot (x - 2/3 + 1/3 \cdot 10^{1/2})^2 + (1/3 + 4/3 \cdot 10^{1/2}) \cdot (x - 2/3 + 1/3 \cdot 10^{1/2}) - 1/9 - 1/9 \cdot 10^{1/2})^{1/2}} \cdot x + \frac{26}{255} \frac{(-1/9 - 1/9 \cdot 10^{1/2})^2}{(-2 \cdot (x - 2/3 + 1/3 \cdot 10^{1/2})^2 + (1/3 + 4/3 \cdot 10^{1/2}) \cdot (x - 2/3 + 1/3 \cdot 10^{1/2}) - 1/9 - 1/9 \cdot 10^{1/2})^{1/2}} \cdot (-1/9 - 1/9 \cdot 10^{1/2})^{1/2} \cdot 10^{1/2} - \frac{1}{2} \frac{(-1/9 - 1/9 \cdot 10^{1/2})^2}{(1 + 10^{1/2})^{1/2}} \cdot \arctan\left(\frac{9}{2} \cdot (-2/9 - 2/9 \cdot 10^{1/2}) + (1/3 + 4/3 \cdot 10^{1/2}) \cdot (x - 2/3 + 1/3 \cdot 10^{1/2})\right) \frac{1}{(1 + 10^{1/2})^{1/2}} - \frac{18 \cdot (x - 2/3 + 1/3 \cdot 10^{1/2})^2 + 9 \cdot (1/3 + 4/3 \cdot 10^{1/2}) \cdot (x - 2/3 + 1/3 \cdot 10^{1/2}) - 1 - 10^{1/2}}{(-1 + 10^{1/2})^{1/2}} - \frac{62}{459} \frac{(-1/9 - 1/9 \cdot 10^{1/2})}{(-2 \cdot (x - 2/3 + 1/3 \cdot 10^{1/2})^2 + (1/3 + 4/3 \cdot 10^{1/2}) \cdot (x - 2/3 + 1/3 \cdot 10^{1/2}) - 1/9 - 1/9 \cdot 10^{1/2})^{3/2}} \cdot x + \frac{26}{765} \frac{(-1/9 - 1/9 \cdot 10^{1/2}) \cdot 10^{1/2}}{(-2 \cdot (x - 2/3 + 1/3 \cdot 10^{1/2})^2 + (1/3 + 4/3 \cdot 10^{1/2}) \cdot (x - 2/3 + 1/3 \cdot 10^{1/2}) - 1/9 - 1/9 \cdot 10^{1/2})^{3/2}} - \frac{26}{255} \frac{(-1/9 + 1/9 \cdot 10^{1/2})^2}{(-2 \cdot (x - 2/3 - 1/3 \cdot 10^{1/2})^2 + (1/3 - 4/3 \cdot 10^{1/2}) \cdot (x - 2/3 - 1/3 \cdot 10^{1/2}) - 1/9 + 1/9 \cdot 10^{1/2})^{1/2}} \cdot 10^{1/2} + \frac{1}{2} \frac{(-1/9 + 1/9 \cdot 10^{1/2})^2}{(-1 + 10^{1/2})^{1/2}} \cdot \operatorname{arctanh}\left(\frac{9}{2} \cdot (-2/9 + 2/9 \cdot 10^{1/2}) + (1/3 - 4/3 \cdot 10^{1/2}) \cdot (x - 2/3 - 1/3 \cdot 10^{1/2})\right) \frac{1}{(-1 + 10^{1/2})^{1/2}} - \frac{18 \cdot (x - 2/3 - 1/3 \cdot 10^{1/2})^2 + 9 \cdot (1/3 - 4/3 \cdot 10^{1/2}) \cdot (x - 2/3 - 1/3 \cdot 10^{1/2}) - 1 + 10^{1/2}}{(-1 + 10^{1/2})^{1/2}} - \frac{26}{765} \frac{(-1/9 + 1/9 \cdot 10^{1/2}) \cdot 10^{1/2}}{(-2 \cdot (x - 2/3 - 1/3 \cdot 10^{1/2})^2 + (1/3 - 4/3 \cdot 10^{1/2}) \cdot (x - 2/3 - 1/3 \cdot 10^{1/2}) - 1/9 + 1/9 \cdot 10^{1/2})^{3/2}} - \frac{62}{153} \frac{(-1/9 + 1/9 \cdot 10^{1/2})^2}{(-2 \cdot (x - 2/3 - 1/3 \cdot 10^{1/2})^2 + (1/3 - 4/3 \cdot 10^{1/2}) \cdot (x - 2/3 - 1/3 \cdot 10^{1/2}) - 1/9 + 1/9 \cdot 10^{1/2})^{1/2}} \cdot x - \frac{62}{459} \frac{(-1/9 + 1/9 \cdot 10^{1/2})}{(-2 \cdot (x - 2/3 - 1/3 \cdot 10^{1/2})^2 + (1/3 - 4/3 \cdot 10^{1/2}) \cdot (x - 2/3 - 1/3 \cdot 10^{1/2}) - 1/9 + 1/9 \cdot 10^{1/2})^{3/2}} \cdot x + \frac{128}{13005} \cdot 10^{1/2} \frac{(-1/9 + 1/9 \cdot 10^{1/2})}{(-2 \cdot (x - 2/3 - 1/3 \cdot 10^{1/2})^2 + (1/3 - 4/3 \cdot 10^{1/2}) \cdot (x - 2/3 - 1/3 \cdot 10^{1/2}) - 1/9 + 1/9 \cdot 10^{1/2})^{1/2}} - \frac{992}{7803} \frac{(-1/9 + 1/9 \cdot 10^{1/2})}{(-2 \cdot (x - 2/3 - 1/3 \cdot 10^{1/2})^2 + (1/3 - 4/3 \cdot 10^{1/2}) \cdot (x - 2/3 - 1/3 \cdot 10^{1/2}) - 1/9 + 1/9 \cdot 10^{1/2})^{1/2}} \cdot x - \frac{128}{13005} \cdot 10^{1/2} \frac{(-1/9 - 1/9 \cdot 10^{1/2})}{(-2 \cdot (x - 2/3 + 1/3 \cdot 10^{1/2})^2 + (1/3 + 4/3 \cdot 10^{1/2}) \cdot (x - 2/3 + 1/3 \cdot 10^{1/2}) - 1/9 - 1/9 \cdot 10^{1/2})^{1/2}} - \frac{992}{7803} \frac{(-1/9 - 1/9 \cdot 10^{1/2})}{(-2 \cdot (x - 2/3 + 1/3 \cdot 10^{1/2})^2 + (1/3 + 4/3 \cdot 10^{1/2}) \cdot (x - 2/3 + 1/3 \cdot 10^{1/2}) - 1/9 - 1/9 \cdot 10^{1/2})^{1/2}} \cdot x$$

maxima [B] time = 1.26, size = 1276, normalized size = 6.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(5/2),x, algorithm="maxima")
[Out] 1/17340*sqrt(10)*(2108*sqrt(10)*x/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3/2) + (-2*x^2 + 3*x + 1)^(3/2)) - 2108*sqrt(10)*x/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3/2) - (-2*x^2 + 3*x + 1)^(3/2)) - 56916*sqrt(10)*x/(2*sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + 11*sqrt(-2*x^2 + 3*x + 1)) + 56916*sqrt(10)*x/(2*sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - 11*sqrt(-2*x^2 + 3*x + 1)) + 1984*sqrt(10)*x/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1)) - 1984*sqrt(10)*x/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) - 70227*sqrt(10)*arcsin(8/17*sqrt(17)*sqrt(10)*x/abs(6*x + 2*sqrt(10) - 4) + 2/17*sqrt(17)*x/abs(6*x + 2*sqrt(10) - 4) - 6/17*sqrt(17)*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 24/17*sqrt(17)/abs(6*x + 2*sqrt(10) - 4))/(2*sqrt(10)*sqrt(sqrt(10) + 1) + 11*sqrt(sqrt(10) + 1)) - 2176*x/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3/2) + (-2*x^2 + 3*x + 1)^(3/2)) - 2176*x/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3/2) - (-2*x^2 + 3*x + 1)^(3/2)) + 58752*x/(2*sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + 11*sqrt(-2*x^2 + 3*x + 1)) + 58752*x/(2*sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - 11*sqrt(-2*x^2 + 3*x + 1)) - 2048*x/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1)) - 2048*x/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) + 561816*arcsin(8/17*sqrt(17)*sqrt(10)*x/abs(6*x + 2*sqrt(10) - 4) + 2/17*sqrt(17)*x/abs(6*x + 2*sqrt(10) - 4) - 6/17*sqrt(17)*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 24/17*sqrt(17)/abs(6*x + 2*sqrt(10) - 4))/(2*sqrt(10)*sqrt(sqrt(10) + 1) + 11*sqrt(sqrt(10) + 1)) - 714*sqrt(10)/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3/2) + (-2*x^2 + 3*x + 1)^(3/2)) + 714*sqrt(10)/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3/2) - (-2*x^2 + 3*x + 1)^(3/2)) + 19278*sqrt(10)/(2*sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + 11*sqrt(-2*x^2 + 3*x + 1)) - 19278*sqrt(10)/(2*sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - 11*sqrt(-2*x^2 + 3*x + 1)) - 1488*sqrt(10)/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1)) + 1488*sqrt(10)/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) - 5304/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3/2) + (-2*x^2 + 3*x + 1)^(3/2)) - 5304/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3/2) - (-2*x^2 + 3*x + 1)^(3/2)) + 143208/(2*sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + 11*sqrt(-2*x^2 + 3*x + 1)) + 143208/(2*sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - 11*sqrt(-2*x^2 + 3*x + 1)) + 1536/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1)) + 1536/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) + 70227*sqrt(10)*log(-2/9*sqrt(10) + 2/3*sqrt(-2*x^2 + 3*x + 1)*sqrt(sqrt(10) - 1)/abs(6*x - 2*sqrt(10) - 4) + 2/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) - 2/9/abs(6*x - 2*sqrt(10) - 4) + 1/18)/(sqrt(10) - 1)^(5/2) + 561816*log(-2/9*sqrt(10) + 2/3*sqrt(-2*x^2 + 3*x + 1)*sqrt(sqrt(10) - 1)/abs(6*x - 2*sqrt(10) - 4) + 2/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) - 2/9/abs(6*x - 2*sqrt(10) - 4) + 1/18)/(sqrt(10) - 1)^(5/2))
mupad [F] time = 0.00, size = -1, normalized size = -0.01
```

$$\int \frac{x+2}{(-2x^2+3x+1)^{5/2}(-3x^2+4x+2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/((3*x - 2*x^2 + 1)^(5/2)*(4*x - 3*x^2 + 2)),x)
[Out] int((x + 2)/((3*x - 2*x^2 + 1)^(5/2)*(4*x - 3*x^2 + 2)), x)
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{x}{12x^6\sqrt{-2x^2+3x+1}-52x^5\sqrt{-2x^2+3x+1}+55x^4\sqrt{-2x^2+3x+1}+22x^3\sqrt{-2x^2+3x+1}-31x^2\sqrt{-2x^2+3x+1}-16x\sqrt{-2x^2+3x+1}-2\sqrt{-2x^2+3x+1}} dx - \int \frac{2}{12x^6\sqrt{-2x^2+3x+1}-52x^5\sqrt{-2x^2+3x+1}+55x^4\sqrt{-2x^2+3x+1}+22x^3\sqrt{-2x^2+3x+1}-31x^2\sqrt{-2x^2+3x+1}-16x\sqrt{-2x^2+3x+1}-2\sqrt{-2x^2+3x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x**2+4*x+2)/(-2*x**2+3*x+1)**(5/2),x)
```



```
[Out] -Integral(x/(12*x**6*sqrt(-2*x**2 + 3*x + 1) - 52*x**5*sqrt(-2*x**2 + 3*x + 1) + 55*x**4*sqrt(-2*x**2 + 3*x + 1) + 22*x**3*sqrt(-2*x**2 + 3*x + 1) - 31*x**2*sqrt(-2*x**2 + 3*x + 1) - 16*x*sqrt(-2*x**2 + 3*x + 1) - 2*sqrt(-2*x**2 + 3*x + 1)), x) - Integral(2/(12*x**6*sqrt(-2*x**2 + 3*x + 1) - 52*x**5*sqrt(-2*x**2 + 3*x + 1) + 55*x**4*sqrt(-2*x**2 + 3*x + 1) + 22*x**3*sqrt(-2*x**2 + 3*x + 1) - 31*x**2*sqrt(-2*x**2 + 3*x + 1) - 16*x*sqrt(-2*x**2 + 3*x + 1) - 2*sqrt(-2*x**2 + 3*x + 1)), x)
```

$$3.28 \quad \int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x+2x^2}} dx$$

Optimal. Leaf size=151

$$\frac{1}{2}\sqrt{1-\frac{7\sqrt{\frac{2}{5}}}{5}} \tanh^{-1}\left(\frac{(17+4\sqrt{10})x+3(4+\sqrt{10})}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}}\right) - \frac{1}{2}\sqrt{1+\frac{7\sqrt{\frac{2}{5}}}{5}} \tanh^{-1}\left(\frac{(17-4\sqrt{10})x+3(4-\sqrt{10})}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)$$

Rubi [A] time = 0.23, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1032, 724, 206}

$$\frac{1}{2}\sqrt{1-\frac{7\sqrt{\frac{2}{5}}}{5}} \tanh^{-1}\left(\frac{(17+4\sqrt{10})x+3(4+\sqrt{10})}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}}\right) - \frac{1}{2}\sqrt{1+\frac{7\sqrt{\frac{2}{5}}}{5}} \tanh^{-1}\left(\frac{(17-4\sqrt{10})x+3(4-\sqrt{10})}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)$$

Antiderivative was successfully verified.

[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x + 2*x^2]), x]

[Out] -(Sqrt[1 + (7*Sqrt[2/5])/5]*ArcTanh[(3*(4 - Sqrt[10]) + (17 - 4*Sqrt[10])*x)/(2*Sqrt[55 - 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/2 + (Sqrt[1 - (7*Sqrt[2/5])/5]*ArcTanh[(3*(4 + Sqrt[10]) + (17 + 4*Sqrt[10])*x)/(2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/2

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1032

Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x+2x^2}} dx &= \frac{1}{5}(5-4\sqrt{10}) \int \frac{1}{(4-2\sqrt{10}-6x)\sqrt{1+3x+2x^2}} dx + \frac{1}{5}(5+4\sqrt{10}) \int \\ &= -\left(\frac{1}{5}(2(5-4\sqrt{10}))\right) \text{Subst}\left(\int \frac{1}{144+72(4-2\sqrt{10})+8(4-2\sqrt{10})^2-x}\right. \\ &= -\frac{1}{10}\sqrt{25+7\sqrt{10}} \tanh^{-1}\left(\frac{3(4-\sqrt{10})+(17-4\sqrt{10})x}{2\sqrt{55-17\sqrt{10}}\sqrt{1+3x+2x^2}}\right) + \frac{1}{10}\sqrt{25-7\sqrt{10}} \tanh^{-1}\left(\frac{3(4+\sqrt{10})+(17+4\sqrt{10})x}{2\sqrt{55+17\sqrt{10}}\sqrt{1+3x+2x^2}}\right) \end{aligned}$$

Mathematica [A] time = 0.36, size = 148, normalized size = 0.98

$$\frac{(5 - 4\sqrt{10}) \tanh^{-1}\left(\frac{-4\sqrt{10}x + 17x - 3\sqrt{10} + 12}{2\sqrt{55 - 17\sqrt{10}}\sqrt{2x^2 + 3x + 1}}\right) + 3\sqrt{285 - 90\sqrt{10}} \tanh^{-1}\left(\frac{(17 + 4\sqrt{10})x + 3(4 + \sqrt{10})}{2\sqrt{55 + 17\sqrt{10}}\sqrt{2x^2 + 3x + 1}}\right)}{10\sqrt{55 - 17\sqrt{10}}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x + 2*x^2]),x]

[Out] ((5 - 4*Sqrt[10])*ArcTanh[(12 - 3*Sqrt[10] + 17*x - 4*Sqrt[10]*x)/(2*Sqrt[55 - 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2]]) + 3*Sqrt[285 - 90*Sqrt[10]]*ArcTanh[(3*(4 + Sqrt[10]) + (17 + 4*Sqrt[10])*x)/(2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])))/(10*Sqrt[55 - 17*Sqrt[10]])

IntegrateAlgebraic [A] time = 0.78, size = 109, normalized size = 0.72

$$\frac{1}{5}\sqrt{25 - 7\sqrt{10}} \tanh^{-1}\left(\frac{\sqrt{1 + \sqrt{\frac{2}{5}}}\sqrt{2x^2 + 3x + 1}}{2x + 1}\right) - \frac{1}{5}\sqrt{25 + 7\sqrt{10}} \tanh^{-1}\left(\frac{\sqrt{1 - \sqrt{\frac{2}{5}}}\sqrt{2x^2 + 3x + 1}}{2x + 1}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x + 2*x^2]),x]

[Out] -1/5*(Sqrt[25 + 7*Sqrt[10]]*ArcTanh[(Sqrt[1 - Sqrt[2/5]]*Sqrt[1 + 3*x + 2*x^2])/(1 + 2*x)]) + (Sqrt[25 - 7*Sqrt[10]]*ArcTanh[(Sqrt[1 + Sqrt[2/5]]*Sqrt[1 + 3*x + 2*x^2])/(1 + 2*x)))/5

fricas [B] time = 0.43, size = 245, normalized size = 1.62

$$\frac{1}{10}\sqrt{7\sqrt{10} + 25} \log\left(\frac{3\sqrt{10}x + (\sqrt{10}x - 4x)\sqrt{7\sqrt{10} + 25 + 6x - 6\sqrt{2x^2 + 3x + 1}} + 6}{x}\right) - \frac{1}{10}\sqrt{7\sqrt{10} + 25} \log\left(\frac{3\sqrt{10}x - (\sqrt{10}x - 4x)\sqrt{7\sqrt{10} + 25 + 6x - 6\sqrt{2x^2 + 3x + 1}} + 6}{x}\right) + \frac{1}{10}\sqrt{-7\sqrt{10} + 25} \log\left(\frac{3\sqrt{10}x + (\sqrt{10}x + 4x)\sqrt{-7\sqrt{10} + 25 - 6x + 6\sqrt{2x^2 + 3x + 1}} - 6}{x}\right) - \frac{1}{10}\sqrt{-7\sqrt{10} + 25} \log\left(\frac{3\sqrt{10}x - (\sqrt{10}x + 4x)\sqrt{-7\sqrt{10} + 25 - 6x + 6\sqrt{2x^2 + 3x + 1}} - 6}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(1/2),x, algorithm="fricas")

[Out] 1/10*sqrt(7*sqrt(10) + 25)*log(-(3*sqrt(10)*x + (sqrt(10)*x - 4*x)*sqrt(7*sqrt(10) + 25) + 6*x - 6*sqrt(2*x^2 + 3*x + 1) + 6)/x) - 1/10*sqrt(7*sqrt(10) + 25)*log(-(3*sqrt(10)*x - (sqrt(10)*x - 4*x)*sqrt(7*sqrt(10) + 25) + 6*x - 6*sqrt(2*x^2 + 3*x + 1) + 6)/x) + 1/10*sqrt(-7*sqrt(10) + 25)*log((3*sqrt(10)*x + (sqrt(10)*x + 4*x)*sqrt(-7*sqrt(10) + 25) - 6*x + 6*sqrt(2*x^2 + 3*x + 1) - 6)/x) - 1/10*sqrt(-7*sqrt(10) + 25)*log((3*sqrt(10)*x - (sqrt(10)*x + 4*x)*sqrt(-7*sqrt(10) + 25) - 6*x + 6*sqrt(2*x^2 + 3*x + 1) - 6)/x)

giac [A] time = 0.48, size = 93, normalized size = 0.62

$$0.169235232112667 \log(-\sqrt{2}x + \sqrt{2x^2 + 3x + 1} + 5.90976932712000) - 0.686556214893333 \log(-\sqrt{2}x + \sqrt{2x^2 + 3x + 1} - 0.176527156327000) + 0.686556214893333 \log(-\sqrt{2}x + \sqrt{2x^2 + 3x + 1} - 0.919278730509000) - 0.169235232112667 \log(-\sqrt{2}x + \sqrt{2x^2 + 3x + 1} - 1.04272727395000)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(1/2),x, algorithm="giac")

[Out] 0.169235232112667*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) + 5.90976932712000) - 0.686556214893333*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 0.176527156327000) + 0.686556214893333*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 0.919278730509000) - 0.169235232112667*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 1.04272727395000)

maple [A] time = 0.05, size = 186, normalized size = 1.23

$$\frac{(-8 + \sqrt{10})\sqrt{10} \operatorname{arctanh}\left(\frac{55-17\sqrt{10} + \frac{9\left(\frac{17}{3} - \frac{4\sqrt{10}}{3}\right)\left(x - \frac{2}{3} + \frac{\sqrt{10}}{3}\right)}{2}}{\sqrt{55-17\sqrt{10}} \sqrt{18\left(x - \frac{2}{3} + \frac{\sqrt{10}}{3}\right)^2 + 9\left(\frac{17}{3} - \frac{4\sqrt{10}}{3}\right)\left(x - \frac{2}{3} + \frac{\sqrt{10}}{3}\right) + 55-17\sqrt{10}}}\right)}{20\sqrt{55-17\sqrt{10}}} + \frac{(8 + \sqrt{10})\sqrt{10} \operatorname{arctanh}\left(\frac{55+17\sqrt{10} + \frac{9\left(\frac{17}{3} + \frac{4\sqrt{10}}{3}\right)\left(x - \frac{2}{3} - \frac{\sqrt{10}}{3}\right)}{2}}{\sqrt{55+17\sqrt{10}} \sqrt{18\left(x - \frac{2}{3} - \frac{\sqrt{10}}{3}\right)^2 + 9\left(\frac{17}{3} + \frac{4\sqrt{10}}{3}\right)\left(x - \frac{2}{3} - \frac{\sqrt{10}}{3}\right) + 55+17\sqrt{10}}}\right)}{20\sqrt{55+17\sqrt{10}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(1/2), x)

[Out] 1/20*(-8+10^(1/2))*10^(1/2)/(55-17*10^(1/2))^(1/2)*arctanh(9/2*(110/9-34/9*10^(1/2)+(17/3-4/3*10^(1/2))*(x-2/3+1/3*10^(1/2)))/(55-17*10^(1/2))^(1/2)/(18*(x-2/3+1/3*10^(1/2))^2+9*(17/3-4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))+55-17*10^(1/2))^(1/2))+1/20*(8+10^(1/2))*10^(1/2)/(55+17*10^(1/2))^(1/2)*arctanh(9/2*(110/9+34/9*10^(1/2)+(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2)))/(55+17*10^(1/2))^(1/2)/(18*(x-2/3-1/3*10^(1/2))^2+9*(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))+55+17*10^(1/2))^(1/2))

maxima [B] time = 1.07, size = 363, normalized size = 2.40

$$\frac{1}{60}\sqrt{10} \left(\frac{3\sqrt{10} \log\left(\frac{2}{3}\sqrt{10} + \frac{2\sqrt{27+3x+1}\sqrt{17\sqrt{10}+55}}{9|x-2\sqrt{10}-4|} + \frac{34\sqrt{10}}{9|x-2\sqrt{10}-4|} + \frac{110}{9|x-2\sqrt{10}-4|} + \frac{17}{18}\right)}{\sqrt{17\sqrt{10}+55}} + \frac{\sqrt{10} \log\left(\frac{2}{3}\sqrt{10} + \frac{2\sqrt{27+3x+1}\sqrt{17\sqrt{10}+55}}{9|x+2\sqrt{10}-4|} + \frac{34\sqrt{10}}{9|x+2\sqrt{10}-4|} + \frac{110}{9|x+2\sqrt{10}-4|} + \frac{17}{18}\right)}{\sqrt{17\sqrt{10}+55}} + \frac{24 \log\left(\frac{2}{3}\sqrt{10} + \frac{2\sqrt{27+3x+1}\sqrt{17\sqrt{10}+55}}{9|x-2\sqrt{10}-4|} + \frac{34\sqrt{10}}{9|x-2\sqrt{10}-4|} + \frac{110}{9|x-2\sqrt{10}-4|} + \frac{17}{18}\right)}{\sqrt{17\sqrt{10}+55}} - \frac{8 \log\left(\frac{2}{3}\sqrt{10} + \frac{2\sqrt{27+3x+1}\sqrt{17\sqrt{10}+55}}{9|x+2\sqrt{10}-4|} + \frac{34\sqrt{10}}{9|x+2\sqrt{10}-4|} + \frac{110}{9|x+2\sqrt{10}-4|} + \frac{17}{18}\right)}{\sqrt{17\sqrt{10}+55}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(1/2), x, algorithm="maxima")

[Out] 1/60*sqrt(10)*(3*sqrt(10)*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1)*sqrt(17*sqrt(10) + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/sqrt(17*sqrt(10) + 55) + sqrt(10)*log(-2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x + 1)*sqrt(-17/9*sqrt(10) + 55/9)/abs(6*x + 2*sqrt(10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 110/9/abs(6*x + 2*sqrt(10) - 4) + 17/18)/sqrt(-17/9*sqrt(10) + 55/9) + 24*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1)*sqrt(17*sqrt(10) + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/sqrt(17*sqrt(10) + 55) - 8*log(-2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x + 1)*sqrt(-17/9*sqrt(10) + 55/9)/abs(6*x + 2*sqrt(10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 110/9/abs(6*x + 2*sqrt(10) - 4) + 17/18)/sqrt(-17/9*sqrt(10) + 55/9))

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x + 2}{\sqrt{2x^2 + 3x + 1} (-3x^2 + 4x + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 2)/((3*x + 2*x^2 + 1)^(1/2)*(4*x - 3*x^2 + 2)), x)

[Out] int((x + 2)/((3*x + 2*x^2 + 1)^(1/2)*(4*x - 3*x^2 + 2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{3x^2\sqrt{2x^2 + 3x + 1} - 4x\sqrt{2x^2 + 3x + 1} - 2\sqrt{2x^2 + 3x + 1}} dx - \int \frac{2}{3x^2\sqrt{2x^2 + 3x + 1} - 4x\sqrt{2x^2 + 3x + 1} - 2\sqrt{2x^2 + 3x + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(-3*x**2+4*x+2)/(2*x**2+3*x+1)**(1/2), x)

[Out] -Integral(x/(3*x**2*sqrt(2*x**2 + 3*x + 1) - 4*x*sqrt(2*x**2 + 3*x + 1) - 2*sqrt(2*x**2 + 3*x + 1)), x) - Integral(2/(3*x**2*sqrt(2*x**2 + 3*x + 1) - 4*x*sqrt(2*x**2 + 3*x + 1) - 2*sqrt(2*x**2 + 3*x + 1)), x)

$$3.29 \quad \int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{3/2}} dx$$

Optimal. Leaf size=174

$$\frac{2(22x+21)}{5\sqrt{2x^2+3x+1}} - \frac{1}{10}\sqrt{\frac{3}{5}(2065+653\sqrt{10})} \tanh^{-1}\left(\frac{(17-4\sqrt{10})x+3(4-\sqrt{10})}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right) + \frac{1}{10}\sqrt{\frac{3}{5}(2065-653\sqrt{10})} \tanh^{-1}\left(\frac{(17+4\sqrt{10})x+3(4+\sqrt{10})}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)$$

Rubi [A] time = 0.25, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1016, 1032, 724, 206}

$$\frac{2(22x+21)}{5\sqrt{2x^2+3x+1}} - \frac{1}{10}\sqrt{\frac{3}{5}(2065+653\sqrt{10})} \tanh^{-1}\left(\frac{(17-4\sqrt{10})x+3(4-\sqrt{10})}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right) + \frac{1}{10}\sqrt{\frac{3}{5}(2065-653\sqrt{10})} \tanh^{-1}\left(\frac{(17+4\sqrt{10})x+3(4+\sqrt{10})}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)$$

Antiderivative was successfully verified.

[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x + 2*x^2)^(3/2)),x]

[Out] (2*(21 + 22*x))/(5*Sqrt[1 + 3*x + 2*x^2]) - (Sqrt[(3*(2065 + 653*Sqrt[10]))/5]*ArcTanh[(3*(4 - Sqrt[10]) + (17 - 4*Sqrt[10])*x)/(2*Sqrt[55 - 17*Sqrt[10]])*Sqrt[1 + 3*x + 2*x^2]])/10 + (Sqrt[(3*(2065 - 653*Sqrt[10]))/5]*ArcTanh[(3*(4 + Sqrt[10]) + (17 + 4*Sqrt[10])*x)/(2*Sqrt[55 + 17*Sqrt[10]])*Sqrt[1 + 3*x + 2*x^2]])/10

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1016

Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*c*d - 2*a*c*e + a*b*f))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1])

Rule 1032

Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{3/2}} dx &= \frac{2(21+22x)}{5\sqrt{1+3x+2x^2}} - \frac{2}{15} \int \frac{-72 + \frac{81x}{2}}{(2+4x-3x^2)\sqrt{1+3x+2x^2}} dx \\ &= \frac{2(21+22x)}{5\sqrt{1+3x+2x^2}} - \frac{1}{5} (9(3-\sqrt{10})) \int \frac{1}{(4+2\sqrt{10}-6x)\sqrt{1+3x+2x^2}} \\ &= \frac{2(21+22x)}{5\sqrt{1+3x+2x^2}} + \frac{1}{5} (18(3-\sqrt{10})) \text{Subst} \left(\int \frac{1}{144+72(4+2\sqrt{10})+} \right. \\ &= \frac{2(21+22x)}{5\sqrt{1+3x+2x^2}} - \frac{1}{10} \sqrt{\frac{3}{5}} (2065+653\sqrt{10}) \tanh^{-1} \left(\frac{3(4-\sqrt{10})+}{2\sqrt{55-17\sqrt{10}}} \right) \end{aligned}$$

Mathematica [A] time = 0.57, size = 172, normalized size = 0.99

$$\frac{1}{50} \left(\frac{\sqrt{30975-9795\sqrt{10}} \sqrt{2x^2+3x+1} \tanh^{-1} \left(\frac{4\sqrt{10}x+17x+3\sqrt{10}+12}{2\sqrt{55+17\sqrt{10}} \sqrt{2x^2+3x+1}} \right) + 440x+420}{\sqrt{2x^2+3x+1}} - \sqrt{30975+9795\sqrt{10}} \tanh^{-1} \left(\frac{-4\sqrt{10}x+17x-3\sqrt{10}+12}{2\sqrt{55-17\sqrt{10}} \sqrt{2x^2+3x+1}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x + 2*x^2)^(3/2)), x]

[Out] (- (Sqrt[30975 + 9795*Sqrt[10]]*ArcTanh[(12 - 3*Sqrt[10] + 17*x - 4*Sqrt[10]*x)/(2*Sqrt[55 - 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])]) + (420 + 440*x + Sqrt[30975 - 9795*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2]*ArcTanh[(12 + 3*Sqrt[10] + 17*x + 4*Sqrt[10]*x)/(2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/Sqrt[1 + 3*x + 2*x^2])/50

IntegrateAlgebraic [A] time = 1.02, size = 148, normalized size = 0.85

$$\frac{2\sqrt{2x^2+3x+1}(22x+21)}{5(x+1)(2x+1)} - \frac{1}{5} \sqrt{\frac{1}{5}} (6195+1959\sqrt{10}) \tanh^{-1} \left(\frac{\sqrt{1-\frac{2}{5}} \sqrt{2x^2+3x+1}}{2x+1} \right) + \frac{9 \tanh^{-1} \left(\frac{\sqrt{1+\frac{2}{5}} \sqrt{2x^2+3x+1}}{2x+1} \right)}{5\sqrt{2065+653\sqrt{10}}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x + 2*x^2)^(3/2)), x]

[Out] (2*(21 + 22*x)*Sqrt[1 + 3*x + 2*x^2])/(5*(1 + x)*(1 + 2*x)) - (Sqrt[(6195 + 1959*Sqrt[10])/5]*ArcTanh[(Sqrt[1 - Sqrt[2/5]]*Sqrt[1 + 3*x + 2*x^2])/(1 + 2*x)])/5 + (9*ArcTanh[(Sqrt[1 + Sqrt[2/5]]*Sqrt[1 + 3*x + 2*x^2])/(1 + 2*x)])/5/Sqrt[2065 + 653*Sqrt[10]])

fricas [B] time = 0.45, size = 365, normalized size = 2.10

$$\frac{\sqrt{(2x^2+3x+1)\sqrt{55+17\sqrt{10}} \tanh^{-1} \left(\frac{4\sqrt{10}x+17x+3\sqrt{10}+12}{2\sqrt{55+17\sqrt{10}} \sqrt{2x^2+3x+1}} \right) + 440x+420}{\sqrt{2x^2+3x+1}} - \sqrt{(2x^2+3x+1)\sqrt{55-17\sqrt{10}} \tanh^{-1} \left(\frac{-4\sqrt{10}x+17x-3\sqrt{10}+12}{2\sqrt{55-17\sqrt{10}} \sqrt{2x^2+3x+1}} \right) + 440x+420}{\sqrt{2x^2+3x+1}}}{50}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/50*(sqrt(5)*(2*x^2 + 3*x + 1)*sqrt(1959*sqrt(10) + 6195)*log(-(45*sqrt(10)
)*x + (41*sqrt(10)*sqrt(5)*x - 130*sqrt(5)*x)*sqrt(1959*sqrt(10) + 6195) +
90*x - 90*sqrt(2*x^2 + 3*x + 1) + 90)/x) - sqrt(5)*(2*x^2 + 3*x + 1)*sqrt(1
959*sqrt(10) + 6195)*log(-(45*sqrt(10)*x - (41*sqrt(10)*sqrt(5)*x - 130*sq
rt(5)*x)*sqrt(1959*sqrt(10) + 6195) + 90*x - 90*sqrt(2*x^2 + 3*x + 1) + 90)/
x) + sqrt(5)*(2*x^2 + 3*x + 1)*sqrt(-1959*sqrt(10) + 6195)*log((45*sqrt(10)
*x + (41*sqrt(10)*sqrt(5)*x + 130*sqrt(5)*x)*sqrt(-1959*sqrt(10) + 6195) -
90*x + 90*sqrt(2*x^2 + 3*x + 1) - 90)/x) - sqrt(5)*(2*x^2 + 3*x + 1)*sqrt(-
1959*sqrt(10) + 6195)*log((45*sqrt(10)*x - (41*sqrt(10)*sqrt(5)*x + 130*sq
rt(5)*x)*sqrt(-1959*sqrt(10) + 6195) - 90*x + 90*sqrt(2*x^2 + 3*x + 1) - 90)
/x) + 840*x^2 + 20*sqrt(2*x^2 + 3*x + 1)*(22*x + 21) + 1260*x + 420)/(2*x^2
+ 3*x + 1)
```

giac [A] time = 0.50, size = 112, normalized size = 0.64

$$\frac{2(22x+21)}{5\sqrt{2x^2+3x+1}} + 0.0140045514133333 \log(-\sqrt{2x^2+3x+1} + 5.90976932712000) - 4.97793168620000 \log(-\sqrt{2x^2+3x+1} - 0.176527156327000) + 4.97793168620000 \log(-\sqrt{2x^2+3x+1} - 0.919278730509000) - 0.0140045514125333 \log(-\sqrt{2x^2+3x+1} - 1.04272727395000)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(3/2),x, algorithm="giac")
```

```
[Out] 2/5*(22*x + 21)/sqrt(2*x^2 + 3*x + 1) + 0.0140045514133333*log(-sqrt(2)*x +
sqrt(2*x^2 + 3*x + 1) + 5.90976932712000) - 4.97793168620000*log(-sqrt(2)*
x + sqrt(2*x^2 + 3*x + 1) - 0.176527156327000) + 4.97793168620000*log(-sqrt
(2)*x + sqrt(2*x^2 + 3*x + 1) - 0.919278730509000) - 0.0140045514125333*log
(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 1.04272727395000)
```

maple [B] time = 0.02, size = 466, normalized size = 2.68

$$\frac{(x + \sqrt{2}) \sqrt{\frac{\frac{2(22x+21)}{5\sqrt{2x^2+3x+1}} + 0.0140045514133333 \log(-\sqrt{2}x + \sqrt{2x^2+3x+1} + 5.90976932712000) - 4.97793168620000 \log(-\sqrt{2}x + \sqrt{2x^2+3x+1} - 0.176527156327000) + 4.97793168620000 \log(-\sqrt{2}x + \sqrt{2x^2+3x+1} - 0.919278730509000) - 0.0140045514125333 \log(-\sqrt{2}x + \sqrt{2x^2+3x+1} - 1.04272727395000)}{(\sqrt{2x^2+3x+1})^{3/2}}}}{(x + \sqrt{2}) \sqrt{\frac{\frac{2(22x+21)}{5\sqrt{2x^2+3x+1}} + 0.0140045514133333 \log(-\sqrt{2}x + \sqrt{2x^2+3x+1} + 5.90976932712000) - 4.97793168620000 \log(-\sqrt{2}x + \sqrt{2x^2+3x+1} - 0.176527156327000) + 4.97793168620000 \log(-\sqrt{2}x + \sqrt{2x^2+3x+1} - 0.919278730509000) - 0.0140045514125333 \log(-\sqrt{2}x + \sqrt{2x^2+3x+1} - 1.04272727395000)}{(\sqrt{2x^2+3x+1})^{3/2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(3/2),x)
```

```
[Out] -1/20*(-8+10^(1/2))*10^(1/2)*(1/3/(55/9-17/9*10^(1/2)))/(2*(x-2/3+1/3*10^(1/
2))^2+(17/3-4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))+55/9-17/9*10^(1/2))^(1/2)-1/
3*(17/3-4/3*10^(1/2))/(55/9-17/9*10^(1/2))*(3+4*x)/(440/9-136/9*10^(1/2)-(1
7/3-4/3*10^(1/2))^2)/(2*(x-2/3+1/3*10^(1/2))^2+(17/3-4/3*10^(1/2))*(x-2/3+1
/3*10^(1/2))+55/9-17/9*10^(1/2))^(1/2)-1/(55/9-17/9*10^(1/2))/(55-17*10^(1/
2))^(1/2)*arctanh(9/2*(110/9-34/9*10^(1/2)+(17/3-4/3*10^(1/2))*(x-2/3+1/3*1
0^(1/2)))/(55-17*10^(1/2))^(1/2)/(18*(x-2/3+1/3*10^(1/2))^2+9*(17/3-4/3*10^
(1/2))*(x-2/3+1/3*10^(1/2))+55-17*10^(1/2))^(1/2))-1/20*(8+10^(1/2))*10^(1
/2)*(1/3/(55/9+17/9*10^(1/2)))/(2*(x-2/3-1/3*10^(1/2))^2+(17/3+4/3*10^(1/2)
)*(x-2/3-1/3*10^(1/2))+55/9+17/9*10^(1/2))^(1/2)-1/3*(17/3+4/3*10^(1/2))/(55
/9+17/9*10^(1/2))*(3+4*x)/(440/9+136/9*10^(1/2)-(17/3+4/3*10^(1/2))^2)/(2*(
x-2/3-1/3*10^(1/2))^2+(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))+55/9+17/9*10
^(1/2))^(1/2)-1/(55/9+17/9*10^(1/2))/(55+17*10^(1/2))^(1/2)*arctanh(9/2*(11
0/9+34/9*10^(1/2)+(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2)))/(55+17*10^(1/2)
)^(1/2)/(18*(x-2/3-1/3*10^(1/2))^2+9*(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/
2))+55+17*10^(1/2))^(1/2))
```

maxima [B] time = 1.10, size = 668, normalized size = 3.84

$$\frac{(x + \sqrt{2}) \sqrt{\frac{\frac{2(22x+21)}{5\sqrt{2x^2+3x+1}} + 0.0140045514133333 \log(-\sqrt{2}x + \sqrt{2x^2+3x+1} + 5.90976932712000) - 4.97793168620000 \log(-\sqrt{2}x + \sqrt{2x^2+3x+1} - 0.176527156327000) + 4.97793168620000 \log(-\sqrt{2}x + \sqrt{2x^2+3x+1} - 0.919278730509000) - 0.0140045514125333 \log(-\sqrt{2}x + \sqrt{2x^2+3x+1} - 1.04272727395000)}{(\sqrt{2x^2+3x+1})^{3/2}}}}{(x + \sqrt{2}) \sqrt{\frac{\frac{2(22x+21)}{5\sqrt{2x^2+3x+1}} + 0.0140045514133333 \log(-\sqrt{2}x + \sqrt{2x^2+3x+1} + 5.90976932712000) - 4.97793168620000 \log(-\sqrt{2}x + \sqrt{2x^2+3x+1} - 0.176527156327000) + 4.97793168620000 \log(-\sqrt{2}x + \sqrt{2x^2+3x+1} - 0.919278730509000) - 0.0140045514125333 \log(-\sqrt{2}x + \sqrt{2x^2+3x+1} - 1.04272727395000)}{(\sqrt{2x^2+3x+1})^{3/2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(3/2),x, algorithm="maxima")
[Out] -1/60*sqrt(10)*(588*sqrt(10)*x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqrt(2*x^2 + 3*x + 1)) - 588*sqrt(10)*x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*x^2 + 3*x + 1)) + 2112*x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqrt(2*x^2 + 3*x + 1)) + 2112*x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*x^2 + 3*x + 1)) - 27*sqrt(10)*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1))*sqrt(17*sqrt(10) + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/(17*sqrt(10) + 55)^(3/2) - sqrt(10)*log(-2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x + 1))*sqrt(-17/9*sqrt(10) + 55/9)/abs(6*x + 2*sqrt(10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 110/9/abs(6*x + 2*sqrt(10) - 4) + 17/18)/(-17/9*sqrt(10) + 55/9)^(3/2) + 450*sqrt(10)/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqrt(2*x^2 + 3*x + 1)) - 450*sqrt(10)/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*x^2 + 3*x + 1)) - 216*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1))*sqrt(17*sqrt(10) + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/(17*sqrt(10) + 55)^(3/2) + 8*log(-2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x + 1))*sqrt(-17/9*sqrt(10) + 55/9)/abs(6*x + 2*sqrt(10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 110/9/abs(6*x + 2*sqrt(10) - 4) + 17/18)/(-17/9*sqrt(10) + 55/9)^(3/2) + 1656/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqrt(2*x^2 + 3*x + 1)) + 1656/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*x^2 + 3*x + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+2}{(2x^2+3x+1)^{3/2}(-3x^2+4x+2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/((3*x + 2*x^2 + 1)^(3/2)*(4*x - 3*x^2 + 2)),x)
[Out] int((x + 2)/((3*x + 2*x^2 + 1)^(3/2)*(4*x - 3*x^2 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{6x^4\sqrt{2x^2+3x+1} + x^3\sqrt{2x^2+3x+1} - 13x^2\sqrt{2x^2+3x+1} - 10x\sqrt{2x^2+3x+1} - 2\sqrt{2x^2+3x+1}} dx - \int \frac{2}{6x^4\sqrt{2x^2+3x+1} + x^3\sqrt{2x^2+3x+1} - 13x^2\sqrt{2x^2+3x+1} - 10x\sqrt{2x^2+3x+1} - 2\sqrt{2x^2+3x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x**2+4*x+2)/(2*x**2+3*x+1)**(3/2),x)
[Out] -Integral(x/(6*x**4*sqrt(2*x**2 + 3*x + 1) + x**3*sqrt(2*x**2 + 3*x + 1) - 13*x**2*sqrt(2*x**2 + 3*x + 1) - 10*x*sqrt(2*x**2 + 3*x + 1) - 2*sqrt(2*x**2 + 3*x + 1)), x) - Integral(2/(6*x**4*sqrt(2*x**2 + 3*x + 1) + x**3*sqrt(2*x**2 + 3*x + 1) - 13*x**2*sqrt(2*x**2 + 3*x + 1) - 10*x*sqrt(2*x**2 + 3*x + 1) - 2*sqrt(2*x**2 + 3*x + 1)), x)
```


$$3.30 \quad \int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{5/2}} dx$$

Optimal. Leaf size=197

$$\frac{2(22x+21)}{15(2x^2+3x+1)^{3/2}} + \frac{2(230x+273)}{15\sqrt{2x^2+3x+1}} - \frac{1}{50} \sqrt{\frac{1}{3}(4885115+1544809\sqrt{10})} \tanh^{-1} \left(\frac{(17-4\sqrt{10})x+3(4-\sqrt{10})}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}} \right) + \frac{1}{50} \sqrt{\frac{1}{3}(4885115-1544809\sqrt{10})} \tanh^{-1} \left(\frac{(17+4\sqrt{10})x+3(4+\sqrt{10})}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}} \right)$$

Rubi [A] time = 0.30, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1016, 1060, 1032, 724, 206}

$$\frac{2(22x+21)}{15(2x^2+3x+1)^{3/2}} + \frac{2(230x+273)}{15\sqrt{2x^2+3x+1}} - \frac{1}{50} \sqrt{\frac{1}{3}(4885115+1544809\sqrt{10})} \tanh^{-1} \left(\frac{(17-4\sqrt{10})x+3(4-\sqrt{10})}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}} \right) + \frac{1}{50} \sqrt{\frac{1}{3}(4885115-1544809\sqrt{10})} \tanh^{-1} \left(\frac{(17+4\sqrt{10})x+3(4+\sqrt{10})}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}} \right)$$

Antiderivative was successfully verified.

[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x + 2*x^2)^(5/2)), x]

[Out] (2*(21 + 22*x))/(15*(1 + 3*x + 2*x^2)^(3/2)) + (2*(273 + 230*x))/(15*Sqrt[1 + 3*x + 2*x^2]) - (Sqrt[(4885115 + 1544809*Sqrt[10])/3]*ArcTanh[(3*(4 - Sqrt[10]) + (17 - 4*Sqrt[10])*x)/(2*Sqrt[55 - 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/50 + (Sqrt[(4885115 - 1544809*Sqrt[10])/3]*ArcTanh[(3*(4 + Sqrt[10]) + (17 + 4*Sqrt[10])*x)/(2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/50

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1016

Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*c*d - 2*a*c*e + a*b*f))*x)/(b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1])

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1060

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))^(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))^(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))^(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{5/2}} dx &= \frac{2(21+22x)}{15(1+3x+2x^2)^{3/2}} - \frac{2}{45} \int \frac{-480 - \frac{813x}{2} + 396x^2}{(2+4x-3x^2)(1+3x+2x^2)^{3/2}} dx \\ &= \frac{2(21+22x)}{15(1+3x+2x^2)^{3/2}} + \frac{2(273+230x)}{15\sqrt{1+3x+2x^2}} + \frac{4}{675} \int \frac{\frac{23355}{2} - \frac{2713}{4}}{(2+4x-3x^2)\sqrt{1+3x+2x^2}} dx \\ &= \frac{2(21+22x)}{15(1+3x+2x^2)^{3/2}} + \frac{2(273+230x)}{15\sqrt{1+3x+2x^2}} - \frac{1}{25} (3(335-106\sqrt{10})) \int \frac{1}{(2+4x-3x^2)\sqrt{1+3x+2x^2}} dx \\ &= \frac{2(21+22x)}{15(1+3x+2x^2)^{3/2}} + \frac{2(273+230x)}{15\sqrt{1+3x+2x^2}} + \frac{1}{25} (6(335-106\sqrt{10})) \operatorname{Subst} \int \frac{1}{(2+4x-3x^2)\sqrt{1+3x+2x^2}} dx \\ &= \frac{2(21+22x)}{15(1+3x+2x^2)^{3/2}} + \frac{2(273+230x)}{15\sqrt{1+3x+2x^2}} - \frac{1}{50} \sqrt{\frac{1}{3} (4885115 + 1544809\sqrt{10})} \end{aligned}$$

Mathematica [A] time = 0.71, size = 190, normalized size = 0.96

$$\frac{1}{450} \left(\sqrt{55-17\sqrt{10}} (7289+2305\sqrt{10}) \tanh^{-1} \left(\frac{(4\sqrt{10}-17)x+3(\sqrt{10}-4)}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}} \right) - \sqrt{55+17\sqrt{10}} (2305\sqrt{10}-7289) \tanh^{-1} \left(\frac{-((17+4\sqrt{10})x)-3(4+\sqrt{10})}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}} \right) + \frac{60(460x^3+1236x^2+1071x+294)}{(2x^2+3x+1)^{3/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x + 2*x^2)^(5/2)),x]

[Out] ((60*(294 + 1071*x + 1236*x^2 + 460*x^3))/(1 + 3*x + 2*x^2)^(3/2) + Sqrt[55 - 17*Sqrt[10]]*(7289 + 2305*Sqrt[10])*ArcTanh[(3*(-4 + Sqrt[10]) + (-17 + 4*Sqrt[10])*x)/(2*Sqrt[55 - 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])] - Sqrt[55 + 17*Sqrt[10]]*(-7289 + 2305*Sqrt[10])*ArcTanh[(-3*(4 + Sqrt[10]) - (17 + 4*Sqrt[10])*x)/(2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/450

IntegrateAlgebraic [A] time = 0.97, size = 160, normalized size = 0.81

$$-\frac{1}{25}\sqrt{\frac{1}{3}(4885115 + 1544809\sqrt{10})} \tanh^{-1}\left(\frac{\sqrt{1-\sqrt{\frac{2}{5}}}\sqrt{2x^2+3x+1}}{2x+1}\right) + \frac{81 \tanh^{-1}\left(\frac{\sqrt{1+\sqrt{\frac{2}{5}}}\sqrt{2x^2+3x+1}}{2x+1}\right)}{5\sqrt{5}(4885115 + 1544809\sqrt{10})} + \frac{2\sqrt{2x^2+3x+1}(460x^3 + 1236x^2 + 1071x + 294)}{15(x+1)^2(2x+1)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x + 2*x^2)^(5/2)),x]

[Out] (2*Sqrt[1 + 3*x + 2*x^2]*(294 + 1071*x + 1236*x^2 + 460*x^3))/(15*(1 + x)^2*(1 + 2*x)^2 - (Sqrt[(4885115 + 1544809*Sqrt[10])/3]*ArcTanh[(Sqrt[1 - Sqrt[2/5]]*Sqrt[1 + 3*x + 2*x^2])/(1 + 2*x)])/25 + (81*ArcTanh[(Sqrt[1 + Sqrt[2/5]]*Sqrt[1 + 3*x + 2*x^2])/(1 + 2*x)])/(5*Sqrt[5*(4885115 + 1544809*Sqrt[10])])

fricas [B] time = 0.43, size = 435, normalized size = 2.21

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(5/2),x, algorithm="fricas")

[Out] 1/150*(23520*x^4 + 70560*x^3 + sqrt(3)*(4*x^4 + 12*x^3 + 13*x^2 + 6*x + 1)*sqrt(1544809*sqrt(10) + 4885115)*log(-(243*sqrt(10)*x + (893*sqrt(10)*sqrt(3)*x - 2824*sqrt(3)*x)*sqrt(1544809*sqrt(10) + 4885115) + 486*x - 486*sqrt(2*x^2 + 3*x + 1) + 486)/x) - sqrt(3)*(4*x^4 + 12*x^3 + 13*x^2 + 6*x + 1)*sqrt(1544809*sqrt(10) + 4885115)*log(-(243*sqrt(10)*x - (893*sqrt(10)*sqrt(3)*x - 2824*sqrt(3)*x)*sqrt(1544809*sqrt(10) + 4885115) + 486*x - 486*sqrt(2*x^2 + 3*x + 1) + 486)/x) + sqrt(3)*(4*x^4 + 12*x^3 + 13*x^2 + 6*x + 1)*sqrt(-1544809*sqrt(10) + 4885115)*log((243*sqrt(10)*x + (893*sqrt(10)*sqrt(3)*x + 2824*sqrt(3)*x)*sqrt(-1544809*sqrt(10) + 4885115) - 486*x + 486*sqrt(2*x^2 + 3*x + 1) - 486)/x) - sqrt(3)*(4*x^4 + 12*x^3 + 13*x^2 + 6*x + 1)*sqrt(-1544809*sqrt(10) + 4885115)*log((243*sqrt(10)*x - (893*sqrt(10)*sqrt(3)*x + 2824*sqrt(3)*x)*sqrt(-1544809*sqrt(10) + 4885115) - 486*x + 486*sqrt(2*x^2 + 3*x + 1) - 486)/x) + 76440*x^2 + 20*(460*x^3 + 1236*x^2 + 1071*x + 294)*sqrt(2*x^2 + 3*x + 1) + 35280*x + 5880)/(4*x^4 + 12*x^3 + 13*x^2 + 6*x + 1)

giac [A] time = 0.54, size = 121, normalized size = 0.61

$$\frac{2(4(115x + 309)x + 1071)x + 294}{15(2x^2 + 3x + 1)^2} + 0.00115890442528267 \log(-\sqrt{2}x + \sqrt{2x^2 + 3x + 1} + 5.90976932712000) - 36.0928986365333 \log(-\sqrt{2}x + \sqrt{2x^2 + 3x + 1} - 0.176527156327000) + 36.0928986365333 \log(-\sqrt{2}x + \sqrt{2x^2 + 3x + 1} - 0.919278730509000) - 0.00115890442528267 \log(-\sqrt{2}x + \sqrt{2x^2 + 3x + 1} - 1.0427272795000)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(5/2),x, algorithm="giac")

[Out] 2/15*((4*(115*x + 309)*x + 1071)*x + 294)/(2*x^2 + 3*x + 1)^(3/2) + 0.00115890443050800*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) + 5.90976932712000) - 36.0928986365333*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 0.176527156327000) + 36.0928986365333*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 0.919278730509000) - 0.00115890442528267*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 1.0427272795000)

maple [B] time = 0.02, size = 878, normalized size = 4.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^{(5/2)}, x)$

[Out]
$$\begin{aligned} & -1/20*(-8+10^{(1/2)})*10^{(1/2)}*(1/9/(55/9-17/9*10^{(1/2)}))/(2*(x-2/3+1/3*10^{(1/2)})^2+(17/3-4/3*10^{(1/2)})*(x-2/3+1/3*10^{(1/2)})+55/9-17/9*10^{(1/2)})^{(3/2)}-1/ \\ & 6*(17/3-4/3*10^{(1/2)})/(55/9-17/9*10^{(1/2)})*(2/3*(4*x+3)/(440/9-136/9*10^{(1/2)}-(17/3-4/3*10^{(1/2)})^2)/(2*(x-2/3+1/3*10^{(1/2)})^2+(17/3-4/3*10^{(1/2)})*(x- \\ & 2/3+1/3*10^{(1/2)})+55/9-17/9*10^{(1/2)})^{(3/2)}+32/3/(440/9-136/9*10^{(1/2)}-(17/3-4/3*10^{(1/2)})^2)^2*(4*x+3)/(2*(x-2/3+1/3*10^{(1/2)})^2+(17/3-4/3*10^{(1/2)})* \\ & (x-2/3+1/3*10^{(1/2)})+55/9-17/9*10^{(1/2)})^{(1/2)})+1/3/(55/9-17/9*10^{(1/2)})*(1 \\ & /((55/9-17/9*10^{(1/2)})/(2*(x-2/3+1/3*10^{(1/2)})^2+(17/3-4/3*10^{(1/2)})*(x-2/3+ \\ & 1/3*10^{(1/2)})+55/9-17/9*10^{(1/2)})^{(1/2)}-(17/3-4/3*10^{(1/2)})/(55/9-17/9*10^{(1/2)}) \\ & *(4*x+3)/(440/9-136/9*10^{(1/2)}-(17/3-4/3*10^{(1/2)})^2)/(2*(x-2/3+1/3*10^{(1/2)})^2+(17/3-4/3*10^{(1/2)})*(x-2/3+1/3*10^{(1/2)})+55/9-17/9*10^{(1/2)})^{(1/2)} \\ &)-3/(55/9-17/9*10^{(1/2)})/(55-17*10^{(1/2)})^{(1/2)}*\text{arctanh}(9/2*(110/9-34/9*10^{(1/2)}+(17/3-4/3*10^{(1/2)})*(x-2/3+1/3*10^{(1/2)})))/(55-17*10^{(1/2)})^{(1/2)} \\ & /((18*(x-2/3+1/3*10^{(1/2)})^2+9*(17/3-4/3*10^{(1/2)})*(x-2/3+1/3*10^{(1/2)})+55-17*10^{(1/2)})^{(1/2)})))-1/20*(8+10^{(1/2)})*10^{(1/2)}*(1/9/(55/9+17/9*10^{(1/2)}))/(2*(x- \\ & 2/3-1/3*10^{(1/2)})^2+(17/3+4/3*10^{(1/2)})*(x-2/3-1/3*10^{(1/2)})+55/9+17/9*10^{(1/2)})^{(3/2)}-1/6*(17/3+4/3*10^{(1/2)})/(55/9+17/9*10^{(1/2)})*(2/3*(4*x+3)/(440/ \\ & 9+136/9*10^{(1/2)}-(17/3+4/3*10^{(1/2)})^2)/(2*(x-2/3-1/3*10^{(1/2)})^2+(17/3+4/3*10^{(1/2)})*(x-2/3-1/3*10^{(1/2)})+55/9+17/9*10^{(1/2)})^{(3/2)}+32/3/(440/9+136/9 \\ & *10^{(1/2)}-(17/3+4/3*10^{(1/2)})^2)^2*(4*x+3)/(2*(x-2/3-1/3*10^{(1/2)})^2+(17/3+4/3*10^{(1/2)})*(x-2/3-1/3*10^{(1/2)})+55/9+17/9*10^{(1/2)})^{(1/2)}+1/3/(55/9+17/ \\ & 9*10^{(1/2)})*(1/(55/9+17/9*10^{(1/2)})/(2*(x-2/3-1/3*10^{(1/2)})^2+(17/3+4/3*10^{(1/2)})*(x-2/3-1/3*10^{(1/2)})+55/9+17/9*10^{(1/2)})^{(1/2)}-(17/3+4/3*10^{(1/2)})/(\\ & 55/9+17/9*10^{(1/2)})*(4*x+3)/(440/9+136/9*10^{(1/2)}-(17/3+4/3*10^{(1/2)})^2)/(2 \\ & *(x-2/3-1/3*10^{(1/2)})^2+(17/3+4/3*10^{(1/2)})*(x-2/3-1/3*10^{(1/2)})+55/9+17/9* \\ & 10^{(1/2)})^{(1/2)}-3/(55/9+17/9*10^{(1/2)})/(55+17*10^{(1/2)})^{(1/2)}*\text{arctanh}(9/2*(\\ & 110/9+34/9*10^{(1/2)}+(17/3+4/3*10^{(1/2)})*(x-2/3-1/3*10^{(1/2)})))/(55+17*10^{(1/2)})^{(1/2)} \\ & /((18*(x-2/3-1/3*10^{(1/2)})^2+9*(17/3+4/3*10^{(1/2)})*(x-2/3-1/3*10^{(1/2)})+55+17*10^{(1/2)})^{(1/2)})))+55+17*10^{(1/2)})^{(1/2)})) \end{aligned}$$

maxima [B] time = 1.24, size = 1276, normalized size = 6.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^{(5/2)}, x, \text{algorithm}="maxima")$

[Out]
$$\begin{aligned} & -1/300*\text{sqrt}(10)*(980*\text{sqrt}(10)*x/(17*\text{sqrt}(10)*(2*x^2 + 3*x + 1)^{(3/2)} + 55*(\\ & 2*x^2 + 3*x + 1)^{(3/2)}) - 980*\text{sqrt}(10)*x/(17*\text{sqrt}(10)*(2*x^2 + 3*x + 1)^{(3/2)} \\ & - 55*(2*x^2 + 3*x + 1)^{(3/2)}) + 5292*\text{sqrt}(10)*x/(374*\text{sqrt}(10)*\text{sqrt}(2*x^2 \\ & + 3*x + 1) + 1183*\text{sqrt}(2*x^2 + 3*x + 1)) - 5292*\text{sqrt}(10)*x/(374*\text{sqrt}(10)*\text{s} \\ & \text{qrt}(2*x^2 + 3*x + 1) - 1183*\text{sqrt}(2*x^2 + 3*x + 1)) - 15680*\text{sqrt}(10)*x/(17*\text{s} \\ & \text{qrt}(10)*\text{sqrt}(2*x^2 + 3*x + 1) + 55*\text{sqrt}(2*x^2 + 3*x + 1)) + 15680*\text{sqrt}(10)* \\ & x/(17*\text{sqrt}(10)*\text{sqrt}(2*x^2 + 3*x + 1) - 55*\text{sqrt}(2*x^2 + 3*x + 1)) + 3520*x/(\\ & 17*\text{sqrt}(10)*(2*x^2 + 3*x + 1)^{(3/2)} + 55*(2*x^2 + 3*x + 1)^{(3/2)}) + 3520*x/(\\ & 17*\text{sqrt}(10)*(2*x^2 + 3*x + 1)^{(3/2)} - 55*(2*x^2 + 3*x + 1)^{(3/2)}) + 19008* \\ & x/(374*\text{sqrt}(10)*\text{sqrt}(2*x^2 + 3*x + 1) + 1183*\text{sqrt}(2*x^2 + 3*x + 1)) + 19008 \\ & *x/(374*\text{sqrt}(10)*\text{sqrt}(2*x^2 + 3*x + 1) - 1183*\text{sqrt}(2*x^2 + 3*x + 1)) - 5632 \\ & 0*x/(17*\text{sqrt}(10)*\text{sqrt}(2*x^2 + 3*x + 1) + 55*\text{sqrt}(2*x^2 + 3*x + 1)) - 56320* \\ & x/(17*\text{sqrt}(10)*\text{sqrt}(2*x^2 + 3*x + 1) - 55*\text{sqrt}(2*x^2 + 3*x + 1)) + 750*\text{sqrt} \\ & (10)/(17*\text{sqrt}(10)*(2*x^2 + 3*x + 1)^{(3/2)} + 55*(2*x^2 + 3*x + 1)^{(3/2)}) - 7 \\ & 50*\text{sqrt}(10)/(17*\text{sqrt}(10)*(2*x^2 + 3*x + 1)^{(3/2)} - 55*(2*x^2 + 3*x + 1)^{(3/2)} \\ &) + 4050*\text{sqrt}(10)/(374*\text{sqrt}(10)*\text{sqrt}(2*x^2 + 3*x + 1) + 1183*\text{sqrt}(2*x^2 + \\ & 3*x + 1)) - 4050*\text{sqrt}(10)/(374*\text{sqrt}(10)*\text{sqrt}(2*x^2 + 3*x + 1) - 1183*\text{sqrt}(\end{aligned}$$

$2x^2 + 3x + 1)) - 11760\sqrt{10}/(17\sqrt{10})\sqrt{2x^2 + 3x + 1} + 55\sqrt{2x^2 + 3x + 1}) + 11760\sqrt{10}/(17\sqrt{10})\sqrt{2x^2 + 3x + 1} - 55\sqrt{2x^2 + 3x + 1}) + 2760/(17\sqrt{10})(2x^2 + 3x + 1)^{(3/2)} + 55(2x^2 + 3x + 1)^{(3/2)}) + 2760/(17\sqrt{10})(2x^2 + 3x + 1)^{(3/2)} - 55(2x^2 + 3x + 1)^{(3/2)}) + 14904/(374\sqrt{10})\sqrt{2x^2 + 3x + 1} + 1183\sqrt{2x^2 + 3x + 1}) + 14904/(374\sqrt{10})\sqrt{2x^2 + 3x + 1} - 1183\sqrt{2x^2 + 3x + 1}) - 42240/(17\sqrt{10})\sqrt{2x^2 + 3x + 1} + 55\sqrt{2x^2 + 3x + 1}) - 42240/(17\sqrt{10})\sqrt{2x^2 + 3x + 1} - 55\sqrt{2x^2 + 3x + 1}) - 1215\sqrt{10}\log(2/9\sqrt{10} + 2/3\sqrt{2x^2 + 3x + 1})\sqrt{17\sqrt{10} + 55}/\text{abs}(6x - 2\sqrt{10} - 4) + 34/9\sqrt{10}/\text{abs}(6x - 2\sqrt{10} - 4) + 110/9/\text{abs}(6x - 2\sqrt{10} - 4) + 17/18)/(17\sqrt{10} + 55)^{(5/2)} - 5\sqrt{10}\log(-2/9\sqrt{10} + 2\sqrt{2x^2 + 3x + 1})\sqrt{-17/9\sqrt{10} + 55/9}/\text{abs}(6x + 2\sqrt{10} - 4) - 34/9\sqrt{10}/\text{abs}(6x + 2\sqrt{10} - 4) + 110/9/\text{abs}(6x + 2\sqrt{10} - 4) + 17/18)/(-17/9\sqrt{10} + 55/9)^{(5/2)} - 9720\log(2/9\sqrt{10} + 2/3\sqrt{2x^2 + 3x + 1})\sqrt{17\sqrt{10} + 55}/\text{abs}(6x - 2\sqrt{10} - 4) + 34/9\sqrt{10}/\text{abs}(6x - 2\sqrt{10} - 4) + 110/9/\text{abs}(6x - 2\sqrt{10} - 4) + 17/18)/(17\sqrt{10} + 55)^{(5/2)} + 40\log(-2/9\sqrt{10} + 2\sqrt{2x^2 + 3x + 1})\sqrt{-17/9\sqrt{10} + 55/9}/\text{abs}(6x + 2\sqrt{10} - 4) - 34/9\sqrt{10}/\text{abs}(6x + 2\sqrt{10} - 4) + 110/9/\text{abs}(6x + 2\sqrt{10} - 4) + 17/18)/(-17/9\sqrt{10} + 55/9)^{(5/2)}$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x + 2}{(2x^2 + 3x + 1)^{5/2} (-3x^2 + 4x + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 2)/((3*x + 2*x^2 + 1)^(5/2)*(4*x - 3*x^2 + 2)), x)

[Out] int((x + 2)/((3*x + 2*x^2 + 1)^(5/2)*(4*x - 3*x^2 + 2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{12x^6\sqrt{2x^2 + 3x + 1} + 20x^5\sqrt{2x^2 + 3x + 1} - 17x^4\sqrt{2x^2 + 3x + 1} - 58x^3\sqrt{2x^2 + 3x + 1} - 47x^2\sqrt{2x^2 + 3x + 1} - 16x\sqrt{2x^2 + 3x + 1} - 2\sqrt{2x^2 + 3x + 1}}{dx} - \int \frac{2}{12x^6\sqrt{2x^2 + 3x + 1} + 20x^5\sqrt{2x^2 + 3x + 1} - 17x^4\sqrt{2x^2 + 3x + 1} - 58x^3\sqrt{2x^2 + 3x + 1} - 47x^2\sqrt{2x^2 + 3x + 1} - 16x\sqrt{2x^2 + 3x + 1} - 2\sqrt{2x^2 + 3x + 1}}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(-3*x**2+4*x+2)/(2*x**2+3*x+1)**(5/2), x)

[Out] -Integral(x/(12*x**6*sqrt(2*x**2 + 3*x + 1) + 20*x**5*sqrt(2*x**2 + 3*x + 1) - 17*x**4*sqrt(2*x**2 + 3*x + 1) - 58*x**3*sqrt(2*x**2 + 3*x + 1) - 47*x**2*sqrt(2*x**2 + 3*x + 1) - 16*x*sqrt(2*x**2 + 3*x + 1) - 2*sqrt(2*x**2 + 3*x + 1)), x) - Integral(2/(12*x**6*sqrt(2*x**2 + 3*x + 1) + 20*x**5*sqrt(2*x**2 + 3*x + 1) - 17*x**4*sqrt(2*x**2 + 3*x + 1) - 58*x**3*sqrt(2*x**2 + 3*x + 1) - 47*x**2*sqrt(2*x**2 + 3*x + 1) - 16*x*sqrt(2*x**2 + 3*x + 1) - 2*sqrt(2*x**2 + 3*x + 1)), x)

$$3.31 \quad \int \frac{1+x}{(4+2x+x^2)\sqrt{5+2x+x^2}} dx$$

Optimal. Leaf size=15

$$-\tanh^{-1}\left(\sqrt{x^2+2x+5}\right)$$

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1024, 206}

$$-\tanh^{-1}\left(\sqrt{x^2+2x+5}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]), x]

[Out] -ArcTanh[Sqrt[5 + 2*x + x^2]]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1024

Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e - b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] && EqQ[h*e - 2*g*f, 0]

Rubi steps

$$\int \frac{1+x}{(4+2x+x^2)\sqrt{5+2x+x^2}} dx = -\left(2 \operatorname{Subst}\left(\int \frac{1}{2-2x^2} dx, x, \sqrt{5+2x+x^2}\right)\right) \\ = -\tanh^{-1}\left(\sqrt{5+2x+x^2}\right)$$

Mathematica [C] time = 0.04, size = 79, normalized size = 5.27

$$\frac{1}{2} \left(-\tanh^{-1}\left(\frac{-i\sqrt{3}x - i\sqrt{3} + 4}{\sqrt{x^2 + 2x + 5}}\right) - \tanh^{-1}\left(\frac{i\sqrt{3}x + i\sqrt{3} + 4}{\sqrt{x^2 + 2x + 5}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]), x]

[Out] (-ArcTanh[(4 - I*Sqrt[3] - I*Sqrt[3]*x)/Sqrt[5 + 2*x + x^2]] - ArcTanh[(4 + I*Sqrt[3] + I*Sqrt[3]*x)/Sqrt[5 + 2*x + x^2]])/2

IntegrateAlgebraic [A] time = 0.30, size = 15, normalized size = 1.00

$$-\tanh^{-1}\left(\sqrt{x^2+2x+5}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]

[Out] -ArcTanh[Sqrt[5 + 2*x + x^2]]

fricas [B] time = 0.41, size = 49, normalized size = 3.27

$$\frac{1}{2} \log\left(x^2 - \sqrt{x^2 + 2x + 5}(x + 2) + 3x + 6\right) - \frac{1}{2} \log\left(x^2 - \sqrt{x^2 + 2x + 5}x + x + 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="fricas")

[Out] 1/2*log(x^2 - sqrt(x^2 + 2*x + 5)*(x + 2) + 3*x + 6) - 1/2*log(x^2 - sqrt(x^2 + 2*x + 5)*x + x + 4)

giac [B] time = 0.35, size = 31, normalized size = 2.07

$$-\frac{1}{2} \log\left(\sqrt{x^2 + 2x + 5} + 1\right) + \frac{1}{2} \log\left(\sqrt{x^2 + 2x + 5} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="giac")

[Out] -1/2*log(sqrt(x^2 + 2*x + 5) + 1) + 1/2*log(sqrt(x^2 + 2*x + 5) - 1)

maple [A] time = 0.02, size = 14, normalized size = 0.93

$$-\operatorname{arctanh}\left(\sqrt{x^2 + 2x + 5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+1)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x)

[Out] -arctanh((x^2+2*x+5)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x + 1}{\sqrt{x^2 + 2x + 5}(x^2 + 2x + 4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="maxima")

[Out] integrate((x + 1)/(sqrt(x^2 + 2*x + 5)*(x^2 + 2*x + 4)), x)

mupad [B] time = 3.76, size = 13, normalized size = 0.87

$$-\operatorname{atanh}\left(\sqrt{x^2 + 2x + 5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 1)/((2*x + x^2 + 4)*(2*x + x^2 + 5)^(1/2)),x)

[Out] -atanh((2*x + x^2 + 5)^(1/2))

sympy [B] time = 7.00, size = 36, normalized size = 2.40

$$\frac{\log\left(-1 + \frac{1}{\sqrt{x^2+2x+5}}\right)}{2} - \frac{\log\left(1 + \frac{1}{\sqrt{x^2+2x+5}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(x**2+2*x+4)/(x**2+2*x+5)**(1/2),x)

[Out] log(-1 + 1/sqrt(x**2 + 2*x + 5))/2 - log(1 + 1/sqrt(x**2 + 2*x + 5))/2

$$3.32 \quad \int \frac{4+x}{(4+2x+x^2)\sqrt{5+2x+x^2}} dx$$

Optimal. Leaf size=44

$$\sqrt{3} \tan^{-1}\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right) - \tanh^{-1}\left(\sqrt{x^2+2x+5}\right)$$

Rubi [A] time = 0.05, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1025, 982, 204, 1024, 206}

$$\sqrt{3} \tan^{-1}\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right) - \tanh^{-1}\left(\sqrt{x^2+2x+5}\right)$$

Antiderivative was successfully verified.

[In] Int[(4 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]

[Out] Sqrt[3]*ArcTan[(1 + x)/(Sqrt[3]*Sqrt[5 + 2*x + x^2])] - ArcTanh[Sqrt[5 + 2*x + x^2]]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 982

Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0]

Rule 1024

Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e - b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] && EqQ[h*e - 2*g*f, 0]

Rule 1025

Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(h*e - 2*g*f)/(2*f), Int[1/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] && NeQ[h*e - 2*g*f, 0]

Rubi steps

$$\begin{aligned} \int \frac{4+x}{(4+2x+x^2)\sqrt{5+2x+x^2}} dx &= \frac{1}{2} \int \frac{2+2x}{(4+2x+x^2)\sqrt{5+2x+x^2}} dx + 3 \int \frac{1}{(4+2x+x^2)\sqrt{5+2x+x^2}} dx \\ &= -\left(2 \operatorname{Subst}\left(\int \frac{1}{2-2x^2} dx, x, \sqrt{5+2x+x^2}\right)\right) - 12 \operatorname{Subst}\left(\int \frac{1}{-24-2x^2} dx, x, \sqrt{5+2x+x^2}\right) \\ &= \sqrt{3} \tan^{-1}\left(\frac{1+x}{\sqrt{3}\sqrt{5+2x+x^2}}\right) - \tanh^{-1}\left(\sqrt{5+2x+x^2}\right) \end{aligned}$$

Mathematica [C] time = 0.05, size = 101, normalized size = 2.30

$$-\frac{1}{2}(1+i\sqrt{3}) \tanh^{-1}\left(\frac{-i\sqrt{3}x-i\sqrt{3}+4}{\sqrt{x^2+2x+5}}\right) - \frac{1}{2}(1-i\sqrt{3}) \tanh^{-1}\left(\frac{i\sqrt{3}x+i\sqrt{3}+4}{\sqrt{x^2+2x+5}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]), x]

[Out] -1/2*((1 + I*Sqrt[3])*ArcTanh[(4 - I*Sqrt[3] - I*Sqrt[3]*x)/Sqrt[5 + 2*x + x^2]]) - ((1 - I*Sqrt[3])*ArcTanh[(4 + I*Sqrt[3] + I*Sqrt[3]*x)/Sqrt[5 + 2*x + x^2]])/2

IntegrateAlgebraic [A] time = 0.35, size = 71, normalized size = 1.61

$$-\sqrt{3} \tan^{-1}\left(\frac{x^2}{\sqrt{3}} - \frac{(x+1)\sqrt{x^2+2x+5}}{\sqrt{3}} + \frac{2x}{\sqrt{3}} + \frac{4}{\sqrt{3}}\right) - \tanh^{-1}\left(\sqrt{x^2+2x+5}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(4 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]), x]

[Out] -(Sqrt[3]*ArcTan[4/Sqrt[3] + (2*x)/Sqrt[3] + x^2/Sqrt[3] - ((1 + x)*Sqrt[5 + 2*x + x^2])/Sqrt[3]]) - ArcTanh[Sqrt[5 + 2*x + x^2]]

fricas [B] time = 0.41, size = 106, normalized size = 2.41

$$-\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(x+2) + \frac{1}{3}\sqrt{3}\sqrt{x^2+2x+5}\right) + \sqrt{3} \arctan\left(-\frac{1}{3}\sqrt{3}x + \frac{1}{3}\sqrt{3}\sqrt{x^2+2x+5}\right) + \frac{1}{2} \log(x^2 - \sqrt{x^2+2x+5}(x+2) + 3x+6) - \frac{1}{2} \log(x^2 - \sqrt{x^2+2x+5}x + x+4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2), x, algorithm="fricas")

[Out] -sqrt(3)*arctan(-1/3*sqrt(3)*(x + 2) + 1/3*sqrt(3)*sqrt(x^2 + 2*x + 5)) + sqrt(3)*arctan(-1/3*sqrt(3)*x + 1/3*sqrt(3)*sqrt(x^2 + 2*x + 5)) + 1/2*log(x^2 - sqrt(x^2 + 2*x + 5)*(x + 2) + 3*x + 6) - 1/2*log(x^2 - sqrt(x^2 + 2*x + 5)*x + x + 4)

giac [B] time = 0.37, size = 108, normalized size = 2.45

$$-\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(x - \sqrt{x^2+2x+5} + 2)\right) + \sqrt{3} \arctan\left(-\frac{1}{3}\sqrt{3}(x - \sqrt{x^2+2x+5})\right) + \frac{1}{2} \log\left(\left(x - \sqrt{x^2+2x+5}\right)^2 + 4x - 4\sqrt{x^2+2x+5} + 7\right) - \frac{1}{2} \log\left(\left(x - \sqrt{x^2+2x+5}\right)^2 + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2), x, algorithm="giac")

[Out] -sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5) + 2)) + sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5))) + 1/2*log((x - sqrt(x^2 + 2*x + 5))^2 + 4*x - 4*sqrt(x^2 + 2*x + 5) + 7) - 1/2*log((x - sqrt(x^2 + 2*x + 5))^2 + 3)

maple [A] time = 0.01, size = 40, normalized size = 0.91

$$-\operatorname{arctanh}\left(\sqrt{x^2+2x+5}\right)+\sqrt{3}\operatorname{arctan}\left(\frac{\sqrt{3}(2x+2)}{6\sqrt{x^2+2x+5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x)`

[Out] `-arctanh((x^2+2*x+5)^(1/2))+3^(1/2)*arctan(1/6*3^(1/2)/(x^2+2*x+5)^(1/2)*(2*x+2))`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+4}{\sqrt{x^2+2x+5}(x^2+2x+4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="maxima")`

[Out] `integrate((x+4)/(sqrt(x^2+2*x+5)*(x^2+2*x+4)),x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x+4}{(x^2+2x+4)\sqrt{x^2+2x+5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x+4)/((2*x+x^2+4)*(2*x+x^2+5)^(1/2)),x)`

[Out] `int((x+4)/((2*x+x^2+4)*(2*x+x^2+5)^(1/2)),x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+4}{(x^2+2x+4)\sqrt{x^2+2x+5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4+x)/(x**2+2*x+4)/(x**2+2*x+5)**(1/2),x)`

[Out] `Integral((x+4)/((x**2+2*x+4)*sqrt(x**2+2*x+5)),x)`

$$3.33 \quad \int \frac{1+2x}{(3+x+x^2)\sqrt{5+x+x^2}} dx$$

Optimal. Leaf size=24

$$-\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{x^2+x+5}}{\sqrt{2}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1024, 206}

$$-\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{x^2+x+5}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x)/((3 + x + x^2)*Sqrt[5 + x + x^2]),x]

[Out] -(Sqrt[2]*ArcTanh[Sqrt[5 + x + x^2]/Sqrt[2]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1024

Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*g, Subst[Int[1/(b*d - a*e - b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] && EqQ[h*e - 2*g*f, 0]

Rubi steps

$$\begin{aligned} \int \frac{1+2x}{(3+x+x^2)\sqrt{5+x+x^2}} dx &= -\left(2 \text{Subst}\left(\int \frac{1}{2-x^2} dx, x, \sqrt{5+x+x^2}\right)\right) \\ &= -\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{5+x+x^2}}{\sqrt{2}}\right) \end{aligned}$$

Mathematica [C] time = 0.06, size = 90, normalized size = 3.75

$$\frac{\tanh^{-1}\left(\frac{-2i\sqrt{11}x-i\sqrt{11}+19}{4\sqrt{2}\sqrt{x^2+x+5}}\right) + \tanh^{-1}\left(\frac{2i\sqrt{11}x+i\sqrt{11}+19}{4\sqrt{2}\sqrt{x^2+x+5}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x)/((3 + x + x^2)*Sqrt[5 + x + x^2]),x]

[Out] -((ArcTanh[(19 - I*Sqrt[11] - (2*I)*Sqrt[11]*x)/(4*Sqrt[2]*Sqrt[5 + x + x^2]]) + ArcTanh[(19 + I*Sqrt[11] + (2*I)*Sqrt[11]*x)/(4*Sqrt[2]*Sqrt[5 + x + x^2])])/Sqrt[2])

IntegrateAlgebraic [A] time = 0.37, size = 24, normalized size = 1.00

$$-\sqrt{2} \tanh^{-1} \left(\frac{\sqrt{x^2 + x + 5}}{\sqrt{2}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 2*x)/((3 + x + x^2)*Sqrt[5 + x + x^2]),x]

[Out] -(Sqrt[2]*ArcTanh[Sqrt[5 + x + x^2]/Sqrt[2]])

fricas [A] time = 0.42, size = 34, normalized size = 1.42

$$\frac{1}{2} \sqrt{2} \log \left(\frac{x^2 - 2\sqrt{2}\sqrt{x^2 + x + 5} + x + 7}{x^2 + x + 3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="fricas")

[Out] 1/2*sqrt(2)*log((x^2 - 2*sqrt(2)*sqrt(x^2 + x + 5) + x + 7)/(x^2 + x + 3))

giac [B] time = 0.36, size = 39, normalized size = 1.62

$$-\frac{1}{2} \sqrt{2} \log \left(\sqrt{2} + \sqrt{x^2 + x + 5} \right) + \frac{1}{2} \sqrt{2} \log \left(-\sqrt{2} + \sqrt{x^2 + x + 5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="giac")

[Out] -1/2*sqrt(2)*log(sqrt(2) + sqrt(x^2 + x + 5)) + 1/2*sqrt(2)*log(-sqrt(2) + sqrt(x^2 + x + 5))

maple [A] time = 0.02, size = 20, normalized size = 0.83

$$-\sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{x^2 + x + 5} \sqrt{2}}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+2*x)/(x^2+x+3)/(x^2+x+5)^(1/2),x)

[Out] -arctanh(1/2*(x^2+x+5)^(1/2)*2^(1/2))*2^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x + 1}{\sqrt{x^2 + x + 5}(x^2 + x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="maxima")

[Out] integrate((2*x + 1)/(sqrt(x^2 + x + 5)*(x^2 + x + 3)), x)

mupad [B] time = 3.78, size = 19, normalized size = 0.79

$$-\sqrt{2} \operatorname{atanh} \left(\frac{\sqrt{2} \sqrt{x^2 + x + 5}}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x + 1)/((x + x^2 + 3)*(x + x^2 + 5)^(1/2)), x)`

[Out] `-2^(1/2)*atanh((2^(1/2)*(x + x^2 + 5)^(1/2))/2)`

sympy [A] time = 6.81, size = 68, normalized size = 2.83

$$2 \left(\begin{array}{l} -\frac{\sqrt{2} \operatorname{acoth}\left(\frac{\sqrt{2}}{\sqrt{x^2+x+5}}\right)}{2} \quad \text{for } \frac{1}{x^2+x+5} > \frac{1}{2} \\ -\frac{\sqrt{2} \operatorname{atanh}\left(\frac{\sqrt{2}}{\sqrt{x^2+x+5}}\right)}{2} \quad \text{for } \frac{1}{x^2+x+5} < \frac{1}{2} \end{array} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)/(x**2+x+3)/(x**2+x+5)**(1/2), x)`

[Out] `2*Piecewise((-sqrt(2)*acoth(sqrt(2)/sqrt(x**2 + x + 5))/2, 1/(x**2 + x + 5) > 1/2), (-sqrt(2)*atanh(sqrt(2)/sqrt(x**2 + x + 5))/2, 1/(x**2 + x + 5) < 1/2))`

$$3.34 \quad \int \frac{x}{(3+x+x^2)\sqrt{5+x+x^2}} dx$$

Optimal. Leaf size=56

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{2}{11}}(2x+1)}{\sqrt{x^2+x+5}}\right)}{\sqrt{22}} - \frac{\tanh^{-1}\left(\frac{\sqrt{x^2+x+5}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1025, 982, 204, 1024, 206}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{2}{11}}(2x+1)}{\sqrt{x^2+x+5}}\right)}{\sqrt{22}} - \frac{\tanh^{-1}\left(\frac{\sqrt{x^2+x+5}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[x/((3 + x + x^2)*Sqrt[5 + x + x^2]),x]

[Out] -(ArcTan[(Sqrt[2/11]*(1 + 2*x))/Sqrt[5 + x + x^2]]/Sqrt[22]) - ArcTanh[Sqrt[5 + x + x^2]/Sqrt[2]]/Sqrt[2]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 982

Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0]

Rule 1024

Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*g, Subst[Int[1/(b*d - a*e - b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] && EqQ[h*e - 2*g*f, 0]

Rule 1025

Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := -Dist[(h*e - 2*g*f)/(2*f), Int[1/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e

- b*f, 0] && NeQ[h*e - 2*g*f, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x}{(3+x+x^2)\sqrt{5+x+x^2}} dx &= -\left(\frac{1}{2} \int \frac{1}{(3+x+x^2)\sqrt{5+x+x^2}} dx\right) + \frac{1}{2} \int \frac{1+2x}{(3+x+x^2)\sqrt{5+x+x^2}} dx \\
 &= \text{Subst}\left(\int \frac{1}{-11-2x^2} dx, x, \frac{1+2x}{\sqrt{5+x+x^2}}\right) - \text{Subst}\left(\int \frac{1}{2-x^2} dx, x, \sqrt{5+x+x^2}\right) \\
 &= -\frac{\tan^{-1}\left(\frac{\sqrt{\frac{2}{11}}(1+2x)}{\sqrt{5+x+x^2}}\right)}{\sqrt{22}} - \frac{\tanh^{-1}\left(\frac{\sqrt{5+x+x^2}}{\sqrt{2}}\right)}{\sqrt{2}}
 \end{aligned}$$

Mathematica [C] time = 0.06, size = 114, normalized size = 2.04

$$\frac{-\left(\left(\sqrt{11}-i\right) \tanh^{-1}\left(\frac{-2i\sqrt{11}x-i\sqrt{11}+19}{4\sqrt{2}\sqrt{x^2+x+5}}\right)\right) - \left(\sqrt{11}+i\right) \tanh^{-1}\left(\frac{2i\sqrt{11}x+i\sqrt{11}+19}{4\sqrt{2}\sqrt{x^2+x+5}}\right)}{2\sqrt{22}}$$

Antiderivative was successfully verified.

[In] Integrate[x/((3 + x + x^2)*Sqrt[5 + x + x^2]), x]

[Out] $(-((-I + \text{Sqrt}[11]) \cdot \text{ArcTanh}[(19 - I \cdot \text{Sqrt}[11] - (2 \cdot I) \cdot \text{Sqrt}[11] \cdot x) / (4 \cdot \text{Sqrt}[2] \cdot \text{Sqrt}[5 + x + x^2])]) - (I + \text{Sqrt}[11]) \cdot \text{ArcTanh}[(19 + I \cdot \text{Sqrt}[11] + (2 \cdot I) \cdot \text{Sqrt}[11] \cdot x) / (4 \cdot \text{Sqrt}[2] \cdot \text{Sqrt}[5 + x + x^2])]) / (2 \cdot \text{Sqrt}[22])$

IntegrateAlgebraic [C] time = 0.24, size = 92, normalized size = 1.64

$$\text{RootSum}\left[\#1^4 - 2\#1^3 + 3\#1^2 - 2\#1 + 23\&, \frac{\#1^2 \log(-\#1 + \sqrt{x^2 + x + 5} - x) - 5 \log(-\#1 + \sqrt{x^2 + x + 5} - x)}{2\#1^3 - 3\#1^2 + 3\#1 - 1} \&\right]$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/((3 + x + x^2)*Sqrt[5 + x + x^2]), x]

[Out] $\text{RootSum}[23 - 2\#1 + 3\#1^2 - 2\#1^3 + \#1^4 \&, (-5 \cdot \text{Log}[-x + \text{Sqrt}[5 + x + x^2] - \#1] + \text{Log}[-x + \text{Sqrt}[5 + x + x^2] - \#1] \cdot \#1^2) / (-1 + 3\#1 - 3\#1^2 + 2\#1^3) \&]$

fricas [B] time = 0.45, size = 307, normalized size = 5.48

1/33*sqrt(11)*sqrt(6)*sqrt(3)*arctan(2/33*sqrt(11)*sqrt(3)*sqrt(sqrt(6)*sqrt(3)*(2*x+1)+6*x^2-sqrt(x^2+x+5)*(2*sqrt(6)*sqrt(3)+6*x+3)+6*x+30)+1/33*sqrt(11)*(2*sqrt(6)*sqrt(3)+6*x+3)-2/11*sqrt(11)*sqrt(x^2+x+5)+1/33*sqrt(11)*sqrt(6)*sqrt(3)*arctan(-1/33*sqrt(11)*(2*sqrt(6)*sqrt(3)-6*x-3)+1/33*sqrt(11)*sqrt(-12*sqrt(6)*sqrt(3)*(2*x+1)+72*x^2+12*sqrt(x^2+x+5)*(2*sqrt(6)*sqrt(3)-6*x-3)+72*x+360)-2/11*sqrt(11)*sqrt(x^2+x+5))+1/12*sqrt(6)*sqrt(3)*log(12*sqrt(6)*sqrt(3)*(2*x+1)+72*x^2-12*sqrt(x^2+x+5)*(2*sqrt(6)*sqrt(3)+6*x+3)+72*x+360)-1/12*sqrt(6)*sqrt(3)*log(-12*sqrt(6)*sqrt(3)*(2*x+1

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^2+x+3)/(x^2+x+5)^(1/2), x, algorithm="fricas")

[Out] $-1/33 \cdot \sqrt{11} \cdot \sqrt{6} \cdot \sqrt{3} \cdot \arctan(2/33 \cdot \sqrt{11} \cdot \sqrt{3} \cdot \sqrt{\sqrt{6} \cdot \sqrt{3} \cdot (2x+1) + 6x^2 - \sqrt{x^2+x+5} \cdot (2\sqrt{6}\sqrt{3} + 6x+3) + 6x+30} + 1/33 \cdot \sqrt{11} \cdot (2\sqrt{6}\sqrt{3} + 6x+3) - 2/11 \cdot \sqrt{11} \cdot \sqrt{x^2+x+5} + 1/33 \cdot \sqrt{11} \cdot \sqrt{6} \cdot \sqrt{3} \cdot \arctan(-1/33 \cdot \sqrt{11} \cdot (2\sqrt{6}\sqrt{3} - 6x-3) + 1/33 \cdot \sqrt{11} \cdot \sqrt{-12\sqrt{6}\sqrt{3} \cdot (2x+1) + 72x^2 + 12\sqrt{x^2+x+5} \cdot (2\sqrt{6}\sqrt{3} - 6x-3) + 72x+360} - 2/11 \cdot \sqrt{11} \cdot \sqrt{x^2+x+5} + 1/12 \cdot \sqrt{6} \cdot \sqrt{3} \cdot \log(12\sqrt{6}\sqrt{3} \cdot (2x+1) + 72x^2 - 12\sqrt{x^2+x+5} \cdot (2\sqrt{6}\sqrt{3} + 6x+3) + 72x+360) - 1/12 \cdot \sqrt{6} \cdot \sqrt{3} \cdot \log(-12\sqrt{6}\sqrt{3} \cdot (2x+1$

) + 72*x^2 + 12*sqrt(x^2 + x + 5)*(2*sqrt(6)*sqrt(3) - 6*x - 3) + 72*x + 36
0)

giac [B] time = 0.32, size = 133, normalized size = 2.38

$$\frac{1}{22}\sqrt{11}\sqrt{2}\arctan\left(\frac{1}{11}\sqrt{11}(2x+2\sqrt{2}-2\sqrt{x^2+x+5}+1)\right)-\frac{1}{22}\sqrt{11}\sqrt{2}\arctan\left(\frac{1}{11}\sqrt{11}(2x-2\sqrt{2}-2\sqrt{x^2+x+5}+1)\right)+\frac{1}{4}\sqrt{2}\log\left(324(2x+2\sqrt{2}-2\sqrt{x^2+x+5}+1)^2+3564\right)-\frac{1}{4}\sqrt{2}\log\left(324(2x-2\sqrt{2}-2\sqrt{x^2+x+5}+1)^2+3564\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="giac")

[Out] 1/22*sqrt(11)*sqrt(2)*arctan(-1/11*sqrt(11)*(2*x + 2*sqrt(2) - 2*sqrt(x^2 + x + 5) + 1)) - 1/22*sqrt(11)*sqrt(2)*arctan(-1/11*sqrt(11)*(2*x - 2*sqrt(2) - 2*sqrt(x^2 + x + 5) + 1)) + 1/4*sqrt(2)*log(324*(2*x + 2*sqrt(2) - 2*sqrt(x^2 + x + 5) + 1)^2 + 3564) - 1/4*sqrt(2)*log(324*(2*x - 2*sqrt(2) - 2*sqrt(x^2 + x + 5) + 1)^2 + 3564)

maple [A] time = 0.01, size = 45, normalized size = 0.80

$$-\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{x^2+x+5}\sqrt{2}}{2}\right)}{2}-\frac{\sqrt{22}\operatorname{arctan}\left(\frac{(2x+1)\sqrt{22}}{11\sqrt{x^2+x+5}}\right)}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x^2+x+3)/(x^2+x+5)^(1/2),x)

[Out] -1/2*arctanh(1/2*(x^2+x+5)^(1/2)*2^(1/2))*2^(1/2)-1/22*arctan(1/11*(1+2*x)*22^(1/2)/(x^2+x+5)^(1/2))*22^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^2+x+5}(x^2+x+3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="maxima")

[Out] integrate(x/(sqrt(x^2 + x + 5)*(x^2 + x + 3)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{(x^2+x+3)\sqrt{x^2+x+5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((x + x^2 + 3)*(x + x^2 + 5)^(1/2)),x)

[Out] int(x/((x + x^2 + 3)*(x + x^2 + 5)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(x^2+x+3)\sqrt{x^2+x+5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x**2+x+3)/(x**2+x+5)**(1/2),x)

[Out] Integral(x/((x**2 + x + 3)*sqrt(x**2 + x + 5)), x)

$$3.35 \quad \int \frac{A+Bx}{\sqrt{d+ex+fx^2} (ae+bex+bf x^2)^2} dx$$

Optimal. Leaf size=249

$$\frac{(Be - 2Af)(8aef - b(4df + e^2)) \tanh^{-1}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{2e^{3/2}f(bd-ae)^{3/2}(be-4af)^{3/2}} - \frac{\sqrt{d+ex+fx^2}(e(Ab-2aB) - bx(Be-2Af))}{e(bd-ae)(be-4af)(ae+bex+bf x^2)}$$

Rubi [A] time = 0.91, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {1016, 1025, 982, 208, 1024}

$$\frac{(Be - 2Af)(8aef - b(4df + e^2)) \tanh^{-1}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{2e^{3/2}f(bd-ae)^{3/2}(be-4af)^{3/2}} - \frac{\sqrt{d+ex+fx^2}(e(Ab-2aB) - bx(Be-2Af))}{e(bd-ae)(be-4af)(ae+bex+bf x^2)} + \frac{B \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex+fx^2}}{\sqrt{bd-ae}}\right)}{2\sqrt{b}f(bd-ae)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x)/(Sqrt[d + e*x + f*x^2]*(a*e + b*e*x + b*f*x^2)^2), x]

[Out] -((((A*b - 2*a*B)*e - b*(B*e - 2*A*f)*x)*Sqrt[d + e*x + f*x^2])/(e*(b*d - a*e)*(b*e - 4*a*f)*(a*e + b*e*x + b*f*x^2))) + ((B*e - 2*A*f)*(8*a*e*f - b*(e^2 + 4*d*f))*ArcTanh[(Sqrt[b*d - a*e]*(e + 2*f*x))/(Sqrt[e]*Sqrt[b*e - 4*a*f]*Sqrt[d + e*x + f*x^2])])/(2*e^(3/2)*(b*d - a*e)^(3/2)*f*(b*e - 4*a*f)^(3/2)) + (B*ArcTanh[(Sqrt[b]*Sqrt[d + e*x + f*x^2])/Sqrt[b*d - a*e]])/(2*Sqrt[b]*(b*d - a*e)^(3/2)*f)

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 982

Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0]

Rule 1016

Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*c*d - 2*a*c*e + a*b*f))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e

- b*f), 0] && !(!IntegerQ[p] && ILtQ[q, -1])

Rule 1024

Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[-2*g, Subst[Int[1/(b*d - a*e - b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] && EqQ[h*e - 2*g*f, 0]

Rule 1025

Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := -Dist[(h*e - 2*g*f)/(2*f), Int[1/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] && NeQ[h*e - 2*g*f, 0]

Rubi steps

$$\int \frac{A + Bx}{\sqrt{d + ex + fx^2} (ae + bex + bfx^2)^2} dx = -\frac{((Ab - 2aB)e - b(Be - 2Af)x)\sqrt{d + ex + fx^2}}{e(bd - ae)(be - 4af)(ae + bex + bfx^2)} - \int \frac{-\frac{1}{2}b(bd - ae)f^2(2bBde - e^2)}{\sqrt{d + ex + fx^2} (ae + bex + bfx^2)^2} dx$$

$$= -\frac{((Ab - 2aB)e - b(Be - 2Af)x)\sqrt{d + ex + fx^2}}{e(bd - ae)(be - 4af)(ae + bex + bfx^2)} - \frac{B \int \frac{e + 2fx}{\sqrt{d + ex + fx^2} (ae + bex + bfx^2)} dx}{4(bd - ae)}$$

$$= -\frac{((Ab - 2aB)e - b(Be - 2Af)x)\sqrt{d + ex + fx^2}}{e(bd - ae)(be - 4af)(ae + bex + bfx^2)} + \frac{(Be) \text{Subst}\left(\int \frac{1}{bde - a} dx\right)}{2}$$

$$= -\frac{((Ab - 2aB)e - b(Be - 2Af)x)\sqrt{d + ex + fx^2}}{e(bd - ae)(be - 4af)(ae + bex + bfx^2)} + \frac{(Be - 2Af)(8aef - 2e^2)}{2e}$$

Mathematica [B] time = 1.49, size = 767, normalized size = 3.08

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(Sqrt[d + e*x + f*x^2]*(a*e + b*e*x + b*f*x^2)^2), x]
[Out] -1/4*(4*b*Sqrt[e]*Sqrt[b*d - a*e]*f*Sqrt[b*e - 4*a*f]*Sqrt[d + x*(e + f*x)]
*(-(B*e*(2*a + b*x)) + A*b*(e + 2*f*x)) - (b^(3/2)*B*e^(5/2)*Sqrt[b*e - 4
*a*f]) + 4*a*Sqrt[b]*B*e^(3/2)*f*Sqrt[b*e - 4*a*f] - 8*a*b*e*f*(B*e - 2*A*f
) + b^2*(B*e - 2*A*f)*(e^2 + 4*d*f))*(a*e + b*x*(e + f*x))*Log[-(Sqrt[b]*Sq
rt[e]*Sqrt[b*e - 4*a*f]) + b*(e + 2*f*x)] + (b^(3/2)*B*e^(5/2)*Sqrt[b*e - 4
*a*f] - 4*a*Sqrt[b]*B*e^(3/2)*f*Sqrt[b*e - 4*a*f] - 8*a*b*e*f*(B*e - 2*A*f)
+ b^2*(B*e - 2*A*f)*(e^2 + 4*d*f))*(a*e + b*x*(e + f*x))*Log[Sqrt[b]*Sqrt[
e]*Sqrt[b*e - 4*a*f] + b*(e + 2*f*x)] - (b^(3/2)*B*e^(5/2)*Sqrt[b*e - 4*a*f
] - 4*a*Sqrt[b]*B*e^(3/2)*f*Sqrt[b*e - 4*a*f] - 8*a*b*e*f*(B*e - 2*A*f) + b
^2*(B*e - 2*A*f)*(e^2 + 4*d*f))*(a*e + b*x*(e + f*x))*Log[Sqrt[b]*(e^(3/2)*
Sqrt[b*e - 4*a*f] + Sqrt[b]*(e^2 - 4*d*f) + 2*Sqrt[e]*f*Sqrt[b*e - 4*a*f]*x
- 4*Sqrt[b*d - a*e]*f*Sqrt[d + x*(e + f*x)])] + (-(b^(3/2)*B*e^(5/2)*Sqrt[
```

$$b^2e - 4a^2f) + 4a\sqrt{b}B^2e^{3/2}f\sqrt{b^2e - 4a^2f} - 8a^2b^2e^2f(B^2e - 2A^2f) + b^2(B^2e - 2A^2f)(e^2 + 4d^2f)(ae + b^2x(e + fx))\text{Log}[\sqrt{b}(e^{3/2}\sqrt{b^2e - 4a^2f} - \sqrt{b}(e^2 - 4d^2f) + 2\sqrt{e}f\sqrt{b^2e - 4a^2f})x + 4\sqrt{bd - ae}f\sqrt{d + x(e + fx)}]]/(b^2e^{3/2}(bd - ae)^{3/2}f(b^2e - 4a^2f)^{3/2}(ae + b^2x(e + fx)))$$

IntegrateAlgebraic [C] time = 2.43, size = 1019, normalized size = 4.09

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B*x)/(Sqrt[d + e*x + f*x^2]*(a*e + b*e*x + b*f*x^2)^2), x]

[Out] -(((-(A*b*e) + 2*a*B*e + b*B*e*x - 2*A*b*f*x)*Sqrt[d + e*x + f*x^2])/(e*(-(b*d) + a*e)*(b^2e - 4*a^2*f)*(a*e + b*e*x + b*f*x^2))) - (4*B*RootSum[-(b*d*e^2) + a*e^3 + b*d^2*f + 2*b*d*e*Sqrt[f]*#1 - 4*a*e^2*Sqrt[f]*#1 + b*e^2*#1^2 - 2*b*d*f*#1^2 + 4*a*e*f*#1^2 - 2*b*e*Sqrt[f]*#1^3 + b*f*#1^4 & , Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1]/(-(b*d*e*Sqrt[f]) + 2*a*e^2*Sqrt[f] - b*e^2*#1 + 2*b*d*f*#1 - 4*a*e*f*#1 + 3*b*e*Sqrt[f]*#1^2 - 2*b*f*#1^3) &])/b - RootSum[-(b*d*e^2) + a*e^3 + b*d^2*f + 2*b*d*e*Sqrt[f]*#1 - 4*a*e^2*Sqrt[f]*#1 + b*e^2*#1^2 - 2*b*d*f*#1^2 + 4*a*e*f*#1^2 - 2*b*e*Sqrt[f]*#1^3 + b*f*#1^4 & , (-5*b^2*B*d*e^2*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1] - A*b^2*e^3*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1] + 6*a*b*B^2e^3*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1] - 4*A*b^2*d*e*f*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1] + 28*a*b*B*d*e*f*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1] + 8*a*A*b^2e^2*f*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1] - 32*a^2*B^2e^2*f*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1] - 4*b^2*B*d*e*Sqrt[f]*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1]*#1 + 2*A*b^2e^2*Sqrt[f]*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1]*#1 + 4*a*b*B^2e^2*Sqrt[f]*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1]*#1 + 8*A*b^2*d*f^(3/2)*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1]*#1 - 16*a*A*b^2e*f^(3/2)*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1]*#1 - b^2*B^2e^2*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1]*#1^2 + 4*a*b*B^2e*f*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1]*#1^2)/(-(b*d*e*Sqrt[f]) + 2*a*e^2*Sqrt[f] - b*e^2*#1 + 2*b*d*f*#1 - 4*a*e*f*#1 + 3*b*e*Sqrt[f]*#1^2 - 2*b*f*#1^3) &]/(2*b^2e*(b*d - a*e)*(b^2e - 4*a^2*f))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2), x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding error%%{%%}{1, [2]%%}, [8,2,0,0,0]%%}+%%{%%}{[%%]{-4, [1]%%}, 0]: [1,0,%%{-1,

```

[1]%%}]%}, [7, 2, 1, 0, 0]%%}+%%{%%{6, [1]%%}, [6, 2, 2, 0, 0]%%}+%%{%%{-4, [2
]%%}, [6, 2, 0, 0, 1]%%}+%%{%%{8, [2]%%}, [6, 1, 1, 1, 0]%%}+%%{%%{-4, 0} : [1, 0,
%%{-1, [1]%%}]%}, [5, 2, 3, 0, 0]%%}+%%{%%{12, [1]%%}, 0} : [1, 0, %%{-1, [1
]%%}]%}, [5, 2, 1, 0, 1]%%}+%%{%%{%%{-24, [1]%%}, 0} : [1, 0, %%{-1, [1]%%}]%}
}, [5, 1, 2, 1, 0]%%}+%%{1, [4, 2, 4, 0, 0]%%}+%%{%%{-14, [1]%%}, [4, 2, 2, 0, 1]%%}
+%%{%%{6, [2]%%}, [4, 2, 0, 0, 2]%%}+%%{%%{26, [1]%%}, [4, 1, 3, 1, 0]%%}+%%{%%
%%{-16, [2]%%}, [4, 1, 1, 1, 1]%%}+%%{%%{16, [2]%%}, [4, 0, 2, 2, 0]%%}+%%{%%{8
, 0} : [1, 0, %%{-1, [1]%%}]%}, [3, 2, 3, 0, 1]%%}+%%{%%{%%{-12, [1]%%}, 0} : [1, 0
, %%{-1, [1]%%}]%}, [3, 2, 1, 0, 2]%%}+%%{%%{%%{-12, 0} : [1, 0, %%{-1, [1]%%}]%},
[3, 1, 4, 1, 0]%%}+%%{%%{%%{32, [1]%%}, 0} : [1, 0, %%{-1, [1]%%}]%}, [3, 1, 2, 1,
1]%%}+%%{%%{%%{-32, [1]%%}, 0} : [1, 0, %%{-1, [1]%%}]%}, [3, 0, 3, 2, 0]%%}+
%%{-2, [2, 2, 4, 0, 1]%%}+%%{%%{10, [1]%%}, [2, 2, 2, 0, 2]%%}+%%{%%{-4, [2]%%}
}, [2, 2, 0, 0, 3]%%}+%%{2, [2, 1, 5, 1, 0]%%}+%%{%%{-28, [1]%%}, [2, 1, 3, 1, 1]%%}+
%%{%%{8, [2]%%}, [2, 1, 1, 1, 2]%%}+%%{%%{24, [1]%%}, [2, 0, 4, 2, 0]%%}+%%{%%
%%{-4, 0} : [1, 0, %%{-1, [1]%%}]%}, [1, 2, 3, 0, 2]%%}+%%{%%{4, [1]%%}, 0} : [1
, 0, %%{-1, [1]%%}]%}, [1, 2, 1, 0, 3]%%}+%%{%%{12, 0} : [1, 0, %%{-1, [1]%%}]%}
}, [1, 1, 4, 1, 1]%%}+%%{%%{%%{-8, [1]%%}, 0} : [1, 0, %%{-1, [1]%%}]%}, [1, 1, 2, 1
, 2]%%}+%%{%%{%%{-8, 0} : [1, 0, %%{-1, [1]%%}]%}, [1, 0, 5, 2, 0]%%}+%%{1, [0, 2, 4,
0, 2]%%}+%%{%%{-2, [1]%%}, [0, 2, 2, 0, 3]%%}+%%{%%{1, [2]%%}, [0, 2, 0, 0, 4]%%
%%}+%%{-2, [0, 1, 5, 1, 1]%%}+%%{%%{2, [1]%%}, [0, 1, 3, 1, 2]%%}+%%{1, [0, 0, 6, 2,
0]%%} / %%{%%{1, [3]%%}, [8, 2, 0, 0, 0]%%}+%%{%%{poly1[%%{-4, [2]%%}, 0} : [
1, 0, %%{-1, [1]%%}]%}, [7, 2, 1, 0, 0]%%}+%%{%%{6, [2]%%}, [6, 2, 2, 0, 0]%%}+%%
%%{%%{-4, [3]%%}, [6, 2, 0, 0, 1]%%}+%%{%%{8, [3]%%}, [6, 1, 1, 1, 0]%%}+%%{%%{p
oly1[%%{-4, [1]%%}, 0} : [1, 0, %%{-1, [1]%%}]%}, [5, 2, 3, 0, 0]%%}+%%{%%{poly1
[%%{12, [2]%%}, 0} : [1, 0, %%{-1, [1]%%}]%}, [5, 2, 1, 0, 1]%%}+%%{%%{poly1[%%
{-24, [2]%%}, 0} : [1, 0, %%{-1, [1]%%}]%}, [5, 1, 2, 1, 0]%%}+%%{%%{1, [1]%%}, [
4, 2, 4, 0, 0]%%}+%%{%%{-14, [2]%%}, [4, 2, 2, 0, 1]%%}+%%{%%{6, [3]%%}, [4, 2, 0
, 0, 2]%%}+%%{%%{26, [2]%%}, [4, 1, 3, 1, 0]%%}+%%{%%{-16, [3]%%}, [4, 1, 1, 1, 1
]%%}+%%{%%{16, [3]%%}, [4, 0, 2, 2, 0]%%}+%%{%%{poly1[%%{8, [1]%%}, 0} : [1, 0
, %%{-1, [1]%%}]%}, [3, 2, 3, 0, 1]%%}+%%{%%{poly1[%%{-12, [2]%%}, 0} : [1, 0, %%
{-1, [1]%%}]%}, [3, 2, 1, 0, 2]%%}+%%{%%{poly1[%%{-12, [1]%%}, 0} : [1, 0, %%{-
1, [1]%%}]%}, [3, 1, 4, 1, 0]%%}+%%{%%{poly1[%%{32, [2]%%}, 0} : [1, 0, %%{-1, [1
]%%}]%}, [3, 1, 2, 1, 1]%%}+%%{%%{poly1[%%{-32, [2]%%}, 0} : [1, 0, %%{-1, [1]%%
}]%}, [3, 0, 3, 2, 0]%%}+%%{%%{-2, [1]%%}, [2, 2, 4, 0, 1]%%}+%%{%%{10, [2]%%}
}, [2, 2, 2, 0, 2]%%}+%%{%%{-4, [3]%%}, [2, 2, 0, 0, 3]%%}+%%{%%{2, [1]%%}, [2, 1
, 5, 1, 0]%%}+%%{%%{-28, [2]%%}, [2, 1, 3, 1, 1]%%}+%%{%%{8, [3]%%}, [2, 1, 1, 1,
2]%%}+%%{%%{24, [2]%%}, [2, 0, 4, 2, 0]%%}+%%{%%{poly1[%%{-4, [1]%%}, 0} : [1
, 0, %%{-1, [1]%%}]%}, [1, 2, 3, 0, 2]%%}+%%{%%{poly1[%%{4, [2]%%}, 0} : [1, 0, %%
{-1, [1]%%}]%}, [1, 2, 1, 0, 3]%%}+%%{%%{poly1[%%{12, [1]%%}, 0} : [1, 0, %%{-1
, [1]%%}]%}, [1, 1, 4, 1, 1]%%}+%%{%%{poly1[%%{-8, [2]%%}, 0} : [1, 0, %%{-1, [1]
%%}]%}, [1, 1, 2, 1, 2]%%}+%%{%%{poly1[%%{-8, [1]%%}, 0} : [1, 0, %%{-1, [1]%%}
]%%}, [1, 0, 5, 2, 0]%%}+%%{%%{1, [1]%%}, [0, 2, 4, 0, 2]%%}+%%{%%{-2, [2]%%}, [
0, 2, 2, 0, 3]%%}+%%{%%{1, [3]%%}, [0, 2, 0, 0, 4]%%}+%%{%%{-2, [1]%%}, [0, 1, 5,
1, 1]%%}+%%{%%{2, [2]%%}, [0, 1, 3, 1, 2]%%}+%%{%%{1, [1]%%}, [0, 0, 6, 2, 0]%%}
} Error: Bad Argument Value

```

maple [B] time = 0.05, size = 3606, normalized size = 14.48

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((B*x+A)/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^{(1/2)}, x)$

[Out]
$$-1/e/(4*a*f-b*e)/(a*e-b*d)/(x+1/2/f*e-1/2/b/f*(-b*e*(4*a*f-b*e))^{(1/2)})*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)^2*f+(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)-1/b*(a*e-b*d))^{(1/2)}*A+1/2/f/(4*a*f-b*e)/(a*e-b*d)/(x+1/2/f*e-1/2/b/f*(-b*e*(4*a*f-b*e))^{(1/2)})*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)^2*f+(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)-1/b*(a*e-b*d))^{(1/2)}*B-1/2/f/e/(4*a*f-b*e)/b/(a*e-b*d)/(x+1/2/f*e-1/2/b/f*(-b*e*(4*a*f-b*e))^{(1/2)})*((x-1/2*(-b*e+(-b$$

$$\begin{aligned}
& *e*(4*a*f-b*e))^{(1/2)}/b/f)^{2*f+(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x-1/2*(-b*e+(-b \\
& *e*(4*a*f-b*e))^{(1/2)}/b/f)-1/b*(a*e-b*d))^{(1/2)}*B*(-b*e*(4*a*f-b*e))^{(1/2)} \\
& +1/2/e/(4*a*f-b*e)/b*(-b*e*(4*a*f-b*e))^{(1/2)/(a*e-b*d)/(-1/b*(a*e-b*d))^{(1 \\
& /2)*\ln((-2/b*(a*e-b*d)+(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x-1/2*(-b*e+(-b*e*(4*a*f \\
& -b*e))^{(1/2)}/b/f)+2*(-1/b*(a*e-b*d))^{(1/2)*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e) \\
&)^{(1/2)}/b/f)^{2*f+(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e) \\
&)^{(1/2)}/b/f)-1/b*(a*e-b*d))^{(1/2)}/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)}/ \\
& b/f))*A-1/4/f/(4*a*f-b*e)/b*(-b*e*(4*a*f-b*e))^{(1/2)/(a*e-b*d)/(-1/b*(a*e-b \\
& *d))^{(1/2)*\ln((-2/b*(a*e-b*d)+(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x-1/2*(-b*e+(-b*e \\
& *(4*a*f-b*e))^{(1/2)}/b/f)+2*(-1/b*(a*e-b*d))^{(1/2)*((x-1/2*(-b*e+(-b*e*(4*a \\
& *f-b*e))^{(1/2)}/b/f)^{2*f+(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x-1/2*(-b*e+(-b*e*(4*a \\
& *f-b*e))^{(1/2)}/b/f)-1/b*(a*e-b*d))^{(1/2)}/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(\\
& 1/2)}/b/f))*B-1/4/f/b/(a*e-b*d)/(-1/b*(a*e-b*d))^{(1/2)*\ln((-2/b*(a*e-b*d)+ \\
& (-b*e*(4*a*f-b*e))^{(1/2)}/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)}/b/f)+2*(- \\
& 1/b*(a*e-b*d))^{(1/2)*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)}/b/f)^{2*f+(-b*e \\
& *(4*a*f-b*e))^{(1/2)}/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)}/b/f)-1/b*(a*e- \\
& b*d))^{(1/2)}/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)}/b/f))*B-2/e/(4*a*f-b*e) \\
& /(-b*e*(4*a*f-b*e))^{(1/2)/(-1/b*(a*e-b*d))^{(1/2)*\ln((-2/b*(a*e-b*d)+(-b*e*(\\
& 4*a*f-b*e))^{(1/2)}/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)}/b/f)+2*(-1/b*(a* \\
& e-b*d))^{(1/2)*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)}/b/f)^{2*f+(-b*e*(4*a*f \\
& -b*e))^{(1/2)}/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)}/b/f)-1/b*(a*e-b*d))^{(\\
& 1/2)}/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)}/b/f))*A*f+1/(4*a*f-b*e)/(-b*e* \\
& (4*a*f-b*e))^{(1/2)/(-1/b*(a*e-b*d))^{(1/2)*\ln((-2/b*(a*e-b*d)+(-b*e*(4*a*f-b \\
& *e))^{(1/2)}/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)}/b/f)+2*(-1/b*(a*e-b*d)) \\
& ^{(1/2)*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)}/b/f)^{2*f+(-b*e*(4*a*f-b*e))^{(\\
& 1/2)}/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)}/b/f)-1/b*(a*e-b*d))^{(1/2)}/(\\
& x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)}/b/f))*B+2/e/(4*a*f-b*e)/(-b*e*(4*a*f- \\
& b*e))^{(1/2)/(-1/b*(a*e-b*d))^{(1/2)*\ln((-2/b*(a*e-b*d)-(-b*e*(4*a*f-b*e))^{(1 \\
& /2)}/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)}/b/f)+2*(-1/b*(a*e-b*d))^{(1/2)* \\
& ((x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)}/b/f)^{2*f-(-b*e*(4*a*f-b*e))^{(1/2)}/b*(\\
& x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)}/b/f)-1/b*(a*e-b*d))^{(1/2)}/(x+1/2*(b*e \\
& +(-b*e*(4*a*f-b*e))^{(1/2)}/b/f))*A*f-1/(4*a*f-b*e)/(-b*e*(4*a*f-b*e))^{(1/2) \\
& /(-1/b*(a*e-b*d))^{(1/2)*\ln((-2/b*(a*e-b*d)-(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x+1/ \\
& 2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)}/b/f)+2*(-1/b*(a*e-b*d))^{(1/2)*((x+1/2*(b*e \\
& +(-b*e*(4*a*f-b*e))^{(1/2)}/b/f)^{2*f-(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x+1/2*(b*e+ \\
& (-b*e*(4*a*f-b*e))^{(1/2)}/b/f)-1/b*(a*e-b*d))^{(1/2)}/(x+1/2*(b*e+(-b*e*(4*a \\
& *f-b*e))^{(1/2)}/b/f))*B-1/e/(4*a*f-b*e)/(a*e-b*d)/(x+1/2/f*e+1/2/b/f*(-b*e* \\
& (4*a*f-b*e))^{(1/2)*((x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)}/b/f)^{2*f-(-b*e*(\\
& 4*a*f-b*e))^{(1/2)}/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)}/b/f)-1/b*(a*e-b*d \\
&))^{(1/2)*A+1/2/f/(4*a*f-b*e)/(a*e-b*d)/(x+1/2/f*e+1/2/b/f*(-b*e*(4*a*f-b*e) \\
&)^{(1/2)*((x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)}/b/f)^{2*f-(-b*e*(4*a*f-b*e)) \\
& ^{(1/2)}/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)}/b/f)-1/b*(a*e-b*d))^{(1/2)*B+ \\
& 1/2/f/e/(4*a*f-b*e)/b/(a*e-b*d)/(x+1/2/f*e+1/2/b/f*(-b*e*(4*a*f-b*e))^{(1/2) \\
&)*((x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)}/b/f)^{2*f-(-b*e*(4*a*f-b*e))^{(1/2)}/ \\
& b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)}/b/f)-1/b*(a*e-b*d))^{(1/2)*B*(-b*e*(\\
& 4*a*f-b*e))^{(1/2)-1/2/e/(4*a*f-b*e)/b*(-b*e*(4*a*f-b*e))^{(1/2)/(a*e-b*d)/(- \\
& 1/b*(a*e-b*d))^{(1/2)*\ln((-2/b*(a*e-b*d)-(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x+1/2*(\\
& b*e+(-b*e*(4*a*f-b*e))^{(1/2)}/b/f)+2*(-1/b*(a*e-b*d))^{(1/2)*((x+1/2*(b*e+(- \\
& b*e*(4*a*f-b*e))^{(1/2)}/b/f)^{2*f-(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x+1/2*(b*e+(-b \\
& *e*(4*a*f-b*e))^{(1/2)}/b/f)-1/b*(a*e-b*d))^{(1/2)}/(x+1/2*(b*e+(-b*e*(4*a*f- \\
& b*e))^{(1/2)}/b/f))*A+1/4/f/(4*a*f-b*e)/b*(-b*e*(4*a*f-b*e))^{(1/2)/(a*e-b*d) \\
& /(-1/b*(a*e-b*d))^{(1/2)*\ln((-2/b*(a*e-b*d)-(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x+1/ \\
& 2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)}/b/f)+2*(-1/b*(a*e-b*d))^{(1/2)*((x+1/2*(b*e \\
& +(-b*e*(4*a*f-b*e))^{(1/2)}/b/f)^{2*f-(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x+1/2*(b*e+ \\
& (-b*e*(4*a*f-b*e))^{(1/2)}/b/f)-1/b*(a*e-b*d))^{(1/2)}/(x+1/2*(b*e+(-b*e*(4*a \\
& *f-b*e))^{(1/2)}/b/f))*B-1/4/f/b/(a*e-b*d)/(-1/b*(a*e-b*d))^{(1/2)*\ln((-2/b*(\\
& a*e-b*d)-(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)}/b \\
& /f)+2*(-1/b*(a*e-b*d))^{(1/2)*((x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)}/b/f)^{2* \\
& f-(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)}/b/f)-1/b
\end{aligned}$$

$(a*e-b*d)^{(1/2)}/(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f))*B$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx + A}{(bf x^2 + bex + ae)^2 \sqrt{fx^2 + ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")

[Out] integrate((B*x + A)/((b*f*x^2 + b*e*x + a*e)^2*sqrt(f*x^2 + e*x + d)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + Bx}{(bf x^2 + bex + ae)^2 \sqrt{fx^2 + ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x)/((a*e + b*e*x + b*f*x^2)^2*(d + e*x + f*x^2)^(1/2)),x)

[Out] int((A + B*x)/((a*e + b*e*x + b*f*x^2)^2*(d + e*x + f*x^2)^(1/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x+A)/(b*f*x**2+b*e*x+a*e)**2/(f*x**2+e*x+d)**(1/2),x)

[Out] Timed out

$$3.36 \quad \int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^2} dx$$

Optimal. Leaf size=48

$$-\frac{2(-2ah + x(2cg - bh) + bg)}{d^2 (b^2 - 4ac) \sqrt{a + bx + cx^2}}$$

Rubi [A] time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {998, 636}

$$-\frac{2(-2ah + x(2cg - bh) + bg)}{d^2 (b^2 - 4ac) \sqrt{a + bx + cx^2}}$$

Antiderivative was successfully verified.

[In] Int[((g + h*x)*Sqrt[a + b*x + c*x^2])/(a*d + b*d*x + c*d*x^2)^2,x]

[Out] (-2*(b*g - 2*a*h + (2*c*g - b*h)*x))/((b^2 - 4*a*c)*d^2*Sqrt[a + b*x + c*x^2])

Rule 636

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0]

Rule 998

Int[((g_.) + (h_.)*(x_)^(m_.))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[(c/f)^p, Int[(g + h*x)^m*(d + e*x + f*x^2)^(p + q), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q}, x] && EqQ[c*d - a*f, 0] && EqQ[b*d - a*e, 0] && (IntegerQ[p] || GtQ[c/f, 0]) && (!IntegerQ[q] || LeafCount[d + e*x + f*x^2] <= LeafCount[a + b*x + c*x^2])

Rubi steps

$$\begin{aligned} \int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^2} dx &= \frac{\int \frac{g+hx}{(a+bx+cx^2)^{3/2}} dx}{d^2} \\ &= -\frac{2(bg - 2ah + (2cg - bh)x)}{(b^2 - 4ac) d^2 \sqrt{a + bx + cx^2}} \end{aligned}$$

Mathematica [A] time = 0.19, size = 46, normalized size = 0.96

$$\frac{4ah - 2bg + 2bhx - 4cgx}{d^2 (b^2 - 4ac) \sqrt{a + x(b + cx)}}$$

Antiderivative was successfully verified.

[In] Integrate[((g + h*x)*Sqrt[a + b*x + c*x^2])/(a*d + b*d*x + c*d*x^2)^2,x]

[Out] (-2*b*g + 4*a*h - 4*c*g*x + 2*b*h*x)/((b^2 - 4*a*c)*d^2*Sqrt[a + x*(b + c*x)])

IntegrateAlgebraic [A] time = 0.42, size = 47, normalized size = 0.98

$$\frac{2(-2ah + bg - bhx + 2cgx)}{d^2(b^2 - 4ac)\sqrt{a + bx + cx^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic(((g + h*x)*Sqrt[a + b*x + c*x^2])/(a*d + b*d*x + c*d*x^2)^2,x)

[Out] (-2*(b*g - 2*a*h + 2*c*g*x - b*h*x))/((b^2 - 4*a*c)*d^2*Sqrt[a + b*x + c*x^2])

fricas [A] time = 0.63, size = 85, normalized size = 1.77

$$\frac{2\sqrt{cx^2 + bx + a}(bg - 2ah + (2cg - bh)x)}{(b^2c - 4ac^2)d^2x^2 + (b^3 - 4abc)d^2x + (ab^2 - 4a^2c)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="fricas")

[Out] -2*sqrt(c*x^2 + b*x + a)*(b*g - 2*a*h + (2*c*g - b*h)*x)/((b^2*c - 4*a*c^2)*d^2*x^2 + (b^3 - 4*a*b*c)*d^2*x + (a*b^2 - 4*a^2*c)*d^2)

giac [A] time = 0.27, size = 81, normalized size = 1.69

$$\frac{2\left(\frac{(2cd^2g-bd^2h)x}{b^2d^4-4acd^4} + \frac{bd^2g-2ad^2h}{b^2d^4-4acd^4}\right)}{\sqrt{cx^2 + bx + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="giac")

[Out] -2*((2*c*d^2*g - b*d^2*h)*x/(b^2*d^4 - 4*a*c*d^4) + (b*d^2*g - 2*a*d^2*h)/(b^2*d^4 - 4*a*c*d^4))/sqrt(c*x^2 + b*x + a)

maple [A] time = 0.00, size = 48, normalized size = 1.00

$$\frac{2(bhx - 2cgx + 2ah - bg)}{\sqrt{cx^2 + bx + a}(4ac - b^2)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^2,x)

[Out] -2/(c*x^2+b*x+a)^(1/2)*(b*h*x-2*c*g*x+2*a*h-b*g)/d^2/(4*a*c-b^2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + bx + a}(hx + g)}{(cdx^2 + bdx + ad)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="maxima")

[Out] integrate(sqrt(c*x^2 + b*x + a)*(h*x + g)/(c*d*x^2 + b*d*x + a*d)^2, x)

mupad [B] time = 3.75, size = 49, normalized size = 1.02

$$\frac{4ah - 2bg + 2bhx - 4cgx}{(b^2d^2 - 4acd^2)\sqrt{cx^2 + bx + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((g + h*x)*(a + b*x + c*x^2)^(1/2))/(a*d + b*d*x + c*d*x^2)^2,x)

[Out] (4*a*h - 2*b*g + 2*b*h*x - 4*c*g*x)/((b^2*d^2 - 4*a*c*d^2)*(a + b*x + c*x^2)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{g}{a\sqrt{a+bx+cx^2}+bx\sqrt{a+bx+cx^2}+cx^2\sqrt{a+bx+cx^2}} dx + \int \frac{hx}{a\sqrt{a+bx+cx^2}+bx\sqrt{a+bx+cx^2}+cx^2\sqrt{a+bx+cx^2}} dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)*(c*x**2+b*x+a)**(1/2)/(c*d*x**2+b*d*x+a*d)**2,x)

[Out] (Integral(g/(a*sqrt(a + b*x + c*x**2) + b*x*sqrt(a + b*x + c*x**2) + c*x**2*sqrt(a + b*x + c*x**2)), x) + Integral(h*x/(a*sqrt(a + b*x + c*x**2) + b*x*sqrt(a + b*x + c*x**2) + c*x**2*sqrt(a + b*x + c*x**2)), x))/d**2

$$3.37 \quad \int \frac{3+2x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx$$

Optimal. Leaf size=17

$$\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1027, 206}

$$\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

[In] Int[(3 + 2*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]

[Out] ArcTanh[x/Sqrt[-3 - 4*x - x^2]]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1027

Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] && EqQ[2*h*d - g*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{3+2x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx &= 3 \text{Subst} \left(\int \frac{1}{3-3x^2} dx, x, \frac{x}{\sqrt{-3-4x-x^2}} \right) \\ &= \tanh^{-1} \left(\frac{x}{\sqrt{-3-4x-x^2}} \right) \end{aligned}$$

Mathematica [C] time = 0.29, size = 165, normalized size = 9.71

$$\frac{1}{6} \left(\sqrt{1-2i\sqrt{2}} (\sqrt{2}+i) \tanh^{-1} \left(\frac{(2-i\sqrt{2})x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}} \sqrt{-x^2-4x-3}} \right) + \sqrt{1+2i\sqrt{2}} (\sqrt{2}-i) \tanh^{-1} \left(\frac{(2+i\sqrt{2})x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}} \sqrt{-x^2-4x-3}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 2*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]

[Out] (Sqrt[1 - (2*I)*Sqrt[2]]*(I + Sqrt[2])*ArcTanh[(2 - (2*I)*Sqrt[2] + (2 - I*Sqrt[2])*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])] + Sqrt[1 + (2*I)*Sqrt[2]]*(-I + Sqrt[2])*ArcTanh[(2 + (2*I)*Sqrt[2] + (2 + I*Sqrt[2])*x)/(Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])])/6

IntegrateAlgebraic [A] time = 0.30, size = 17, normalized size = 1.00

$$\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(3 + 2*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]

[Out] ArcTanh[x/Sqrt[-3 - 4*x - x^2]]

fricas [B] time = 0.42, size = 56, normalized size = 3.29

$$-\frac{1}{4} \log\left(-\frac{2\sqrt{-x^2-4x-3}x+4x+3}{x^2}\right) + \frac{1}{4} \log\left(\frac{2\sqrt{-x^2-4x-3}x-4x-3}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+2*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")

[Out] -1/4*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) + 1/4*log((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)

giac [B] time = 0.23, size = 98, normalized size = 5.76

$$\frac{1}{2} \log\left(\frac{2(\sqrt{-x^2-4x-3}-1)}{x+2} + \frac{3(\sqrt{-x^2-4x-3}-1)^2}{(x+2)^2} + 1\right) - \frac{1}{2} \log\left(\frac{2(\sqrt{-x^2-4x-3}-1)}{x+2} + \frac{(\sqrt{-x^2-4x-3}-1)^2}{(x+2)^2} + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+2*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")

[Out] 1/2*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) - 1/2*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)

maple [B] time = 0.02, size = 94, normalized size = 5.53

$$\frac{\sqrt{3} \sqrt{4} \sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2} - 12} \operatorname{arctanh}\left(\frac{3x}{(-x-\frac{3}{2}) \sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2} - 12}}\right)}{6 \sqrt{\frac{\frac{x^2}{(-x-\frac{3}{2})^2} - 4}{\left(\frac{x}{-x-\frac{3}{2}} + 1\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3+2*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)

[Out] -1/6*3^(1/2)*4^(1/2)/((x^2/(-3/2-x)^2-4)/(x/(-3/2-x)+1)^2)^(1/2)/(x/(-3/2-x)+1)*(3*x^2/(-3/2-x)^2-12)^(1/2)*arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+3}{(2x^2+4x+3)\sqrt{-x^2-4x-3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+2*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")

[Out] integrate((2*x + 3)/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{2x + 3}{\sqrt{-x^2 - 4x - 3} (2x^2 + 4x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x + 3)/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)

[Out] int((2*x + 3)/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x + 3}{\sqrt{-(x + 1)(x + 3)} (2x^2 + 4x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+2*x)/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)

[Out] Integral((2*x + 3)/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)

$$3.38 \quad \int \frac{3+4x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx$$

Optimal. Leaf size=86

$$\sqrt{2} \tan^{-1} \left(\frac{1 - \frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}} \right) - \sqrt{2} \tan^{-1} \left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}} + 1}{\sqrt{2}} \right) + \tanh^{-1} \left(\frac{x}{\sqrt{-x^2-4x-3}} \right)$$

Rubi [A] time = 0.18, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {1028, 986, 12, 1026, 1161, 618, 204, 1027, 206}

$$\sqrt{2} \tan^{-1} \left(\frac{1 - \frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}} \right) - \sqrt{2} \tan^{-1} \left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}} + 1}{\sqrt{2}} \right) + \tanh^{-1} \left(\frac{x}{\sqrt{-x^2-4x-3}} \right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] Sqrt[2]*ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]] - Sqrt[2]*ArcTan
[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]] + ArcTanh[x/Sqrt[-3 - 4*x - x^
2]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1026

```
Int[(x_)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*
(x_)^2]), x_Symbol] := Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 -
4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1027

```
Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_
)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
EqQ[2*h*d - g*e, 0]
```

Rule 1028

```
Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_
)*(x_) + (f_)*(x_)^2]), x_Symbol] := -Dist[(2*h*d - g*e)/e, Int[1/((a + b
*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g
, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
&& NeQ[2*h*d - g*e, 0]
```

Rule 1161

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{3+4x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx &= -\left(3 \int \frac{1}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx\right) - \int \frac{-6-4x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx \\
&= \frac{1}{2} \int \frac{-6-4x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx - \frac{1}{2} \int \frac{4x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx \\
&= 2 \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + 2 \int \frac{x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx - 3S \\
&= \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + 16 \text{Subst}\left(\int \frac{1+3x^2}{-4-8x^2-36x^4} dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right) \\
&= \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - \frac{2}{3} \text{Subst}\left(\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2} dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right) \\
&= \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{4}{3} \text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2} dx, x, \frac{2}{3}\left(-1+\frac{3+x}{\sqrt{-3-4x-x^2}}\right)\right) \\
&= \sqrt{2} \tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) - \sqrt{2} \tan^{-1}\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) + \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)
\end{aligned}$$

Mathematica [C] time = 0.11, size = 150, normalized size = 1.74

$$-\frac{1}{2}i \left(\sqrt{1+2i\sqrt{2}} \tanh^{-1} \left(\frac{(2-i\sqrt{2})x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}} \right) - \sqrt{1-2i\sqrt{2}} \tanh^{-1} \left(\frac{(2+i\sqrt{2})x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}}\sqrt{-x^2-4x-3}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 4*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]

[Out] (-1/2*I)*(Sqrt[1 + (2*I)*Sqrt[2]]*ArcTanh[(2 - (2*I)*Sqrt[2] + (2 - I*Sqrt[2])*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]]) - Sqrt[1 - (2*I)*Sqrt[2]]*ArcTanh[(2 + (2*I)*Sqrt[2] + (2 + I*Sqrt[2])*x)/(Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]])

IntegrateAlgebraic [A] time = 0.35, size = 56, normalized size = 0.65

$$\tanh^{-1} \left(\frac{x}{\sqrt{-x^2-4x-3}} \right) - \sqrt{2} \tan^{-1} \left(\frac{\sqrt{2}x + \frac{3}{\sqrt{2}}}{\sqrt{-x^2-4x-3}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(3 + 4*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]

[Out] -(Sqrt[2]*ArcTan[(3/Sqrt[2] + Sqrt[2]*x)/Sqrt[-3 - 4*x - x^2]]) + ArcTanh[x/Sqrt[-3 - 4*x - x^2]]

fricas [A] time = 0.43, size = 132, normalized size = 1.53

$$\frac{1}{2}\sqrt{2} \arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right) + \frac{1}{2}\sqrt{2} \arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right) - \frac{1}{4} \log\left(-\frac{2\sqrt{-x^2-4x-3}x+4x+3}{x^2}\right) + \frac{1}{4} \log\left(\frac{2\sqrt{-x^2-4x-3}x-4x-3}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+4*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")

[Out] 1/2*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) + 1/2*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) - 1/4*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) + 1/4*log((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)

giac [B] time = 0.27, size = 163, normalized size = 1.90

$$\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3(\sqrt{-x^2-4x-3}-1)}{x+2}+1\right)\right) + \sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}\right)\right) + \frac{1}{2} \log\left(\frac{2(\sqrt{-x^2-4x-3}-1)}{x+2} + \frac{3(\sqrt{-x^2-4x-3}-1)^2}{(x+2)^2} + 1\right) - \frac{1}{2} \log\left(\frac{2(\sqrt{-x^2-4x-3}-1)}{x+2} + \frac{(\sqrt{-x^2-4x-3}-1)^2}{(x+2)^2} + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+4*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")

[Out] sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/2*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) - 1/2*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)

maple [A] time = 0.01, size = 123, normalized size = 1.43

$$\frac{\sqrt{3} \sqrt{4} \sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2} - 12} \left(-\operatorname{arctanh} \left(\frac{3x}{(-x-\frac{3}{2}) \sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2} - 12}} \right) + \sqrt{2} \operatorname{arctan} \left(\frac{\sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2} - 12} \sqrt{2}}{6} \right) \right)}{6 \sqrt{\frac{\frac{x^2}{(-x-\frac{3}{2})^2} - 4}{\left(\frac{x}{-x-\frac{3}{2}} + 1\right)^2}} \left(\frac{x}{-x-\frac{3}{2}} + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x+3)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)`

[Out] `1/6*3^(1/2)*4^(1/2)*(3/(-x-3/2)^2*x^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2))-arctanh(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(1/2)*x))/((1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^(1/2)/(1/(-x-3/2)*x+1)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x + 3}{(2x^2 + 4x + 3)\sqrt{-x^2 - 4x - 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3+4*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")`

[Out] `integrate((4*x + 3)/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4x + 3}{\sqrt{-x^2 - 4x - 3} (2x^2 + 4x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x + 3)/((-4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)`

[Out] `int((4*x + 3)/((-4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x + 3}{\sqrt{-(x + 1)(x + 3)} (2x^2 + 4x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3+4*x)/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)`

[Out] `Integral((4*x + 3)/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)`

$$3.39 \quad \int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^{3/2}} dx$$

Optimal. Leaf size=136

$$\frac{h\sqrt{a+bx+cx^2} \log(a+bx+cx^2)}{2cd\sqrt{ad+bdx+cdx^2}} - \frac{\sqrt{a+bx+cx^2} (2cg-bh) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{cd\sqrt{b^2-4ac}\sqrt{ad+bdx+cdx^2}}$$

Rubi [A] time = 0.11, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {999, 634, 618, 206, 628}

$$\frac{h\sqrt{a+bx+cx^2} \log(a+bx+cx^2)}{2cd\sqrt{ad+bdx+cdx^2}} - \frac{\sqrt{a+bx+cx^2} (2cg-bh) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{cd\sqrt{b^2-4ac}\sqrt{ad+bdx+cdx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)*Sqrt[a + b*x + c*x^2])/(a*d + b*d*x + c*d*x^2)^(3/2), x]
[Out] -(((2*c*g - b*h)*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]*d*Sqrt[a*d + b*d*x + c*d*x^2])) + (h*Sqrt[a + b*x + c*x^2]*Log[a + b*x + c*x^2])/(2*c*d*Sqrt[a*d + b*d*x + c*d*x^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 999

```
Int[((g_.) + (h_.)*(x_)^(m_.))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x + c*x^2)^FracPart[p])/(d^IntPart[p]*(d + e*x + f*x^2)^FracPart[p]), Int[(g + h*x)^m*(d + e*x + f*x^2)^(p + q), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q}, x] && EqQ[c*d - a*f, 0] && EqQ[b*d - a*e, 0] && !IntegerQ[p] && !IntegerQ[q] && !GtQ[c/f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^{3/2}} dx &= \frac{\sqrt{a+bx+cx^2} \int \frac{g+hx}{ad+bdx+cdx^2} dx}{\sqrt{ad+bdx+cdx^2}} \\
&= \frac{\left(h\sqrt{a+bx+cx^2}\right) \int \frac{bd+2cdx}{ad+bdx+cdx^2} dx + \left((2cdg-bdh)\sqrt{a+bx+cx^2}\right) \int \frac{1}{ad+bdx+cdx^2}}{2cd\sqrt{ad+bdx+cdx^2}} \\
&= \frac{h\sqrt{a+bx+cx^2} \log(a+bx+cx^2)}{2cd\sqrt{ad+bdx+cdx^2}} - \frac{\left((2cdg-bdh)\sqrt{a+bx+cx^2}\right) \text{Subst}\left(\int \frac{1}{b^2}\right)}{cd\sqrt{ad+bdx+cdx^2}} \\
&= -\frac{(2cg-bh)\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}d\sqrt{ad+bdx+cdx^2}} + \frac{h\sqrt{a+bx+cx^2} \log(a+bx+cx^2)}{2cd\sqrt{ad+bdx+cdx^2}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 108, normalized size = 0.79

$$\frac{(a+x(b+cx))^{3/2} \left((4cg-2bh) \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right) + h\sqrt{4ac-b^2} \log(a+x(b+cx)) \right)}{2c\sqrt{4ac-b^2} (d(a+x(b+cx)))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((g+h*x)*Sqrt[a+b*x+c*x^2])/(a*d+b*d*x+c*d*x^2)^(3/2),x]

[Out] ((a+x*(b+c*x))^(3/2)*((4*c*g-2*b*h)*ArcTan[(b+2*c*x)/Sqrt[-b^2+4*a*c]]+Sqrt[-b^2+4*a*c]*h*Log[a+x*(b+c*x)]))/(2*c*Sqrt[-b^2+4*a*c]*(d*(a+x*(b+c*x)))^(3/2))

IntegrateAlgebraic [A] time = 0.28, size = 127, normalized size = 0.93

$$\frac{d^{3/2}(a+x(b+cx))^{3/2} \left(\frac{(2cg-bh) \tan^{-1}\left(\frac{2cx}{\sqrt{4ac-b^2}} + \frac{b}{\sqrt{4ac-b^2}}\right)}{cd^{3/2}\sqrt{4ac-b^2}} + \frac{h \log(a+bx+cx^2)}{2cd^{3/2}} \right)}{(d(a+x(b+cx)))^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((g+h*x)*Sqrt[a+b*x+c*x^2])/(a*d+b*d*x+c*d*x^2)^(3/2),x]

[Out] (d^(3/2)*(a+x*(b+c*x))^(3/2)*(((2*c*g-b*h)*ArcTan[b/Sqrt[-b^2+4*a*c]]+(2*c*x)/Sqrt[-b^2+4*a*c]))/(c*Sqrt[-b^2+4*a*c]*d^(3/2))+ (h*Log[a+b*x+c*x^2])/(2*c*d^(3/2)))/(d*(a+x*(b+c*x)))^(3/2)

fricas [F] time = 50.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cdx^2+bdx+ad}\sqrt{cx^2+bx+a}(hx+g)}{c^2d^2x^4+2bcd^2x^3+2abd^2x+(b^2+2ac)d^2x^2+a^2d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(c*d*x^2+b*d*x+a*d)*sqrt(c*x^2+b*x+a)*(h*x+g)/(c^2*d^2*x^4+2*b*c*d^2*x^3+2*a*b*d^2*x+(b^2+2*a*c)*d^2*x^2+a^2*d^2),x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + bx + a} (hx + g)}{(cdx^2 + bdx + ad)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^(3/2),x, algorithm m="giac")

[Out] integrate(sqrt(c*x^2 + b*x + a)*(h*x + g)/(c*d*x^2 + b*d*x + a*d)^(3/2), x)

maple [A] time = 0.03, size = 121, normalized size = 0.89

$$\frac{\sqrt{(cx^2 + bx + a)} d \left(-2bh \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right) + 4cg \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right) + \sqrt{4ac-b^2} h \ln(cx^2 + bx + a) \right)}{2\sqrt{cx^2 + bx + a} \sqrt{4ac-b^2} cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^(3/2),x)

[Out] 1/2/(c*x^2+b*x+a)^(1/2)*(d*(c*x^2+b*x+a)^(1/2)*(h*ln(c*x^2+b*x+a)*(4*a*c-b^2)^(1/2)-2*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*h+4*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c*g)/d^2/c/(4*a*c-b^2)^(1/2)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^(3/2),x, algorithm m="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g + hx) \sqrt{cx^2 + bx + a}}{(cdx^2 + bdx + ad)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((g + h*x)*(a + b*x + c*x^2)^(1/2))/(a*d + b*d*x + c*d*x^2)^(3/2),x)

[Out] int(((g + h*x)*(a + b*x + c*x^2)^(1/2))/(a*d + b*d*x + c*d*x^2)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g + hx) \sqrt{a + bx + cx^2}}{(d(a + bx + cx^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)*(c*x**2+b*x+a)**(1/2)/(c*d*x**2+b*d*x+a*d)**(3/2),x)

[Out] Integral((g + h*x)*sqrt(a + b*x + c*x**2)/(d*(a + b*x + c*x**2))**(3/2), x)

3.40 $\int x^2 \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2} dx$

Optimal. Leaf size=212

$$\frac{ac^2\sqrt{a^2 + 2abx + b^2x^2} \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a + bx)} - \frac{\sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2} (8bc - 15adx)}{60d^2(a + bx)} - \frac{acx\sqrt{a^2 + 2abx + b^2x^2}}{8d(a + bx)}$$

Rubi [A] time = 0.12, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {1001, 833, 780, 195, 217, 206}

$$\frac{ac^2\sqrt{a^2 + 2abx + b^2x^2} \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a + bx)} - \frac{\sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2} (8bc - 15adx)}{60d^2(a + bx)} - \frac{acx\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{8d(a + bx)} + \frac{bx^2\sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2}}{5d(a + bx)}$$

Antiderivative was successfully verified.

[In] Int[x^2*sqrt[a^2 + 2*a*b*x + b^2*x^2]*sqrt[c + d*x^2],x]

[Out] -(a*c*x*sqrt[a^2 + 2*a*b*x + b^2*x^2]*sqrt[c + d*x^2])/(8*d*(a + b*x)) + (b*x^2*sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + d*x^2)^(3/2))/(5*d*(a + b*x)) - ((8*b*c - 15*a*d*x)*sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + d*x^2)^(3/2))/(60*d^2*(a + b*x)) - (a*c^2*sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(sqrt[d]*x)/sqrt[c + d*x^2]])/(8*d^(3/2)*(a + b*x))

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 780

Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p + 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p + 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !LeQ[p, -1]

Rule 833

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

) && !(IGtQ[m, 0] && EqQ[f, 0])

Rule 1001

Int[((g_.) + (h_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(p_.)*((d_.) + (f_.)*(x_.)^2)^(q_.), x_Symbol] := Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, g, h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int x^2 \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2} dx &= \frac{\sqrt{a^2 + 2abx + b^2x^2} \int x^2 (2ab + 2b^2x) \sqrt{c + dx^2} dx}{2ab + 2b^2x} \\ &= \frac{bx^2 \sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2}}{5d(a + bx)} + \frac{\sqrt{a^2 + 2abx + b^2x^2} \int x (-4b^2c + 2b^2dx) \sqrt{c + dx^2} dx}{5d(2ab + 2b^2x)} \\ &= \frac{bx^2 \sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2}}{5d(a + bx)} - \frac{(8bc - 15adx) \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{60d^2(a + bx)} \\ &= -\frac{acx \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{8d(a + bx)} + \frac{bx^2 \sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2}}{5d(a + bx)} \\ &= -\frac{acx \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{8d(a + bx)} + \frac{bx^2 \sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2}}{5d(a + bx)} \\ &= -\frac{acx \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{8d(a + bx)} + \frac{bx^2 \sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2}}{5d(a + bx)} \end{aligned}$$

Mathematica [A] time = 0.15, size = 129, normalized size = 0.61

$$\frac{\sqrt{(a + bx)^2} \sqrt{c + dx^2} \left(\sqrt{\frac{dx^2}{c} + 1} (15adx(c + 2dx^2) + 8b(-2c^2 + cdx^2 + 3d^2x^4)) - 15ac^{3/2} \sqrt{d} \sinh^{-1} \left(\frac{\sqrt{dx}}{\sqrt{c}} \right) \right)}{120d^2(a + bx) \sqrt{\frac{dx^2}{c} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2], x]

[Out] (Sqrt[(a + b*x)^2]*Sqrt[c + d*x^2]*(Sqrt[1 + (d*x^2)/c]*(15*a*d*x*(c + 2*d*x^2) + 8*b*(-2*c^2 + c*d*x^2 + 3*d^2*x^4)) - 15*a*c^(3/2)*Sqrt[d]*ArcSinh[(Sqrt[d]*x)/Sqrt[c]]))/(120*d^2*(a + b*x)*Sqrt[1 + (d*x^2)/c])

IntegrateAlgebraic [A] time = 0.29, size = 111, normalized size = 0.52

$$\frac{\sqrt{(a + bx)^2} \left(\frac{\sqrt{c+dx^2} (15acdx + 30ad^2x^3 - 16bc^2 + 8bcdx^2 + 24bd^2x^4)}{120d^2} + \frac{ac^2 \log(\sqrt{c+dx^2} - \sqrt{d}x)}{8d^{3/2}} \right)}{a + bx}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2], x]

[Out] $(\text{Sqrt}[(a + b*x)^2] * ((\text{Sqrt}[c + d*x^2] * (-16*b*c^2 + 15*a*c*d*x + 8*b*c*d*x^2 + 30*a*d^2*x^3 + 24*b*d^2*x^4)) / (120*d^2) + (a*c^2 * \text{Log}[-(\text{Sqrt}[d]*x) + \text{Sqrt}[c + d*x^2]])) / (8*d^{(3/2)})) / (a + b*x)$

fricas [A] time = 0.43, size = 175, normalized size = 0.83

$$\left[\frac{15ac\sqrt{d} \log\left(-2dx^2 + 2\sqrt{dx^2+c}\sqrt{dx-c}\right) + 2(24bd^2x^4 + 30ad^2x^3 + 8bcdx^2 + 15acdx - 16bc^2)\sqrt{dx^2+c}}{240d^2}, \frac{15ac\sqrt{-d} \arctan\left(\frac{\sqrt{-d}}{\sqrt{dx^2+c}}\right) + (24bd^2x^4 + 30ad^2x^3 + 8bcdx^2 + 15acdx - 16bc^2)\sqrt{dx^2+c}}{120d^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="fricas")`

[Out] $[1/240*(15*a*c^2*\text{sqrt}(d)*\log(-2*d*x^2 + 2*\text{sqrt}(d*x^2 + c)*\text{sqrt}(d)*x - c) + 2*(24*b*d^2*x^4 + 30*a*d^2*x^3 + 8*b*c*d*x^2 + 15*a*c*d*x - 16*b*c^2)*\text{sqrt}(d*x^2 + c))/d^2, 1/120*(15*a*c^2*\text{sqrt}(-d)*\arctan(\text{sqrt}(-d)*x/\text{sqrt}(d*x^2 + c)) + (24*b*d^2*x^4 + 30*a*d^2*x^3 + 8*b*c*d*x^2 + 15*a*c*d*x - 16*b*c^2)*\text{sqrt}(d*x^2 + c))/d^2]$

giac [A] time = 0.25, size = 117, normalized size = 0.55

$$\frac{ac^2 \log\left(\left|-\sqrt{d}x + \sqrt{dx^2+c}\right|\right) \text{sgn}(bx+a)}{8d^{\frac{3}{2}}} + \frac{1}{120} \sqrt{dx^2+c} \left(\left(2 \left(3(4bx \text{sgn}(bx+a) + 5a \text{sgn}(bx+a))x + \frac{4bc \text{sgn}(bx+a)}{d} \right) x + \frac{15ac \text{sgn}(bx+a)}{d} \right) x - \frac{16bc^2 \text{sgn}(bx+a)}{d^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="giac")`

[Out] $1/8*a*c^2*\log(\text{abs}(-\text{sqrt}(d)*x + \text{sqrt}(d*x^2 + c)))*\text{sgn}(b*x + a)/d^{(3/2)} + 1/120*\text{sqrt}(d*x^2 + c)*((2*(3*(4*b*x*\text{sgn}(b*x + a) + 5*a*\text{sgn}(b*x + a))*x + 4*b*c*\text{sgn}(b*x + a)/d)*x + 15*a*c*\text{sgn}(b*x + a)/d)*x - 16*b*c^2*\text{sgn}(b*x + a)/d^2)$

maple [C] time = 0.05, size = 103, normalized size = 0.49

$$\frac{\left(15ac^2d \ln\left(\sqrt{d}x + \sqrt{dx^2+c}\right) + 15\sqrt{dx^2+c}ac d^{\frac{3}{2}}x - 24(dx^2+c)^{\frac{3}{2}}b d^{\frac{3}{2}}x^2 - 30(dx^2+c)^{\frac{3}{2}}a d^{\frac{3}{2}}x + 16(dx^2+c)^{\frac{3}{2}}bc\sqrt{d}\right) \text{csgn}(bx+a)}{120d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x)`

[Out] $-1/120*\text{csgn}(b*x+a)*(-24*d^{(3/2)}*(d*x^2+c)^{(3/2)}*x^2*b-30*d^{(3/2)}*(d*x^2+c)^{(3/2)}*x*a+16*d^{(1/2)}*(d*x^2+c)^{(3/2)}*b*c+15*d^{(3/2)}*(d*x^2+c)^{(1/2)}*x*a*c+15*\ln(x*d^{(1/2)}+(d*x^2+c)^{(1/2)})*a*c^2*d)/d^{(5/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx^2+c} \sqrt{(bx+a)^2} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)*x^2, x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \sqrt{(a+bx)^2} \sqrt{dx^2+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2),x)`

[Out] `int(x^2*((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{c + dx^2} \sqrt{(a + bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2), x)`

[Out] `Integral(x**2*sqrt(c + d*x**2)*sqrt((a + b*x)**2), x)`

3.41 $\int x\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2} dx$

Optimal. Leaf size=161

$$\frac{bc^2\sqrt{a^2 + 2abx + b^2x^2} \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a + bx)} - \frac{bcx\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{8d(a + bx)} + \frac{(4a + 3bx)\sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2}}{12d(a + bx)}$$

Rubi [A] time = 0.07, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1001, 780, 195, 217, 206}

$$\frac{bc^2\sqrt{a^2 + 2abx + b^2x^2} \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a + bx)} - \frac{bcx\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{8d(a + bx)} + \frac{(4a + 3bx)\sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2}}{12d(a + bx)}$$

Antiderivative was successfully verified.

[In] Int[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]

[Out] -(b*c*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(8*d*(a + b*x)) + ((4*a + 3*b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + d*x^2)^(3/2))/(12*d*(a + b*x)) - (b*c^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*d^(3/2)*(a + b*x))

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 780

Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p + 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p + 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !LeQ[p, -1]

Rule 1001

Int[((g_.) + (h_.)*(x_)^(m_.))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x_Symbol] := Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, g, h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
\int x\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2} dx &= \frac{\sqrt{a^2 + 2abx + b^2x^2} \int x(2ab + 2b^2x) \sqrt{c + dx^2} dx}{2ab + 2b^2x} \\
&= \frac{(4a + 3bx)\sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2}}{12d(a + bx)} - \frac{(b^2c\sqrt{a^2 + 2abx + b^2x^2})}{2d(2ab + 2b^2x)} \\
&= -\frac{bcx\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{8d(a + bx)} + \frac{(4a + 3bx)\sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2}}{12d(a + bx)} \\
&= -\frac{bcx\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{8d(a + bx)} + \frac{(4a + 3bx)\sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2}}{12d(a + bx)} \\
&= -\frac{bcx\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{8d(a + bx)} + \frac{(4a + 3bx)\sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2}}{12d(a + bx)}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 117, normalized size = 0.73

$$\frac{\sqrt{(a + bx)^2} \sqrt{c + dx^2} \left(\sqrt{d} \sqrt{\frac{dx^2}{c}} + 1 \left(8a(c + dx^2) + 3bx(c + 2dx^2) \right) - 3bc^{3/2} \sinh^{-1} \left(\frac{\sqrt{dx}}{\sqrt{c}} \right) \right)}{24d^{3/2}(a + bx) \sqrt{\frac{dx^2}{c} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2], x]

[Out] (Sqrt[(a + b*x)^2]*Sqrt[c + d*x^2]*(Sqrt[d]*Sqrt[1 + (d*x^2)/c]*(8*a*(c + d*x^2) + 3*b*x*(c + 2*d*x^2)) - 3*b*c^(3/2)*ArcSinh[(Sqrt[d]*x)/Sqrt[c]]))/(24*d^(3/2)*(a + b*x)*Sqrt[1 + (d*x^2)/c])

IntegrateAlgebraic [A] time = 0.26, size = 96, normalized size = 0.60

$$\frac{\sqrt{(a + bx)^2} \left(\frac{\sqrt{c+dx^2} (8ac+8adx^2+3bcx+6bdx^3)}{24d} + \frac{bc^2 \log(\sqrt{c+dx^2} - \sqrt{d}x)}{8d^{3/2}} \right)}{a + bx}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2], x]

[Out] (Sqrt[(a + b*x)^2]*((Sqrt[c + d*x^2]*(8*a*c + 3*b*c*x + 8*a*d*x^2 + 6*b*d*x^3))/(24*d) + (b*c^2*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(8*d^(3/2))))/(a + b*x)

fricas [A] time = 0.43, size = 157, normalized size = 0.98

$$\left[\frac{3bc^2\sqrt{d} \log(-2dx^2 + 2\sqrt{dx^2+c}\sqrt{d}x - c) + 2(6bd^2x^3 + 8ad^2x^2 + 3bcdx + 8acd)\sqrt{dx^2+c}}{48d^2}, \frac{3bc^2\sqrt{-d} \arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right) + (6bd^2x^3 + 8ad^2x^2 + 3bcdx + 8acd)\sqrt{dx^2+c}}{24d^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2), x, algorithm="fricas")

[Out] [1/48*(3*b*c^2*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(6*b*d^2*x^3 + 8*a*d^2*x^2 + 3*b*c*d*x + 8*a*c*d)*sqrt(d*x^2 + c))/d^2, 1/24*(3*b*c^2*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (6*b*d^2*x^3 + 8*a*d^2*x^2 + 3*b*c*d*x + 8*a*c*d)*sqrt(d*x^2 + c))/d^2]

giac [A] time = 0.21, size = 98, normalized size = 0.61

$$\frac{bc^2 \log\left(\left|-\sqrt{d}x + \sqrt{dx^2 + c}\right|\right) \operatorname{sgn}(bx + a)}{8d^{\frac{3}{2}}} + \frac{1}{24} \sqrt{dx^2 + c} \left(\left(2(3bx \operatorname{sgn}(bx + a) + 4a \operatorname{sgn}(bx + a))x + \frac{3bc \operatorname{sgn}(bx + a)}{d} \right)x + \frac{8ac \operatorname{sgn}(bx + a)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="giac")

[Out] 1/8*b*c^2*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a)/d^(3/2) + 1/24*sqrt(d*x^2 + c)*((2*(3*b*x*sgn(b*x + a) + 4*a*sgn(b*x + a))*x + 3*b*c*sgn(b*x + a)/d)*x + 8*a*c*sgn(b*x + a)/d)

maple [C] time = 0.01, size = 83, normalized size = 0.52

$$\frac{\left(-3bc^2 \ln\left(\sqrt{d}x + \sqrt{dx^2 + c}\right) - 3\sqrt{dx^2 + c}bc\sqrt{d}x + 6(dx^2 + c)^{\frac{3}{2}}b\sqrt{d}x + 8(dx^2 + c)^{\frac{3}{2}}a\sqrt{d}\right) \operatorname{csgn}(bx + a)}{24d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x)

[Out] 1/24*csgn(b*x+a)*(6*d^(1/2)*(d*x^2+c)^(3/2)*x*b+8*a*(d*x^2+c)^(3/2)*d^(1/2)-3*d^(1/2)*(d*x^2+c)^(1/2)*x*b-c-3*ln(d^(1/2)*x+(d*x^2+c)^(1/2))*b*c^2)/d^(3/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx^2 + c} \sqrt{(bx + a)^2} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)*x, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \sqrt{(a + bx)^2} \sqrt{dx^2 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2),x)

[Out] int(x*((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sqrt{c + dx^2} \sqrt{(a + bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2),x)

[Out] Integral(x*sqrt(c + d*x**2)*sqrt((a + b*x)**2), x)

3.42 $\int \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2} dx$

Optimal. Leaf size=148

$$\frac{b\sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2}}{3d(a + bx)} + \frac{ax\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{2(a + bx)} + \frac{ac\sqrt{a^2 + 2abx + b^2x^2} \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}(a + bx)}$$

Rubi [A] time = 0.06, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {970, 641, 195, 217, 206}

$$\frac{b\sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2}}{3d(a + bx)} + \frac{ax\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{2(a + bx)} + \frac{ac\sqrt{a^2 + 2abx + b^2x^2} \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}(a + bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] (a*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(2*(a + b*x)) + (b*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + d*x^2)^(3/2))/(3*d*(a + b*x)) + (a*c*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*Sqrt[d]*(a + b*x))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 970

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2} dx &= \frac{\sqrt{a^2 + 2abx + b^2x^2} \int (2ab + 2b^2x) \sqrt{c + dx^2} dx}{2ab + 2b^2x} \\
&= \frac{b\sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2}}{3d(a + bx)} + \frac{(2ab\sqrt{a^2 + 2abx + b^2x^2}) \int \sqrt{c + dx^2} dx}{2ab + 2b^2x} \\
&= \frac{ax\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{2(a + bx)} + \frac{b\sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2}}{3d(a + bx)} + \frac{(abc)}{3d} \\
&= \frac{ax\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{2(a + bx)} + \frac{b\sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2}}{3d(a + bx)} + \frac{(abc)}{3d} \\
&= \frac{ax\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{2(a + bx)} + \frac{b\sqrt{a^2 + 2abx + b^2x^2} (c + dx^2)^{3/2}}{3d(a + bx)} + \frac{ac\sqrt{d}}{3d}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 85, normalized size = 0.57

$$\frac{\sqrt{(a + bx)^2} \left(\sqrt{c + dx^2} (3adx + 2b(c + dx^2)) + 3ac\sqrt{d} \log \left(\sqrt{d} \sqrt{c + dx^2} + dx \right) \right)}{6d(a + bx)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2], x]

[Out] (Sqrt[(a + b*x)^2]*(Sqrt[c + d*x^2]*(3*a*d*x + 2*b*(c + d*x^2)) + 3*a*c*Sqrt[d]*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]]))/(6*d*(a + b*x))

IntegrateAlgebraic [A] time = 0.29, size = 87, normalized size = 0.59

$$\frac{\sqrt{(a + bx)^2} \left(\frac{\sqrt{c+dx^2}(3adx+2bc+2bdx^2)}{6d} - \frac{ac \log(\sqrt{c+dx^2} - \sqrt{d}x)}{2\sqrt{d}} \right)}{a + bx}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2], x]

[Out] (Sqrt[(a + b*x)^2]*((Sqrt[c + d*x^2]*(2*b*c + 3*a*d*x + 2*b*d*x^2))/(6*d) - (a*c*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(2*Sqrt[d])))/(a + b*x)

fricas [A] time = 0.43, size = 128, normalized size = 0.86

$$\left[\frac{3ac\sqrt{d} \log(-2dx^2 - 2\sqrt{dx^2+c}\sqrt{d}x - c) + 2(2bdx^2 + 3adx + 2bc)\sqrt{dx^2+c}}{12d}, -\frac{3ac\sqrt{-d} \arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right) - (2bdx^2 + 3adx + 2bc)\sqrt{dx^2+c}}{6d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2), x, algorithm="fricas")

[Out] [1/12*(3*a*c*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(2*b*d*x^2 + 3*a*d*x + 2*b*c)*sqrt(d*x^2 + c))/d, -1/6*(3*a*c*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (2*b*d*x^2 + 3*a*d*x + 2*b*c)*sqrt(d*x^2 + c))/d]

giac [A] time = 0.27, size = 79, normalized size = 0.53

$$-\frac{ac \log \left(\left| -\sqrt{d}x + \sqrt{dx^2 + c} \right| \right) \operatorname{sgn}(bx + a)}{2\sqrt{d}} + \frac{1}{6} \sqrt{dx^2 + c} \left((2bx \operatorname{sgn}(bx + a) + 3a \operatorname{sgn}(bx + a))x + \frac{2bc \operatorname{sgn}(bx + a)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="giac")

[Out] $-1/2*a*c*\log(\text{abs}(-\sqrt{d}*x + \sqrt{d*x^2 + c}))*\text{sgn}(b*x + a)/\sqrt{d} + 1/6*\sqrt{d*x^2 + c}*((2*b*x*\text{sgn}(b*x + a) + 3*a*\text{sgn}(b*x + a))*x + 2*b*c*\text{sgn}(b*x + a)/d)$

maple [C] time = 0.01, size = 65, normalized size = 0.44

$$\frac{\left(3acd \ln\left(\sqrt{d} x + \sqrt{d x^2 + c}\right) + 3\sqrt{d x^2 + c} a d^{\frac{3}{2}} x + 2\left(d x^2 + c\right)^{\frac{3}{2}} b \sqrt{d}\right) \text{csgn}(b x + a)}{6 d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x)

[Out] $1/6*\text{csgn}(b*x+a)*(2*b*(d*x^2+c)^(3/2)*d^(1/2)+3*d^(3/2)*(d*x^2+c)^(1/2)*x*a+3*\ln(d^(1/2)*x+(d*x^2+c)^(1/2))*a*c*d)/d^(3/2)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d x^2 + c} \sqrt{(b x + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{(a + b x)^2} \sqrt{d x^2 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2),x)

[Out] int(((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c + d x^2} \sqrt{(a + b x)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2),x)

[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x)**2), x)

$$3.43 \quad \int \frac{\sqrt{a^2+2abx+b^2x^2} \sqrt{c+dx^2}}{x} dx$$

Optimal. Leaf size=160

$$\frac{\sqrt{a^2+2abx+b^2x^2}(2a+bx)\sqrt{c+dx^2}}{2(a+bx)} + \frac{bc\sqrt{a^2+2abx+b^2x^2} \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}(a+bx)} - \frac{a\sqrt{c}\sqrt{a^2+2abx+b^2x^2} \tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx}$$

Rubi [A] time = 0.12, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {1001, 815, 844, 217, 206, 266, 63, 208}

$$\frac{\sqrt{a^2+2abx+b^2x^2}(2a+bx)\sqrt{c+dx^2}}{2(a+bx)} + \frac{bc\sqrt{a^2+2abx+b^2x^2} \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}(a+bx)} - \frac{a\sqrt{c}\sqrt{a^2+2abx+b^2x^2} \tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x,x]

[Out] ((2*a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(2*(a + b*x)) + (b*c*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*Sqrt[d]*(a + b*x)) - (a*Sqrt[c]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(a + b*x)

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 815

Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p

+ 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 844

Int[((d_.) + (e_.)*(x_.))^(m_.)*((f_.) + (g_.)*(x_.))*((a_.) + (c_.)*(x_.)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]

Rule 1001

Int[((g_.) + (h_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(p_.)*((d_.) + (f_.)*(x_.)^2)^(q_.), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, g, h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{x} dx &= \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(2ab+2b^2x)\sqrt{c+dx^2}}{x} dx}{2ab + 2b^2x} \\ &= \frac{(2a + bx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{2(a + bx)} + \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{4abcd+2b^2}{x\sqrt{c+dx^2}}}{2d(2ab + 2b^2x)} \\ &= \frac{(2a + bx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{2(a + bx)} + \frac{(2abc\sqrt{a^2 + 2abx + b^2x^2}) \int \frac{1}{x}}{2ab + 2b^2x} \\ &= \frac{(2a + bx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{2(a + bx)} + \frac{(abc\sqrt{a^2 + 2abx + b^2x^2}) \operatorname{Subst}\left(\int \frac{1}{u}, u, \frac{c+dx^2}{x}\right)}{2ab + 2b^2x} \\ &= \frac{(2a + bx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{2(a + bx)} + \frac{bc\sqrt{a^2 + 2abx + b^2x^2} \tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{x}\right)}{2\sqrt{d}(a + bx)} \\ &= \frac{(2a + bx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{2(a + bx)} + \frac{bc\sqrt{a^2 + 2abx + b^2x^2} \tanh^{-1}\left(\frac{\sqrt{dx^2+c}}{x}\right)}{2\sqrt{d}(a + bx)} \end{aligned}$$

Mathematica [A] time = 0.14, size = 139, normalized size = 0.87

$$\frac{\sqrt{(a + bx)^2} \left(\sqrt{d} \sqrt{\frac{dx^2}{c} + 1} \left((2a + bx)\sqrt{c + dx^2} - 2a\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right) \right) + b\sqrt{c} \sqrt{c + dx^2} \sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right) \right)}{2\sqrt{d}(a + bx)\sqrt{\frac{dx^2}{c} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x,x]

[Out] (Sqrt[(a + b*x)^2]*(b*Sqrt[c]*Sqrt[c + d*x^2]*ArcSinh[(Sqrt[d]*x)/Sqrt[c]] + Sqrt[d]*Sqrt[1 + (d*x^2)/c]*((2*a + b*x)*Sqrt[c + d*x^2] - 2*a*Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])))/(2*Sqrt[d]*(a + b*x)*Sqrt[1 + (d*x^2)/c])

IntegrateAlgebraic [A] time = 0.31, size = 114, normalized size = 0.71

$$\frac{\sqrt{(a + bx)^2} \left(\frac{1}{2}(2a + bx)\sqrt{c + dx^2} + 2a\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{d}x}{\sqrt{c}} - \frac{\sqrt{c+dx^2}}{\sqrt{c}} \right) - \frac{bc \log(\sqrt{c+dx^2} - \sqrt{d}x)}{2\sqrt{d}} \right)}{a + bx}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x,x]

[Out] (Sqrt[(a + b*x)^2]*(((2*a + b*x)*Sqrt[c + d*x^2])/2 + 2*a*Sqrt[c]*ArcTanh[(Sqrt[d]*x)/Sqrt[c] - Sqrt[c + d*x^2]/Sqrt[c]] - (b*c*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(2*Sqrt[d])))/(a + b*x)

fricas [A] time = 0.45, size = 341, normalized size = 2.13

$$\frac{bc\sqrt{d} \log(-2dx^2 - 2\sqrt{dx^2+c}\sqrt{d}x - c) + 2a\sqrt{d} \log\left(\frac{d^2 + \sqrt{dx^2+c}d}{2d}\right) + 2(bdx + 2ad)\sqrt{dx^2+c} - bc\sqrt{-d} \arctan\left(\frac{\sqrt{-d}}{\sqrt{dx^2+c}}\right) - a\sqrt{d} \log\left(\frac{d^2 + \sqrt{dx^2+c}d}{2d}\right) - (bdx + 2ad)\sqrt{dx^2+c} + 4a\sqrt{-d} \arctan\left(\frac{\sqrt{-d}}{\sqrt{dx^2+c}}\right) + bc\sqrt{d} \log(-2dx^2 - 2\sqrt{dx^2+c}\sqrt{d}x - c) + 2(bdx + 2ad)\sqrt{dx^2+c} - bc\sqrt{-d} \arctan\left(\frac{\sqrt{-d}}{\sqrt{dx^2+c}}\right) - 2a\sqrt{-d} \arctan\left(\frac{\sqrt{-d}}{\sqrt{dx^2+c}}\right) - (bdx + 2ad)\sqrt{dx^2+c}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x,x, algorithm="fricas")

[Out] [1/4*(b*c*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*a*sqrt(c)*d*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(b*d*x + 2*a*d)*sqrt(d*x^2 + c))/d, -1/2*(b*c*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - a*sqrt(c)*d*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - (b*d*x + 2*a*d)*sqrt(d*x^2 + c))/d, 1/4*(4*a*sqrt(-c)*d*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + b*c*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(b*d*x + 2*a*d)*sqrt(d*x^2 + c))/d, -1/2*(b*c*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - 2*a*sqrt(-c)*d*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (b*d*x + 2*a*d)*sqrt(d*x^2 + c))/d]

giac [A] time = 0.25, size = 102, normalized size = 0.64

$$\frac{2ac \arctan\left(-\frac{\sqrt{d}x - \sqrt{dx^2+c}}{\sqrt{-c}}\right) \operatorname{sgn}(bx + a)}{\sqrt{-c}} - \frac{bc \log\left(\left|-\sqrt{d}x + \sqrt{dx^2+c}\right|\right) \operatorname{sgn}(bx + a)}{2\sqrt{d}} + \frac{1}{2} \sqrt{dx^2+c} (bx \operatorname{sgn}(bx + a) + 2a \operatorname{sgn}(bx + a))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x,x, algorithm="giac")

[Out] 2*a*c*arctan(-sqrt(d)*x - sqrt(d*x^2 + c))/sqrt(-c)*sgn(b*x + a)/sqrt(-c) - 1/2*b*c*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a)/sqrt(d) + 1/2*sqrt(d*x^2 + c)*(b*x*sgn(b*x + a) + 2*a*sgn(b*x + a))

maple [C] time = 0.01, size = 94, normalized size = 0.59

$$\frac{\left(2a\sqrt{c}\sqrt{d}\ln\left(\frac{2c+2\sqrt{d}x^2+c\sqrt{c}}{x}\right) - bc\ln\left(\sqrt{d}x + \sqrt{dx^2+c}\right) - \sqrt{dx^2+c}b\sqrt{d}x - 2\sqrt{dx^2+c}a\sqrt{d}\right) \operatorname{csgn}(bx + a)}{2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x,x)

[Out] -1/2*csgn(b*x+a)*(2*c^(1/2)*d^(1/2)*ln(2*(c^(1/2)*(d*x^2+c)^(1/2)+c)/x)*a-d^(1/2)*(d*x^2+c)^(1/2)*x*b-2*d^(1/2)*(d*x^2+c)^(1/2)*a-ln(d^(1/2)*x+(d*x^2+c)^(1/2))*b*c)/d^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2 + c} \sqrt{(bx + a)^2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x,x, algorithm="maxima")

[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)/x, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{(a + bx)^2} \sqrt{dx^2 + c}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x,x)

[Out] int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^2} \sqrt{(a + bx)^2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2)/x,x)

[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x)**2)/x, x)

$$3.44 \quad \int \frac{\sqrt{a^2+2abx+b^2x^2} \sqrt{c+dx^2}}{x^2} dx$$

Optimal. Leaf size=156

$$-\frac{\sqrt{a^2+2abx+b^2x^2}(a-bx)\sqrt{c+dx^2}}{x(a+bx)} + \frac{a\sqrt{d}\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c+dx^2}}\right)}{a+bx} - \frac{b\sqrt{c}\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx}$$

Rubi [A] time = 0.11, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {1001, 813, 844, 217, 206, 266, 63, 208}

$$-\frac{\sqrt{a^2+2abx+b^2x^2}(a-bx)\sqrt{c+dx^2}}{x(a+bx)} + \frac{a\sqrt{d}\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c+dx^2}}\right)}{a+bx} - \frac{b\sqrt{c}\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x^2,x]

[Out] -(((a - b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(x*(a + b*x))) + (a*Sqrt[d]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(a + b*x) - (b*Sqrt[c]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(a + b*x)

Rule 63

Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 813

Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1) + e*g*(m + 1)*x*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/

$(e^{2(m+1)(m+2p+2)}), \text{Int}[(d+e*x)^{(m+1)}*(a+c*x^2)^{(p-1)}*\text{Simp}[g*(2*a*e+2*a*e*m)+(g*(2*c*d+4*c*d*p)-2*c*e*f*(m+2*p+2))*x, x], x] /;$ FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2+a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !RationalQ[m])) && NeQ[m, -1] && !ILtQ[m+2*p+1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 844

$\text{Int}[(d+e*x)^m*(f+g*x)^n*(a+c*x^2)^p, x] := \text{Dist}[g/e, \text{Int}[(d+e*x)^{m+1}*(a+c*x^2)^p, x], x] + \text{Dist}[(e*f-d*g)/e, \text{Int}[(d+e*x)^m*(a+c*x^2)^p, x], x] /;$ FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2+a*e^2, 0] && !IGtQ[m, 0]

Rule 1001

$\text{Int}[(g+h*x)^m*(a+b*x+c*x^2)^p*(d+e*x+f*x^2)^q, x] := \text{Dist}[(a+b*x+c*x^2)^{\text{FracPart}[p]} / ((4*c)^{\text{IntPart}[p]}*(b+2*c*x)^{(2*\text{FracPart}[p])}), \text{Int}[(g+h*x)^m*(b+2*c*x)^{(2*p)}*(d+f*x^2)^q, x], x] /;$ FreeQ[{a, b, c, d, f, g, h, m, p, q}, x] && EqQ[b^2-4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^2} dx &= \frac{\sqrt{a^2+2abx+b^2x^2} \int \frac{(2ab+2b^2x)\sqrt{c+dx^2}}{x^2} dx}{2ab+2b^2x} \\ &= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x(a+bx)} - \frac{\sqrt{a^2+2abx+b^2x^2} \int \frac{-4b^2c-4abx}{x\sqrt{c+dx^2}} dx}{2(2ab+2b^2x)} \\ &= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x(a+bx)} + \frac{(2b^2c\sqrt{a^2+2abx+b^2x^2}) \int \frac{-1}{x} dx}{2ab+2b^2x} \\ &= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x(a+bx)} + \frac{(b^2c\sqrt{a^2+2abx+b^2x^2}) \text{Subst}(\int \frac{-1}{u} du, x, a+bx)}{2ab+2b^2x} \\ &= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x(a+bx)} + \frac{a\sqrt{d}\sqrt{a^2+2abx+b^2x^2} \tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx} \\ &= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x(a+bx)} + \frac{a\sqrt{d}\sqrt{a^2+2abx+b^2x^2} \tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx} \end{aligned}$$

Mathematica [A] time = 0.28, size = 118, normalized size = 0.76

$$\frac{\sqrt{(a+bx)^2} \left(\frac{(bx-a)\sqrt{c+dx^2}}{x} + \frac{a\sqrt{c}\sqrt{d}\sqrt{\frac{dx^2}{c}+1} \sinh^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{\sqrt{c+dx^2}} - b\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right) \right)}{a+bx}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a^2+2*a*b*x+b^2*x^2]*Sqrt[c+d*x^2])/x^2,x]

[Out] (Sqrt[(a + b*x)^2]*((-a + b*x)*Sqrt[c + d*x^2])/x + (a*Sqrt[c]*Sqrt[d]*Sqrt[1 + (d*x^2)/c]*ArcSinh[(Sqrt[d]*x)/Sqrt[c]])/Sqrt[c + d*x^2] - b*Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]/(a + b*x)

IntegrateAlgebraic [A] time = 0.31, size = 111, normalized size = 0.71

$$\frac{\sqrt{(a + bx)^2} \left(\frac{(bx-a)\sqrt{c+dx^2}}{x} - a\sqrt{d} \log\left(\sqrt{c + dx^2} - \sqrt{d}x\right) + 2b\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}} - \frac{\sqrt{c+dx^2}}{\sqrt{c}}\right) \right)}{a + bx}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x^2,x]

[Out] (Sqrt[(a + b*x)^2]*((-a + b*x)*Sqrt[c + d*x^2])/x + 2*b*Sqrt[c]*ArcTanh[(Sqrt[d]*x)/Sqrt[c] - Sqrt[c + d*x^2]/Sqrt[c]] - a*Sqrt[d]*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]]/(a + b*x)

fricas [A] time = 0.44, size = 333, normalized size = 2.13

$$\frac{a\sqrt{d}x \log(-2dx^2 - 2\sqrt{d^2+c}\sqrt{d}x - c) + b\sqrt{c}x \log\left(\frac{d^2-2\sqrt{d^2+c}dx - c}{d^2}\right) + 2\sqrt{d^2+c}(bx-a) - 2a\sqrt{-d}x \arctan\left(\frac{\sqrt{d}x}{\sqrt{d^2+c}}\right) - b\sqrt{c}x \log\left(\frac{d^2-2\sqrt{d^2+c}dx - c}{d^2}\right) - 2\sqrt{d^2+c}(bx-a) - 2b\sqrt{c}x \arctan\left(\frac{\sqrt{d}x}{\sqrt{d^2+c}}\right) + a\sqrt{d}x \log(-2dx^2 - 2\sqrt{d^2+c}\sqrt{d}x - c) + 2\sqrt{d^2+c}(bx-a) - a\sqrt{-d}x \arctan\left(\frac{\sqrt{d}x}{\sqrt{d^2+c}}\right) - b\sqrt{c}x \arctan\left(\frac{\sqrt{d}x}{\sqrt{d^2+c}}\right) - \sqrt{d^2+c}(bx-a)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^2,x, algorithm="fricas")

[Out] [1/2*(a*sqrt(d)*x*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + b*sqrt(c)*x*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*sqrt(d*x^2 + c)*(b*x - a))/x, -1/2*(2*a*sqrt(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - b*sqrt(c)*x*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*sqrt(d*x^2 + c)*(b*x - a))/x, 1/2*(2*b*sqrt(-c)*x*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + a*sqrt(d)*x*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*sqrt(d*x^2 + c)*(b*x - a))/x, -(a*sqrt(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - b*sqrt(-c)*x*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - sqrt(d*x^2 + c)*(b*x - a))/x]

giac [A] time = 0.26, size = 126, normalized size = 0.81

$$\frac{2bc \arctan\left(-\frac{\sqrt{d}x - \sqrt{dx^2+c}}{\sqrt{-c}}\right) \operatorname{sgn}(bx+a)}{\sqrt{-c}} - a\sqrt{d} \log\left(\left|-\sqrt{d}x + \sqrt{dx^2+c}\right|\right) \operatorname{sgn}(bx+a) + \sqrt{dx^2+c} b \operatorname{sgn}(bx+a) + \frac{2ac\sqrt{d} \operatorname{sgn}(bx+a)}{\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2 - c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^2,x, algorithm="giac")

[Out] 2*b*c*arctan(-(sqrt(d)*x - sqrt(d*x^2 + c))/sqrt(-c))*sgn(b*x + a)/sqrt(-c) - a*sqrt(d)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a) + sqrt(d*x^2 + c)*b*sgn(b*x + a) + 2*a*c*sqrt(d)*sgn(b*x + a)/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)

maple [C] time = 0.01, size = 118, normalized size = 0.76

$$\frac{\left(acdx \ln\left(\sqrt{d}x + \sqrt{dx^2+c}\right) - bc^{\frac{3}{2}}\sqrt{d}x \ln\left(\frac{2c+2\sqrt{dx^2+c}\sqrt{c}}{x}\right) + \sqrt{dx^2+c}ad^{\frac{3}{2}}x^2 + \sqrt{dx^2+c}bc\sqrt{d}x - (dx^2+c)^{\frac{3}{2}}a\sqrt{d} \right) \operatorname{csgn}(bx+a)}{c\sqrt{d}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^2,x)

[Out] csgn(b*x+a)*(d^(3/2)*(d*x^2+c)^(1/2)*x^2*a-d^(1/2)*c^(3/2)*ln(2*(c+(d*x^2+c)^(1/2)*c^(1/2))/x)*x*b-(d*x^2+c)^(3/2)*a*d^(1/2)+(d*x^2+c)^(1/2)*b*c*d^(1/2)*x+ln(d^(1/2)*x+(d*x^2+c)^(1/2))*x*a*c*d)/c/x/d^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2 + c} \sqrt{(bx + a)^2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^2,x, algorithm="maxima")

[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)/x^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{(a + bx)^2} \sqrt{dx^2 + c}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x^2,x)

[Out] int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^2} \sqrt{(a + bx)^2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2)/x**2,x)

[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x)**2)/x**2, x)

$$3.45 \quad \int \frac{\sqrt{a^2+2abx+b^2x^2} \sqrt{c+dx^2}}{x^3} dx$$

Optimal. Leaf size=161

$$-\frac{\sqrt{a^2+2abx+b^2x^2}(a+2bx)\sqrt{c+dx^2}}{2x^2(a+bx)} + \frac{b\sqrt{d}\sqrt{a^2+2abx+b^2x^2} \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{a+bx} - \frac{ad\sqrt{a^2+2abx+b^2x^2} \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{c}(a+bx)}$$

Rubi [A] time = 0.12, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {1001, 811, 844, 217, 206, 266, 63, 208}

$$-\frac{\sqrt{a^2+2abx+b^2x^2}(a+2bx)\sqrt{c+dx^2}}{2x^2(a+bx)} + \frac{b\sqrt{d}\sqrt{a^2+2abx+b^2x^2} \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{a+bx} - \frac{ad\sqrt{a^2+2abx+b^2x^2} \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{c}(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x^3,x]

[Out] -((a + 2*b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(2*x^2*(a + b*x)) + (b*Sqrt[d]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(a + b*x) - (a*d*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*Sqrt[c]*(a + b*x))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 811

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +

$2*c*d*p*(e*f - d*g)*x)/(e^{2*(m+1)*(m+2)*(c*d^2 + a*e^2)}, x] - \text{Dist}[p/(e^{2*(m+1)*(m+2)*(c*d^2 + a*e^2)}, \text{Int}[(d + e*x)^{(m+2)*(a + c*x^2)^{(p-1)*\text{Simp}[2*a*c*e*(e*f - d*g)*(m+2) - c*(2*c*d*(d*g*(2*p+1) - e*f*(m+2*p+2)) - 2*a*e^2*g*(m+1)]*x, x}], x] /; \text{FreeQ}\{a, c, d, e, f, g\}, x] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{GtQ}[p, 0] \&\& \text{LtQ}[m, -2] \&\& \text{LtQ}[m + 2*p, 0] \&\& !\text{LtQ}[m + 2*p + 3, 0]$

Rule 844

$\text{Int}[(d + e*x)^m * (a + c*x^2)^p, x] := \text{Dist}[g/e, \text{Int}[(d + e*x)^{(m+1)*(a + c*x^2)^p}, x] + \text{Dist}[(e*f - d*g)/e, \text{Int}[(d + e*x)^m * (a + c*x^2)^p, x], x] /; \text{FreeQ}\{a, c, d, e, f, g, m, p\}, x] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& !\text{IGtQ}[m, 0]$

Rule 1001

$\text{Int}[(g + h*x)^m * (a + b*x + c*x^2)^p, x] := \text{Dist}[(a + b*x + c*x^2)^{\text{FracPart}[p]} / ((4*c)^{\text{IntPart}[p]} * (b + 2*c*x)^{(2*\text{FracPart}[p])}), \text{Int}[(g + h*x)^m * (b + 2*c*x)^{(2*p)*(d + f*x^2)^q}, x], x] /; \text{FreeQ}\{a, b, c, d, f, g, h, m, p, q\}, x] \&\& \text{EqQ}[b^2 - 4*a*c, 0]$

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{x^3} dx &= \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(2ab + 2b^2x)\sqrt{c + dx^2}}{x^3} dx}{2ab + 2b^2x} \\ &= -\frac{(a + 2bx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{2x^2(a + bx)} - \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{-4abcd - 4ab^2c}{x\sqrt{c + dx^2}} dx}{4c(2ab + 2b^2x)} \\ &= -\frac{(a + 2bx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{2x^2(a + bx)} + \frac{(abd\sqrt{a^2 + 2abx + b^2x^2}) \int \frac{1}{x} dx}{2ab + 2b^2x} \\ &= -\frac{(a + 2bx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{2x^2(a + bx)} + \frac{(abd\sqrt{a^2 + 2abx + b^2x^2}) \text{Su}}{2(2ab + 2b^2x)} \\ &= -\frac{(a + 2bx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{2x^2(a + bx)} + \frac{b\sqrt{d} \sqrt{a^2 + 2abx + b^2x^2} \tan^{-1}\left(\frac{\sqrt{dx^2 + c}}{a + bx}\right)}{a + bx} \\ &= -\frac{(a + 2bx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + dx^2}}{2x^2(a + bx)} + \frac{b\sqrt{d} \sqrt{a^2 + 2abx + b^2x^2} \tan^{-1}\left(\frac{\sqrt{dx^2 + c}}{a + bx}\right)}{a + bx} \end{aligned}$$

Mathematica [A] time = 0.12, size = 126, normalized size = 0.78

$$\frac{\sqrt{(a + bx)^2} \sqrt{c + dx^2} \left(c(a + 2bx) \sqrt{\frac{dx^2}{c} + 1} + adx^2 \tanh^{-1}\left(\sqrt{\frac{dx^2}{c} + 1}\right) - 2b\sqrt{c} \sqrt{d} x^2 \sinh^{-1}\left(\frac{\sqrt{dx^2 + c}}{\sqrt{c}}\right) \right)}{2cx^2(a + bx) \sqrt{\frac{dx^2}{c} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x^3,x]

[Out] $-1/2*(\text{Sqrt}[(a + b*x)^2]*\text{Sqrt}[c + d*x^2]*(c*(a + 2*b*x)*\text{Sqrt}[1 + (d*x^2)/c] - 2*b*\text{Sqrt}[c]*\text{Sqrt}[d]*x^2*\text{ArcSinh}[(\text{Sqrt}[d]*x)/\text{Sqrt}[c]] + a*d*x^2*\text{ArcTanh}[\text{Sqrt}[1 + (d*x^2)/c]])/(c*x^2*(a + b*x)*\text{Sqrt}[1 + (d*x^2)/c])$

IntegrateAlgebraic [A] time = 0.41, size = 115, normalized size = 0.71

$$\frac{\sqrt{(a + bx)^2} \left(\frac{(-a - 2bx)\sqrt{c + dx^2}}{2x^2} + \frac{ad \tanh^{-1}\left(\frac{\sqrt{d}x - \sqrt{c + dx^2}}{\sqrt{c}}\right)}{\sqrt{c}} - b\sqrt{d} \log\left(\sqrt{c + dx^2} - \sqrt{d}x\right) \right)}{a + bx}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[a^2 + 2*a*b*x + b^2*x^2])*Sqrt[c + d*x^2])/x^3,x]

[Out] $(\text{Sqrt}[(a + b*x)^2]*(((-a - 2*b*x)*\text{Sqrt}[c + d*x^2])/(2*x^2) + (a*d*\text{ArcTanh}[(\text{Sqrt}[d]*x)/\text{Sqrt}[c] - \text{Sqrt}[c + d*x^2]/\text{Sqrt}[c]])/\text{Sqrt}[c] - b*\text{Sqrt}[d]*\text{Log}[-(\text{Sqrt}[d]*x) + \text{Sqrt}[c + d*x^2]]))/(a + b*x)$

fricas [A] time = 0.44, size = 377, normalized size = 2.34

$$\frac{2bc\sqrt{d}x^2 \log(-2d^2 - 2\sqrt{d^2+c}\sqrt{d}x - c) + a\sqrt{c}d^2 \log\left(\frac{d^2 + \sqrt{d^2+c}d^2x}{d^2}\right) - 2(2bcx + a)\sqrt{d^2+c} - 4bc\sqrt{-d}x^2 \arctan\left(\frac{\sqrt{d}x}{\sqrt{d^2+c}}\right) - a\sqrt{c}d^2 \log\left(\frac{d^2 - 2\sqrt{d^2+c}d^2x}{d^2}\right) + 2(2bcx + a)\sqrt{d^2+c} - a\sqrt{-d}d^2 \arctan\left(\frac{\sqrt{d}x}{\sqrt{d^2+c}}\right) + bc\sqrt{d}x^2 \log(-2d^2 - 2\sqrt{d^2+c}\sqrt{d}x - c) - (2bcx + a)\sqrt{d^2+c} - 2bc\sqrt{-d}d^2 \arctan\left(\frac{\sqrt{d}x}{\sqrt{d^2+c}}\right) - a\sqrt{-d}d^2 \arctan\left(\frac{\sqrt{d}x}{\sqrt{d^2+c}}\right) + (2bcx + a)\sqrt{d^2+c}}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^3,x, algorithm="fricas")

[Out] $[1/4*(2*b*c*\text{sqrt}(d)*x^2*\log(-2*d*x^2 - 2*\text{sqrt}(d*x^2 + c)*\text{sqrt}(d)*x - c) + a*\text{sqrt}(c)*d*x^2*\log(-(d*x^2 - 2*\text{sqrt}(d*x^2 + c)*\text{sqrt}(c) + 2*c)/x^2) - 2*(2*b*c*x + a*c)*\text{sqrt}(d*x^2 + c))/(c*x^2), -1/4*(4*b*c*\text{sqrt}(-d)*x^2*\arctan(\text{sqrt}(-d)*x/\text{sqrt}(d*x^2 + c)) - a*\text{sqrt}(c)*d*x^2*\log(-(d*x^2 - 2*\text{sqrt}(d*x^2 + c)*\text{sqrt}(c) + 2*c)/x^2) + 2*(2*b*c*x + a*c)*\text{sqrt}(d*x^2 + c))/(c*x^2), 1/2*(a*\text{sqrt}(-c)*d*x^2*\arctan(\text{sqrt}(-c)/\text{sqrt}(d*x^2 + c)) + b*c*\text{sqrt}(d)*x^2*\log(-2*d*x^2 - 2*\text{sqrt}(d*x^2 + c)*\text{sqrt}(d)*x - c) - (2*b*c*x + a*c)*\text{sqrt}(d*x^2 + c))/(c*x^2), -1/2*(2*b*c*\text{sqrt}(-d)*x^2*\arctan(\text{sqrt}(-d)*x/\text{sqrt}(d*x^2 + c)) - a*\text{sqrt}(-c)*d*x^2*\arctan(\text{sqrt}(-c)/\text{sqrt}(d*x^2 + c)) + (2*b*c*x + a*c)*\text{sqrt}(d*x^2 + c))/(c*x^2)]$

giac [A] time = 0.28, size = 199, normalized size = 1.24

$$\frac{ad \arctan\left(\frac{\sqrt{d}x - \sqrt{d^2+c}}{\sqrt{c}}\right) \text{sgn}(bx + a)}{\sqrt{c}} - b\sqrt{d} \log\left(\left|-\sqrt{d}x + \sqrt{d^2+c}\right|\right) \text{sgn}(bx + a) + \frac{(\sqrt{d}x - \sqrt{d^2+c})^3 ad \text{sgn}(bx + a) + 2(\sqrt{d}x - \sqrt{d^2+c})^2 bc\sqrt{d} \text{sgn}(bx + a) + (\sqrt{d}x - \sqrt{d^2+c}) acd \text{sgn}(bx + a) - 2bc^2\sqrt{d} \text{sgn}(bx + a)}{\left((\sqrt{d}x - \sqrt{d^2+c})^2 - c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^3,x, algorithm="giac")

[Out] $a*d*\arctan(-(\text{sqrt}(d)*x - \text{sqrt}(d*x^2 + c))/\text{sqrt}(-c))*\text{sgn}(b*x + a)/\text{sqrt}(-c) - b*\text{sqrt}(d)*\log(\text{abs}(-\text{sqrt}(d)*x + \text{sqrt}(d*x^2 + c)))*\text{sgn}(b*x + a) + ((\text{sqrt}(d)*x - \text{sqrt}(d*x^2 + c))^3*a*d*\text{sgn}(b*x + a) + 2*(\text{sqrt}(d)*x - \text{sqrt}(d*x^2 + c))^2*b*c*\text{sqrt}(d)*\text{sgn}(b*x + a) + (\text{sqrt}(d)*x - \text{sqrt}(d*x^2 + c))*a*c*d*\text{sgn}(b*x + a) - 2*b*c^2*\text{sqrt}(d)*\text{sgn}(b*x + a))/((\text{sqrt}(d)*x - \text{sqrt}(d*x^2 + c))^2 - c)^2$

maple [C] time = 0.02, size = 141, normalized size = 0.88

$$\frac{\left(a\sqrt{c}d^{\frac{3}{2}}x^2 \ln\left(\frac{2c+2\sqrt{d}x^2+c\sqrt{c}}{x}\right) - 2bcdx^2 \ln\left(\sqrt{d}x + \sqrt{d^2+c}\right) - 2\sqrt{d}x^2+c b d^{\frac{3}{2}}x^3 - \sqrt{d}x^2+c a d^{\frac{3}{2}}x^2 + 2(d^2+c)^{\frac{3}{2}}b\sqrt{d}x + (d^2+c)^{\frac{3}{2}}a\sqrt{d}\right) \text{csgn}(bx + a)}{2c\sqrt{d}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^3,x)

[Out] $-1/2 * \text{csgn}(b*x+a) * (c^{(1/2)} * \ln(2 * (c + (d*x^2+c)^{(1/2)} * c^{(1/2)})) / x) * d^{(3/2)} * x^2 * a - 2 * d^{(3/2)} * (d*x^2+c)^{(1/2)} * x^3 * b + 2 * (d*x^2+c)^{(3/2)} * b * d^{(1/2)} * x - (d*x^2+c)^{(1/2)} * a * d^{(3/2)} * x^2 - 2 * \ln(d^{(1/2)} * x + (d*x^2+c)^{(1/2)}) * x^2 * b * c * d + (d*x^2+c)^{(3/2)} * a * d^{(1/2)} / x^2 / c / d^{(1/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2 + c} \sqrt{(bx + a)^2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^3,x, algorithm="maxima")

[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)/x^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{(a + bx)^2} \sqrt{dx^2 + c}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x^3,x)

[Out] int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^2} \sqrt{(a + bx)^2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2)/x**3,x)

[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x)**2)/x**3, x)

$$3.46 \quad \int x^2 \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2} dx$$

Optimal. Leaf size=317

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} (2dx + e) \sqrt{c + dx^2 + ex} (2ad(4cd - 5e^2) - b(12cde - 7e^3)) \sqrt{a^2 + 2abx + b^2x^2} (c + dx^2 + ex)}{128d^4(a + bx)}$$

Rubi [A] time = 0.33, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 38, number of rules / integrand size = 0.158, Rules used = {1000, 832, 779, 612, 621, 206}

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} (c + dx^2 + ex)^{3/2} (-6dx(10ad - 7be) + 50ade + 32bcd - 35be^2)}{240d^3(a + bx)} - \frac{\sqrt{a^2 + 2abx + b^2x^2} (2dx + e) \sqrt{c + dx^2 + ex} (2ad(4cd - 5e^2) - b(12cde - 7e^3))}{128d^4(a + bx)} - \frac{\sqrt{a^2 + 2abx + b^2x^2} (4cd - e^2) (8acd^2 - 10ade^2 - 12cde + 7be^2) \tanh^{-1}\left(\frac{2bx + e}{2\sqrt{c + dx^2 + ex}}\right)}{256d^3(a + bx)} + \frac{bx^2 \sqrt{a^2 + 2abx + b^2x^2} (c + dx^2 + ex)^{3/2}}{5d(a + bx)}$$

Antiderivative was successfully verified.

[In] Int[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]

[Out] -((2*a*d*(4*c*d - 5*e^2) - b*(12*c*d*e - 7*e^3))*(e + 2*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/(128*d^4*(a + b*x)) + (b*x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + e*x + d*x^2)^(3/2))/(5*d*(a + b*x)) - ((32*b*c*d + 50*a*d*e - 35*b*e^2 - 6*d*(10*a*d - 7*b*e)*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + e*x + d*x^2)^(3/2))/(240*d^3*(a + b*x)) - (((4*c*d - e^2)*(8*a*c*d^2 - 12*b*c*d*e - 10*a*d*e^2 + 7*b*e^3)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(256*d^(9/2)*(a + b*x))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 779

Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp[((b*e*g*(p + 2) - c*(e*f + d*g))*(2*p + 3) - 2*c*e*g*(p + 1)*x*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

Rule 832

Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x] /; FreeQ[{

$a, b, c, d, e, f, g, p\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a$
 $*e^2, 0] \&\& \text{GtQ}[m, 0] \&\& \text{NeQ}[m + 2*p + 2, 0] \&\& (\text{IntegerQ}[m] \mid\mid \text{IntegerQ}[p]$
 $\mid\mid \text{IntegersQ}[2*m, 2*p]) \&\& !(\text{IGtQ}[m, 0] \&\& \text{EqQ}[f, 0])$

Rule 1000

$\text{Int}[(g_.) + (h_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(p_.)*((d_.)$
 $) + (e_.)*(x_.) + (f_.)*(x_.)^2)^(q_.), x_Symbol] :> \text{Dist}[(a + b*x + c*x^2)^{\text{FracPart}[p]}$
 $/((4*c)^{\text{IntPart}[p]}*(b + 2*c*x)^{(2*\text{FracPart}[p])}), \text{Int}[(g + h*x)^m*(b + 2*c*x)^{(2*p)}$
 $*(d + e*x + f*x^2)^q, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h, m, p, q\}, x] \&\& \text{EqQ}[b^2 - 4*a*c, 0]$

Rubi steps

$$\int x^2 \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2} dx = \frac{\sqrt{a^2 + 2abx + b^2x^2} \int x^2 (2ab + 2b^2x) \sqrt{c + ex + dx^2} dx}{2ab + 2b^2x}$$

$$= \frac{bx^2 \sqrt{a^2 + 2abx + b^2x^2} (c + ex + dx^2)^{3/2}}{5d(a + bx)} + \frac{\sqrt{a^2 + 2abx + b^2x^2} \int}{5d(a + bx)}$$

$$= \frac{bx^2 \sqrt{a^2 + 2abx + b^2x^2} (c + ex + dx^2)^{3/2}}{5d(a + bx)} - \frac{(32bcd + 50ade - 35b^2d^2)}{5d^2(a + bx)}$$

$$= -\frac{(8acd^2 - 12bcde - 10ade^2 + 7be^3)(e + 2dx)\sqrt{a^2 + 2abx + b^2x^2}}{128d^4(a + bx)}$$

$$= -\frac{(8acd^2 - 12bcde - 10ade^2 + 7be^3)(e + 2dx)\sqrt{a^2 + 2abx + b^2x^2}}{128d^4(a + bx)}$$

$$= -\frac{(8acd^2 - 12bcde - 10ade^2 + 7be^3)(e + 2dx)\sqrt{a^2 + 2abx + b^2x^2}}{128d^4(a + bx)}$$

Mathematica [A] time = 0.27, size = 198, normalized size = 0.62

$$\frac{\sqrt{(a + bx)^2} \left(-\frac{5(2ad(4cd - 5c^2) + b(7e^3 - 12cde)) \left((4cd - e^2) \tanh^{-1} \left(\frac{2dx + e}{2\sqrt{d}\sqrt{c + x(dx + e)}} \right) + 2\sqrt{d}(2dx + e)\sqrt{c + x(dx + e)} \right)}{256d^2} + \frac{(c + x(dx + e))^{3/2} (10ad(6dx - 5e) - 32bcd + 7be(5e - 6dx))}{48d^2} + bx^2(c + x(dx + e))^{3/2} \right)}{5d(a + bx)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2], x]

[Out] (Sqrt[(a + b*x)^2]*(b*x^2*(c + x*(e + d*x))^(3/2) + ((c + x*(e + d*x))^(3/2) * (-32*b*c*d + 7*b*e*(5*e - 6*d*x) + 10*a*d*(-5*e + 6*d*x)))/(48*d^2) - (5*(2*a*d*(4*c*d - 5*e^2) + b*(-12*c*d*e + 7*e^3))*(2*Sqrt[d]*(e + 2*d*x)*Sqrt[c + x*(e + d*x)] + (4*c*d - e^2)*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + x*(e + d*x)])]))/(256*d^(7/2)))/(5*d*(a + b*x))

IntegrateAlgebraic [A] time = 1.00, size = 263, normalized size = 0.83

$$\frac{\sqrt{(a + bx)^2} \left(\frac{(32a^2d^3 - 48acd^2e + 10ad^4 - 48b^2d^2e + 40bcde^3 - 7be^5) \log(-2\sqrt{d}\sqrt{c + d^2x + dx + e})}{256d^2} + \frac{\sqrt{c + d^2x + dx + e} (240acd^3x - 520acd^2e + 480ad^4x^3 + 80ad^2ex^2 - 100ad^2e^2x + 150ad^2e^3 - 256bd^2d^2 + 128bcd^2e^2 - 232bc^2ex + 460bc^2e^2 + 384b^4x^4 + 48bd^3ex^3 - 56bd^2e^2x^2 + 70bd^2ex - 105bd^4)}{1920d^4} \right)}{a + bx}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2], x]

```
[Out] (Sqrt[(a + b*x)^2]*((Sqrt[c + e*x + d*x^2]*(-256*b*c^2*d^2 - 520*a*c*d^2*e
+ 460*b*c*d*e^2 + 150*a*d*e^3 - 105*b*e^4 + 240*a*c*d^3*x - 232*b*c*d^2*e*x
- 100*a*d^2*e^2*x + 70*b*d*e^3*x + 128*b*c*d^3*x^2 + 80*a*d^3*e*x^2 - 56*b
*d^2*e^2*x^2 + 480*a*d^4*x^3 + 48*b*d^3*e*x^3 + 384*b*d^4*x^4)))/(1920*d^4)
+ ((32*a*c^2*d^3 - 48*b*c^2*d^2*e - 48*a*c*d^2*e^2 + 40*b*c*d*e^3 + 10*a*d*
e^4 - 7*b*e^5)*Log[e + 2*d*x - 2*Sqrt[d]*Sqrt[c + e*x + d*x^2]])/(256*d^(9/
2))))/(a + b*x)
```

fricas [A] time = 0.47, size = 517, normalized size = 1.63



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/7680*(15*(32*a*c^2*d^3 - 48*b*c^2*d^2*e - 48*a*c*d^2*e^2 + 40*b*c*d*e^3
+ 10*a*d*e^4 - 7*b*e^5)*sqrt(d)*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e
*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) - 4*(384*b*d^5*x^4 - 256*b*c^2*d
^3 - 520*a*c*d^3*e + 460*b*c*d^2*e^2 + 150*a*d^2*e^3 - 105*b*d*e^4 + 48*(10
*a*d^5 + b*d^4*e)*x^3 + 8*(16*b*c*d^4 + 10*a*d^4*e - 7*b*d^3*e^2)*x^2 + 2*(
120*a*c*d^4 - 116*b*c*d^3*e - 50*a*d^3*e^2 + 35*b*d^2*e^3)*x)*sqrt(d*x^2 +
e*x + c))/d^5, 1/3840*(15*(32*a*c^2*d^3 - 48*b*c^2*d^2*e - 48*a*c*d^2*e^2 +
40*b*c*d*e^3 + 10*a*d*e^4 - 7*b*e^5)*sqrt(-d)*arctan(1/2*sqrt(d*x^2 + e*x
+ c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(384*b*d^5*x^4 - 256
*b*c^2*d^3 - 520*a*c*d^3*e + 460*b*c*d^2*e^2 + 150*a*d^2*e^3 - 105*b*d*e^4
+ 48*(10*a*d^5 + b*d^4*e)*x^3 + 8*(16*b*c*d^4 + 10*a*d^4*e - 7*b*d^3*e^2)*x
^2 + 2*(120*a*c*d^4 - 116*b*c*d^3*e - 50*a*d^3*e^2 + 35*b*d^2*e^3)*x)*sqrt(
d*x^2 + e*x + c))/d^5]
```

giac [A] time = 0.31, size = 368, normalized size = 1.16



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(d*x^2 + x*e + c)*(2*(4*(6*(8*b*x*sgn(b*x + a) + (10*a*d^4*sgn(b
*x + a) + b*d^3*e*sgn(b*x + a))/d^4)*x + (16*b*c*d^3*sgn(b*x + a) + 10*a*d^
3*e*sgn(b*x + a) - 7*b*d^2*e^2*sgn(b*x + a))/d^4)*x + (120*a*c*d^3*sgn(b*x
+ a) - 116*b*c*d^2*e*sgn(b*x + a) - 50*a*d^2*e^2*sgn(b*x + a) + 35*b*d*e^3*
sgn(b*x + a))/d^4)*x - (256*b*c^2*d^2*sgn(b*x + a) + 520*a*c*d^2*e*sgn(b*x
+ a) - 460*b*c*d*e^2*sgn(b*x + a) - 150*a*d*e^3*sgn(b*x + a) + 105*b*e^4*sg
n(b*x + a))/d^4) + 1/256*(32*a*c^2*d^3*sgn(b*x + a) - 48*b*c^2*d^2*e*sgn(b
*x + a) - 48*a*c*d^2*e^2*sgn(b*x + a) + 40*b*c*d*e^3*sgn(b*x + a) + 10*a*d*e
^4*sgn(b*x + a) - 7*b*e^5*sgn(b*x + a))*log(abs(-2*(sqrt(d)*x - sqrt(d*x^2
+ x*e + c))*sqrt(d) - e))/d^(9/2)
```

maple [C] time = 0.02, size = 530, normalized size = 1.67



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x)
```

```
[Out] 1/3840*csgn(b*x+a)*(768*(d*x^2+e*x+c)^(3/2)*d^(9/2)*x^2*b+960*(d*x^2+e*x+c)
^(3/2)*d^(9/2)*x*a-672*(d*x^2+e*x+c)^(3/2)*d^(7/2)*x*b*e-800*(d*x^2+e*x+c)^(
3/2)*d^(7/2)*a*e-512*(d*x^2+e*x+c)^(3/2)*d^(7/2)*b*c+560*(d*x^2+e*x+c)^(3/
2)*d^(5/2)*b*e^2-480*(d*x^2+e*x+c)^(1/2)*d^(9/2)*x*a*c+600*(d*x^2+e*x+c)^(1
/2)*d^(7/2)*x*a*e^2+720*(d*x^2+e*x+c)^(1/2)*d^(7/2)*x*b*c*e-420*(d*x^2+e*x+
```

$$c)^{(1/2)} * d^{(5/2)} * x * b * e^3 - 240 * (d * x^2 + e * x + c)^{(1/2)} * d^{(7/2)} * a * c * e + 300 * (d * x^2 + e * x + c)^{(1/2)} * d^{(5/2)} * a * e^3 + 360 * (d * x^2 + e * x + c)^{(1/2)} * d^{(5/2)} * b * c * e^2 - 210 * (d * x^2 + e * x + c)^{(1/2)} * d^{(3/2)} * b * e^4 - 480 * \ln(1/2 * (2 * (d * x^2 + e * x + c)^{(1/2)} * d^{(1/2)} + 2 * d * x + e) / d^{(1/2)}) * a * c^2 * d^4 + 720 * \ln(1/2 * (2 * (d * x^2 + e * x + c)^{(1/2)} * d^{(1/2)} + 2 * d * x + e) / d^{(1/2)}) * a * c * d^3 * e^2 - 150 * \ln(1/2 * (2 * (d * x^2 + e * x + c)^{(1/2)} * d^{(1/2)} + 2 * d * x + e) / d^{(1/2)}) * a * d^2 * e^4 + 720 * \ln(1/2 * (2 * (d * x^2 + e * x + c)^{(1/2)} * d^{(1/2)} + 2 * d * x + e) / d^{(1/2)}) * b * c^2 * d^3 * e - 600 * \ln(1/2 * (2 * (d * x^2 + e * x + c)^{(1/2)} * d^{(1/2)} + 2 * d * x + e) / d^{(1/2)}) * b * c * d^2 * e^3 + 105 * \ln(1/2 * (2 * (d * x^2 + e * x + c)^{(1/2)} * d^{(1/2)} + 2 * d * x + e) / d^{(1/2)}) * b * d * e^5) / d^{(11/2)}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx^2 + ex + c} \sqrt{(bx + a)^2} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)*x^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \sqrt{(a + bx)^2} \sqrt{dx^2 + ex + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2),x)

[Out] int(x^2*((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{c + dx^2 + ex} \sqrt{(a + bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2),x)

[Out] Integral(x**2*sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2), x)

3.47 $\int x\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2} dx$

Optimal. Leaf size=227

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} (4cd - e^2) (8ade + 4bcd - 5be^2) \tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right) \sqrt{a^2 + 2abx + b^2x^2} (2dx + e) \sqrt{c + ex + dx^2}}{128d^{7/2}(a + bx) 64d^3(a + bx)}$$

Rubi [A] time = 0.13, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {1000, 779, 612, 621, 206}

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} (2dx + e) \sqrt{c + dx^2 + ex} (8ade + 4bcd - 5be^2)}{64d^3(a + bx)} - \frac{\sqrt{a^2 + 2abx + b^2x^2} (4cd - e^2) (8ade + 4bcd - 5be^2) \tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{128d^{7/2}(a + bx)} + \frac{\sqrt{a^2 + 2abx + b^2x^2} (c + dx^2 + ex)^{3/2} (8ad + 6bdx - 5be)}{24d^2(a + bx)}$$

Antiderivative was successfully verified.

[In] Int[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]

[Out] -((4*b*c*d + 8*a*d*e - 5*b*e^2)*(e + 2*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/(64*d^3*(a + b*x)) + ((8*a*d - 5*b*e + 6*b*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + e*x + d*x^2)^(3/2))/(24*d^2*(a + b*x)) - ((4*c*d - e^2)*(4*b*c*d + 8*a*d*e - 5*b*e^2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(128*d^(7/2)*(a + b*x))

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 779

Int[((d_) + (e_)*(x_))*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := -Simp[((b*e*g*(p + 2) - c*(e*f + d*g))*(2*p + 3) - 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

Rule 1000

Int[((g_) + (h_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
\int x\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2} dx &= \frac{\sqrt{a^2 + 2abx + b^2x^2} \int x(2ab + 2b^2x) \sqrt{c + ex + dx^2} dx}{2ab + 2b^2x} \\
&= \frac{(8ad - 5be + 6bdx)\sqrt{a^2 + 2abx + b^2x^2} (c + ex + dx^2)^{3/2}}{24d^2(a + bx)} - \frac{b(4bcd + 8ade - 5be^2)}{64d^3(a + bx)} \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2} \\
&= -\frac{(4bcd + 8ade - 5be^2)(e + 2dx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{64d^3(a + bx)} \\
&= -\frac{(4bcd + 8ade - 5be^2)(e + 2dx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{64d^3(a + bx)} \\
&= -\frac{(4bcd + 8ade - 5be^2)(e + 2dx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{64d^3(a + bx)}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 147, normalized size = 0.65

$$\frac{\sqrt{(a + bx)^2} \left((c + x(dx + e))^{3/2}(8ad + 6bdx - 5be) - \frac{3(8ade + 4bcd - 5be^2) \left((4cd - e^2) \tanh^{-1} \left(\frac{2dx + e}{2\sqrt{d}\sqrt{c + x(dx + e)}} \right) + 2\sqrt{d}(2dx + e)\sqrt{c + x(dx + e)} \right)}{16d^{3/2}} \right)}{24d^2(a + bx)}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2], x]

[Out] (Sqrt[(a + b*x)^2]*((8*a*d - 5*b*e + 6*b*d*x)*(c + x*(e + d*x))^(3/2) - (3*(4*b*c*d + 8*a*d*e - 5*b*e^2)*(2*Sqrt[d]*(e + 2*d*x)*Sqrt[c + x*(e + d*x)] + (4*c*d - e^2)*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + x*(e + d*x)])))/(16*d^(3/2))))/(24*d^2*(a + b*x))

IntegrateAlgebraic [A] time = 0.70, size = 196, normalized size = 0.86

$$\frac{\sqrt{(a + bx)^2} \left(\frac{(32acd^2e - 8ade^3 + 16bc^2d^2 - 24bcd^2e + 5be^4) \log(-2\sqrt{d}\sqrt{c + dx^2 + ex + 2dx + e})}{128d^{7/2}} + \frac{\sqrt{c + dx^2 + ex} (64acd^2 + 64ad^3x^2 + 16ad^2ex - 24ad^2e + 24bcd^2x - 52bcde + 48bd^3x^3 + 8bd^2ex^2 - 10bde^2x + 15be^3)}{192d^3} \right)}{a + bx}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2], x]

[Out] (Sqrt[(a + b*x)^2]*((Sqrt[c + e*x + d*x^2]*(64*a*c*d^2 - 52*b*c*d*e - 24*a*d*e^2 + 15*b*e^3 + 24*b*c*d^2*x + 16*a*d^2*e*x - 10*b*d*e^2*x + 64*a*d^3*x^2 + 8*b*d^2*e*x^2 + 48*b*d^3*x^3))/(192*d^3) + ((16*b*c^2*d^2 + 32*a*c*d^2*e - 24*b*c*d*e^2 - 8*a*d*e^3 + 5*b*e^4)*Log[e + 2*d*x - 2*Sqrt[d]*Sqrt[c + e*x + d*x^2]])/(128*d^(7/2))))/(a + b*x)

fricas [A] time = 0.45, size = 391, normalized size = 1.72

$$\frac{5(16ac^2d^2 + 32ad^2e - 24ad^2e^3 - 8ad^3e^4 + 5b^2d^2e \log(bd^2 + bde - 4\sqrt{d}\sqrt{c + dx^2 + ex + 2dx + e}) + (48acd^2 + 64ad^3x^2 + 16ad^2ex - 24ad^2e + 24bcd^2x - 52bcde + 48bd^3x^3 + 8bd^2ex^2 - 10bde^2x + 15be^3) \sqrt{c + dx^2 + ex})}{192d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2), x, algorithm="fricas")

[Out] [1/768*(3*(16*b*c^2*d^2 + 32*a*c*d^2*e - 24*b*c*d*e^2 - 8*a*d*e^3 + 5*b*e^4)*sqrt(d)*log(8*d^2*x^2 + 8*d*e*x - 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt

t(d) + 4*c*d + e^2) + 4*(48*b*d^4*x^3 + 64*a*c*d^3 - 52*b*c*d^2*e - 24*a*d^2*e^2 + 15*b*d*e^3 + 8*(8*a*d^4 + b*d^3*e)*x^2 + 2*(12*b*c*d^3 + 8*a*d^3*e - 5*b*d^2*e^2)*x)*sqrt(d*x^2 + e*x + c))/d^4, 1/384*(3*(16*b*c^2*d^2 + 32*a*c*d^2*e - 24*b*c*d*e^2 - 8*a*d*e^3 + 5*b*e^4)*sqrt(-d)*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(48*b*d^4*x^3 + 64*a*c*d^3 - 52*b*c*d^2*e - 24*a*d^2*e^2 + 15*b*d*e^3 + 8*(8*a*d^4 + b*d^3*e)*x^2 + 2*(12*b*c*d^3 + 8*a*d^3*e - 5*b*d^2*e^2)*x)*sqrt(d*x^2 + e*x + c))/d^4]

giac [A] time = 0.28, size = 268, normalized size = 1.18

$$\frac{1}{192} \sqrt{dx^2 + ex + c} \left(2 \left(4 \left(6 \operatorname{sgn}(bx + a) + \frac{8af \operatorname{sgn}(bx + a) + bf^2 \operatorname{sgn}(bx + a)}{d} \right) x + \frac{12bcf \operatorname{sgn}(bx + a) + 8af^2 \operatorname{sgn}(bx + a) - 5bd^2 \operatorname{sgn}(bx + a)}{d} \right) + \frac{64ac^2 \operatorname{sgn}(bx + a) - 52bcd \operatorname{sgn}(bx + a) - 24ad^2 \operatorname{sgn}(bx + a) + 15b^2 \operatorname{sgn}(bx + a)}{d} \right) + \frac{(16b^2c^2 \operatorname{sgn}(bx + a) + 32ac^2 \operatorname{sgn}(bx + a) - 24bcd^2 \operatorname{sgn}(bx + a) - 8af^2 \operatorname{sgn}(bx + a) + 5bf^2 \operatorname{sgn}(bx + a)) \log \left(\left| \frac{-2(\sqrt{dx^2 + ex + c})\sqrt{d} - d}{128d^2} \right| \right)}{128d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="giac")

[Out] 1/192*sqrt(d*x^2 + x*e + c)*(2*(4*(6*b*x*sgn(b*x + a) + (8*a*d^3*sgn(b*x + a) + b*d^2*e*sgn(b*x + a))/d^3)*x + (12*b*c*d^2*sgn(b*x + a) + 8*a*d^2*e*sgn(b*x + a) - 5*b*d*e^2*sgn(b*x + a))/d^3)*x + (64*a*c*d^2*sgn(b*x + a) - 52*b*c*d*e*sgn(b*x + a) - 24*a*d*e^2*sgn(b*x + a) + 15*b*e^3*sgn(b*x + a))/d^3) + 1/128*(16*b*c^2*d^2*sgn(b*x + a) + 32*a*c*d^2*e*sgn(b*x + a) - 24*b*c*d*e^2*sgn(b*x + a) - 8*a*d*e^3*sgn(b*x + a) + 5*b*e^4*sgn(b*x + a))*log(abs(-2*(sqrt(d)*x - sqrt(d*x^2 + x*e + c))*sqrt(d) - e))/d^(7/2)

maple [C] time = 0.01, size = 381, normalized size = 1.68

$$\frac{(96cd^2x^2 \ln \left(\frac{2dx^2 + ex + c}{2d} \right) + 24cd^2x \ln \left(\frac{2dx^2 + ex + c}{2d} \right) - 48cd^2 \ln \left(\frac{2dx^2 + ex + c}{2d} \right) + 72bd^2x \ln \left(\frac{2dx^2 + ex + c}{2d} \right) - 136bd^2 \ln \left(\frac{2dx^2 + ex + c}{2d} \right) - 96\sqrt{d^2 + ex + c} \sqrt{dx^2 + ex + c} + 48\sqrt{d^2 + ex + c} \sqrt{bx + a} + 48\sqrt{d^2 + ex + c} \sqrt{bx + a} - 48\sqrt{d^2 + ex + c} \sqrt{bx + a} + 96\sqrt{d^2 + ex + c} \sqrt{bx + a} - 24\sqrt{d^2 + ex + c} \sqrt{bx + a} + 96(\frac{d^2 + ex + c}{d})^2 \sqrt{bx + a} + 30\sqrt{d^2 + ex + c} \sqrt{bx + a} + 128(\frac{d^2 + ex + c}{d})^2 \sqrt{bx + a} - 80(\frac{d^2 + ex + c}{d})^2 \sqrt{bx + a}) \operatorname{sgn}(bx + a)}{384d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x)

[Out] 1/384*c*sgn(b*x+a)*(96*d^(7/2)*(d*x^2+e*x+c)^(3/2)*x*b+128*d^(7/2)*(d*x^2+e*x+c)^(3/2)*a-80*d^(5/2)*(d*x^2+e*x+c)^(3/2)*b*e-96*d^(7/2)*(d*x^2+e*x+c)^(1/2)*x*a*e-48*d^(7/2)*(d*x^2+e*x+c)^(1/2)*x*b*c+60*d^(5/2)*(d*x^2+e*x+c)^(1/2)*x*b*e^2-48*d^(5/2)*(d*x^2+e*x+c)^(1/2)*a*e^2-24*d^(5/2)*(d*x^2+e*x+c)^(1/2)*b*c*e+30*d^(3/2)*(d*x^2+e*x+c)^(1/2)*b*e^3-96*ln(1/2*(2*d*x+e+2*(d*x^2+e*x+c)^(1/2)*d^(1/2))/d^(1/2))*a*c*d^3*e+24*ln(1/2*(2*d*x+e+2*(d*x^2+e*x+c)^(1/2)*d^(1/2))/d^(1/2))*a*d^2*e^3-48*ln(1/2*(2*d*x+e+2*(d*x^2+e*x+c)^(1/2)*d^(1/2))/d^(1/2))*b*c^2*d^3+72*ln(1/2*(2*d*x+e+2*(d*x^2+e*x+c)^(1/2)*d^(1/2))/d^(1/2))*b*c*d^2*e^2-15*ln(1/2*(2*d*x+e+2*(d*x^2+e*x+c)^(1/2)*d^(1/2))/d^(1/2))*b*d*e^4)/d^(9/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx^2 + ex + c} \sqrt{(bx + a)^2} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)*x, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x \sqrt{(a + bx)^2} \sqrt{dx^2 + ex + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2),x)

[Out] `int(x*((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{c + dx^2 + ex}\sqrt{(a + bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2), x)`

[Out] `Integral(x*sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2), x)`

3.48 $\int \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2} dx$

Optimal. Leaf size=198

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} (4cd - e^2) (2ad - be) \tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{16d^{5/2}(a+bx)} + \frac{\sqrt{a^2 + 2abx + b^2x^2} (2dx + e)(2ad - be)\sqrt{c + dx^2 + dx^2}}{8d^2(a+bx)}$$

Rubi [A] time = 0.10, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {969, 640, 612, 621, 206}

$$\frac{\sqrt{a^2 + 2abx + b^2x^2} (4cd - e^2) (2ad - be) \tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{16d^{5/2}(a+bx)} + \frac{\sqrt{a^2 + 2abx + b^2x^2} (2dx + e)(2ad - be)\sqrt{c + dx^2 + ex}}{8d^2(a+bx)} + \frac{b\sqrt{a^2 + 2abx + b^2x^2} (c + dx^2 + ex)^{3/2}}{3d(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]

[Out] ((2*a*d - b*e)*(e + 2*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/(8*d^2*(a + b*x)) + (b*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + e*x + d*x^2)^(3/2))/(3*d*(a + b*x)) + ((2*a*d - b*e)*(4*c*d - e^2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(16*d^(5/2)*(a + b*x))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 969

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p])*(b + 2*c*x)^(2*FracPart[p]), Int[(b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2} dx &= \frac{\sqrt{a^2 + 2abx + b^2x^2} \int (2ab + 2b^2x) \sqrt{c + ex + dx^2} dx}{2ab + 2b^2x} \\
&= \frac{b\sqrt{a^2 + 2abx + b^2x^2} (c + ex + dx^2)^{3/2}}{3d(a + bx)} + \frac{(b(2ad - be)\sqrt{a^2 + 2abx + b^2x^2})}{d(2ab + 2b^2x)} \\
&= \frac{(2ad - be)(e + 2dx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{8d^2(a + bx)} + \frac{b\sqrt{a^2 + 2abx + b^2x^2}}{d(2ab + 2b^2x)} \\
&= \frac{(2ad - be)(e + 2dx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{8d^2(a + bx)} + \frac{b\sqrt{a^2 + 2abx + b^2x^2}}{d(2ab + 2b^2x)} \\
&= \frac{(2ad - be)(e + 2dx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{8d^2(a + bx)} + \frac{b\sqrt{a^2 + 2abx + b^2x^2}}{d(2ab + 2b^2x)}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 134, normalized size = 0.68

$$\frac{\sqrt{(a + bx)^2} \left(2\sqrt{d} \sqrt{c + x(dx + e)} (6ad(2dx + e) + b(8cd + 8d^2x^2 + 2dex - 3e^2)) + 3(4cd - e^2)(2ad - be) \tanh^{-1} \left(\frac{2dx + e}{2\sqrt{d} \sqrt{c + x(dx + e)}} \right) \right)}{48d^{5/2}(a + bx)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2], x]

[Out] (Sqrt[(a + b*x)^2]*(2*Sqrt[d]*Sqrt[c + x*(e + d*x)]*(6*a*d*(e + 2*d*x) + b*(8*c*d - 3*e^2 + 2*d*e*x + 8*d^2*x^2)) + 3*(2*a*d - b*e)*(4*c*d - e^2)*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + x*(e + d*x)])])/(48*d^(5/2)*(a + b*x))

IntegrateAlgebraic [A] time = 0.60, size = 150, normalized size = 0.76

$$\frac{\sqrt{(a + bx)^2} \left(\frac{\sqrt{c+dx^2+ex} (12ad^2x+6ade+8bcd+8bd^2x^2+2bdex-3be^2)}{24d^2} + \frac{(-8acd^2+2ade^2+4bcde-be^3) \log(-2d^{5/2}\sqrt{c+dx^2+ex}+2d^3x+d^2e)}{16d^{5/2}} \right)}{a + bx}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2], x]

[Out] (Sqrt[(a + b*x)^2]*((Sqrt[c + e*x + d*x^2]*(8*b*c*d + 6*a*d*e - 3*b*e^2 + 12*a*d^2*x + 2*b*d*e*x + 8*b*d^2*x^2))/(24*d^2) + ((-8*a*c*d^2 + 4*b*c*d*e + 2*a*d*e^2 - b*e^3)*Log[d^2*e + 2*d^3*x - 2*d^(5/2)*Sqrt[c + e*x + d*x^2]])/(16*d^(5/2))))/(a + b*x)

fricas [A] time = 0.45, size = 287, normalized size = 1.45

$$\frac{3(8acd^2 - 4bcde - 2ade^2 + be^3)\sqrt{d} \log(8d^2x^2 + 8dex + 4\sqrt{dx^2 + ex + c}(2dx + e)\sqrt{d} + 4cd + e^2) + 4(8bd^3x^2 + 8bcd^2 + 6ad^2e - 3bd^2e + 2(6ad^2 + bd^2e)\sqrt{dx^2 + ex + c} - 3(8acd^2 - 4bcde - 2ade^2 + be^3)\sqrt{-d} \arctan\left(\frac{\sqrt{dx^2 + ex + c}}{2\sqrt{d}\sqrt{c + dx + e}}\right) - 2(8bd^3x^2 + 8bcd^2 + 6ad^2e - 3bd^2e + 2(6ad^2 + bd^2e)\sqrt{dx^2 + ex + c})}{96d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2), x, algorithm="fricas")

[Out] [1/96*(3*(8*a*c*d^2 - 4*b*c*d*e - 2*a*d*e^2 + b*e^3)*sqrt(d)*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + 4*(8*b*d^3*x^2 + 8*b*c*d^2 + 6*a*d^2*e - 3*b*d*e^2 + 2*(6*a*d^3 + b*d^2*e)*x)*sqrt(d*x^2 + e*x + c))/d^3, -1/48*(3*(8*a*c*d^2 - 4*b*c*d*e - 2*a*d*e^2 + b*e^3)*sqrt(-d)*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d))/(d^2*

$$x^2 + d \cdot e \cdot x + c \cdot d) - 2 \cdot (8 \cdot b \cdot d^3 \cdot x^2 + 8 \cdot b \cdot c \cdot d^2 + 6 \cdot a \cdot d^2 \cdot e - 3 \cdot b \cdot d \cdot e^2 + 2 \cdot (6 \cdot a \cdot d^3 + b \cdot d^2 \cdot e) \cdot x) \cdot \sqrt{d \cdot x^2 + e \cdot x + c}) / d^3]$$

giac [A] time = 0.30, size = 185, normalized size = 0.93

$$\frac{1}{24} \sqrt{dx^2 + ex + c} \left(2 \left(4 b \operatorname{sgn}(bx + a) + \frac{6 ad^2 \operatorname{sgn}(bx + a) + b d \operatorname{sgn}(bx + a)}{d^2} \right) x + \frac{8 bcd \operatorname{sgn}(bx + a) + 6 ad \operatorname{sgn}(bx + a) - 3 b e^2 \operatorname{sgn}(bx + a)}{d^2} \right) - \frac{(8 ad^2 \operatorname{sgn}(bx + a) - 4 bcd \operatorname{sgn}(bx + a) - 2 ad e^2 \operatorname{sgn}(bx + a) + b e^3 \operatorname{sgn}(bx + a)) \log \left(\left| -2 \left(\sqrt{dx^2 + ex + c} \right) \sqrt{d} - d \right| \right)}{16 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2), x, algorithm="giac")

[Out] 1/24*sqrt(d*x^2 + x*e + c)*(2*(4*b*x*sgn(b*x + a) + (6*a*d^2*sgn(b*x + a) + b*d*e*sgn(b*x + a))/d^2)*x + (8*b*c*d*sgn(b*x + a) + 6*a*d*e*sgn(b*x + a) - 3*b*e^2*sgn(b*x + a))/d^2) - 1/16*(8*a*c*d^2*sgn(b*x + a) - 4*b*c*d*e*sgn(b*x + a) - 2*a*d*e^2*sgn(b*x + a) + b*e^3*sgn(b*x + a))*log(abs(-2*(sqrt(d*x^2 + x*e + c))*sqrt(d) - e))/d^(5/2)

maple [C] time = 0.01, size = 257, normalized size = 1.30

$$\frac{(24ac d^3 \ln \left(\frac{2dx + e + \sqrt{dx^2 + ex + c}}{2\sqrt{d}} \right) - 6a d^2 \ln \left(\frac{2bx + a + \sqrt{dx^2 + ex + c}}{2\sqrt{d}} \right) - 12bc d^2 \ln \left(\frac{2dx + e + \sqrt{dx^2 + ex + c}}{2\sqrt{d}} \right) + 3bd^2 \ln \left(\frac{2bx + a + \sqrt{dx^2 + ex + c}}{2\sqrt{d}} \right) + 24\sqrt{dx^2 + ex + c} a d^2 x - 12\sqrt{dx^2 + ex + c} b d^2 ex + 12\sqrt{dx^2 + ex + c} a d^2 e - 6\sqrt{dx^2 + ex + c} b d^2 e^2 + 16(d x^2 + ex + c)^{\frac{3}{2}} b d^3) \operatorname{sgn}(bx + a)}{48 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2), x)

[Out] 1/48*csgn(b*x+a)*(16*d^(5/2)*(d*x^2+e*x+c)^(3/2)*b+24*d^(7/2)*(d*x^2+e*x+c)^(1/2)*x*a-12*d^(5/2)*(d*x^2+e*x+c)^(1/2)*x*b*e+12*d^(5/2)*(d*x^2+e*x+c)^(1/2)*a*e-6*d^(3/2)*(d*x^2+e*x+c)^(1/2)*b*e^2+24*ln(1/2*(2*d*x+e+2*(d*x^2+e*x+c)^(1/2)*d^(1/2))/d^(1/2))*a*c*d^3-6*ln(1/2*(2*d*x+e+2*(d*x^2+e*x+c)^(1/2)*d^(1/2))/d^(1/2))*a*d^2*e^2-12*ln(1/2*(2*d*x+e+2*(d*x^2+e*x+c)^(1/2)*d^(1/2))/d^(1/2))*b*c*d^2*e+3*ln(1/2*(2*d*x+e+2*(d*x^2+e*x+c)^(1/2)*d^(1/2))/d^(1/2))*b*d*e^3)/d^(7/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx^2 + ex + c} \sqrt{(bx + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{(a + bx)^2} \sqrt{dx^2 + ex + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2), x)

[Out] int(((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c + dx^2 + ex} \sqrt{(a + bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2), x)

[Out] Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2), x)

$$3.49 \quad \int \frac{\sqrt{a^2+2abx+b^2x^2} \sqrt{c+ex+dx^2}}{x} dx$$

Optimal. Leaf size=211

$$\frac{\sqrt{a^2+2abx+b^2x^2} (4ade+4bcd-be^2) \tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{8d^{3/2}(a+bx)} + \frac{\sqrt{a^2+2abx+b^2x^2} \sqrt{c+dx^2+ex} (4ad+2bdx+be)}{4d(a+bx)}$$

Rubi [A] time = 0.22, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1000, 814, 843, 621, 206, 724}

$$\frac{\sqrt{a^2+2abx+b^2x^2} (4ade+4bcd-be^2) \tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{8d^{3/2}(a+bx)} + \frac{\sqrt{a^2+2abx+b^2x^2} \sqrt{c+dx^2+ex} (4ad+2bdx+be)}{4d(a+bx)} - \frac{a\sqrt{c}\sqrt{a^2+2abx+b^2x^2} \tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{a+bx}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x,x]

[Out] ((4*a*d + b*e + 2*b*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/((4*d*(a + b*x)) + ((4*b*c*d + 4*a*d*e - b*e^2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(8*d^(3/2)*(a + b*x)) - (a*Sqrt[c]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + e*x + d*x^2])])/(a + b*x)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 814

Int[((d_.) + (e_.)*(x_)^m)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^p, x_Symbol] := Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c*d*p) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1000

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_.), x_Symbol] := Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{x} dx = \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(2ab + 2b^2x) \sqrt{c + ex + dx^2}}{x} dx}{2ab + 2b^2x}$$

$$= \frac{(4ad + be + 2bdx) \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{4d(a + bx)} - \frac{\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{4d(a + bx)}$$

$$= \frac{(4ad + be + 2bdx) \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{4d(a + bx)} + \frac{(2abc \sqrt{a^2 + 2abx + b^2x^2}) \sqrt{c + ex + dx^2}}{4d(a + bx)}$$

$$= \frac{(4ad + be + 2bdx) \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{4d(a + bx)} - \frac{(4abc \sqrt{a^2 + 2abx + b^2x^2}) \sqrt{c + ex + dx^2}}{4d(a + bx)}$$

$$= \frac{(4ad + be + 2bdx) \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{4d(a + bx)} + \frac{(4bcd + 4ade) \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{4d(a + bx)}$$

Mathematica [A] time = 0.20, size = 149, normalized size = 0.71

$$\frac{\sqrt{(a + bx)^2} \left((4ade + 4bcd - be^2) \tanh^{-1} \left(\frac{2dx + e}{2\sqrt{d} \sqrt{c + x(dx + e)}} \right) + 2\sqrt{d} \left(\sqrt{c + x(dx + e)} (4ad + b(2dx + e)) - 4a\sqrt{c} d \tanh^{-1} \left(\frac{2c + ex}{2\sqrt{c} \sqrt{c + x(dx + e)}} \right) \right) \right)}{8d^{3/2}(a + bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x, x]
```

```
[Out] (Sqrt[(a + b*x)^2]*((4*b*c*d + 4*a*d*e - b*e^2)*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + x*(e + d*x)])] + 2*Sqrt[d]*(Sqrt[c + x*(e + d*x)]*(4*a*d + b*(e + 2*d*x)) - 4*a*Sqrt[c]*d*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + x*(e + d*x)])]))) / (8*d^(3/2)*(a + b*x))
```

IntegrateAlgebraic [A] time = 0.64, size = 154, normalized size = 0.73

$$\frac{\sqrt{(a + bx)^2} \left(\frac{(-4ade - 4bcd + be^2) \log(-2d^{3/2} \sqrt{c + dx^2 + ex} + 2d^2 x + de)}{8d^{3/2}} + \frac{\sqrt{c + dx^2 + ex} (4ad + 2bdx + be)}{4d} + 2a\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{d}x}{\sqrt{c}} - \frac{\sqrt{c + dx^2 + ex}}{\sqrt{c}} \right) \right)}{a + bx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x, x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(((4*a*d + b*e + 2*b*d*x)*Sqrt[c + e*x + d*x^2])/(4*d) +
2*a*Sqrt[c]*ArcTanh[(Sqrt[d]*x)/Sqrt[c] - Sqrt[c + e*x + d*x^2]/Sqrt[c]] +
((-4*b*c*d - 4*a*d*e + b*e^2)*Log[d*e + 2*d^2*x - 2*d^(3/2)*Sqrt[c + e*x +
d*x^2])/(8*d^(3/2))))/(a + b*x)
```

fricas [A] time = 1.14, size = 651, normalized size = 3.09

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/16*(8*a*sqrt(c)*d^2*log((8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*
x + c)*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) - (4*b*c*d + 4*a*d*e - b*e^2)*sqrt
(d)*log(8*d^2*x^2 + 8*d*e*x - 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) +
4*c*d + e^2) + 4*(2*b*d^2*x + 4*a*d^2 + b*d*e)*sqrt(d*x^2 + e*x + c))/d^2,
1/8*(4*a*sqrt(c)*d^2*log((8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*x
+ c)*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) - (4*b*c*d + 4*a*d*e - b*e^2)*sqrt(
-d)*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x
+ c*d)) + 2*(2*b*d^2*x + 4*a*d^2 + b*d*e)*sqrt(d*x^2 + e*x + c))/d^2, 1/16*
(16*a*sqrt(-c)*d^2*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c
*d*x^2 + c*e*x + c^2)) - (4*b*c*d + 4*a*d*e - b*e^2)*sqrt(d)*log(8*d^2*x^2
+ 8*d*e*x - 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + 4*
(2*b*d^2*x + 4*a*d^2 + b*d*e)*sqrt(d*x^2 + e*x + c))/d^2, 1/8*(8*a*sqrt(-c)
*d^2*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c*d*x^2 + c*e*x
+ c^2)) - (4*b*c*d + 4*a*d*e - b*e^2)*sqrt(-d)*arctan(1/2*sqrt(d*x^2 + e*x
+ c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(2*b*d^2*x + 4*a*d^
2 + b*d*e)*sqrt(d*x^2 + e*x + c))/d^2]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(b*x
+a)]Warning, replacing 0 by `u`, a substitution variable should perhaps be
purged.Warning, replacing 0 by `u`, a substitution variable should perhaps
be purged.Warning, replacing 0 by `u`, a substitution variable should pe
rhaps be purged.index.cc index_m operator + Error: Bad Argument Value
```

maple [C] time = 0.01, size = 214, normalized size = 1.01

$$\frac{(8a\sqrt{c}d^{\frac{5}{2}}\ln\left(\frac{cx+2c+2\sqrt{d^2+ex+c}\sqrt{c}}{x}\right) - 4ad^2e\ln\left(\frac{2dx+c+2\sqrt{d^2+ex+c}\sqrt{d}}{2\sqrt{d}}\right) - 4bc d^2\ln\left(\frac{2dx+c+2\sqrt{d^2+ex+c}\sqrt{d}}{2\sqrt{d}}\right) + bd^2\ln\left(\frac{2dx+c+2\sqrt{d^2+ex+c}\sqrt{d}}{2\sqrt{d}}\right) - 4\sqrt{d}x^2+ex+c b d^{\frac{5}{2}}x - 8\sqrt{d}x^2+ex+c a d^{\frac{5}{2}} - 2\sqrt{d}x^2+ex+c b d^{\frac{3}{2}}e) \operatorname{csgn}(bx+a)}{8d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x,x)
```

```
[Out] -1/8*csgn(b*x+a)*(8*c^(1/2)*d^(5/2)*ln((2*c+e*x+2*c^(1/2)*(d*x^2+e*x+c)^(1/2)
)/x)*a-4*d^(5/2)*(d*x^2+e*x+c)^(1/2)*x*b-8*d^(5/2)*(d*x^2+e*x+c)^(1/2)*a-
2*d^(3/2)*(d*x^2+e*x+c)^(1/2)*b*e-4*d^2*ln(1/2*(2*d*x+e+2*(d*x^2+e*x+c)^(1/2)
)*d^(1/2))/d^(1/2))*a*e-4*ln(1/2*(2*d*x+e+2*(d*x^2+e*x+c)^(1/2)*d^(1/2))/d
^(1/2))*b*c*d^2+ln(1/2*(2*d*x+e+2*(d*x^2+e*x+c)^(1/2)*d^(1/2))/d^(1/2))*b*d
*e^2)/d^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2 + ex + c} \sqrt{(bx + a)^2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x,x, algorithm="maxima")

[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)/x, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{(a + bx)^2} \sqrt{dx^2 + ex + c}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x,x)

[Out] int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^2 + ex} \sqrt{(a + bx)^2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2)/x,x)

[Out] Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2)/x, x)

$$3.50 \quad \int \frac{\sqrt{a^2+2abx+b^2x^2} \sqrt{c+ex+dx^2}}{x^2} dx$$

Optimal. Leaf size=202

$$\frac{\sqrt{a^2+2abx+b^2x^2}(a-bx)\sqrt{c+dx^2+ex}}{x(a+bx)} + \frac{\sqrt{a^2+2abx+b^2x^2}(2ad+be)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{2\sqrt{d}(a+bx)} - \frac{\sqrt{a^2+2abx+b^2x^2}(ae+2bc)\tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{2\sqrt{c}(a+bx)}$$

Rubi [A] time = 0.19, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1000, 812, 843, 621, 206, 724}

$$\frac{\sqrt{a^2+2abx+b^2x^2}(a-bx)\sqrt{c+dx^2+ex}}{x(a+bx)} + \frac{\sqrt{a^2+2abx+b^2x^2}(2ad+be)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{2\sqrt{d}(a+bx)} - \frac{\sqrt{a^2+2abx+b^2x^2}(ae+2bc)\tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{2\sqrt{c}(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^2,x]

[Out] -(((a - b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/(x*(a + b*x))) + ((2*a*d + b*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(2*Sqrt[d]*(a + b*x)) - ((2*b*c + a*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + e*x + d*x^2])])/(2*Sqrt[c]*(a + b*x))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 812

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m + 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !RationalQ[m])) && NeQ[m, -1] && !LtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1000

```
Int[((g_.) + (h_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Dist[(a + b*x + c*x^2)^Fr acPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{x^2} dx = \frac{\sqrt{a^2 + 2abx + b^2x^2} \int \frac{(2ab+2b^2x)\sqrt{c+ex+dx^2}}{x^2} dx}{2ab + 2b^2x}$$

$$= -\frac{(a - bx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{x(a + bx)} - \frac{\sqrt{a^2 + 2abx + b^2x^2} \int -}{2(2ab + 2b^2x)}$$

$$= -\frac{(a - bx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{x(a + bx)} + \frac{(b(2bc + ae)\sqrt{a^2 + 2abx + b^2x^2})}{2ab}$$

$$= -\frac{(a - bx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{x(a + bx)} - \frac{(2b(2bc + ae)\sqrt{a^2 + 2abx + b^2x^2})}{2ab}$$

$$= -\frac{(a - bx)\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{x(a + bx)} + \frac{(2ad + be)\sqrt{a^2 + 2abx + b^2x^2}}{2ab}$$

Mathematica [A] time = 0.19, size = 155, normalized size = 0.77

$$\frac{\sqrt{(a + bx)^2} \left(\sqrt{c} x(2ad + be) \tanh^{-1} \left(\frac{2dx + e}{2\sqrt{d} \sqrt{c + x(dx + e)}} \right) + \sqrt{d} \left(2\sqrt{c} (bx - a) \sqrt{c + x(dx + e)} - x(ae + 2bc) \tanh^{-1} \left(\frac{2c + ex}{2\sqrt{c} \sqrt{c + x(dx + e)}} \right) \right) \right)}{2\sqrt{c} \sqrt{d} x(a + bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^2,x]
[Out] (Sqrt[(a + b*x)^2]*(Sqrt[c]*(2*a*d + b*e)*x*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + x*(e + d*x)])] + Sqrt[d]*(2*Sqrt[c]*(-a + b*x)*Sqrt[c + x*(e + d*x)]) - (2*b*c + a*e)*x*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + x*(e + d*x)])]))/(2*Sqrt[c]*Sqrt[d]*x*(a + b*x))
```

IntegrateAlgebraic [A] time = 0.62, size = 136, normalized size = 0.67

$$\frac{\sqrt{(a + bx)^2} \left(\frac{(bx-a)\sqrt{c+dx^2+ex}}{x} + \frac{(-2ad-be) \log(-2\sqrt{d} \sqrt{c+dx^2+ex} + 2dx+e)}{2\sqrt{d}} + \frac{(-ae-2bc) \tanh^{-1} \left(\frac{\sqrt{c+dx^2+ex} - \sqrt{d}x}{\sqrt{c}} \right)}{\sqrt{c}} \right)}{a + bx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^2,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*((-a + b*x)*Sqrt[c + e*x + d*x^2])/x + ((-2*b*c - a*e)*
ArcTanh[(-(Sqrt[d]*x) + Sqrt[c + e*x + d*x^2])/Sqrt[c]])/Sqrt[c] + ((-2*a*d
- b*e)*Log[e + 2*d*x - 2*Sqrt[d]*Sqrt[c + e*x + d*x^2]])/(2*Sqrt[d]))/(a
+ b*x)
```

fricas [A] time = 0.69, size = 647, normalized size = 3.20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/4*((2*a*c*d + b*c*e)*sqrt(d)*x*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 +
e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + (2*b*c*d + a*d*e)*sqrt(c)*x*log((8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) + 4*(b*c*d*x - a*c*d)*sqrt(d*x^2 + e*x + c))/(c*d*x), -1/4*(2*(2*a*c*d + b*c*e)*sqrt(-d)*x*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) - (2*b*c*d + a*d*e)*sqrt(c)*x*log((8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) - 4*(b*c*d*x - a*c*d)*sqrt(d*x^2 + e*x + c))/(c*d*x), 1/4*(2*(2*b*c*d + a*d*e)*sqrt(-c)*x*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c*d*x^2 + c*e*x + c^2)) + (2*a*c*d + b*c*e)*sqrt(d)*x*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + 4*(b*c*d*x - a*c*d)*sqrt(d*x^2 + e*x + c))/(c*d*x), 1/2*((2*b*c*d + a*d*e)*sqrt(-c)*x*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c*d*x^2 + c*e*x + c^2)) - (2*a*c*d + b*c*e)*sqrt(-d)*x*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(b*c*d*x - a*c*d)*sqrt(d*x^2 + e*x + c))/(c*d*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(b*x
+a)]Unable to divide, perhaps due to rounding error%%{2, [4,4,0]%%}+%%{%%
%{-16, [1]%%}, [4,2,1]%%}+%%{%%{32, [2]%%}, [4,0,2]%%}+%%{-4, [2,4,1]%%}
+%%{%%{32, [1]%%}, [2,2,2]%%}+%%{%%{-64, [2]%%}, [2,0,3]%%}+%%{2, [0,4,
2]%%}+%%{%%{-16, [1]%%}, [0,2,3]%%}+%%{%%{32, [2]%%}, [0,0,4]%%} / %%
{4, [4,0,0]%%}+%%{-8, [2,0,1]%%}+%%{4, [0,0,2]%%} Error: Bad Argument Val
ue
```

maple [C] time = 0.01, size = 249, normalized size = 1.23

$$\frac{(2ac dx \ln\left(\frac{2dx+2\sqrt{d^2+ex+c}\sqrt{d}}{2\sqrt{d}}\right) - a\sqrt{c} d^{\frac{3}{2}} \operatorname{erf}\ln\left(\frac{ex+2+2\sqrt{d^2+ex+c}\sqrt{c}}{x}\right) - 2bc^{\frac{3}{2}} d^{\frac{3}{2}} \ln\left(\frac{ex+2+2\sqrt{d^2+ex+c}\sqrt{c}}{x}\right) + bc \operatorname{erf}\ln\left(\frac{2dx+2\sqrt{d^2+ex+c}\sqrt{d}}{2\sqrt{d}}\right) + 2\sqrt{d^2+ex+c} a d^{\frac{5}{2}} + 2\sqrt{d^2+ex+c} a d^{\frac{3}{2}} ex + c bc d^{\frac{3}{2}} x - 2(d^2+ex+c)^{\frac{3}{2}} a d^{\frac{3}{2}}) \operatorname{csgn}(bx+a)}{2cd^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^2,x)
```

```
[Out] 1/2*csgn(b*x+a)*(2*d^(5/2)*(d*x^2+e*x+c)^(1/2)*x^2*a-2*d^(3/2)*c^(3/2)*ln((
e*x+2*c+2*(d*x^2+e*x+c)^(1/2)*c^(1/2))/x)*x*b-d^(3/2)*c^(1/2)*ln((e*x+2*c+2
*(d*x^2+e*x+c)^(1/2)*c^(1/2))/x)*x*a*e-2*d^(3/2)*(d*x^2+e*x+c)^(3/2)*a+2*d^(
3/2)*(d*x^2+e*x+c)^(1/2)*x*a*e+2*d^(3/2)*(d*x^2+e*x+c)^(1/2)*x*b*c+2*ln(1/
2*(2*d*x+e+2*(d*x^2+e*x+c)^(1/2)*d^(1/2))/d^(1/2))*x*a*c*d^2+ln(1/2*(2*d*x+
e+2*(d*x^2+e*x+c)^(1/2)*d^(1/2))/d^(1/2))*d*x*b*c*e)/c/x/d^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2 + ex + c} \sqrt{(bx + a)^2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^2,x, algorithm="maxima")

[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)/x^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{(a + bx)^2} \sqrt{dx^2 + ex + c}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^2,x)

[Out] int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^2 + ex} \sqrt{(a + bx)^2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2)/x**2,x)

[Out] Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2)/x**2, x)

$$3.51 \quad \int \frac{\sqrt{a^2+2abx+b^2x^2} \sqrt{c+ex+dx^2}}{x^3} dx$$

Optimal. Leaf size=215

$$\frac{\sqrt{a^2+2abx+b^2x^2} (4acd - ae^2 + 4bce) \tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{8c^{3/2}(a+bx)} - \frac{\sqrt{a^2+2abx+b^2x^2} \sqrt{c+dx^2+ex} (xae+4c)}{4cx^2(a+bx)}$$

Rubi [A] time = 0.18, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1000, 810, 843, 621, 206, 724}

$$\frac{\sqrt{a^2+2abx+b^2x^2} (4acd - ae^2 + 4bce) \tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{8c^{3/2}(a+bx)} - \frac{\sqrt{a^2+2abx+b^2x^2} \sqrt{c+dx^2+ex} (xae+4bc)+2ac}{4cx^2(a+bx)} + \frac{b\sqrt{d}\sqrt{a^2+2abx+b^2x^2} \tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{a+bx}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^3,x]

[Out] -((2*a*c + (4*b*c + a*e)*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/(4*c*x^2*(a + b*x)) + (b*Sqrt[d]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTan h[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(a + b*x) - ((4*a*c*d + 4*b*c*e - a*e^2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTan h[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + e*x + d*x^2])])/(8*c^(3/2)*(a + b*x))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 810

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x))/((e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m - 2*p) + e*f*(m + 2*p + 2)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] && !ILtQ[m + 2*p + 3, 0]

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1000

```
Int[((g_.) + (h_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{x^3} dx &= \frac{\sqrt{a^2 + 2abx + b^2x^2}}{2ab + 2b^2x} \int \frac{(2ab + 2b^2x) \sqrt{c + ex + dx^2}}{x^3} dx \\ &= -\frac{(2ac + (4bc + ae)x) \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{4cx^2(a + bx)} - \frac{\sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{4cx^2(a + bx)} \\ &= -\frac{(2ac + (4bc + ae)x) \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{4cx^2(a + bx)} + \frac{(2b^2d \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2})}{4cx^2(a + bx)} \\ &= -\frac{(2ac + (4bc + ae)x) \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{4cx^2(a + bx)} + \frac{(4b^2d \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2})}{4cx^2(a + bx)} \\ &= -\frac{(2ac + (4bc + ae)x) \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{4cx^2(a + bx)} + \frac{b\sqrt{d} \sqrt{a^2 + 2abx + b^2x^2} \sqrt{c + ex + dx^2}}{4cx^2(a + bx)} \end{aligned}$$

Mathematica [A] time = 0.22, size = 161, normalized size = 0.75

$$\frac{\sqrt{(a + bx)^2} \left(x^2 (4acd - ae^2 + 4bce) \tanh^{-1} \left(\frac{2c + ex}{2\sqrt{c} \sqrt{c + x(dx + e)}} \right) + 2\sqrt{c} \sqrt{c + x(dx + e)} (2ac + aex + 4bcx) - 8bc^{3/2} \sqrt{d} x^2 \tanh^{-1} \left(\frac{2dx + e}{2\sqrt{d} \sqrt{c + x(dx + e)}} \right) \right)}{8c^{3/2} x^2 (a + bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^3,x]
```

```
[Out] -1/8*(Sqrt[(a + b*x)^2]*(2*Sqrt[c]*(2*a*c + 4*b*c*x + a*e*x)*Sqrt[c + x*(e + d*x)] - 8*b*c^(3/2)*Sqrt[d]*x^2*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + x*(e + d*x)])]) + (4*a*c*d + 4*b*c*e - a*e^2)*x^2*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + x*(e + d*x)])])/(c^(3/2)*x^2*(a + b*x))
```

IntegrateAlgebraic [A] time = 0.81, size = 150, normalized size = 0.70

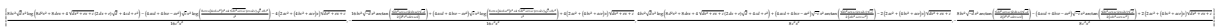
$$\frac{\sqrt{(a + bx)^2} \left(\frac{(-4acd + ae^2 - 4bce) \tanh^{-1} \left(\frac{\sqrt{c + dx^2 + ex} - \sqrt{d}x}{\sqrt{c}} \right)}{4c^{3/2}} + \frac{\sqrt{c + dx^2 + ex} (-2ac - aex - 4bcx)}{4cx^2} - b\sqrt{d} \log \left(-2\sqrt{d} \sqrt{c + dx^2 + ex} + 2dx + e \right) \right)}{a + bx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^3,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(((−2*a*c − 4*b*c*x − a*e*x)*Sqrt[c + e*x + d*x^2]))/(4*c*x^2) + ((−4*a*c*d − 4*b*c*e + a*e^2)*ArcTanh[(−(Sqrt[d]*x) + Sqrt[c + e*x + d*x^2])/Sqrt[c]])/(4*c^(3/2)) − b*Sqrt[d]*Log[e + 2*d*x − 2*Sqrt[d]*Sqrt[c + e*x + d*x^2]))/(a + b*x)
```

fricas [A] time = 0.86, size = 693, normalized size = 3.22

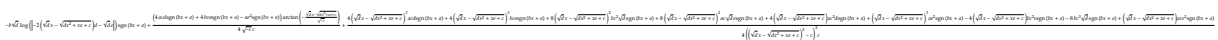


Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/16*(8*b*c^2*sqrt(d)*x^2*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e*x + c))*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) − (4*a*c*d + 4*b*c*e − a*e^2)*sqrt(c)*x^2*log((8*c*e*x + (4*c*d + e^2)*x^2 + 4*sqrt(d*x^2 + e*x + c))*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) − 4*(2*a*c^2 + (4*b*c^2 + a*c*e)*x)*sqrt(d*x^2 + e*x + c)/(c^2*x^2), −1/16*(16*b*c^2*sqrt(−d)*x^2*arctan(1/2*sqrt(d*x^2 + e*x + c))*(2*d*x + e)*sqrt(−d)/(d^2*x^2 + d*e*x + c*d)) + (4*a*c*d + 4*b*c*e − a*e^2)*sqrt(c)*x^2*log((8*c*e*x + (4*c*d + e^2)*x^2 + 4*sqrt(d*x^2 + e*x + c))*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) + 4*(2*a*c^2 + (4*b*c^2 + a*c*e)*x)*sqrt(d*x^2 + e*x + c)/(c^2*x^2), 1/8*(4*b*c^2*sqrt(d)*x^2*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e*x + c))*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + (4*a*c*d + 4*b*c*e − a*e^2)*sqrt(−c)*x^2*arctan(1/2*sqrt(d*x^2 + e*x + c))*(e*x + 2*c)*sqrt(−c)/(c*d*x^2 + c*e*x + c^2)) − 2*(2*a*c^2 + (4*b*c^2 + a*c*e)*x)*sqrt(d*x^2 + e*x + c)/(c^2*x^2), −1/8*(8*b*c^2*sqrt(−d)*x^2*arctan(1/2*sqrt(d*x^2 + e*x + c))*(2*d*x + e)*sqrt(−d)/(d^2*x^2 + d*e*x + c*d)) − (4*a*c*d + 4*b*c*e − a*e^2)*sqrt(−c)*x^2*arctan(1/2*sqrt(d*x^2 + e*x + c))*(e*x + 2*c)*sqrt(−c)/(c*d*x^2 + c*e*x + c^2)) + 2*(2*a*c^2 + (4*b*c^2 + a*c*e)*x)*sqrt(d*x^2 + e*x + c)/(c^2*x^2)]
```

giac [B] time = 0.41, size = 450, normalized size = 2.09

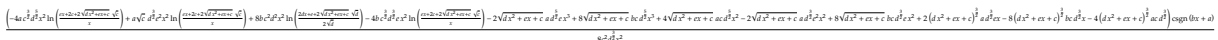


Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] −b*sqrt(d)*log(abs(−2*(sqrt(d)*x − sqrt(d*x^2 + x*e + c))*d − sqrt(d)*e))*sgn(b*x + a) + 1/4*(4*a*c*d*sgn(b*x + a) + 4*b*c*e*sgn(b*x + a) − a*e^2*sgn(b*x + a))*arctan(−(sqrt(d)*x − sqrt(d*x^2 + x*e + c))/sqrt(−c))/(sqrt(−c)*c) + 1/4*(4*(sqrt(d)*x − sqrt(d*x^2 + x*e + c))^3*a*c*d*sgn(b*x + a) + 4*(sqrt(d)*x − sqrt(d*x^2 + x*e + c))^3*b*c*e*sgn(b*x + a) + 8*(sqrt(d)*x − sqrt(d*x^2 + x*e + c))^2*b*c^2*sqrt(d)*sgn(b*x + a) + 8*(sqrt(d)*x − sqrt(d*x^2 + x*e + c))^2*a*c*sqrt(d)*e*sgn(b*x + a) + 4*(sqrt(d)*x − sqrt(d*x^2 + x*e + c))*a*c^2*d*sgn(b*x + a) + (sqrt(d)*x − sqrt(d*x^2 + x*e + c))^3*a*e^2*sgn(b*x + a) − 4*(sqrt(d)*x − sqrt(d*x^2 + x*e + c))*b*c^2*e*sgn(b*x + a) − 8*b*c^3*sqrt(d)*sgn(b*x + a) + (sqrt(d)*x − sqrt(d*x^2 + x*e + c))*a*c*e^2*sgn(b*x + a))/(((sqrt(d)*x − sqrt(d*x^2 + x*e + c))^2 − c)^2*c)
```

maple [C] time = 0.02, size = 358, normalized size = 1.67



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^3,x)
```

```
[Out] 1/8*c*sgn(b*x+a)*(-4*d^(5/2)*c^(3/2)*ln((e*x+2*c+2*(d*x^2+e*x+c)^(1/2))*c^(1/2)/2)/x)*x^2*a-2*d^(5/2)*(d*x^2+e*x+c)^(1/2)*x^3*a*e+8*d^(5/2)*(d*x^2+e*x+c)^(1/2)*x^3*b*c-4*d^(3/2)*c^(3/2)*ln((e*x+2*c+2*(d*x^2+e*x+c)^(1/2))*c^(1/2))/
```

$x) * x^2 * b * e + 4 * d^{(5/2)} * (d * x^2 + e * x + c)^{(1/2)} * x^2 * a * c + d^{(3/2)} * c^{(1/2)} * \ln((e * x + 2 * c + 2 * (d * x^2 + e * x + c)^{(1/2)} * c^{(1/2)}) / x) * x^2 * a * e^2 + 2 * d^{(3/2)} * (d * x^2 + e * x + c)^{(3/2)} * x * a * e - 8 * d^{(3/2)} * (d * x^2 + e * x + c)^{(3/2)} * x * b * c - 2 * d^{(3/2)} * (d * x^2 + e * x + c)^{(1/2)} * x^2 * a * e^2 + 8 * d^{(3/2)} * (d * x^2 + e * x + c)^{(1/2)} * x^2 * b * c * e + 8 * \ln(1/2 * (2 * d * x + e + 2 * (d * x^2 + e * x + c)^{(1/2)} * d^{(1/2)})) / d^{(1/2)} * x^2 * b * c^2 * d^2 - 4 * d^{(3/2)} * (d * x^2 + e * x + c)^{(3/2)} * a * c) / x^2 / c^2 / d^{(3/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2 + ex + c} \sqrt{(bx + a)^2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^3,x, algorithm="maxima")

[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)/x^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{(a + bx)^2} \sqrt{dx^2 + ex + c}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^3,x)

[Out] int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^2 + ex} \sqrt{(a + bx)^2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2)/x**3,x)

[Out] Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2)/x**3, x)

$$3.52 \quad \int \frac{x^2 \sqrt{a+cx^2}}{d+ex+fx^2} dx$$

Optimal. Leaf size=452

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(af^2 + 2c(e^2 - df)) \left(e(e - \sqrt{e^2 - 4df})(af^2 + c(e^2 - 2df)) - 2df(af^2 + c(e^2 - df))\right) \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}f^3} - \frac{\sqrt{2}f^3\sqrt{e^2 - 4df}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df})}}{\sqrt{2}f^3\sqrt{e^2 - 4df}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df})}}$$

Rubi [A] time = 1.97, antiderivative size = 452, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, number of rules / integrand size = 0.222, Rules used = {1069, 1080, 217, 206, 1034, 725}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(af^2 + 2c(e^2 - df))}{2\sqrt{c}f^3} - \frac{(e - \sqrt{e^2 - 4df})(af^2 + c(e^2 - 2df)) - 2df(af^2 + c(e^2 - df)) \tanh^{-1}\left(\frac{2af - c(\sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df}) - 2df + e^2}}\right)}{\sqrt{2}f^3\sqrt{e^2 - 4df}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df}) - 2df + e^2}} + \frac{(e(\sqrt{e^2 - 4df} + e)(af^2 + c(e^2 - 2df)) - 2df(af^2 + c(e^2 - df))) \tanh^{-1}\left(\frac{2af - c(\sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df}) - 2df + e^2}}\right)}{\sqrt{2}f^3\sqrt{e^2 - 4df}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df}) - 2df + e^2}} - \frac{\sqrt{a+cx^2}(2e - fx)}{2f^2}$$

Antiderivative was successfully verified.

[In] Int[(x^2*Sqrt[a + c*x^2])/(d + e*x + f*x^2),x]

[Out] -((2*e - f*x)*Sqrt[a + c*x^2])/(2*f^2) + ((a*f^2 + 2*c*(e^2 - d*f))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c]*f^3) - ((e*(e - Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - 2*d*f)) - 2*d*f*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f) - e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f) - e*Sqrt[e^2 - 4*d*f]]) + ((e*(e + Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - 2*d*f)) - 2*d*f*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f) + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f) + e*Sqrt[e^2 - 4*d*f]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 725

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 1034

Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rule 1069

Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[(C*(-(c*e*(2*p + q + 2))) + 2*c*C*f

```
(p + q + 1)*x*(a + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)
)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[
(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(-(a*e))*(C*(c*e)*(q + 1) -
c*(C*e)*(2*p + 2*q + 3)) + (p + q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2))
+ f*(-2*A*f)*(2*p + 2*q + 3)))] + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e)
)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(-4*a*c)))]*x + (p*(c*e)*(C*(c*e)*
(q + 1) - c*(C*e)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C
*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3))))*x^2, x]
, x], x] /; FreeQ[{a, c, d, e, f, A, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && Gt
Q[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !IGtQ[p, 0] && !
IGtQ[q, 0]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + f*x^2],
x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rubi steps

$$\int \frac{x^2 \sqrt{a + cx^2}}{d + ex + fx^2} dx = -\frac{(2e - fx)\sqrt{a + cx^2}}{2f^2} - \frac{\int \frac{acdf - ce(2cd - af)x - c(af^2 + 2c(e^2 - df))x^2}{\sqrt{a + cx^2}(d + ex + fx^2)} dx}{2cf^2}$$

$$= -\frac{(2e - fx)\sqrt{a + cx^2}}{2f^2} - \frac{\int \frac{acdf^2 + cd(af^2 + 2c(e^2 - df)) + (-cef(2cd - af) + ce(af^2 + 2c(e^2 - df)))x}{\sqrt{a + cx^2}(d + ex + fx^2)} dx}{2cf^3} + \frac{(af^2 + 2c(e^2 - df)) \text{Subst}\left(\int \frac{1}{1 - cx^2} dx, x, \frac{x}{\sqrt{a + cx^2}}\right)}{2f^3}$$

$$= -\frac{(2e - fx)\sqrt{a + cx^2}}{2f^2} + \frac{(af^2 + 2c(e^2 - df)) \text{Subst}\left(\int \frac{1}{1 - cx^2} dx, x, \frac{x}{\sqrt{a + cx^2}}\right)}{2f^3} + \frac{(e(e - \sqrt{e^2 - 4df})) (af^2 + 2c(e^2 - df))}{2\sqrt{c} f^3}$$

$$= -\frac{(2e - fx)\sqrt{a + cx^2}}{2f^2} + \frac{(af^2 + 2c(e^2 - df)) \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a + cx^2}}\right)}{2\sqrt{c} f^3} - \frac{(e(e - \sqrt{e^2 - 4df})) (af^2 + 2c(e^2 - df))}{2\sqrt{c} f^3}$$

Mathematica [A] time = 2.50, size = 516, normalized size = 1.14

$$\frac{\frac{\int \frac{x^2 \sqrt{a + cx^2}}{d + ex + fx^2} dx}{\sqrt{a + cx^2}}}{4f^2} + \frac{\frac{\int \frac{x^2 \sqrt{a + cx^2}}{d + ex + fx^2} dx}{\sqrt{a + cx^2}}}{4f^2} + \frac{\frac{\int \frac{x^2 \sqrt{a + cx^2}}{d + ex + fx^2} dx}{\sqrt{a + cx^2}}}{4f^2} + \frac{\frac{\int \frac{x^2 \sqrt{a + cx^2}}{d + ex + fx^2} dx}{\sqrt{a + cx^2}}}{4f^2} - 2\sqrt{a + cx^2} \left(\frac{2e - fx}{\sqrt{a + cx^2}} + e \right) - 2\sqrt{a + cx^2} \left(\frac{2e - fx}{\sqrt{a + cx^2}} + e \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[a + c*x^2])/(d + e*x + f*x^2), x]
[Out] (-2*(e + (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2] - 2*(e + (-e^2 +
2*d*f)/Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2] + (2*f*(a*x + c*x^3 + (a^(3/2))*Sqr
rt[1 + (c*x^2)/a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[c])/Sqrt[a + c*x^2] +
((e + (-e^2 + 2*d*f)/Sqrt[e^2 - 4*d*f])*(-(Sqrt[c]*(-e + Sqrt[e^2 - 4*d*f]
))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]) + Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f
+ e*Sqrt[e^2 - 4*d*f]])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqr
```

$t[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f])]*\text{Sqrt}[a + c*x^2])]/f + ((e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f])*(\text{Sqrt}[c]*(e + \text{Sqrt}[e^2 - 4*d*f]))*\text{ArcTanh}[(\text{Sqrt}[c]*x)/\text{Sqrt}[a + c*x^2]] + \text{Sqrt}[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f])]*\text{ArcTanh}[(2*a*f - c*(e + \text{Sqrt}[e^2 - 4*d*f])*x)/(\text{Sqrt}[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f])]*\text{Sqrt}[a + c*x^2])])]/(f*\text{Sqrt}[e^2 - 4*d*f]))/(4*f^2)$

IntegrateAlgebraic [C] time = 0.67, size = 496, normalized size = 1.10

RootSum[...]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*Sqrt[a + c*x^2])/(d + e*x + f*x^2),x]
[Out] ((-2*e + f*x)*Sqrt[a + c*x^2])/(2*f^2) + ((-2*c*e^2 + 2*c*d*f - a*f^2)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(2*Sqrt[c]*f^3) + RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (a*c*e^3 *Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - 2*a*c*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + a^2*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*c^(3/2)*d*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 2*c^(3/2)*d^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + 2*a*Sqrt[c]*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - c*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 + 2*c*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - a*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]/f^3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.04, size = 7739, normalized size = 17.12

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x)
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see 'assume?' for more details) Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \sqrt{cx^2 + a}}{fx^2 + ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(a + c*x^2)^(1/2))/(d + e*x + f*x^2), x)

[Out] int((x^2*(a + c*x^2)^(1/2))/(d + e*x + f*x^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sqrt{a + cx^2}}{d + ex + fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(c*x**2+a)**(1/2)/(f*x**2+e*x+d), x)

[Out] Integral(x**2*sqrt(a + c*x**2)/(d + e*x + f*x**2), x)

$$3.53 \quad \int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2} dx$$

Optimal. Leaf size=395

$$\frac{(2cdef - (e - \sqrt{e^2 - 4df})(af^2 + c(e^2 - df))) \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2}f^2\sqrt{e^2 - 4df}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} + \frac{(2cdef - (\sqrt{e^2 - 4df})(af^2 + c(e^2 - df))) \tanh^{-1}\left(\frac{2af - cx(\sqrt{e^2 - 4df} + e)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2}f^2\sqrt{e^2 - 4df}\sqrt{2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)}} - \frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{f^2} + \frac{\sqrt{a+cx^2}}{f}$$

Rubi [A] time = 0.93, antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, number of rules / integrand size = 0.240, Rules used = {1020, 1080, 217, 206, 1034, 725}

$$\frac{(2cdef - (e - \sqrt{e^2 - 4df})(af^2 + c(e^2 - df))) \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2}f^2\sqrt{e^2 - 4df}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} + \frac{(2cdef - (\sqrt{e^2 - 4df})(af^2 + c(e^2 - df))) \tanh^{-1}\left(\frac{2af - cx(\sqrt{e^2 - 4df} + e)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2}f^2\sqrt{e^2 - 4df}\sqrt{2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)}} - \frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{f^2} + \frac{\sqrt{a+cx^2}}{f}$$

Antiderivative was successfully verified.

[In] Int[(x*Sqrt[a + c*x^2])/(d + e*x + f*x^2), x]

[Out] Sqrt[a + c*x^2]/f - (Sqrt[c]*e*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/f^2 - ((2*c*d*e*f - (e - Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((2*c*d*e*f - (e + Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])]*Sqrt[a + c*x^2]])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 725

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 1020

Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x] /; FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2} dx &= \frac{\sqrt{a+cx^2}}{f} + \frac{\int \frac{-((cd-af)x)-cex^2}{\sqrt{a+cx^2}(d+ex+fx^2)} dx}{f} \\ &= \frac{\sqrt{a+cx^2}}{f} + \frac{\int \frac{cde+(ce^2+f(-cd+af))x}{\sqrt{a+cx^2}(d+ex+fx^2)} dx}{f^2} - \frac{(ce) \int \frac{1}{\sqrt{a+cx^2}} dx}{f^2} \\ &= \frac{\sqrt{a+cx^2}}{f} - \frac{(ce) \operatorname{Subst}\left(\int \frac{1}{1-cx^2} dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{f^2} + \frac{(2cdef - (e - \sqrt{e^2 - 4df})(af^2 + c(e^2 - a))}{f^2\sqrt{e^2 - 4d}} \\ &= \frac{\sqrt{a+cx^2}}{f} - \frac{\sqrt{c}e \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f^2} - \frac{(2cdef - (e - \sqrt{e^2 - 4df})(af^2 + c(e^2 - df))) \operatorname{Subst}\left(\int \frac{1}{1-cx^2} dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{f^2\sqrt{e^2 - 4d}} \\ &= \frac{\sqrt{a+cx^2}}{f} - \frac{\sqrt{c}e \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f^2} - \frac{(2cdef - (e - \sqrt{e^2 - 4df})(af^2 + c(e^2 - df))) \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{2}f^2\sqrt{e^2 - 4df}\sqrt{2af^2 + c(e^2 - 2d)}} \end{aligned}$$

Mathematica [A] time = 1.52, size = 422, normalized size = 1.07

$$\frac{\frac{(\sqrt{e^2-4df}-e)\sqrt{4ef^2-2c(\sqrt{e^2-4df}+2df+e^2)} \operatorname{tanh}^{-1}\left(\frac{2ef+e(\sqrt{e^2-4df}+e)}{\sqrt{a+cx^2}\sqrt{4ef^2-2c(\sqrt{e^2-4df}+2df+e^2)}}\right)}{\sqrt{e^2-4df}} + \frac{e\sqrt{4ef^2+2c(\sqrt{e^2-4df}-2df+e^2)} \operatorname{tanh}^{-1}\left(\frac{2ef-e(\sqrt{e^2-4df}+e)}{\sqrt{a+cx^2}\sqrt{4ef^2+2c(\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{e^2-4df}} + \sqrt{4af^2+2c(e\sqrt{e^2-4df}-2df+e^2)} \operatorname{tanh}^{-1}\left(\frac{2ef-e(\sqrt{e^2-4df}+e)}{\sqrt{a+cx^2}\sqrt{4ef^2+2c(\sqrt{e^2-4df}-2df+e^2)}}\right) + 4\sqrt{c}e \operatorname{tanh}^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right) - 4f\sqrt{a+cx^2}}{4f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[a + c*x^2])/(d + e*x + f*x^2), x]
[Out] -1/4*(-4*f*Sqrt[a + c*x^2] + 4*Sqrt[c]*e*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] + ((-e + Sqrt[e^2 - 4*d*f])*Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/Sqrt[e^2 - 4*d*f] + Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]]) + (e*Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/Sqrt[e^2 - 4*d*f])/f^2
```

IntegrateAlgebraic [C] time = 0.53, size = 383, normalized size = 0.97

$$\operatorname{RootSum}\left[\#1^4 f - 2\#1^3 \sqrt{c} e - 2\#1^2 a f + 4\#1^2 c d + 2\#1 e \sqrt{c} + a^2 f \&, \frac{e^2 d \log(-\#1 + \sqrt{a+cx^2} - \sqrt{c}) + e^2 (-c) \log(-\#1 + \sqrt{a+cx^2} - \sqrt{c}) + e^2 f^2 \log(-\#1 + \sqrt{a+cx^2} - \sqrt{c}) + e^2 f^2 \log(-\#1 + \sqrt{a+cx^2} - \sqrt{c}) + 2e^2 d \log(-\#1 + \sqrt{a+cx^2} - \sqrt{c}) + 2c^2 \log(-\#1 + \sqrt{a+cx^2} - \sqrt{c})}{2\#1^3 - 3\#1^2 \sqrt{c} - 2\#1 a f + 4\#1^2 c d + a^2 f \&}, \frac{\sqrt{c} e \log(\sqrt{a+cx^2} - \sqrt{c})}{f^2} + \frac{\sqrt{a+cx^2}}{f}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*Sqrt[a + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] Sqrt[a + c*x^2]/f + (Sqrt[c]*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/f^2 - RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (a*c*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - a*c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + a^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*c^(3/2)*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - c*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 + c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - a*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]/f^2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.02, size = 5581, normalized size = 14.13

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x \sqrt{c x^2 + a}}{f x^2 + e x + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + c*x^2)^(1/2))/(d + e*x + f*x^2),x)
```

```
[Out] int((x*(a + c*x^2)^(1/2))/(d + e*x + f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x*sqrt(a + c*x**2)/(d + e*x + f*x**2), x)
```


$$3.54 \quad \int \frac{\sqrt{a+cx^2}}{d+ex+fx^2} dx$$

Optimal. Leaf size=298

$$\frac{\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)} \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2}f\sqrt{e^2 - 4df}} + \frac{\sqrt{2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)} \tanh^{-1}\left(\frac{2af - cx(e + \sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2}f\sqrt{e^2 - 4df}} + \frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f}$$

Rubi [A] time = 0.38, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {991, 217, 206, 1034, 725}

$$\frac{\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)} \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2}f\sqrt{e^2 - 4df}} + \frac{\sqrt{2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)} \tanh^{-1}\left(\frac{2af - cx(e + \sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2}f\sqrt{e^2 - 4df}} + \frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + c*x^2]/(d + e*x + f*x^2), x]

[Out] (Sqrt[c]*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/f - (Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])])*Sqrt[a + c*x^2]])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]) + (Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])*Sqrt[a + c*x^2]])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 725

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 991

Int[Sqrt[(a_) + (c_.)*(x_)^2]/(((d_) + (e_.)*(x_) + (f_.)*(x_)^2), x_Symbol] := Dist[c/f, Int[1/Sqrt[a + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*f + c*e*x)/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)), x], x] /; FreeQ[{a, c, d, e, f}, x] && NeQ[e^2 - 4*d*f, 0]

Rule 1034

Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{a+cx^2}}{d+ex+fx^2} dx &= -\frac{\int \frac{cd-af+ce x}{\sqrt{a+cx^2}(d+ex+fx^2)} dx}{f} + \frac{c \int \frac{1}{\sqrt{a+cx^2}} dx}{f} \\
 &= \frac{c \operatorname{Subst}\left(\int \frac{1}{1-cx^2} dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{f} + \frac{(2f(cd-af) - ce(e + \sqrt{e^2-4df})) \int \frac{1}{(e+\sqrt{e^2-4df}+2fx)\sqrt{a+cx^2}} dx}{f\sqrt{e^2-4df}} \\
 &= \frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f} - \frac{(2f(cd-af) - ce(e + \sqrt{e^2-4df})) \operatorname{Subst}\left(\int \frac{1}{4af^2+c(e+\sqrt{e^2-4df})^2-x^2} dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{f\sqrt{e^2-4df}} \\
 &= \frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f} - \frac{\sqrt{2af^2+c(e^2-2df-e\sqrt{e^2-4df})} \tanh^{-1}\left(\frac{2af-c(e-\sqrt{e^2-4df})}{\sqrt{2}\sqrt{2af^2+c(e^2-2df-e\sqrt{e^2-4df})}}\right)}{\sqrt{2}f\sqrt{e^2-4df}}
 \end{aligned}$$

Mathematica [A] time = 0.36, size = 282, normalized size = 0.95

$$\frac{-\sqrt{4af^2-2c(e\sqrt{e^2-4df}+2df-e^2)} \tanh^{-1}\left(\frac{2af+cx(\sqrt{e^2-4df}+e)}{\sqrt{a+cx^2}\sqrt{4af^2-2c(e\sqrt{e^2-4df}+2df-e^2)}}\right) + \sqrt{4af^2+2c(e\sqrt{e^2-4df}-2df+e^2)} \tanh^{-1}\left(\frac{2af-cx(\sqrt{e^2-4df}+e)}{\sqrt{a+cx^2}\sqrt{4af^2+2c(e\sqrt{e^2-4df}-2df+e^2)}}\right) + 2\sqrt{c}\sqrt{e^2-4df} \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2f\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + c*x^2]/(d + e*x + f*x^2), x]

[Out] (2*Sqrt[c]*Sqrt[e^2 - 4*d*f]*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]]) + Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]))]/(2*f*Sqrt[e^2 - 4*d*f])

IntegrateAlgebraic [C] time = 0.45, size = 267, normalized size = 0.90

$$\frac{\operatorname{RootSum}\left[\#1^4 f - 2\#1^3 \sqrt{c} e - 2\#1^2 a f + 4\#1^2 c d + 2\#1 a \sqrt{c} e + a^2 f \&, \frac{\#1^2(-c) \log(-\#1 + \sqrt{a+cx^2} - \sqrt{c}x) + 2\#1 c^{3/2} d \log(-\#1 + \sqrt{a+cx^2} - \sqrt{c}x) + a c e \log(-\#1 + \sqrt{a+cx^2} - \sqrt{c}x) - 2\#1 a \sqrt{c} f \log(-\#1 + \sqrt{a+cx^2} - \sqrt{c}x) \&}{2\#1^3 f - 3\#1^2 \sqrt{c} e - 2\#1 a f + 4\#1 c d + a \sqrt{c}}\right]}{f} - \frac{\sqrt{c} \log(\sqrt{a+cx^2} - \sqrt{c}x)}{f}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a + c*x^2]/(d + e*x + f*x^2), x]

[Out] -((Sqrt[c]*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/f) + RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 &, (a*c*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*c^(3/2)*d*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 2*a*Sqrt[c]*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - c*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) &]/f

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(1/2)/(f*x^2+e*x+d), x, algorithm="fricas")

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.01, size = 3249, normalized size = 10.90

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+a)^(1/2)/(f*x^2+e*x+d), x)

[Out]
$$\begin{aligned} & -1/2/(-4*d*f+e^2)^{(1/2)}*(4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c-4*c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f+2*((-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}+1/2/(-4*d*f+e^2)^{(1/2)}*c^{(1/2)}/f*\ln((-1/2*c*(e+(-4*d*f+e^2)^{(1/2)}))/f+c*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)/c^{(1/2)}+ \\ & ((x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c-c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f+1/2*((-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}*e+1/2*c^{(1/2)}/f*\ln((-1/2*c*(e+(-4*d*f+e^2)^{(1/2)}))/f+c*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)/c^{(1/2)}+ \\ & ((x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c-c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f+1/2*((-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}+1/2/f^2*2^{(1/2)}/(((4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}* \\ & \ln(((4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f+1/2*2^{(1/2)}*(((4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}* \\ & (4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c-4*c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f+2*((-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}/ \\ & (x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)*c*e+1/(-4*d*f+e^2)^{(1/2)}*2^{(1/2)}/(((4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}* \\ & \ln(((4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f+1/2*2^{(1/2)}*(((4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}* \\ & (4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c-4*c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f+2*((-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}/ \\ & (x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)*c*d+1/2/(-4*d*f+e^2)^{(1/2)}/f^2*2^{(1/2)}/(((4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}* \\ & \ln(((4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f+1/2*2^{(1/2)}*(((4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}* \\ & (4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c-4*c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f+2*((-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}/ \\ & (x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)*c*e^2+1/2/(-4*d*f+e^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c-4*c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f+2*(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}+1/2*c^{(1/2)}/f*\ln((-1/2*c*(e+(-4*d*f+e^2)^{(1/2)}))/f+c*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)/c^{(1/2)}+ \\ & ((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c-c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f+1/2*(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}-1/2/(-4*d*f+e^2)^{(1/2)}*c^{(1/2)}/f*\ln((-1/2*c*(e+(-4*d*f+e^2)^{(1/2)}))/f+c*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)/c^{(1/2)}+ \\ & ((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c-c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f+1/2*(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)} \end{aligned}$$

```

e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))*e+1/2/f^2*2^(1/2)/((-(-4*d*f+e^2)^(1/2)
)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((( -(-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/
2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1
/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(
x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f
+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*c*e-1/(-4*d*f+e^2)^(
1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln
((( -(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/
2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2
*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)
^(1/2))/f))*a+1/(-4*d*f+e^2)^(1/2)/f*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((( -(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f
+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/
2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x
-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f
^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*c*d-1/2/(-4*d*f+e^2)^(1/2)/f^
2*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((
-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))
/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-
4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(
1/2))/f))*c*e^2

```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^2 + a}}{fx^2 + ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c*x^2)^(1/2)/(d + e*x + f*x^2),x)

[Out] int((a + c*x^2)^(1/2)/(d + e*x + f*x^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^2}}{d + ex + fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+a)**(1/2)/(f*x**2+e*x+d),x)

[Out] Integral(sqrt(a + c*x**2)/(d + e*x + f*x**2), x)

$$3.55 \quad \int \frac{\sqrt{a+cx^2}}{x(d+ex+fx^2)} dx$$

Optimal. Leaf size=358

$$\frac{((e - \sqrt{e^2 - 4df})(cd - af) + 2aef) \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}} - \frac{((\sqrt{e^2-4df}+e)(cd-af))}{\sqrt{2}d\sqrt{e^2-4df}}$$

Rubi [A] time = 1.31, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6728, 266, 50, 63, 208, 1020, 1034, 725, 206}

$$\frac{((e - \sqrt{e^2 - 4df})(cd - af) + 2aef) \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}} - \frac{((\sqrt{e^2-4df}+e)(cd-af)) \tanh^{-1}\left(\frac{2af - cx(\sqrt{e^2-4df}+e)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}} - \frac{\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + c*x^2]/(x*(d + e*x + f*x^2)), x]

[Out] ((2*a*e*f + (c*d - a*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]]) - ((2*a*e*f + (c*d - a*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]]) - (Sqrt[a]*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b,
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (c_)*(x_)^2]), x_Symbol] := -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1020

```
Int[((g_) + (h_)*(x_))*((a_) + (c_)*(x_)^2)^(p_)*((d_) + (e_)*(x_) + (f
_)*(x_)^2)^(q_), x_Symbol] := Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q +
1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 1034

```
Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (f
_)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(2*n_)), x_Symbol] := With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+cx^2}}{x(d+ex+fx^2)} dx &= \int \left(\frac{\sqrt{a+cx^2}}{dx} + \frac{(-e-fx)\sqrt{a+cx^2}}{d(d+ex+fx^2)} \right) dx \\
&= \frac{\int \frac{\sqrt{a+cx^2}}{x} dx}{d} + \frac{\int \frac{(-e-fx)\sqrt{a+cx^2}}{d+ex+fx^2} dx}{d} \\
&= -\frac{\sqrt{a+cx^2}}{d} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x} dx, x, x^2\right)}{2d} + \frac{\int \frac{-aef+f(cd-af)x}{\sqrt{a+cx^2}(d+ex+fx^2)} dx}{df} \\
&= \frac{a \text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}} dx, x, x^2\right)}{2d} - \frac{(2aef + (cd-af)(e - \sqrt{e^2-4df})) \int \frac{1}{(e - \sqrt{e^2-4df} + 2fx)\sqrt{a+cx^2}} dx}{d\sqrt{e^2-4df}} \\
&= \frac{a \text{Subst}\left(\int \frac{1}{\frac{-a}{c} + \frac{x^2}{c}} dx, x, \sqrt{a+cx^2}\right)}{cd} + \frac{(2aef + (cd-af)(e - \sqrt{e^2-4df})) \text{Subst}\left(\int \frac{1}{4af^2} dx, x, \sqrt{a+cx^2}\right)}{d\sqrt{e^2-4df}} \\
&= \frac{(2aef + (cd-af)(e - \sqrt{e^2-4df})) \tanh^{-1}\left(\frac{2af - c(e - \sqrt{e^2-4df})x}{\sqrt{2} \sqrt{2af^2 + c(e^2 - 2df - e\sqrt{e^2-4df})} \sqrt{a+cx^2}}\right)}{\sqrt{2} d \sqrt{e^2-4df} \sqrt{2af^2 + c(e^2 - 2df - e\sqrt{e^2-4df})}} - \frac{(2aef + (cd-af)(e - \sqrt{e^2-4df}))}{d\sqrt{e^2-4df}}
\end{aligned}$$

Mathematica [A] time = 0.65, size = 314, normalized size = 0.88

$$\frac{(\sqrt{e^2 - 4df} + e) \sqrt{4af^2 - 2c(e\sqrt{e^2 - 4df} + 2df - e^2)} \tanh^{-1}\left(\frac{2af + c(\sqrt{e^2 - 4df} - e)}{\sqrt{a+cx^2} \sqrt{4af^2 - 2c(e\sqrt{e^2 - 4df} + 2df - e^2)}}\right) + (\sqrt{e^2 - 4df} - e) \sqrt{4af^2 + 2c(e\sqrt{e^2 - 4df} - 2df + e^2)} \tanh^{-1}\left(\frac{2af - c(\sqrt{e^2 - 4df} + e)}{\sqrt{a+cx^2} \sqrt{4af^2 + 2c(e\sqrt{e^2 - 4df} - 2df + e^2)}}\right) - 4\sqrt{a} f \sqrt{e^2 - 4df} \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{4df\sqrt{e^2 - 4df}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + c*x^2]/(x*(d + e*x + f*x^2)),x]

[Out] ((e + Sqrt[e^2 - 4*d*f])*Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]]) + (-e + Sqrt[e^2 - 4*d*f])*Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]]) - 4*Sqrt[a]*f*Sqrt[e^2 - 4*d*f]*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(4*d*f*Sqrt[e^2 - 4*d*f])

IntegrateAlgebraic [C] time = 0.49, size = 306, normalized size = 0.85

$$\frac{\text{RootSum}\left[\#1^4 f - 2\#1^3 \sqrt{c} e - 2\#1^2 a f + 4\#1^2 c d + 2\#1 a \sqrt{c} e + a^2 f \&, \frac{\#1^2 c d \log(-\#1 + \sqrt{a+cx^2} - \sqrt{c} x) - \#1^2 a f \log(-\#1 + \sqrt{a+cx^2} - \sqrt{c} x) + a^2 f \log(-\#1 + \sqrt{a+cx^2} - \sqrt{c} x) - a c d \log(-\#1 + \sqrt{a+cx^2} - \sqrt{c} x) + 2\#1 a \sqrt{c} e \log(-\#1 + \sqrt{a+cx^2} - \sqrt{c} x)}{2\#1^3 f - 3\#1^2 \sqrt{c} e - 2\#1 a f + 4\#1^2 c d + a^2 f \&}\right]}{d} + \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{c} x - \sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a + c*x^2]/(x*(d + e*x + f*x^2)),x]

[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[c]*x)/Sqrt[a] - Sqrt[a + c*x^2]/Sqrt[a]])/d + RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (- (a*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]) + a^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*a*Sqrt[c]*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + c*d*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - a*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) &]/d

fricas [B] time = 38.14, size = 2266, normalized size = 6.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] [-1/4*(sqrt(2)*d*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))))/(d^2*e^2 - 4*d^3*f))*log((2*a*c*d*e*x - a^2*e^2 + sqrt(2)*(d^3*e^2 - 4*d^4*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))))/(d^2*e^2 - 4*d^3*f)) - (a*d^2*e^2 - 4*a*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/x) - sqrt(2)*d*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))))/(d^2*e^2 - 4*d^3*f))*log((2*a*c*d*e*x - a^2*e^2 - sqrt(2)*(d^3*e^2 - 4*d^4*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))))/(d^2*e^2 - 4*d^3*f)) - (a*d^2*e^2 - 4*a*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/x) - sqrt(2)*d*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))))/(d^2*e^2 - 4*d^3*f))*log((2*a*c*d*e*x - a^2*e^2 + sqrt(2)*(d^3*e^2 - 4*d^4*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))))/(d^2*e^2 - 4*d^3*f)) + (a*d^2*e^2 - 4*a*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/x) + sqrt(2)*d*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))))/(d^2*e^2 - 4*d^3*f))*log((2*a*c*d*e*x - a^2*e^2 - sqrt(2)*(d^3*e^2 - 4*d^4*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))))/(d^2*e^2 - 4*d^3*f)) + (a*d^2*e^2 - 4*a*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/x) - 2*sqrt(a)*log(-(c*x^2 - 2*sqrt(a

$$\begin{aligned}
& c*x^2 + a)*\sqrt{a} + 2*a)/x^2))/d, -1/4*(\sqrt{2}*d*\sqrt{((2*c*d^2 + a*e^2 - 2*a*d*f + (d^2*e^2 - 4*d^3*f)*\sqrt{a^2*e^2/(d^4*e^2 - 4*d^5*f))})/(d^2*e^2 - 4*d^3*f))} \\
& * \log((2*a*c*d*e*x - a^2*e^2 + \sqrt{2}*(d^3*e^2 - 4*d^4*f)*\sqrt{a^2*e^2/(d^4*e^2 - 4*d^5*f)})*\sqrt{c*x^2 + a}*\sqrt{((2*c*d^2 + a*e^2 - 2*a*d*f + (d^2*e^2 - 4*d^3*f)*\sqrt{a^2*e^2/(d^4*e^2 - 4*d^5*f))})/(d^2*e^2 - 4*d^3*f)} \\
&) - (a*d^2*e^2 - 4*a*d^3*f)*\sqrt{a^2*e^2/(d^4*e^2 - 4*d^5*f)})/x) - \sqrt{2} \\
&)*d*\sqrt{((2*c*d^2 + a*e^2 - 2*a*d*f + (d^2*e^2 - 4*d^3*f)*\sqrt{a^2*e^2/(d^4*e^2 - 4*d^5*f))})/(d^2*e^2 - 4*d^3*f)} \\
&)*\log((2*a*c*d*e*x - a^2*e^2 - \sqrt{2}*(d^3*e^2 - 4*d^4*f)*\sqrt{a^2*e^2/(d^4*e^2 - 4*d^5*f)})*\sqrt{c*x^2 + a}*\sqrt{((2*c*d^2 + a*e^2 - 2*a*d*f + (d^2*e^2 - 4*d^3*f)*\sqrt{a^2*e^2/(d^4*e^2 - 4*d^5*f))})/(d^2*e^2 - 4*d^3*f)} \\
&) - (a*d^2*e^2 - 4*a*d^3*f)*\sqrt{a^2*e^2/(d^4*e^2 - 4*d^5*f)})/x) - \sqrt{2} \\
&)*d*\sqrt{((2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f)*\sqrt{a^2*e^2/(d^4*e^2 - 4*d^5*f))})/(d^2*e^2 - 4*d^3*f)} \\
&)*\log((2*a*c*d*e*x - a^2*e^2 + \sqrt{2}*(d^3*e^2 - 4*d^4*f)*\sqrt{a^2*e^2/(d^4*e^2 - 4*d^5*f)})*\sqrt{c*x^2 + a}*\sqrt{((2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f)*\sqrt{a^2*e^2/(d^4*e^2 - 4*d^5*f))})/(d^2*e^2 - 4*d^3*f)} \\
&) + (a*d^2*e^2 - 4*a*d^3*f)*\sqrt{a^2*e^2/(d^4*e^2 - 4*d^5*f)})/x) + \sqrt{2} \\
&)*d*\sqrt{((2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f)*\sqrt{a^2*e^2/(d^4*e^2 - 4*d^5*f))})/(d^2*e^2 - 4*d^3*f)} \\
&)*\log((2*a*c*d*e*x - a^2*e^2 - \sqrt{2}*(d^3*e^2 - 4*d^4*f)*\sqrt{a^2*e^2/(d^4*e^2 - 4*d^5*f)})*\sqrt{c*x^2 + a}*\sqrt{((2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f)*\sqrt{a^2*e^2/(d^4*e^2 - 4*d^5*f))})/(d^2*e^2 - 4*d^3*f)} \\
&) + (a*d^2*e^2 - 4*a*d^3*f)*\sqrt{a^2*e^2/(d^4*e^2 - 4*d^5*f)})/x) - 4*\sqrt{-a}*\arctan(\sqrt{-a}/\sqrt{c*x^2 + a}))/d]
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 2Error index.cc index_gcd E
rror: Bad Argument Value

maple [B] time = 0.02, size = 3544, normalized size = 9.90

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+a)^(1/2)/x/(f*x^2+e*x+d),x)

[Out] $f/(e+(-4*d*f+e^2)^{(1/2)})/(-4*d*f+e^2)^{(1/2)}*(4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-4*c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)+2*((-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}-1/(e+(-4*d*f+e^2)^{(1/2)})*c^{(1/2)}*\ln((-1/2*c*(e+(-4*d*f+e^2)^{(1/2)})/f+c*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f))/c^{(1/2)}+((x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)+1/2*((-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}-1/(e+(-4*d*f+e^2)^{(1/2)})/(-4*d*f+e^2)^{(1/2)}*c^{(1/2)}*\ln((-1/2*c*(e+(-4*d*f+e^2)^{(1/2)})/f+c*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f))/c^{(1/2)}+((x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)+1/2*((-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}*e-1/f/(e+(-4*d*f+e^2)^{(1/2)})*2^{(1/2)}/(((4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}*\ln((((4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)+1/2*2^{(1/2)}*((4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}*(4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-4*c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)+2*((4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}/(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f))*c*e-2*f/(e+(-4*d*f+e^2)$

$$\begin{aligned}
& \wedge(1/2)) / (-4*d*f+e^2)^{(1/2)} * 2^{(1/2)} / (((-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f \\
& +c*e^2) / f^2)^{(1/2)} * \ln(((-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f+c*e^2) / f^2-c \\
& *(e+(-4*d*f+e^2)^{(1/2)}) / f * (x+1/2*(e+(-4*d*f+e^2)^{(1/2)}) / f) + 1/2*2^{(1/2)} * (((-4 \\
& *d*f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f+c*e^2) / f^2)^{(1/2)} * (4*(x+1/2*(e+(-4*d*f \\
& +e^2)^{(1/2)}) / f)^2*c-4*c*(e+(-4*d*f+e^2)^{(1/2)}) / f * (x+1/2*(e+(-4*d*f+e^2)^{(1/2) \\
&) / f) + 2*((-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f+c*e^2) / f^2)^{(1/2)} / (x+1/2* \\
& (e+(-4*d*f+e^2)^{(1/2)}) / f) * a+2 / (e+(-4*d*f+e^2)^{(1/2)}) / (-4*d*f+e^2)^{(1/2)} * 2^{(\\
& 1/2)} / (((-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f+c*e^2) / f^2)^{(1/2)} * \ln(((-4*d \\
& *f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f+c*e^2) / f^2-c*(e+(-4*d*f+e^2)^{(1/2)}) / f * (x+ \\
& 1/2*(e+(-4*d*f+e^2)^{(1/2)}) / f) + 1/2*2^{(1/2)} * (((-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2- \\
& 2*c*d*f+c*e^2) / f^2)^{(1/2)} * (4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}) / f)^2*c-4*c*(e+(- \\
& 4*d*f+e^2)^{(1/2)}) / f * (x+1/2*(e+(-4*d*f+e^2)^{(1/2)}) / f) + 2*((-4*d*f+e^2)^{(1/2)} * \\
& c*e+2*a*f^2-2*c*d*f+c*e^2) / f^2)^{(1/2)} / (x+1/2*(e+(-4*d*f+e^2)^{(1/2)}) / f) * c* \\
& d-1/f / (e+(-4*d*f+e^2)^{(1/2)}) / (-4*d*f+e^2)^{(1/2)} * 2^{(1/2)} / (((-4*d*f+e^2)^{(1/2) \\
&) * c*e+2*a*f^2-2*c*d*f+c*e^2) / f^2)^{(1/2)} * \ln(((-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2 \\
& -2*c*d*f+c*e^2) / f^2-c*(e+(-4*d*f+e^2)^{(1/2)}) / f * (x+1/2*(e+(-4*d*f+e^2)^{(1/2) \\
&) / f) + 1/2*2^{(1/2)} * (((-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f+c*e^2) / f^2)^{(1/2) \\
& } * (4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}) / f)^2*c-4*c*(e+(-4*d*f+e^2)^{(1/2)}) / f * (x+1/ \\
& 2*(e+(-4*d*f+e^2)^{(1/2)}) / f) + 2*((-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f+c*e^2 \\
&) / f^2)^{(1/2)} / (x+1/2*(e+(-4*d*f+e^2)^{(1/2)}) / f) * c*e^2+f / (-e+(-4*d*f+e^2)^{(1 \\
& /2)}) / (-4*d*f+e^2)^{(1/2)} * (4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}) / f)^2*c-4*c*(e-(-4 \\
& *d*f+e^2)^{(1/2)}) / f * (x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}) / f) + 2*(-(-4*d*f+e^2)^{(1/2) \\
& } * c*e+2*a*f^2-2*c*d*f+c*e^2) / f^2)^{(1/2)} + 1 / (-e+(-4*d*f+e^2)^{(1/2)}) * c^{(1/2)} * \ln \\
& ((-1/2*c*(e-(-4*d*f+e^2)^{(1/2)}) / f + c*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}) / f)) / c^{(1 \\
& /2)} + ((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}) / f)^2*c-c*(e-(-4*d*f+e^2)^{(1/2)}) / f * (x-1/ \\
& 2*(-e+(-4*d*f+e^2)^{(1/2)}) / f) + 1/2*(-(-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f+c \\
& *e^2) / f^2)^{(1/2)} - 1 / (-e+(-4*d*f+e^2)^{(1/2)}) / (-4*d*f+e^2)^{(1/2)} * c^{(1/2)} * \ln((\\
& -1/2*c*(e-(-4*d*f+e^2)^{(1/2)}) / f + c*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}) / f)) / c^{(1/2) \\
& } + ((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}) / f)^2*c-c*(e-(-4*d*f+e^2)^{(1/2)}) / f * (x-1/2* \\
& (-e+(-4*d*f+e^2)^{(1/2)}) / f) + 1/2*(-(-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f+c*e \\
& ^2) / f^2)^{(1/2)} * e+1/f / (-e+(-4*d*f+e^2)^{(1/2)}) * 2^{(1/2)} / (((-4*d*f+e^2)^{(1/2) \\
& } * c*e+2*a*f^2-2*c*d*f+c*e^2) / f^2)^{(1/2)} * \ln(((-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2 \\
& -2*c*d*f+c*e^2) / f^2-c*(e-(-4*d*f+e^2)^{(1/2)}) / f * (x-1/2*(-e+(-4*d*f+e^2)^{(1/2) \\
&) / f) + 1/2*2^{(1/2)} * (((-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f+c*e^2) / f^2)^{(1/ \\
& 2)} * (4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}) / f)^2*c-4*c*(e-(-4*d*f+e^2)^{(1/2)}) / f * (x \\
& -1/2*(-e+(-4*d*f+e^2)^{(1/2)}) / f) + 2*(-(-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f+ \\
& c*e^2) / f^2)^{(1/2)} / (x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}) / f) * c*e-2*f / (-e+(-4*d*f+e \\
& ^2)^{(1/2)}) / (-4*d*f+e^2)^{(1/2)} * 2^{(1/2)} / (((-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c \\
& *d*f+c*e^2) / f^2)^{(1/2)} * \ln(((-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f+c*e^2) / \\
& f^2-c*(e-(-4*d*f+e^2)^{(1/2)}) / f * (x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}) / f) + 1/2*2^{(1/2) \\
& } * (((-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f+c*e^2) / f^2)^{(1/2)} * (4*(x-1/2*(-e \\
& +(-4*d*f+e^2)^{(1/2)}) / f)^2*c-4*c*(e-(-4*d*f+e^2)^{(1/2)}) / f * (x-1/2*(-e+(-4*d*f \\
& +e^2)^{(1/2)}) / f) + 2*(-(-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f+c*e^2) / f^2)^{(1/2) \\
&) / (x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}) / f) * a+2 / (-e+(-4*d*f+e^2)^{(1/2)}) / (-4*d*f+e \\
& ^2)^{(1/2)} * 2^{(1/2)} / (((-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f+c*e^2) / f^2)^{(1/ \\
& 2)} * \ln(((-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f+c*e^2) / f^2-c*(e-(-4*d*f+e^2) \\
& ^{(1/2)}) / f * (x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}) / f) + 1/2*2^{(1/2)} * (((-4*d*f+e^2)^{(1 \\
& /2)} * c*e+2*a*f^2-2*c*d*f+c*e^2) / f^2)^{(1/2)} * (4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2) \\
&) / f)^2*c-4*c*(e-(-4*d*f+e^2)^{(1/2)}) / f * (x-1/2*(-e+(-4*d*f+e^2)^{(1/2) \\
&) / f) + 2*(-(-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f+c*e^2) / f^2)^{(1/2)} / (x-1/2*(-e+(-4*d* \\
& f+e^2)^{(1/2)}) / f) * c*d-1/f / (-e+(-4*d*f+e^2)^{(1/2)}) / (-4*d*f+e^2)^{(1/2)} * 2^{(1/2) \\
&) / (((-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f+c*e^2) / f^2)^{(1/2)} * \ln(((-4*d*f \\
& +e^2)^{(1/2)} * c*e+2*a*f^2-2*c*d*f+c*e^2) / f^2-c*(e-(-4*d*f+e^2)^{(1/2)}) / f * (x-1/ \\
& 2*(-e+(-4*d*f+e^2)^{(1/2)}) / f) + 1/2*2^{(1/2)} * (((-4*d*f+e^2)^{(1/2)} * c*e+2*a*f^2- \\
& 2*c*d*f+c*e^2) / f^2)^{(1/2)} * (4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}) / f)^2*c-4*c*(e-(- \\
& 4*d*f+e^2)^{(1/2)}) / f * (x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}) / f) + 2*(-(-4*d*f+e^2)^{(1/ \\
& 2)} * c*e+2*a*f^2-2*c*d*f+c*e^2) / f^2)^{(1/2)} / (x-1/2*(-e+(-4*d*f+e^2)^{(1/2) \\
&) / f) * c*e^2+4*f / (-e+(-4*d*f+e^2)^{(1/2)}) / (e+(-4*d*f+e^2)^{(1/2)}) * a^{(1/2)} * \ln((2*a+ \\
& 2*a^{(1/2)} * (c*x^2+a)^{(1/2)}) / x) - 4*f / (-e+(-4*d*f+e^2)^{(1/2)}) / (e+(-4*d*f+e^2)^{(
\end{aligned}$$

$1/2)) * (c * x^2 + a)^{1/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + a}}{(fx^2 + ex + d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] integrate(sqrt(c*x^2 + a)/((f*x^2 + e*x + d)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^2 + a}}{x(fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c*x^2)^(1/2)/(x*(d + e*x + f*x^2)),x)

[Out] int((a + c*x^2)^(1/2)/(x*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^2}}{x(d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+a)**(1/2)/x/(f*x**2+e*x+d),x)

[Out] Integral(sqrt(a + c*x**2)/(x*(d + e*x + f*x**2)), x)

$$3.56 \quad \int \frac{\sqrt{a+cx^2}}{x^2(d+ex+fx^2)} dx$$

Optimal. Leaf size=382

$$\frac{f\left(a\left(e\sqrt{e^2-4df}-2df+e^2\right)+2cd^2\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{f\left(a\left(-e\sqrt{e^2-4df}-2df+e^2\right)+2cd^2\right)\tanh^{-1}\left(\frac{2af-cx\left(e+\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2} - \frac{\sqrt{a+cx^2}}{dx}$$

Rubi [A] time = 1.42, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {6728, 277, 217, 206, 266, 50, 63, 208, 1020, 1080, 1034, 725}

$$\frac{f\left(a\left(e\sqrt{e^2-4df}-2df+e^2\right)+2cd^2\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{f\left(a\left(-e\sqrt{e^2-4df}-2df+e^2\right)+2cd^2\right)\tanh^{-1}\left(\frac{2af-cx\left(e+\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2} - \frac{\sqrt{a+cx^2}}{dx}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + c*x^2]/(x^2*(d + e*x + f*x^2)), x]

[Out] -(Sqrt[a + c*x^2]/(d*x)) - (f*(2*c*d^2 + a*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]]) + (f*(2*c*d^2 + a*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]]) + (Sqrt[a]*e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d^2

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 206

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 217

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] /; \text{FreeQ}\{a, b\}, x] \&\& !\text{GtQ}[a, 0]$

Rule 266

$\text{Int}[(x_)^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p}, x], x, x^n], x] /; \text{FreeQ}\{a, b, m, n, p\}, x] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 277

$\text{Int}[((c_)*(x_))^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(c*x)^{(m + 1)}*(a + b*x^n)^p/(c*(m + 1)), x] - \text{Dist}[(b*n*p)/(c^n*(m + 1)), \text{Int}[(c*x)^{(m + n)}*(a + b*x^n)^{(p - 1)}, x], x] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{GtQ}[p, 0] \&\& \text{LtQ}[m, -1] \&\& !\text{ILtQ}[(m + n*p + n + 1)/n, 0] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 725

$\text{Int}[1/(((d_) + (e_)*(x_))*\text{Sqrt}[(a_) + (c_)*(x_)^2]), x_Symbol] \rightarrow -\text{Subst}[\text{Int}[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/\text{Sqrt}[a + c*x^2]] /; \text{FreeQ}\{a, c, d, e\}, x]$

Rule 1020

$\text{Int}[((g_.) + (h_)*(x_))*((a_) + (c_)*(x_)^2)^{(p_)}*((d_) + (e_)*(x_) + (f_)*(x_)^2)^{(q_)}, x_Symbol] \rightarrow \text{Simp}[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^{(q + 1)})/(2*f*(p + q + 1)), x] + \text{Dist}[1/(2*f*(p + q + 1)), \text{Int}[(a + c*x^2)^{(p - 1)}*(d + e*x + f*x^2)^q*\text{Simp}[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /; \text{FreeQ}\{a, c, d, e, f, g, h, q\}, x] \&\& \text{NeQ}[e^2 - 4*d*f, 0] \&\& \text{GtQ}[p, 0] \&\& \text{NeQ}[p + q + 1, 0]$

Rule 1034

$\text{Int}[((g_.) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*\text{Sqrt}[(d_) + (f_)*(x_)^2]), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(2*c*g - h*(b - q))/q, \text{Int}[1/((b - q + 2*c*x)*\text{Sqrt}[d + f*x^2]), x], x] - \text{Dist}[(2*c*g - h*(b + q))/q, \text{Int}[1/((b + q + 2*c*x)*\text{Sqrt}[d + f*x^2]), x], x]] /; \text{FreeQ}\{a, b, c, d, f, g, h\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{PosQ}[b^2 - 4*a*c]$

Rule 1080

$\text{Int}[((A_.) + (B_)*(x_) + (C_)*(x_)^2)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*\text{Sqrt}[(d_.) + (f_)*(x_)^2]), x_Symbol] \rightarrow \text{Dist}[C/c, \text{Int}[1/\text{Sqrt}[d + f*x^2], x], x] + \text{Dist}[1/c, \text{Int}[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*\text{Sqrt}[d + f*x^2]), x], x] /; \text{FreeQ}\{a, b, c, d, f, A, B, C\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 6728

$\text{Int}[(u_)/((a_.) + (b_)*(x_)^{(n_)} + (c_)*(x_)^{(2n_)}), x_Symbol] \rightarrow \text{With}\{v = \text{RationalFunctionExpand}[u/(a + b*x^n + c*x^{2n}), x]\}, \text{Int}[v, x] /; \text{SumQ}[v] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{EqQ}[n2, 2*n] \&\& \text{IGtQ}[n, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+cx^2}}{x^2(d+ex+fx^2)} dx &= \int \left(\frac{\sqrt{a+cx^2}}{dx^2} - \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{(e^2-df+efx)\sqrt{a+cx^2}}{d^2(d+ex+fx^2)} \right) dx \\
&= \frac{\int \frac{(e^2-df+efx)\sqrt{a+cx^2}}{d+ex+fx^2} dx}{d^2} + \frac{\int \frac{\sqrt{a+cx^2}}{x^2} dx}{d} - \frac{e \int \frac{\sqrt{a+cx^2}}{x} dx}{d^2} \\
&= \frac{e\sqrt{a+cx^2}}{d^2} - \frac{\sqrt{a+cx^2}}{dx} + \frac{c \int \frac{1}{\sqrt{a+cx^2}} dx}{d} - \frac{e \operatorname{Subst} \left(\int \frac{\sqrt{a+cx}}{x} dx, x, x^2 \right)}{2d^2} + \frac{\int \frac{af(e^2-df)}{\sqrt{a+cx^2}} dx}{d^2} \\
&= -\frac{\sqrt{a+cx^2}}{dx} - \frac{c \int \frac{1}{\sqrt{a+cx^2}} dx}{d} + \frac{c \operatorname{Subst} \left(\int \frac{1}{1-cx^2} dx, x, \frac{x}{\sqrt{a+cx^2}} \right)}{d} - \frac{(ae) \operatorname{Subst} \left(\int \frac{1}{x\sqrt{a+cx^2}} dx, x, x^2 \right)}{2d^2} \\
&= -\frac{\sqrt{a+cx^2}}{dx} + \frac{\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}} \right)}{d} - \frac{c \operatorname{Subst} \left(\int \frac{1}{1-cx^2} dx, x, \frac{x}{\sqrt{a+cx^2}} \right)}{d} - \frac{(ae) \operatorname{Subst} \left(\int \frac{1}{x\sqrt{a+cx^2}} dx, x, x^2 \right)}{2d^2} \\
&= -\frac{\sqrt{a+cx^2}}{dx} + \frac{\sqrt{a} e \tanh^{-1} \left(\frac{\sqrt{a+cx^2}}{\sqrt{a}} \right)}{d^2} + \frac{(f(2cd^2 + a(e^2 - 2df - e\sqrt{e^2 - 4df}))) \operatorname{Subst} \left(\int \frac{1}{\sqrt{a+cx^2}} dx, x, x^2 \right)}{d^2} \\
&= -\frac{\sqrt{a+cx^2}}{dx} - \frac{f(2cd^2 + a(e^2 - 2df + e\sqrt{e^2 - 4df})) \tanh^{-1} \left(\frac{2af - c(e - \sqrt{e^2 - 4df})}{\sqrt{2} \sqrt{2af^2 + c(e^2 - 2df - e\sqrt{e^2 - 4df})}} \right)}{\sqrt{2} d^2 \sqrt{e^2 - 4df} \sqrt{2af^2 + c(e^2 - 2df - e\sqrt{e^2 - 4df})}}
\end{aligned}$$

Mathematica [A] time = 3.03, size = 569, normalized size = 1.49

$$\frac{\sqrt{a+cx^2} \operatorname{ArcTanh} \left(\frac{\sqrt{a+cx^2}}{\sqrt{a}} \right) + \frac{c \operatorname{Subst} \left(\int \frac{1}{1-cx^2} dx, x, \frac{x}{\sqrt{a+cx^2}} \right)}{d} - \frac{(ae) \operatorname{Subst} \left(\int \frac{1}{x\sqrt{a+cx^2}} dx, x, x^2 \right)}{2d^2} + \frac{(f(2cd^2 + a(e^2 - 2df - e\sqrt{e^2 - 4df}))) \operatorname{Subst} \left(\int \frac{1}{\sqrt{a+cx^2}} dx, x, x^2 \right)}{d^2} - \frac{f(2cd^2 + a(e^2 - 2df + e\sqrt{e^2 - 4df})) \tanh^{-1} \left(\frac{2af - c(e - \sqrt{e^2 - 4df})}{\sqrt{2} \sqrt{2af^2 + c(e^2 - 2df - e\sqrt{e^2 - 4df})}} \right)}{\sqrt{2} d^2 \sqrt{e^2 - 4df} \sqrt{2af^2 + c(e^2 - 2df - e\sqrt{e^2 - 4df})}}}{d^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + c*x^2]/(x^2*(d + e*x + f*x^2)), x]

[Out] (2*(e + (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2] + 2*(e + (-e^2 + 2*d*f)/Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2] - (4*d*(a + c*x^2 - Sqrt[a]*Sqrt[c]*x*Sqrt[1 + (c*x^2)/a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]]))/(x*Sqrt[a + c*x^2]) + ((e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*(Sqrt[c]*(-e + Sqrt[e^2 - 4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]))/(f*Sqrt[e^2 - 4*d*f]) + ((e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])*(Sqrt[c]*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] + Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]))/(f*Sqrt[e^2 - 4*d*f]) - 4*e*(Sqrt[a + c*x^2] - Sqrt[a]*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(4*d^2)

IntegrateAlgebraic [C] time = 0.58, size = 343, normalized size = 0.90

$$\frac{\operatorname{RootSum} \left[\#1^4 f - 2\#1^3 \sqrt{c} e - 2\#1^2 a f + 4\#1^2 c d + 2\#1 a \sqrt{c} e + a^2 f \&, \frac{-\#1^2 e f \log \left(\#1 + \sqrt{a+c\#1^2} - \sqrt{c} \right) + e^2 f \log \left(\#1 + \sqrt{a+c\#1^2} - \sqrt{c} \right) + 2\#1^2 e^2 \log \left(\#1 + \sqrt{a+c\#1^2} - \sqrt{c} \right) + 2\#1 a \sqrt{c} f \log \left(\#1 + \sqrt{a+c\#1^2} - \sqrt{c} \right) + 2\#1 a \sqrt{c}^2 \log \left(\#1 + \sqrt{a+c\#1^2} - \sqrt{c} \right)}{2\#1^3 - 3\#1^2 \sqrt{c} e - 2\#1 a f + 4\#1^2 c d + a^2 f} \& \right]}{d^2} - \frac{2\sqrt{a} e \tanh^{-1} \left(\frac{\sqrt{c}x}{\sqrt{a}} \right)}{d^2} - \frac{\sqrt{a+cx^2}}{dx}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a + c*x^2]/(x^2*(d + e*x + f*x^2)), x]

```
[Out] -(Sqrt[a + c*x^2]/(d*x)) - (2*Sqrt[a]*e*ArcTanh[(Sqrt[c]*x)/Sqrt[a] - Sqrt[
a + c*x^2]/Sqrt[a]])/d^2 - RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 -
2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (a^2*e*f*Log[-(Sqrt[c]*x) + Sqrt
[a + c*x^2] - #1] + 2*c^(3/2)*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*
#1 + 2*a*Sqrt[c]*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 2*a*Sqrt
[c]*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - a*e*f*Log[-(Sqrt[c]*x
) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt
[c]*e*#1^2 + 2*f*#1^3) & ]/d^2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="fricas")
```

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Evaluation time: 5.11Error index.cc index_gc
d Error: Bad Argument Value
```

maple [B] time = 0.02, size = 3703, normalized size = 9.69

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/x^2/(f*x^2+e*x+d),x)
```

```
[Out] -2*f^2/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)
)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/
f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2^(1/2)+2*f/(e+(-4*d
*f+e^2)^(1/2))^2*c^(1/2)*ln((-1/2*c*(e+(-4*d*f+e^2)^(1/2))/f+c*(x+1/2*(e+(-
4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-
4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)
)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2^(1/2))+2*f/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d
*f+e^2)^(1/2)*c^(1/2)*ln((-1/2*c*(e+(-4*d*f+e^2)^(1/2))/f+c*(x+1/2*(e+(-4*d
*f+e^2)^(1/2))/f))/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d
*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-2*c*d*f+c*e^2)/f^2^(1/2))*e+2/(e+(-4*d*f+e^2)^(1/2))^2*2^(1/2)/
((( -4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2^(1/2)*ln(((( -4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e
+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((( -4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*
f+c*e^2)/f^2^(1/2)*4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+
e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-2*c*d*f+c*e^2)/f^2^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c*e+4*f^
2/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((( -4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2^(1/2)*ln(((( -4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2
*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/
f)+1/2*2^(1/2)*((( -4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2^(1/2)*
(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/
```


maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + a}}{(fx^2 + ex + d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] integrate(sqrt(c*x^2 + a)/((f*x^2 + e*x + d)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^2 + a}}{x^2 (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c*x^2)^(1/2)/(x^2*(d + e*x + f*x^2)),x)

[Out] int((a + c*x^2)^(1/2)/(x^2*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^2}}{x^2 (d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+a)**(1/2)/x**2/(f*x**2+e*x+d),x)

[Out] Integral(sqrt(a + c*x**2)/(x**2*(d + e*x + f*x**2)), x)

$$3.57 \quad \int \frac{\sqrt{a+cx^2}}{x^3(d+ex+fx^2)} dx$$

Optimal. Leaf size=507

$$\frac{\sqrt{a} (e^2 - df) \tanh^{-1} \left(\frac{\sqrt{a+cx^2}}{\sqrt{a}} \right) + \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{f(a(e^2\sqrt{e^2-4df} - df\sqrt{e^2-4df} - 3def + e^3) + cd^2(\sqrt{e^2-4df} - e\sqrt{a+cx^2}))}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c}(-e\sqrt{a+cx^2})}}{d^3} + \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{f(a(e^2\sqrt{e^2-4df} - df\sqrt{e^2-4df} - 3def + e^3) + cd^2(\sqrt{e^2-4df} - e\sqrt{a+cx^2}))}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c}(-e\sqrt{a+cx^2})}}$$

Rubi [A] time = 1.88, antiderivative size = 507, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 13, integrand size = 27, number of rules / integrand size = 0.482, Rules used = {6728, 266, 47, 63, 208, 277, 217, 206, 50, 1020, 1080, 1034, 725}

$$\frac{f(a(e^2\sqrt{e^2-4df} - df\sqrt{e^2-4df} - 3def + e^3) + cd^2(\sqrt{e^2-4df} - e\sqrt{a+cx^2})) \tanh^{-1} \left(\frac{\sqrt{a+cx^2}}{\sqrt{a}} \right) + \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{f(a(e^2\sqrt{e^2-4df} - df\sqrt{e^2-4df} - 3def + e^3) + cd^2(\sqrt{e^2-4df} - e\sqrt{a+cx^2}))}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c}(-e\sqrt{a+cx^2})}}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c}(-e\sqrt{a+cx^2})} + \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{f(a(e^2\sqrt{e^2-4df} - df\sqrt{e^2-4df} - 3def + e^3) + cd^2(\sqrt{e^2-4df} - e\sqrt{a+cx^2}))}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c}(-e\sqrt{a+cx^2})}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + c*x^2]/(x^3*(d + e*x + f*x^2)),x]

[Out] $-\text{Sqrt}[a + c*x^2]/(2*d*x^2) + (e*\text{Sqrt}[a + c*x^2])/(d^2*x) + (f*(c*d^2*(e + \text{Sqrt}[e^2 - 4*d*f]) + a*(e^3 - 3*d*e*f + e^2*\text{Sqrt}[e^2 - 4*d*f] - d*f*\text{Sqrt}[e^2 - 4*d*f]))*\text{ArcTanh}[(2*a*f - c*(e - \text{Sqrt}[e^2 - 4*d*f])*x)/(\text{Sqrt}[2]*\text{Sqrt}[2*a*f^2 + c*(e^2 - 2*d*f - e*\text{Sqrt}[e^2 - 4*d*f])])* \text{Sqrt}[a + c*x^2]])/(\text{Sqrt}[2]*d^3*\text{Sqrt}[e^2 - 4*d*f]*\text{Sqrt}[2*a*f^2 + c*(e^2 - 2*d*f - e*\text{Sqrt}[e^2 - 4*d*f])]) - (f*(c*d^2*(e - \text{Sqrt}[e^2 - 4*d*f]) + a*(e^3 - 3*d*e*f - e^2*\text{Sqrt}[e^2 - 4*d*f] + d*f*\text{Sqrt}[e^2 - 4*d*f]))*\text{ArcTanh}[(2*a*f - c*(e + \text{Sqrt}[e^2 - 4*d*f])*x)/(\text{Sqrt}[2]*\text{Sqrt}[2*a*f^2 + c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f])])* \text{Sqrt}[a + c*x^2]])/(\text{Sqrt}[2]*d^3*\text{Sqrt}[e^2 - 4*d*f]*\text{Sqrt}[2*a*f^2 + c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f])]) - (c*\text{ArcTanh}[\text{Sqrt}[a + c*x^2]/\text{Sqrt}[a]])/(2*\text{Sqrt}[a]*d) - (\text{Sqrt}[a]*(e^2 - d*f)*\text{ArcTanh}[\text{Sqrt}[a + c*x^2]/\text{Sqrt}[a]])/d^3$

Rule 47

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c, d, m, n, x]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 217

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 266

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 277

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 725

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (c_)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 1020

Int[((g_) + (h_)*(x_))*((a_) + (c_)*(x_)^2)^(p_)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x] /; FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]

Rule 1034

Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (f_)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rule 1080

Int[((A_) + (B_)*(x_) + (C_)*(x_)^2)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (f_)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a

*c, 0]

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{x^3(d+ex+fx^2)} dx = \int \left(\frac{\sqrt{a+cx^2}}{dx^3} - \frac{e\sqrt{a+cx^2}}{d^2x^2} + \frac{(e^2-df)\sqrt{a+cx^2}}{d^3x} + \frac{(-e(e^2-2df)-f(e^2-df)x)}{d^3(d+ex+fx^2)} \right) dx$$

$$= \frac{\int \frac{(-e(e^2-2df)-f(e^2-df)x)\sqrt{a+cx^2}}{d+ex+fx^2} dx}{d^3} + \frac{\int \frac{\sqrt{a+cx^2}}{x^3} dx}{d} - \frac{e \int \frac{\sqrt{a+cx^2}}{x^2} dx}{d^2} + \frac{(e^2-df) \int \frac{\sqrt{a+cx^2}}{x} dx}{d^3}$$

$$= -\frac{(e^2-df)\sqrt{a+cx^2}}{d^3} + \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x^2} dx, x, x^2\right)}{2d} - \frac{(ce) \int \frac{1}{\sqrt{a+cx^2}} dx}{d^2}$$

$$= -\frac{\sqrt{a+cx^2}}{2dx^2} + \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{c \text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}} dx, x, x^2\right)}{4d} + \frac{(ce) \int \frac{1}{\sqrt{a+cx^2}} dx}{d^2} - \frac{(ce) \int \frac{1}{\sqrt{a+cx^2}} dx}{d^2}$$

$$= -\frac{\sqrt{a+cx^2}}{2dx^2} + \frac{e\sqrt{a+cx^2}}{d^2x} - \frac{\sqrt{c} e \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{d^2} + \frac{\text{Subst}\left(\int \frac{1}{\frac{a}{c} + x^2} dx, x, \sqrt{a+cx^2}\right)}{2d}$$

$$= -\frac{\sqrt{a+cx^2}}{2dx^2} + \frac{e\sqrt{a+cx^2}}{d^2x} - \frac{c \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}d} - \frac{\sqrt{a}(e^2-df) \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3} + \dots$$

$$= -\frac{\sqrt{a+cx^2}}{2dx^2} + \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{f(cd^2(e+\sqrt{e^2-4df})+a(e^3-3def+e^2\sqrt{e^2-4df}))}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2}}$$

Mathematica [A] time = 2.61, size = 642, normalized size = 1.27

```
2*(e^2-d*f-(e*(e^2-3*d*f)))/Sqrt[e^2-4*d*f]*Sqrt[a+c*x^2] - 2*(e^2-d*f+(e*(e^2-3*d*f)))/Sqrt[e^2-4*d*f]*Sqrt[a+c*x^2] + (4*d*e*(a+c*x^2-Sqrt[a]*Sqrt[c]*x*Sqrt[1+(c*x^2)/a])*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/(x*Sqrt[a+c*x^2]) + ((e^2-d*f+(e*(e^2-3*d*f)))/Sqrt[e^2-4*d*f])*(-(Sqrt[c]*(-e+Sqrt[e^2-4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a+c*x^2]]) + Sqrt[4*a*f^2-2*c*(-e^2+2*d*f+e*Sqrt[e^2-4*d*f]])*ArcTanh[(2*a*f+c*(-e+Sqrt[e^2-4*d*f]))*(x)/(Sqrt[4*a*f^2-2*c*(-e^2+2*d*f+e*Sqrt[e^2-4*d*f]])*Sqrt[a+c*x^2])])/f + ((e^2-d*f-(e*(e^2-3*d*f)))/Sqrt[e^2-4*d*f])*(Sqrt[c]*(e+Sqrt[e^2-4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a+c*x^2]] + Sqrt[4*a*f^2+2*c*(e^2-2*d*f+e*Sqrt[e^2-4*d*f]])*ArcTanh[(2*a*f-c*(e+Sqrt[e^2-4*d*f]))*(x)/(Sqrt[4*a*f^2+2*c*(e^2-2*d*f+e*Sqrt[e^2-4*d*f]])*Sqrt[a+c*x^2])])/f + 4*(e^2-d*f)*(Sqrt[a+c*x^2])
```

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(x^3*(d + e*x + f*x^2)), x]
[Out] (-2*(e^2 - d*f - (e*(e^2 - 3*d*f)))/Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2] - 2*(e^2 - d*f + (e*(e^2 - 3*d*f)))/Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2] + (4*d*e*(a + c*x^2 - Sqrt[a]*Sqrt[c]*x*Sqrt[1 + (c*x^2)/a])*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/(x*Sqrt[a + c*x^2]) + ((e^2 - d*f + (e*(e^2 - 3*d*f)))/Sqrt[e^2 - 4*d*f])*(-(Sqrt[c]*(-e + Sqrt[e^2 - 4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]) + Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f]))*(x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2])])/f + ((e^2 - d*f - (e*(e^2 - 3*d*f)))/Sqrt[e^2 - 4*d*f])*(Sqrt[c]*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] + Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f]))*(x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2])])/f + 4*(e^2 - d*f)*(Sqrt[a + c*x^2])
```

2) - Sqrt[a]*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]) - (2*d^2*(a + c*x^2 + c*x^2*Sqrt[1 + (c*x^2)/a]*ArcTanh[Sqrt[1 + (c*x^2)/a]]))/(x^2*Sqrt[a + c*x^2]))/(4*d^3)

IntegrateAlgebraic [C] time = 0.99, size = 541, normalized size = 1.07

RootSum[...]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + c*x^2]/(x^3*(d + e*x + f*x^2)),x]
[Out] ((-d + 2*e*x)*Sqrt[a + c*x^2])/(2*d^2*x^2) + (2*Sqrt[a]*(-e^2 + d*f)*ArcTanh[(-Sqrt[c]*x) + Sqrt[a + c*x^2])/Sqrt[a]]/d^3 + (c*ArcTanh[(Sqrt[c]*x)/Sqrt[a] - Sqrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d) - RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (-a*c*d^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]) - a^2*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + a^2*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - 2*c^(3/2)*d^2*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 2*a*Sqrt[c]*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + 4*a*Sqrt[c]*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + c*d^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 + a*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - a*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]/d^3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^3/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^3/(f*x^2+e*x+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.02, size = 3993, normalized size = 7.88

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/x^3/(f*x^2+e*x+d),x)
[Out] 4*f^3/(e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2)))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2)))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)-4*f^2/(e+(-4*d*f+e^2)^(1/2))^3*c^(1/2)*ln((-1/2*c*(e+(-4*d*f+e^2)^(1/2))/f+c*(x+1/2*(e+(-4*d*f+e^2)^(1/2)))/f))/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2)))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2)))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))-4*f^2/(e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*c^(1/2)*ln((-1/2*c*(e+(-4*d*f+e^2)^(1/2))/f+c*(x+1/2*(e+(-4*d*f+e^2)^(1/2)))/f))
```

$$\begin{aligned}
& 4*d*f+e^2)^{(1/2)})/f)/c^{(1/2)}+((x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)+1/2*((-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}*e-4*f/(e+(-4*d*f+e^2)^{(1/2)})^3*2^{(1/2)}/(((-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}*\ln((((-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)+1/2*2^{(1/2)}*((-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}*(4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-4*c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)+2*((-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)})/(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)*c*e-8*f^3/(e+(-4*d*f+e^2)^{(1/2)})^3/(-4*d*f+e^2)^{(1/2)}*2^{(1/2)}/(((-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}*\ln((((-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)+1/2*2^{(1/2)}*((-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}*(4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-4*c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)+2*((-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)})/(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)*a+8*f^2/(e+(-4*d*f+e^2)^{(1/2)})^3/(-4*d*f+e^2)^{(1/2)}*2^{(1/2)}/(((-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}*\ln((((-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)+1/2*2^{(1/2)}*((-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}*(4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-4*c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)+2*((-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)})/(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)*c*d-4*f/(e+(-4*d*f+e^2)^{(1/2)})^3/(-4*d*f+e^2)^{(1/2)}*2^{(1/2)}/(((-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}*\ln((((-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)+1/2*2^{(1/2)}*((-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}*(4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-4*c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)+2*((-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)})/(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)*c*e^2+16*f^2*e/(-e+(-4*d*f+e^2)^{(1/2)})^2/(e+(-4*d*f+e^2)^{(1/2)})^2/a/x*(c*x^2+a)^{(3/2)}-16*f^2*e/(-e+(-4*d*f+e^2)^{(1/2)})^2/(e+(-4*d*f+e^2)^{(1/2)})^2*c/a*x*(c*x^2+a)^{(1/2)}-16*f^2*e/(-e+(-4*d*f+e^2)^{(1/2)})^2/(e+(-4*d*f+e^2)^{(1/2)})^2*c^{(1/2)}*\ln(c^{(1/2)}*x+(c*x^2+a)^{(1/2)}+2*f/(-e+(-4*d*f+e^2)^{(1/2)})/(e+(-4*d*f+e^2)^{(1/2)})/a/x^2*(c*x^2+a)^{(3/2)}+2*f/(-e+(-4*d*f+e^2)^{(1/2)})/(e+(-4*d*f+e^2)^{(1/2)})*c/a^{(1/2)}*\ln((2*a+2*a^{(1/2)}*(c*x^2+a)^{(1/2)})/x)-2*f/(-e+(-4*d*f+e^2)^{(1/2)})/(e+(-4*d*f+e^2)^{(1/2)})*c/a*(c*x^2+a)^{(1/2)}+4*f^3/(-e+(-4*d*f+e^2)^{(1/2)})^3/(-4*d*f+e^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-4*c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)+2*(-(-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}+4*f^2/(-e+(-4*d*f+e^2)^{(1/2)})^3*c^{(1/2)}*\ln((-1/2*c*(e+(-4*d*f+e^2)^{(1/2)})/f+c*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f))/c^{(1/2)}+((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)+1/2*(-(-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}*e+4*f/(-e+(-4*d*f+e^2)^{(1/2)})^3*2^{(1/2)}/((-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}*\ln((((-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)+1/2*2^{(1/2)}*((-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-4*c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)+2*(-(-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)})/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c*e-8*f^3/(-e+(-4*d*f+e^2)^{(1/2)})^3/(-4*d*f+e^2)^{(1/2)}*2^{(1/2)}/(((-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}*\ln((((-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)+1/2*2^{(1/2)}*((-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-4*c*(e+(-4*d*f+e^2)^{(1/2)})/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)+2*(-(-4*d*f+e^2)^{(1/2)})*c
\end{aligned}$$

$$e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}/f))*a+8*f^2/(-e+(-4*d*f+e^2)^{(1/2)})^3/(-4*d*f+e^2)^{(1/2)}*2^{(1/2)}/((-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}*\ln(((-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^{(1/2)}/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}/f)+1/2*2^{(1/2)}*((-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}/f))^2*c-4*c*(e-(-4*d*f+e^2)^{(1/2)}/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}/f)+2*(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)})/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}/f))*c*d-4*f/(-e+(-4*d*f+e^2)^{(1/2)})^3/(-4*d*f+e^2)^{(1/2)}*2^{(1/2)}/((-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}*\ln(((-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^{(1/2)}/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}/f)+1/2*2^{(1/2)}*((-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}/f))^2*c-4*c*(e-(-4*d*f+e^2)^{(1/2)}/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}/f)+2*(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)})/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}/f))*c*e^2-64*f^4/(-e+(-4*d*f+e^2)^{(1/2)})^3/(e+(-4*d*f+e^2)^{(1/2)})^3*a^{(1/2)}*\ln((2*a+2*a^{(1/2)}*(c*x^2+a)^{(1/2)})/x)*d+64*f^3/(-e+(-4*d*f+e^2)^{(1/2)})^3/(e+(-4*d*f+e^2)^{(1/2)})^3*a^{(1/2)}*\ln((2*a+2*a^{(1/2)}*(c*x^2+a)^{(1/2)})/x)*e^2+64*f^4/(-e+(-4*d*f+e^2)^{(1/2)})^3/(e+(-4*d*f+e^2)^{(1/2)})^3*(c*x^2+a)^{(1/2)}*d-64*f^3/(-e+(-4*d*f+e^2)^{(1/2)})^3/(e+(-4*d*f+e^2)^{(1/2)})^3*(c*x^2+a)^{(1/2)}*e^2$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + a}}{(fx^2 + ex + d)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(1/2)/x^3/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] integrate(sqrt(c*x^2 + a)/((f*x^2 + e*x + d)*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^2 + a}}{x^3 (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c*x^2)^(1/2)/(x^3*(d + e*x + f*x^2)),x)

[Out] int((a + c*x^2)^(1/2)/(x^3*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^2}}{x^3 (d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+a)**(1/2)/x**3/(f*x**2+e*x+d),x)

[Out] Integral(sqrt(a + c*x**2)/(x**3*(d + e*x + f*x**2)), x)

$$3.58 \quad \int \frac{x^2(a+cx^2)^{3/2}}{d+ex+fx^2} dx$$

Optimal. Leaf size=795

$$\frac{(4e-3fx)(cx^2+a)^{3/2}}{12f^2} - \frac{(8e(af^2+c(e^2-2df))-f(3af^2+4c(e^2-df))x)\sqrt{cx^2+a}}{8f^4} + \frac{(3a^2f^4+12ac(e^2-df))\sqrt{cx^2+a}}{8f^4}$$

Rubi [A] time = 4.26, antiderivative size = 795, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {1069, 1068, 1080, 217, 206, 1034, 725}

Antiderivative was successfully verified.

[In] Int[(x^2*(a + c*x^2)^(3/2))/(d + e*x + f*x^2), x]

[Out] -((8*e*(a*f^2 + c*(e^2 - 2*d*f)) - f*(3*a*f^2 + 4*c*(e^2 - d*f))*x)*Sqrt[a + c*x^2])/(8*f^4) - ((4*e - 3*f*x)*(a + c*x^2)^(3/2))/(12*f^2) + ((3*a^2*f^4 + 12*a*c*f^2*(e^2 - d*f) + 8*c^2*(e^4 - 3*d*e^2*f + d^2*f^2))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*Sqrt[c]*f^5) - ((a^2*f^4*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + 2*a*c*f^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2 - e^3*Sqrt[e^2 - 4*d*f]) + 2*d*e*f*Sqrt[e^2 - 4*d*f]) + c^2*(e^6 - 6*d*e^4*f + 9*d^2*e^2*f^2 - 2*d^3*f^3 - e^5*Sqrt[e^2 - 4*d*f] + 4*d*e^3*f*Sqrt[e^2 - 4*d*f] - 3*d^2*e*f^2*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f]))*x]/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]))/(Sqrt[2]*f^5*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((a^2*f^4*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + 2*a*c*f^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2 + e^3*Sqrt[e^2 - 4*d*f] - 2*d*e*f*Sqrt[e^2 - 4*d*f]) + c^2*(e^6 - 6*d*e^4*f + 9*d^2*e^2*f^2 - 2*d^3*f^3 + e^5*Sqrt[e^2 - 4*d*f] - 4*d*e^3*f*Sqrt[e^2 - 4*d*f] + 3*d^2*e*f^2*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f]))*x]/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]))/(Sqrt[2]*f^5*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 725

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 1034

Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -

$h*(b + q)/q$, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rule 1068

Int[((a_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x_)^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*p + 2*q + 3) + C*(-(c*e*(2*p + q + 2))) + 2*c*C*f*(p + q + 1)*x)*(a + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(-(a*e))*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3)))] + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(-4*a*c)))*x + (p*(c*e)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3)))]*x^2, x], x] /; FreeQ[{a, c, d, e, f, A, B, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]

Rule 1069

Int[((a_) + (c_)*(x_)^2)^(p_)*((A_) + (C_)*(x_)^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] :> Simp[((C*(-(c*e*(2*p + q + 2))) + 2*c*C*f*(p + q + 1)*x)*(a + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(-(a*e))*(C*(c*e)*(q + 1) - c*(C*e)*(2*p + 2*q + 3)) + (p + q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(-2*A*f)*(2*p + 2*q + 3)))] + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(-4*a*c)))*x + (p*(c*e)*(C*(c*e)*(q + 1) - c*(C*e)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))]*x^2, x], x] /; FreeQ[{a, c, d, e, f, A, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]

Rule 1080

Int[((A_) + (B_)*(x_) + (C_)*(x_)^2)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (f_)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^2 (a + cx^2)^{3/2}}{d + ex + fx^2} dx &= -\frac{(4e - 3fx)(a + cx^2)^{3/2}}{12f^2} - \frac{\int \frac{\sqrt{a+cx^2}(3acd f - 3ce(4cd - af)x - 3c(3af^2 + 4c(e^2 - df))x^2)}{d+ex+fx^2} dx}{12cf^2} \\
&= -\frac{(8e(a f^2 + c(e^2 - 2df)) - f(3af^2 + 4c(e^2 - df))x)\sqrt{a + cx^2}}{8f^4} - \frac{(4e - 3fx)(a + cx^2)^{3/2}}{12f^2} \\
&= -\frac{(8e(a f^2 + c(e^2 - 2df)) - f(3af^2 + 4c(e^2 - df))x)\sqrt{a + cx^2}}{8f^4} - \frac{(4e - 3fx)(a + cx^2)^{3/2}}{12f^2} \\
&= -\frac{(8e(a f^2 + c(e^2 - 2df)) - f(3af^2 + 4c(e^2 - df))x)\sqrt{a + cx^2}}{8f^4} - \frac{(4e - 3fx)(a + cx^2)^{3/2}}{12f^2} \\
&= -\frac{(8e(a f^2 + c(e^2 - 2df)) - f(3af^2 + 4c(e^2 - df))x)\sqrt{a + cx^2}}{8f^4} - \frac{(4e - 3fx)(a + cx^2)^{3/2}}{12f^2} \\
&= -\frac{(8e(a f^2 + c(e^2 - 2df)) - f(3af^2 + 4c(e^2 - df))x)\sqrt{a + cx^2}}{8f^4} - \frac{(4e - 3fx)(a + cx^2)^{3/2}}{12f^2}
\end{aligned}$$

Mathematica [A] time = 3.45, size = 793, normalized size = 1.00

Antiderivative was successfully verified.

[In] Integrate[(x^2*(a + c*x^2)^(3/2))/(d + e*x + f*x^2), x]

[Out]
$$\begin{aligned}
&(-4*(e + (e^2 - 2*d*f)/\text{Sqrt}[e^2 - 4*d*f])*(a + c*x^2)^{3/2} - 4*(e + (-e^2 + 2*d*f)/\text{Sqrt}[e^2 - 4*d*f])*(a + c*x^2)^{3/2} + 3*f*\text{Sqrt}[a + c*x^2]*(5*a*x \\
&+ 2*c*x^3 + (3*a^{3/2}*\text{ArcSinh}[(\text{Sqrt}[c]*x)/\text{Sqrt}[a]])/(\text{Sqrt}[c]*\text{Sqrt}[1 + (c*x^2)/a])) - (3*(e + (-e^2 + 2*d*f)/\text{Sqrt}[e^2 - 4*d*f])*((2*\text{Sqrt}[c]*(-e + \text{Sqrt}[e^2 - 4*d*f])*\text{Sqrt}[a + c*x^2]*(\text{Sqrt}[c]*x*\text{Sqrt}[1 + (c*x^2)/a] + \text{Sqrt}[a]*\text{ArcSinh}[(\text{Sqrt}[c]*x)/\text{Sqrt}[a]]))/\text{Sqrt}[1 + (c*x^2)/a] + (2*(2*a*f^2 + c*(e^2 - 2*d*f - e*\text{Sqrt}[e^2 - 4*d*f]))*(2*f*\text{Sqrt}[a + c*x^2] + \text{Sqrt}[c]*(-e + \text{Sqrt}[e^2 - 4*d*f])*\text{ArcTanh}[(\text{Sqrt}[c]*x)/\text{Sqrt}[a + c*x^2]] - \text{Sqrt}[2*c*e^2 - 4*c*d*f + 4*a*f^2 - 2*c*e*\text{Sqrt}[e^2 - 4*d*f]]*\text{ArcTanh}[(2*a*f + c*(-e + \text{Sqrt}[e^2 - 4*d*f])]*x)/(\text{Sqrt}[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]])*\text{Sqrt}[a + c*x^2])))/f^2))/ (2*f) + (3*(e + (e^2 - 2*d*f)/\text{Sqrt}[e^2 - 4*d*f])*((2*\text{Sqrt}[c]*(e + \text{Sqrt}[e^2 - 4*d*f])*\text{Sqrt}[a + c*x^2]*(\text{Sqrt}[c]*x*\text{Sqrt}[1 + (c*x^2)/a] + \text{Sqrt}[a]*\text{ArcSinh}[(\text{Sqrt}[c]*x)/\text{Sqrt}[a]]))/\text{Sqrt}[1 + (c*x^2)/a] + (2*(2*a*f^2 + c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]))*(-2*f*\text{Sqrt}[a + c*x^2] + \text{Sqrt}[c]*(e + \text{Sqrt}[e^2 - 4*d*f])*\text{ArcTanh}[(\text{Sqrt}[c]*x)/\text{Sqrt}[a + c*x^2]] + \text{Sqrt}[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]])*\text{ArcTanh}[(2*a*f - c*(e + \text{Sqrt}[e^2 - 4*d*f])]*x)/(\text{Sqrt}[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]])*\text{Sqrt}[a + c*x^2])))/f^2))/ (2*f)) / (24*f^2)
\end{aligned}$$

IntegrateAlgebraic [C] time = 1.22, size = 960, normalized size = 1.21

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
[Out] (Sqrt[a + c*x^2]*(-24*c*e^3 + 48*c*d*e*f - 32*a*e*f^2 + 12*c*e^2*f*x - 12*c*d*f^2*x + 15*a*f^3*x - 8*c*e*f^2*x^2 + 6*c*f^3*x^3))/(24*f^4) + ((-8*c^2*e^4 + 24*c^2*d*e^2*f - 8*c^2*d^2*f^2 - 12*a*c*e^2*f^2 + 12*a*c*d*f^3 - 3*a^2*f^4)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(8*Sqrt[c]*f^5) + RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (a*c^2*e^5*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - 4*a*c^2*d*e^3*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 3*a*c^2*d^2*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*a^2*c*e^3*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - 4*a^2*c*d*e*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + a^3*e*f^4*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*c^(5/2)*d*e^4*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 6*c^(5/2)*d^2*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + 2*c^(5/2)*d^3*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + 4*a*c^(3/2)*d*e^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 4*a*c^(3/2)*d^2*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + 2*a^2*Sqrt[c]*d*f^4*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - c^2*e^5*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 + 4*c^2*d*e^3*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - 3*c^2*d^2*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - 2*a*c*e^3*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 + 4*a*c*d*e*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - a^2*e*f^4*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]/f^5
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage2

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage2
```

maple [B] time = 0.03, size = 19148, normalized size = 24.09

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* h
```

elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details) Is $4*d*f-e^2$ positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 (cx^2 + a)^{3/2}}{fx^2 + ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(a + c*x^2)^(3/2))/(d + e*x + f*x^2), x)

[Out] int((x^2*(a + c*x^2)^(3/2))/(d + e*x + f*x^2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(c*x**2+a)**(3/2)/(f*x**2+e*x+d), x)

[Out] Timed out

$$3.59 \quad \int \frac{x(a+cx^2)^{3/2}}{d+ex+fx^2} dx$$

Optimal. Leaf size=553

$$\frac{(2cdef(2af^2 + c(e^2 - 2df)) - (e - \sqrt{e^2 - 4df})(a^2f^4 + 2acf^2(e^2 - df) + c^2(d^2f^2 - 3de^2f + e^4))) \tanh^{-1}\left(\frac{\sqrt{2}f^4\sqrt{e^2 - 4df}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}{\sqrt{2}f^4\sqrt{e^2 - 4df}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2}f^4\sqrt{e^2 - 4df}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}$$

Rubi [A] time = 2.43, antiderivative size = 553, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1020, 1068, 1080, 217, 206, 1034, 725}

$$\frac{(2cde(f^2 + c(e^2 - 2df)) - (e - \sqrt{e^2 - 4df})(a^2f^4 + 2acf^2(e^2 - df) + c^2(d^2f^2 - 3de^2f + e^4))) \tanh^{-1}\left(\frac{\sqrt{2}f^4\sqrt{e^2 - 4df}}{\sqrt{2}f^4\sqrt{e^2 - 4df}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2}f^4\sqrt{e^2 - 4df}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}$$

Antiderivative was successfully verified.

[In] Int[(x*(a + c*x^2)^(3/2))/(d + e*x + f*x^2), x]

[Out] ((2*(a*f^2 + c*(e^2 - d*f)) - c*e*f*x)*Sqrt[a + c*x^2])/(2*f^3) + (a + c*x^2)^(3/2)/(3*f) - (Sqrt[c]*e*(3*a*f^2 + 2*c*(e^2 - 2*d*f))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*f^4) - ((2*c*d*e*f*(2*a*f^2 + c*(e^2 - 2*d*f)) - (e - Sqrt[e^2 - 4*d*f])*(a^2*f^4 + 2*a*c*f^2*(e^2 - d*f) + c^2*(e^4 - 3*d*e^2*f + d^2*f^2)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f) - e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[2]*f^4*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f) - e*Sqrt[e^2 - 4*d*f]]) + ((2*c*d*e*f*(2*a*f^2 + c*(e^2 - 2*d*f)) - (e + Sqrt[e^2 - 4*d*f])*(a^2*f^4 + 2*a*c*f^2*(e^2 - d*f) + c^2*(e^4 - 3*d*e^2*f + d^2*f^2)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f) + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[2]*f^4*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f) + e*Sqrt[e^2 - 4*d*f]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 725

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 1020

Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x] /; FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]

Rule 1034

Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rule 1068

Int[((a_.) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((B*c*f*(2*p + 2*q + 3) + C*(-c*e*(2*p + q + 2))) + 2*c*C*f*(p + q + 1)*x*(a + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(-a*e)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))] + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(-4*a*c)))*x + (p*(c*e)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3)))]*x^2, x], x] /; FreeQ[{a, c, d, e, f, A, B, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]

Rule 1080

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{x(a+cx^2)^{3/2}}{d+ex+fx^2} dx &= \frac{(a+cx^2)^{3/2}}{3f} + \frac{\int \frac{\sqrt{a+cx^2}(-3(cd-af)x-3cex^2)}{d+ex+fx^2} dx}{3f} \\ &= \frac{(2(af^2+c(e^2-df))-cef x)\sqrt{a+cx^2}}{2f^3} + \frac{(a+cx^2)^{3/2}}{3f} - \frac{\int \frac{-3ac^2def+3c(ace^2f+2(cd-af)(ce^2-df))\sqrt{a+cx^2}}{\sqrt{a+cx^2}} dx}{3f} \\ &= \frac{(2(af^2+c(e^2-df))-cef x)\sqrt{a+cx^2}}{2f^3} + \frac{(a+cx^2)^{3/2}}{3f} - \frac{\int \frac{-3ac^2def^2-3c^2de(3af^2+2c(e^2-2df))\sqrt{a+cx^2}}{\sqrt{a+cx^2}} dx}{3f} \\ &= \frac{(2(af^2+c(e^2-df))-cef x)\sqrt{a+cx^2}}{2f^3} + \frac{(a+cx^2)^{3/2}}{3f} - \frac{(ce(3af^2+2c(e^2-2df)))\sqrt{a+cx^2}}{2f} \\ &= \frac{(2(af^2+c(e^2-df))-cef x)\sqrt{a+cx^2}}{2f^3} + \frac{(a+cx^2)^{3/2}}{3f} - \frac{\sqrt{c}e(3af^2+2c(e^2-2df))\sqrt{a+cx^2}}{2f^4} \\ &= \frac{(2(af^2+c(e^2-df))-cef x)\sqrt{a+cx^2}}{2f^3} + \frac{(a+cx^2)^{3/2}}{3f} - \frac{\sqrt{c}e(3af^2+2c(e^2-2df))\sqrt{a+cx^2}}{2f^4} \end{aligned}$$

Mathematica [A] time = 2.01, size = 755, normalized size = 1.37

Antiderivative was successfully verified.

[In] Integrate[(x*(a + c*x^2)^(3/2))/(d + e*x + f*x^2), x]

[Out] $(8*f^3*(-e + \sqrt{e^2 - 4*d*f})*(a + c*x^2)^{(5/2)}*\sqrt{1 + (c*x^2)/a} + 8*f^3*(e + \sqrt{e^2 - 4*d*f})*(a + c*x^2)^{(5/2)}*\sqrt{1 + (c*x^2)/a} + 3*(e - \sqrt{e^2 - 4*d*f})*(2*\sqrt{c}*f^2*(e - \sqrt{e^2 - 4*d*f})*\sqrt{a + c*x^2}*(a*\sqrt{c}*x*(1 + (c*x^2)/a)^{(3/2)} + \sqrt{a}*(a + c*x^2)*\text{ArcSinh}[(\sqrt{c}*x)/\sqrt{a}])) - a*(4*a*f^2 + c*(e - \sqrt{e^2 - 4*d*f})^2)*(1 + (c*x^2)/a)^{(3/2)}*(2*f*\sqrt{a + c*x^2} + \sqrt{c}*(-e + \sqrt{e^2 - 4*d*f})*\text{ArcTanh}[(\sqrt{c}*x)/\sqrt{a + c*x^2}]) - \sqrt{2*c*e^2 - 4*c*d*f + 4*a*f^2 - 2*c*e*\sqrt{e^2 - 4*d*f}}*\text{ArcTanh}[(2*a*f + c*(-e + \sqrt{e^2 - 4*d*f})*x)/(\sqrt{4*a*f^2 - 2*c*(e^2 + 2*d*f + e*\sqrt{e^2 - 4*d*f})}*\sqrt{a + c*x^2})]) - 3*(e + \sqrt{e^2 - 4*d*f})*(2*\sqrt{c}*f^2*(e + \sqrt{e^2 - 4*d*f})*\sqrt{a + c*x^2}*(a*\sqrt{c}*x*(1 + (c*x^2)/a)^{(3/2)} + \sqrt{a}*(a + c*x^2)*\text{ArcSinh}[(\sqrt{c}*x)/\sqrt{a}]) - a*(4*a*f^2 + c*(e + \sqrt{e^2 - 4*d*f})^2)*(1 + (c*x^2)/a)^{(3/2)}*(2*f*\sqrt{a + c*x^2} - \sqrt{c}*(e + \sqrt{e^2 - 4*d*f})*\text{ArcTanh}[(\sqrt{c}*x)/\sqrt{a + c*x^2}]) - \sqrt{4*a*f^2 + 2*c*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f})}*\text{ArcTanh}[(2*a*f - c*(e + \sqrt{e^2 - 4*d*f})*x)/(\sqrt{4*a*f^2 + 2*c*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*d*f})}*\sqrt{a + c*x^2})]))/(48*a*f^4*\sqrt{e^2 - 4*d*f}*(1 + (c*x^2)/a)^{(3/2)})$

IntegrateAlgebraic [C] time = 0.89, size = 771, normalized size = 1.39

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x*(a + c*x^2)^(3/2))/(d + e*x + f*x^2), x]

[Out] $(\sqrt{a + c*x^2}*(6*c*e^2 - 6*c*d*f + 8*a*f^2 - 3*c*e*f*x + 2*c*f^2*x^2))/(6*f^3 + ((2*c^{(3/2)}*e^3 - 4*c^{(3/2)}*d*e*f + 3*a*\sqrt{c}*e*f^2)*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + c*x^2}])/(2*f^4) - \text{RootSum}[a^2*f + 2*a*\sqrt{c}*e*\#1 + 4*c*d*\#1^2 - 2*a*f*\#1^2 - 2*\sqrt{c}*e*\#1^3 + f*\#1^4 \& , (a*c^2*e^4*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + c*x^2} - \#1] - 3*a*c^2*d*e^2*f*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + c*x^2} - \#1] + a*c^2*d^2*f^2*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + c*x^2} - \#1] + 2*a^2*c*e^2*f^2*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + c*x^2} - \#1] - 2*a^2*c*d*f^3*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + c*x^2} - \#1] + a^3*f^4*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + c*x^2} - \#1] + 2*c^{(5/2)}*d*e^3*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + c*x^2} - \#1]*\#1 - 4*c^{(5/2)}*d^2*e*f*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + c*x^2} - \#1]*\#1 + 4*a*c^{(3/2)}*d*e*f^2*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + c*x^2} - \#1]*\#1 - c^2*e^4*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + c*x^2} - \#1]*\#1^2 + 3*c^2*d*e^2*f*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + c*x^2} - \#1]*\#1^2 - c^2*d^2*f^2*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + c*x^2} - \#1]*\#1^2 - 2*a*c*e^2*f^2*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + c*x^2} - \#1]*\#1^2 + 2*a*c*d*f^3*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + c*x^2} - \#1]*\#1^2 - a^2*f^4*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + c*x^2} - \#1]*\#1^2)/(a*\sqrt{c}*e + 4*c*d*\#1 - 2*a*f*\#1 - 3*\sqrt{c}*e*\#1^2 + 2*f*\#1^3) \&]/f^4$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^2+a)^(3/2)/(f*x^2+e*x+d), x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage2

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] sage2

maple [B] time = 0.02, size = 14709, normalized size = 26.60

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)

[Out] result too large to display

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see 'assume?' for more details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x(c x^2 + a)^{3/2}}{f x^2 + e x + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x)

[Out] int((x*(a + c*x^2)^(3/2))/(d + e*x + f*x^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + cx^2)^{3/2}}{d + ex + fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)

[Out] Integral(x*(a + c*x**2)**(3/2)/(d + e*x + f*x**2), x)

$$3.60 \quad \int \frac{(a+cx^2)^{3/2}}{d+ex+fx^2} dx$$

Optimal. Leaf size=484

$$\frac{(ce(e - \sqrt{e^2 - 4df})(2af^2 + c(e^2 - 2df)) - 2f(-a^2f^3 + 2acdf^2 + c^2d(e^2 - df))) \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2 + c(e^2 - 2df) - 2df + e^2}}\right)}{\sqrt{2}f^3\sqrt{e^2 - 4df}\sqrt{2af^2 + c(e^2 - 2df) - 2df + e^2}}$$

Rubi [A] time = 4.24, antiderivative size = 482, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {979, 1080, 217, 206, 1034, 725}

$$\frac{(-2d^2f^4 - ce(e - \sqrt{e^2 - 4df})(2af^2 + c(e^2 - 2df)) + 4acdf^3 + 2c^2df(e^2 - df)) \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2 + c(e^2 - 2df) - 2df + e^2}}\right) - (-2d^2f^4 - ce(\sqrt{e^2 - 4df} + c)(2af^2 + c(e^2 - 2df)) + 4acdf^3 + 2c^2df(e^2 - df)) \tanh^{-1}\left(\frac{2af - c(\sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2 + c(e^2 - 2df) - 2df + e^2}}\right) + \sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{a+cx^2}}\right)(3af^2 + 2c(e^2 - df))}{\sqrt{2}f^3\sqrt{e^2 - 4df}\sqrt{2af^2 + c(e^2 - 2df) - 2df + e^2} - \sqrt{2}f^3\sqrt{e^2 - 4df}\sqrt{2af^2 + c(e^2 - 2df) - 2df + e^2} + \frac{c\sqrt{a+cx^2}(2e - f)}{2f^3}}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^2)^(3/2)/(d + e*x + f*x^2), x]

[Out] -(c*(2*e - f*x)*Sqrt[a + c*x^2])/(2*f^2) + (Sqrt[c]*(3*a*f^2 + 2*c*(e^2 - d*f))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*f^3) + ((4*a*c*d*f^3 - 2*a^2*f^4 + 2*c^2*d*f*(e^2 - d*f) - c*e*(e - Sqrt[e^2 - 4*d*f])*(2*a*f^2 + c*(e^2 - 2*d*f)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]]) - ((4*a*c*d*f^3 - 2*a^2*f^4 + 2*c^2*d*f*(e^2 - d*f) - c*e*(e + Sqrt[e^2 - 4*d*f])*(2*a*f^2 + c*(e^2 - 2*d*f)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 725

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 979

Int[((a_.) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := -Simp[(c*(e*(2*p + q) - 2*f*(p + q)*x)*(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^(q + 1))/(2*f^2*(p + q)*(2*p + 2*q + 1)), x] - Dist[1/(2*f^2*(p + q)*(2*p + 2*q + 1)), Int[(a + c*x^2)^(p - 2)*(d + e*x + f*x^2)^q*Simp[-(a*c*e^2*(1 - p)*(2*p + q) + a*(p + q)*(-2*a*f^2*(2*p + 2*q + 1) + c*(2*d*f - e^2*(2*p + q)) + (2*(c*d - a*f)*(c*e)*(1 - p)*(2*p + q) + 4*a*c*e*f*(1 - p)*(p + q))*x + (p*c^2*e^2*(1 - p) - c*(p + q)*(2*a*f^2*(4*p + 2*q - 1) + c*(2*d*f*(1 - 2*p) + e^2*(3*p + q - 1)))]*x^2, x], x] /; FreeQ[{a, c, d, e, f, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 1] && NeQ[p + q, 0] && N

eQ[2*p + 2*q + 1, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]

Rule 1034

Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rule 1080

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a + cx^2)^{3/2}}{d + ex + fx^2} dx &= -\frac{c(2e - fx)\sqrt{a + cx^2}}{2f^2} - \frac{\int \frac{af(cd - 2af) - ce(2cd - af)x - c(3af^2 + 2c(e^2 - df))x^2}{\sqrt{a + cx^2}(d + ex + fx^2)} dx}{2f^2} \\ &= -\frac{c(2e - fx)\sqrt{a + cx^2}}{2f^2} - \frac{\int \frac{af^2(cd - 2af) + cd(3af^2 + 2c(e^2 - df)) + (-cef(2cd - af) + ce(3af^2 + 2c(e^2 - df)))x}{\sqrt{a + cx^2}(d + ex + fx^2)} dx}{2f^3} \\ &= -\frac{c(2e - fx)\sqrt{a + cx^2}}{2f^2} + \frac{(c(3af^2 + 2c(e^2 - df))) \text{Subst}\left(\int \frac{1}{1 - cx^2} dx, x, \frac{x}{\sqrt{a + cx^2}}\right)}{2f^3} - \frac{(2f(a + cx^2))^{3/2}}{2f^3} \\ &= -\frac{c(2e - fx)\sqrt{a + cx^2}}{2f^2} + \frac{\sqrt{c}(3af^2 + 2c(e^2 - df)) \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a + cx^2}}\right)}{2f^3} + \frac{(2f(a + cx^2))^{3/2}}{2f^3} \\ &= -\frac{c(2e - fx)\sqrt{a + cx^2}}{2f^2} + \frac{\sqrt{c}(3af^2 + 2c(e^2 - df)) \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a + cx^2}}\right)}{2f^3} - \frac{(ce(e - \sqrt{e^2 - 4df}))^{3/2}}{2f^3} \end{aligned}$$

Mathematica [A] time = 1.07, size = 603, normalized size = 1.25

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^2)^(3/2)/(d + e*x + f*x^2), x]
 [Out] ((2*Sqrt[c]*(-e + Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/a] + Sqrt[a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]]))/Sqrt[1 + (c*x^2)/a] + (2*Sqrt[c]*(e + Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/a] + Sqrt[a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]]))/Sqrt[1 + (c*x^2)/a] + (2*(2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))*(2*f*Sqrt[a + c*x^2] + Sqrt[c]*(-e + Sqrt[e^2 - 4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - Sqrt[2*c*e^2 - 4*c*d*f + 4*a*f^2 - 2*c*e*Sqrt[e^2 - 4*d*f]]*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]])])

f]])*Sqrt[a + c*x^2])))/f^2 + (2*(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*(-2*f*Sqrt[a + c*x^2] + Sqrt[c]*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] + Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))]*Sqrt[a + c*x^2])))/f^2)/(8*f*Sqrt[e^2 - 4*d*f])

IntegrateAlgebraic [C] time = 0.70, size = 557, normalized size = 1.15

RootSum[1/2*(c*(2*e - f*x)*Sqrt[a + c*x^2])/f^2 + ((-2*c^(3/2)*e^2 + 2*c^(3/2)*d*f - 3*a*Sqrt[c]*f^2)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(2*f^3) + RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (a*c^2*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - 2*a*c^2*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*a^2*c*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*c^(5/2)*d*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 2*c^(5/2)*d^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + 4*a*c^(3/2)*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 2*a^2*Sqrt[c]*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - c^2*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 + 2*c^2*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - 2*a*c*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) &]/f^3

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + c*x^2)^(3/2)/(d + e*x + f*x^2),x]
[Out] -1/2*(c*(2*e - f*x)*Sqrt[a + c*x^2])/f^2 + ((-2*c^(3/2)*e^2 + 2*c^(3/2)*d*f - 3*a*Sqrt[c]*f^2)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(2*f^3) + RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (a*c^2*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - 2*a*c^2*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*a^2*c*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*c^(5/2)*d*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 2*c^(5/2)*d^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + 4*a*c^(3/2)*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 2*a^2*Sqrt[c]*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - c^2*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 + 2*c^2*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - 2*a*c*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]/f^3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage2

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
[Out] sage2
```

maple [B] time = 0.01, size = 8954, normalized size = 18.50

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^2 + a)^{3/2}}{fx^2 + ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c*x^2)^(3/2)/(d + e*x + f*x^2),x)

[Out] int((a + c*x^2)^(3/2)/(d + e*x + f*x^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + cx^2)^{3/2}}{d + ex + fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)

[Out] Integral((a + c*x**2)**(3/2)/(d + e*x + f*x**2), x)

$$3.61 \quad \int \frac{(a+cx^2)^{3/2}}{x(d+ex+fx^2)} dx$$

Optimal. Leaf size=496

$$\frac{a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right) \left(2ef(c^2d^2 - a^2f^2) - (e - \sqrt{e^2 - 4df})(c^2de^2 - f(cd - af)^2)\right) \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{d \sqrt{2}df^2\sqrt{e^2 - 4df}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}$$

Rubi [A] time = 2.57, antiderivative size = 496, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {6728, 266, 50, 63, 208, 1020, 1080, 217, 206, 1034, 725}

$$\frac{(2ef(c^2d^2 - a^2f^2) - (e - \sqrt{e^2 - 4df})(c^2de^2 - f(cd - af)^2)) \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}\right) + (2ef(c^2d^2 - a^2f^2) - (\sqrt{e^2 - 4df} + e)(c^2de^2 - f(cd - af)^2)) \tanh^{-1}\left(\frac{2af - cx(\sqrt{e^2 - 4df} + e)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}\right) - \frac{a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d} - \frac{c^{3/2} \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{f} + \frac{\sqrt{a+cx^2}(cd - af)}{df} + \frac{e\sqrt{a+cx^2}}{d}}{\sqrt{2}df^2\sqrt{e^2 - 4df}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^2)^(3/2)/(x*(d + e*x + f*x^2)), x]

[Out] (a*Sqrt[a + c*x^2])/d + ((c*d - a*f)*Sqrt[a + c*x^2])/(d*f) - (c^(3/2)*e*ArcTanH[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/f^2 - ((2*e*f*(c^2*d^2 - a^2*f^2) - (c^2*d*e^2 - f*(c*d - a*f)^2)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanH[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[2]*d*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]]) + ((2*e*f*(c^2*d^2 - a^2*f^2) - (c^2*d*e^2 - f*(c*d - a*f)^2)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanH[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[2]*d*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]]) - (a^(3/2)*ArcTanH[Sqrt[a + c*x^2]/Sqrt[a]])/d

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanH[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanH[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

$\text{Int}[1/\text{Sqrt}[(a_) + (b_.)*(x_)^2], x_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] \text{ /; FreeQ}[\{a, b\}, x] \ \&\& \ !\text{GtQ}[a, 0]$

Rule 266

$\text{Int}[(x_)^{(m_.)}*((a_) + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p}, x], x, x^n], x] \text{ /; FreeQ}[\{a, b, m, n, p\}, x] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 725

$\text{Int}[1/(((d_) + (e_.)*(x_))*\text{Sqrt}[(a_) + (c_.)*(x_)^2]), x_Symbol] \rightarrow -\text{Subst}[\text{Int}[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/\text{Sqrt}[a + c*x^2]] \text{ /; FreeQ}[\{a, c, d, e\}, x]$

Rule 1020

$\text{Int}[(g_.) + (h_.)*(x_)]*((a_) + (c_.)*(x_)^2)^{(p_.)}*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^{(q_.)}, x_Symbol] \rightarrow \text{Simp}[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^{q+1})/(2*f*(p + q + 1)), x] + \text{Dist}[1/(2*f*(p + q + 1)), \text{Int}[(a + c*x^2)^{(p-1)}*(d + e*x + f*x^2)^q*\text{Simp}[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x] \text{ /; FreeQ}[\{a, c, d, e, f, g, h, q\}, x] \ \&\& \ \text{NeQ}[e^2 - 4*d*f, 0] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{NeQ}[p + q + 1, 0]$

Rule 1034

$\text{Int}[(g_.) + (h_.)*(x_)]/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*\text{Sqrt}[(d_) + (f_.)*(x_)^2]), x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(2*c*g - h*(b - q))/q, \text{Int}[1/((b - q + 2*c*x)*\text{Sqrt}[d + f*x^2]), x], x] - \text{Dist}[(2*c*g - h*(b + q))/q, \text{Int}[1/((b + q + 2*c*x)*\text{Sqrt}[d + f*x^2]), x], x]] \text{ /; FreeQ}[\{a, b, c, d, f, g, h\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{PosQ}[b^2 - 4*a*c]$

Rule 1080

$\text{Int}[(A_.) + (B_.)*(x_) + (C_.)*(x_)^2]/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*\text{Sqrt}[(d_.) + (f_.)*(x_)^2]), x_Symbol] \rightarrow \text{Dist}[C/c, \text{Int}[1/\text{Sqrt}[d + f*x^2], x], x] + \text{Dist}[1/c, \text{Int}[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*\text{Sqrt}[d + f*x^2]), x], x] \text{ /; FreeQ}[\{a, b, c, d, f, A, B, C\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 6728

$\text{Int}[(u_)/((a_.) + (b_.)*(x_)^{(n_.)} + (c_.)*(x_)^{(n2_.)}), x_Symbol] \rightarrow \text{With}[\{v = \text{RationalFunctionExpand}[u/(a + b*x^n + c*x^{2*n}), x]\}, \text{Int}[v, x] \text{ /; SumQ}[v]] \text{ /; FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{EqQ}[n2, 2*n] \ \&\& \ \text{IGtQ}[n, 0]$

Rubi steps

$$\begin{aligned}
 \int \frac{(a + cx^2)^{3/2}}{x(d + ex + fx^2)} dx &= \int \left(\frac{(a + cx^2)^{3/2}}{dx} + \frac{(-e - fx)(a + cx^2)^{3/2}}{d(d + ex + fx^2)} \right) dx \\
 &= \frac{\int \frac{(a+cx^2)^{3/2}}{x} dx}{d} + \frac{\int \frac{(-e-fx)(a+cx^2)^{3/2}}{d+ex+fx^2} dx}{d} \\
 &= -\frac{(a + cx^2)^{3/2}}{3d} + \frac{\text{Subst}\left(\int \frac{(a+cx)^{3/2}}{x} dx, x, x^2\right)}{2d} + \frac{\int \frac{(-3aef+3f(cd-af)x)\sqrt{a+cx^2}}{d+ex+fx^2} dx}{3df} \\
 &= \frac{(cd - af)\sqrt{a + cx^2}}{df} + \frac{a \text{Subst}\left(\int \frac{\sqrt{a+cx}}{x} dx, x, x^2\right)}{2d} + \frac{\int \frac{-3a^2ef^2-3f(cd-af)^2x-3c^2defx^2}{\sqrt{a+cx^2}(d+ex+fx^2)} dx}{3df^2} \\
 &= \frac{a\sqrt{a + cx^2}}{d} + \frac{(cd - af)\sqrt{a + cx^2}}{df} + \frac{a^2 \text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}} dx, x, x^2\right)}{2d} + \frac{\int \frac{3c^2d^2ef-3a^2ef^3+(3c^2d^2ef-3a^2ef^3+3c^2d^2ef-3a^2ef^3)}{\sqrt{a+cx^2}(d+ex+fx^2)} dx}{3d} \\
 &= \frac{a\sqrt{a + cx^2}}{d} + \frac{(cd - af)\sqrt{a + cx^2}}{df} + \frac{a^2 \text{Subst}\left(\int \frac{1}{-\frac{a}{c} + \frac{x^2}{c}} dx, x, \sqrt{a + cx^2}\right)}{cd} - \frac{(c^2e) \text{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}} dx, x, \sqrt{a+cx^2}\right)}{d} \\
 &= \frac{a\sqrt{a + cx^2}}{d} + \frac{(cd - af)\sqrt{a + cx^2}}{df} - \frac{c^{3/2}e \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{f^2} - \frac{a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d} - \frac{(2ef(c^2d^2 - a^2f^2) - (c^2de - a^2ef^2))\sqrt{a+cx^2}}{d^2} \\
 &= \frac{a\sqrt{a + cx^2}}{d} + \frac{(cd - af)\sqrt{a + cx^2}}{df} - \frac{c^{3/2}e \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{f^2} - \frac{(2ef(c^2d^2 - a^2f^2) - (c^2de - a^2ef^2))\sqrt{a+cx^2}}{d^2}
 \end{aligned}$$

Mathematica [A] time = 1.58, size = 746, normalized size = 1.50

$$\frac{a^2 \sqrt{a+cx^2}}{d} + \frac{(cd-af)\sqrt{a+cx^2}}{df} - \frac{c^{3/2}e \operatorname{tanh}^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{f^2} - \frac{a^{3/2} \operatorname{tanh}^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d} - \frac{(2ef(c^2d^2 - a^2f^2) - (c^2de - a^2ef^2))\sqrt{a+cx^2}}{d^2}$$

Antiderivative was successfully verified.

```

[In] Integrate[(a + c*x^2)^(3/2)/(x*(d + e*x + f*x^2)), x]
[Out] (c*Sqrt[a + c*x^2])/f - (c^(3/2)*e*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/f^2 - ((c*d*(-e + Sqrt[e^2 - 4*d*f]) - a*f*(e + Sqrt[e^2 - 4*d*f]))*Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])])*Sqrt[a + c*x^2]])/(4*d*f^2*Sqrt[e^2 - 4*d*f]) - (c*Sqrt[a*f^2 + (c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))/2]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])*Sqrt[a + c*x^2]])/(2*f^2) + (a*Sqrt[a*f^2 + (c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))/2]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])*Sqrt[a + c*x^2]])/(2*d*f) - (c*e*Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])*Sqrt[a + c*x^2]])/(4*f^2*Sqrt[e^2 - 4*d*f]) - (a*e*Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])*Sqrt[a + c*x^2]])/(4*d*f*Sqrt[e^2 - 4*d*f]) - (a^(3/2)*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d

```

IntegrateAlgebraic [C] time = 0.68, size = 559, normalized size = 1.13

$$\frac{a^2 \sqrt{a+cx^2}}{d} + \frac{(cd-af)\sqrt{a+cx^2}}{df} - \frac{c^{3/2}e \operatorname{tanh}^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{f^2} - \frac{a^{3/2} \operatorname{tanh}^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d} - \frac{(2ef(c^2d^2 - a^2f^2) - (c^2de - a^2ef^2))\sqrt{a+cx^2}}{d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + c*x^2)^(3/2)/(x*(d + e*x + f*x^2)),x]
[Out] (c*Sqrt[a + c*x^2])/f + (2*a^(3/2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a] - Sqrt[a + c
*x^2]/Sqrt[a]])/d + (c^(3/2)*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/f^2 + R
ootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^
3 + f*#1^4 & , (-a*c^2*d*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]) + a
*c^2*d^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - 2*a^2*c*d*f^2*Log[-(S
qrt[c]*x) + Sqrt[a + c*x^2] - #1] + a^3*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x
^2] - #1] - 2*c^(5/2)*d^2*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + 2
*a^2*Sqrt[c]*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + c^2*d*e^2*
Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - c^2*d^2*f*Log[-(Sqrt[c]*x)
+ Sqrt[a + c*x^2] - #1]*#1^2 + 2*a*c*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^
2] - #1]*#1^2 - a^2*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*S
qrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]/(d*f^2)
fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x/(f*x^2+e*x+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
maple [B] time = 0.02, size = 9728, normalized size = 19.61
```

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)/x/(f*x^2+e*x+d),x)
[Out] result too large to display
maxima [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{(cx^2 + a)^{\frac{3}{2}}}{(fx^2 + ex + d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate((c*x^2 + a)^(3/2)/((f*x^2 + e*x + d)*x), x)
mupad [F] time = 0.00, size = -1, normalized size = -0.00
```

$$\int \frac{(cx^2 + a)^{\frac{3}{2}}}{x(fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + c*x^2)^(3/2)/(x*(d + e*x + f*x^2)), x)`

[Out] `int((a + c*x^2)^(3/2)/(x*(d + e*x + f*x^2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + cx^2)^{\frac{3}{2}}}{x(d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+a)**(3/2)/x/(f*x**2+e*x+d), x)`

[Out] `Integral((a + c*x**2)**(3/2)/(x*(d + e*x + f*x**2)), x)`

$$3.62 \quad \int \frac{(a+cx^2)^{3/2}}{x^2(d+ex+fx^2)} dx$$

Optimal. Leaf size=604

$$\frac{a^{3/2}e \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right) \left(a^2 f^2 (e\sqrt{e^2 - 4df} - 2df + e^2) + 4acd^2 f^2 + c^2 d^2 (-e\sqrt{e^2 - 4df} - 2df + e^2)\right) \tanh^{-1}\left(\frac{\sqrt{2} d^2 f \sqrt{e^2 - 4df} \sqrt{2af^2 + c} (-e\sqrt{e^2 - 4df} - 2df + e^2)}{\sqrt{2} d^2 f \sqrt{e^2 - 4df} \sqrt{2af^2 + c} (-e\sqrt{e^2 - 4df} - 2df + e^2)}\right)}{d^2}$$

Rubi [A] time = 2.81, antiderivative size = 604, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.518$, Rules used = {6728, 277, 195, 217, 206, 266, 50, 63, 208, 1020, 1068, 1080, 1034, 725}

$$\frac{(f^2(\sqrt{e^2-4df}-2f+e^2)+4acd^2f^2+c^2d^2(-\sqrt{e^2-4df}-2f+e^2))\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{2}d^2f\sqrt{e^2-4df}\sqrt{2af^2+c}} + \frac{(f^2(-\sqrt{e^2-4df}-2f+e^2)+4acd^2f^2+c^2d^2(\sqrt{e^2-4df}-2f+e^2))\tanh^{-1}\left(\frac{\sqrt{2}d^2f\sqrt{e^2-4df}\sqrt{2af^2+c}(-e\sqrt{e^2-4df}-2df+e^2)}{\sqrt{2}d^2f\sqrt{e^2-4df}\sqrt{2af^2+c}(-e\sqrt{e^2-4df}-2df+e^2)}\right)}{\sqrt{2}d^2f\sqrt{e^2-4df}\sqrt{2af^2+c}(-e\sqrt{e^2-4df}-2df+e^2)} + \frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2} + \frac{e\sqrt{a+cx^2}}{d^2} + \frac{\sqrt{a+cx^2}(2d-cd)}{2d^2} + \frac{\sqrt{e^2-4df}\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2df} + \frac{(e+cd)^2}{d^2} + \frac{3e\sqrt{a+cx^2}}{2d} + \frac{3e\sqrt{e^2-4df}\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2d}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^2)^(3/2)/(x^2*(d + e*x + f*x^2)), x]

[Out] -((a*e*Sqrt[a + c*x^2])/d^2) + (3*c*x*Sqrt[a + c*x^2])/(2*d) + ((2*a*e - c*d*x)*Sqrt[a + c*x^2])/(2*d^2) - (a + c*x^2)^(3/2)/(d*x) + (3*a*Sqrt[c]*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*d) + (Sqrt[c]*(2*c*d - 3*a*f)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*d*f) - (((4*a*c*d^2*f^2 + c^2*d^2*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])) + a^2*f^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[2]*d^2*f*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]]) + (((4*a*c*d^2*f^2 + a^2*f^2*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])) + c^2*d^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[2]*d^2*f*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]]) + (a^(3/2)*e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d^2

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 195

Int[((a_.) + (b_.)*(x_))^(n_)]^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 206

$\text{Int}[(a_ + (b_ \cdot)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1 \cdot \text{ArcTanh}[\text{Rt}[-b, 2] \cdot x] / \text{Rt}[a, 2]) / (\text{Rt}[a, 2] \cdot \text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 208

$\text{Int}[(a_ + (b_ \cdot)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2] \cdot \text{ArcTanh}[x / \text{Rt}[-(a/b), 2]]) / a, x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

Rule 217

$\text{Int}[1/\text{Sqrt}[a_ + (b_ \cdot)(x_)^2], x_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(1 - b \cdot x^2), x], x, x/\text{Sqrt}[a + b \cdot x^2]] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ !\text{GtQ}[a, 0]$

Rule 266

$\text{Int}[(x_)^{m_} \cdot ((a_ + (b_ \cdot)(x_)^{n_})^{p_}), x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1) \cdot (a + b \cdot x)^p}, x], x, x^n], x] /; \text{FreeQ}\{a, b, m, n, p\}, x] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 277

$\text{Int}[(c_ \cdot)(x_)^{m_} \cdot ((a_ + (b_ \cdot)(x_)^{n_})^{p_}), x_Symbol] \rightarrow \text{Simp}[(c \cdot x)^{m + 1} \cdot (a + b \cdot x^n)^p / (c \cdot (m + 1)), x] - \text{Dist}[(b \cdot n \cdot p) / (c \cdot n \cdot (m + 1)), \text{Int}[(c \cdot x)^{m + n} \cdot (a + b \cdot x^n)^{p - 1}, x], x] /; \text{FreeQ}\{a, b, c\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ !\text{LtQ}[(m + n \cdot p + n + 1)/n, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 725

$\text{Int}[1/(((d_) + (e_ \cdot)(x_)) \cdot \text{Sqrt}[a_ + (c_ \cdot)(x_)^2]), x_Symbol] \rightarrow -\text{Subst}[\text{Int}[1/(c \cdot d^2 + a \cdot e^2 - x^2), x], x, (a \cdot e - c \cdot d \cdot x) / \text{Sqrt}[a + c \cdot x^2]] /; \text{FreeQ}\{a, c, d, e\}, x]$

Rule 1020

$\text{Int}[(g_) + (h_ \cdot)(x_) \cdot ((a_ + (c_ \cdot)(x_)^2)^{p_}) \cdot ((d_) + (e_ \cdot)(x_) + (f_ \cdot)(x_)^2)^{q_}), x_Symbol] \rightarrow \text{Simp}[(h \cdot (a + c \cdot x^2)^p \cdot (d + e \cdot x + f \cdot x^2)^{q + 1}) / (2 \cdot f \cdot (p + q + 1)), x] + \text{Dist}[1 / (2 \cdot f \cdot (p + q + 1)), \text{Int}[(a + c \cdot x^2)^{p - 1} \cdot (d + e \cdot x + f \cdot x^2)^q \cdot \text{Simp}[a \cdot h \cdot e \cdot p - a \cdot (h \cdot e - 2 \cdot g \cdot f) \cdot (p + q + 1) - 2 \cdot h \cdot p \cdot (c \cdot d - a \cdot f) \cdot x - (h \cdot c \cdot e \cdot p + c \cdot (h \cdot e - 2 \cdot g \cdot f) \cdot (p + q + 1)) \cdot x^2, x], x], x] /; \text{FreeQ}\{a, c, d, e, f, g, h, q\}, x] \ \&\& \ \text{NeQ}[e^2 - 4 \cdot d \cdot f, 0] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{NeQ}[p + q + 1, 0]$

Rule 1034

$\text{Int}[(g_) + (h_ \cdot)(x_) / (((a_) + (b_ \cdot)(x_) + (c_ \cdot)(x_)^2) \cdot \text{Sqrt}[(d_) + (f_ \cdot)(x_)^2]), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[b^2 - 4 \cdot a \cdot c, 2]\}, \text{Dist}[(2 \cdot c \cdot g - h \cdot (b - q)) / q, \text{Int}[1 / ((b - q + 2 \cdot c \cdot x) \cdot \text{Sqrt}[d + f \cdot x^2]), x], x] - \text{Dist}[(2 \cdot c \cdot g - h \cdot (b + q)) / q, \text{Int}[1 / ((b + q + 2 \cdot c \cdot x) \cdot \text{Sqrt}[d + f \cdot x^2]), x], x]] /; \text{FreeQ}\{a, b, c, d, f, g, h\}, x] \ \&\& \ \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0] \ \&\& \ \text{PosQ}[b^2 - 4 \cdot a \cdot c]$

Rule 1068

$\text{Int}[(a_) + (c_ \cdot)(x_)^2)^{p_} \cdot ((A_) + (B_ \cdot)(x_) + (C_ \cdot)(x_)^2) \cdot ((d_) + (e_ \cdot)(x_) + (f_ \cdot)(x_)^2)^{q_}), x_Symbol] \rightarrow \text{Simp}[(B \cdot c \cdot f \cdot (2 \cdot p + 2 \cdot q + 3) + C \cdot (-c \cdot e \cdot (2 \cdot p + q + 2))) + 2 \cdot c \cdot C \cdot f \cdot (p + q + 1) \cdot x \cdot (a + c \cdot x^2)^p \cdot (d + e \cdot x + f \cdot x^2)^{q + 1}) / (2 \cdot c \cdot f \cdot 2 \cdot (p + q + 1) \cdot (2 \cdot p + 2 \cdot q + 3)), x] - \text{Dist}[1 / (2 \cdot c \cdot f$

```

^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^
q*Simp[p*(-(a*e))*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p +
q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3
))] + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(C*e*f*p*(-4*a*c)))*x + (p*(c*e)*(C*(c*e)*(q + 1) - c*(C*e - B*
f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C*(e^2 - 4*d*f)*
(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3)))]*x^2, x], x], x]
/; FreeQ[{a, c, d, e, f, A, B, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0
] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q
, 0]

```

Rule 1080

```

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + f*x^2],
x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]

```

Rule 6728

```

Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{(a + cx^2)^{3/2}}{x^2(d + ex + fx^2)} dx &= \int \left(\frac{(a + cx^2)^{3/2}}{dx^2} - \frac{e(a + cx^2)^{3/2}}{d^2x} + \frac{(e^2 - df + efx)(a + cx^2)^{3/2}}{d^2(d + ex + fx^2)} \right) dx \\
&= \frac{\int \frac{(e^2 - df + efx)(a + cx^2)^{3/2}}{d + ex + fx^2} dx}{d^2} + \frac{\int \frac{(a + cx^2)^{3/2}}{x^2} dx}{d} - \frac{e \int \frac{(a + cx^2)^{3/2}}{x} dx}{d^2} \\
&= \frac{e(a + cx^2)^{3/2}}{3d^2} - \frac{(a + cx^2)^{3/2}}{dx} + \frac{(3c) \int \sqrt{a + cx^2} dx}{d} - \frac{e \operatorname{Subst} \left(\int \frac{(a + cx)^{3/2}}{x} dx, x, x^2 \right)}{2d^2} \\
&= \frac{3cx\sqrt{a + cx^2}}{2d} + \frac{(2ae - cdx)\sqrt{a + cx^2}}{2d^2} - \frac{(a + cx^2)^{3/2}}{dx} + \frac{(3ac) \int \frac{1}{\sqrt{a + cx^2}} dx}{2d} - \frac{(ae) \operatorname{Subst} \left(\int \frac{1}{x} dx, x, x^2 \right)}{2d^2} \\
&= -\frac{ae\sqrt{a + cx^2}}{d^2} + \frac{3cx\sqrt{a + cx^2}}{2d} + \frac{(2ae - cdx)\sqrt{a + cx^2}}{2d^2} - \frac{(a + cx^2)^{3/2}}{dx} + \frac{(3ac) \operatorname{Subst} \left(\int \frac{1}{\sqrt{a + cx^2}} dx, x, x^2 \right)}{2d} \\
&= -\frac{ae\sqrt{a + cx^2}}{d^2} + \frac{3cx\sqrt{a + cx^2}}{2d} + \frac{(2ae - cdx)\sqrt{a + cx^2}}{2d^2} - \frac{(a + cx^2)^{3/2}}{dx} + \frac{3a\sqrt{c} \operatorname{tanh}^{-1} \left(\frac{\sqrt{a + cx^2}}{\sqrt{a}} \right)}{2d} \\
&= -\frac{ae\sqrt{a + cx^2}}{d^2} + \frac{3cx\sqrt{a + cx^2}}{2d} + \frac{(2ae - cdx)\sqrt{a + cx^2}}{2d^2} - \frac{(a + cx^2)^{3/2}}{dx} + \frac{3a\sqrt{c} \operatorname{tanh}^{-1} \left(\frac{\sqrt{a + cx^2}}{\sqrt{a}} \right)}{2d} \\
&= -\frac{ae\sqrt{a + cx^2}}{d^2} + \frac{3cx\sqrt{a + cx^2}}{2d} + \frac{(2ae - cdx)\sqrt{a + cx^2}}{2d^2} - \frac{(a + cx^2)^{3/2}}{dx} + \frac{3a\sqrt{c} \operatorname{tanh}^{-1} \left(\frac{\sqrt{a + cx^2}}{\sqrt{a}} \right)}{2d}
\end{aligned}$$

Mathematica [C] time = 4.16, size = 885, normalized size = 1.47

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(3/2)/(x^2*(d + e*x + f*x^2)),x]
```

```
[Out] (-x*(2*Sqrt[a]*Sqrt[c]*d*f*Sqrt[e^2 - 4*d*f]*Sqrt[a + c*x^2]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]] + Sqrt[1 + (c*x^2)/a]*(2*c*d*f*Sqrt[e^2 - 4*d*f]*x*Sqrt[a + c*x^2] - 4*Sqrt[c]*d*(c*d - a*f)*Sqrt[e^2 - 4*d*f]*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]) + (2*c*d^2 + a*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])] * ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f]))*x]/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]) + Sqrt[2]*a*e*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])] * ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f]))*x]/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]) - 2*c*d^2*Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])] * ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f]))*x]/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]) - a*e^2*Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])] * ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f]))*x]/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]) + 2*a*d*f*Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])] * ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f]))*x]/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]) - 4*a^(3/2)*e*f*Sqrt[e^2 - 4*d*f]*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])) - 4*a*d*f*Sqrt[e^2 - 4*d*f]*Sqrt[a + c*x^2]*Hypergeometric2F1[-3/2, -1/2, 1/2, -((c*x^2)/a)]/(4*d^2*f*Sqrt[e^2 - 4*d*f]*x*Sqrt[1 + (c*x^2)/a])
```

IntegrateAlgebraic [C] time = 0.74, size = 502, normalized size = 0.83

```
RootSum[4^1/2 f^2 - 2d^1/2 c - 2d^1/2 e^2 + 4d^1/2 d + 2d^1/2 c + e^2/f, ...]
```

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + c*x^2)^(3/2)/(x^2*(d + e*x + f*x^2)),x]
```

```
[Out] -((a*Sqrt[a + c*x^2])/(d*x)) - (2*a^(3/2)*e*ArcTanh[(Sqrt[c]*x)/Sqrt[a] - Sqrt[a + c*x^2]/Sqrt[a]])/d^2 - (c^(3/2)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/f - RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (-a*c^2*d^2*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]) + a^3*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - 2*c^(5/2)*d^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + 4*a*c^(3/2)*d^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + 2*a^2*Sqrt[c]*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 2*a^2*Sqrt[c]*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + c^2*d^2*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - a^2*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]/(d^2*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x^2/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(3/2)/x^2/(f*x^2+e*x+d),x, algorithm="giac")

[Out] sage2

maple [B] time = 0.02, size = 9912, normalized size = 16.41

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+a)^(3/2)/x^2/(f*x^2+e*x+d),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2 + a)^{\frac{3}{2}}}{(fx^2 + ex + d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(3/2)/x^2/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] integrate((c*x^2 + a)^(3/2)/((f*x^2 + e*x + d)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^2 + a)^{3/2}}{x^2 (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c*x^2)^(3/2)/(x^2*(d + e*x + f*x^2)),x)

[Out] int((a + c*x^2)^(3/2)/(x^2*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + cx^2)^{\frac{3}{2}}}{x^2 (d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+a)**(3/2)/x**2/(f*x**2+e*x+d),x)

[Out] Integral((a + c*x**2)**(3/2)/(x**2*(d + e*x + f*x**2)), x)

3.63 $\int \frac{(a+cx^2)^{3/2}}{x^3(d+ex+fx^2)} dx$

Optimal. Leaf size=668

$$-\frac{a^{3/2}(e^2-df)\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3} + \frac{(a^2f(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3)+2acd^2f(\sqrt{e^2-4df}+e)+c\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}+e)})}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}+e)}}$$

Rubi [A] time = 3.46, antiderivative size = 668, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 15, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6728, 266, 47, 50, 63, 208, 277, 195, 217, 206, 1020, 1068, 1080, 1034, 725}

$\frac{(e^2f\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3)+2acd^2f(\sqrt{e^2-4df}+e)+c\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}+e)}}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}+e)}}$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(3/2)/(x^3*(d + e*x + f*x^2)), x]
[Out] (3*c*Sqrt[a + c*x^2])/(2*d) + (a*(e^2 - d*f)*Sqrt[a + c*x^2])/d^3 - (3*c*e*x*Sqrt[a + c*x^2])/(2*d^2) - ((2*(c*d^2 + a*(e^2 - d*f)) - c*d*e*x)*Sqrt[a + c*x^2])/(2*d^3) - (a + c*x^2)^(3/2)/(2*d*x^2) + (e*(a + c*x^2)^(3/2))/(d^2*x) + ((c^2*d^3*(e - Sqrt[e^2 - 4*d*f]) + 2*a*c*d^2*f*(e + Sqrt[e^2 - 4*d*f]) + a^2*f*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])])*Sqrt[a + c*x^2]])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - ((2*a*c*d^2*f*(e - Sqrt[e^2 - 4*d*f]) + c^2*d^3*(e + Sqrt[e^2 - 4*d*f]) + a^2*f*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f] + d*f*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])*Sqrt[a + c*x^2]])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) - (3*Sqrt[a]*c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*d) - (a^(3/2)*(e^2 - d*f)*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d^3
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) &&
!(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &&
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(
b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

$[b*c - a*d, 0] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 195

$\text{Int}[(a_ + (b_ \cdot)(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(x*(a + b*x^n)^p)/(n*p + 1), x] + \text{Dist}[(a*n*p)/(n*p + 1), \text{Int}[(a + b*x^n)^{p-1}, x], x] /;$ FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 206

$\text{Int}[(a_ + (b_ \cdot)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

$\text{Int}[(a_ + (b_ \cdot)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /;$ FreeQ[{a, b}, x] && NegQ[a/b]

Rule 217

$\text{Int}[1/\text{Sqrt}[a_ + (b_ \cdot)(x_)^2], x_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] /;$ FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 266

$\text{Int}[(x_)^{(m_)}*((a_ + (b_ \cdot)(x_)^{(n_)})^{(p_)}), x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p}, x], x, x^n], x] /;$ FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 277

$\text{Int}[(c_ \cdot)(x_)^{(m_)}*((a_ + (b_ \cdot)(x_)^{(n_)})^{(p_)}), x_Symbol] \rightarrow \text{Simp}[(c*x)^{(m+1)}*(a + b*x^n)^p/(c*(m+1)), x] - \text{Dist}[(b*n*p)/(c^n*(m+1)), \text{Int}[(c*x)^{(m+n)}*(a + b*x^n)^{p-1}, x], x] /;$ FreeQ[{a, b, c}, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 725

$\text{Int}[1/(((d_) + (e_ \cdot)(x_))*\text{Sqrt}[a_ + (c_ \cdot)(x_)^2]), x_Symbol] \rightarrow -\text{Subst}[\text{Int}[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/\text{Sqrt}[a + c*x^2]] /;$ FreeQ[{a, c, d, e}, x]

Rule 1020

$\text{Int}[(g_) + (h_ \cdot)(x_)]*((a_ + (c_ \cdot)(x_)^2)^{(p_)}*((d_) + (e_ \cdot)(x_) + (f_ \cdot)(x_)^2)^{(q_)}), x_Symbol] \rightarrow \text{Simp}[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^q + 1)/(2*f*(p + q + 1)), x] + \text{Dist}[1/(2*f*(p + q + 1)), \text{Int}[(a + c*x^2)^{p-1}*(d + e*x + f*x^2)^q*\text{Simp}[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x] /;$ FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1068

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((B*c*f*(2*p + 2*q + 3) + C*(-(c*e*(2*p + q + 2))) + 2*c*C*f*(p + q + 1)*x)*(a + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(-(a*e))*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))] + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(-4*a*c)))*x + (p*(c*e)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3)))]*x^2, x], x] /; FreeQ[{a, c, d, e, f, A, B, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+cx^2)^{3/2}}{x^3(d+ex+fx^2)} dx &= \int \left(\frac{(a+cx^2)^{3/2}}{dx^3} - \frac{e(a+cx^2)^{3/2}}{d^2x^2} + \frac{(e^2-df)(a+cx^2)^{3/2}}{d^3x} + \frac{(-e(e^2-2df)-f(e^2-df))}{d^3(d+ex+fx^2)} \right) dx \\
&= \frac{\int \frac{(-e(e^2-2df)-f(e^2-df)x)(a+cx^2)^{3/2}}{d+ex+fx^2} dx}{d^3} + \frac{\int \frac{(a+cx^2)^{3/2}}{x^3} dx}{d} - \frac{e \int \frac{(a+cx^2)^{3/2}}{x^2} dx}{d^2} + \frac{(e^2-df) \int \frac{(a+cx^2)^{3/2}}{x} dx}{d^3} \\
&= -\frac{(e^2-df)(a+cx^2)^{3/2}}{3d^3} + \frac{e(a+cx^2)^{3/2}}{d^2x} + \frac{\text{Subst}\left(\int \frac{(a+cx)^{3/2}}{x^2} dx, x, x^2\right)}{2d} - \frac{(3ce) \int \sqrt{a+cx^2} dx}{d^2} \\
&= -\frac{3cex\sqrt{a+cx^2}}{2d^2} - \frac{(2(cd^2+a(e^2-df))-cdex)\sqrt{a+cx^2}}{2d^3} - \frac{(a+cx^2)^{3/2}}{2dx^2} + \frac{e(a+cx^2)^{3/2}}{d^2} \\
&= \frac{3c\sqrt{a+cx^2}}{2d} + \frac{a(e^2-df)\sqrt{a+cx^2}}{d^3} - \frac{3cex\sqrt{a+cx^2}}{2d^2} - \frac{(2(cd^2+a(e^2-df))-cdex)\sqrt{a+cx^2}}{2d^3} \\
&= \frac{3c\sqrt{a+cx^2}}{2d} + \frac{a(e^2-df)\sqrt{a+cx^2}}{d^3} - \frac{3cex\sqrt{a+cx^2}}{2d^2} - \frac{(2(cd^2+a(e^2-df))-cdex)\sqrt{a+cx^2}}{2d^3} \\
&= \frac{3c\sqrt{a+cx^2}}{2d} + \frac{a(e^2-df)\sqrt{a+cx^2}}{d^3} - \frac{3cex\sqrt{a+cx^2}}{2d^2} - \frac{(2(cd^2+a(e^2-df))-cdex)\sqrt{a+cx^2}}{2d^3} \\
&= \frac{3c\sqrt{a+cx^2}}{2d} + \frac{a(e^2-df)\sqrt{a+cx^2}}{d^3} - \frac{3cex\sqrt{a+cx^2}}{2d^2} - \frac{(2(cd^2+a(e^2-df))-cdex)\sqrt{a+cx^2}}{2d^3}
\end{aligned}$$

Mathematica [C] time = 3.22, size = 904, normalized size = 1.35

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^2)^(3/2)/(x^3*(d + e*x + f*x^2)), x]

[Out]
$$\begin{aligned}
&(-5*(e^2 - d*f - (e*(e^2 - 3*d*f)))/\text{Sqrt}[e^2 - 4*d*f])*(a + c*x^2)^{3/2} - 5 \\
&*(e^2 - d*f + (e*(e^2 - 3*d*f)))/\text{Sqrt}[e^2 - 4*d*f]*(a + c*x^2)^{3/2} + (15* \\
&(-e^2 + d*f - (e*(e^2 - 3*d*f)))/\text{Sqrt}[e^2 - 4*d*f])*((2*\text{Sqrt}[c]*(-e + \text{Sqrt}[e \\
&^2 - 4*d*f])*\text{Sqrt}[a + c*x^2]*(\text{Sqrt}[c]*x*\text{Sqrt}[1 + (c*x^2)/a] + \text{Sqrt}[a]*\text{ArcSi} \\
&\text{nh}[(\text{Sqrt}[c]*x)/\text{Sqrt}[a]]))/\text{Sqrt}[1 + (c*x^2)/a] + (2*(2*a*f^2 + c*(e^2 - 2*d* \\
&f - e*\text{Sqrt}[e^2 - 4*d*f]))*(2*f*\text{Sqrt}[a + c*x^2] + \text{Sqrt}[c]*(-e + \text{Sqrt}[e^2 - 4 \\
&*d*f])*\text{ArcTanh}[(\text{Sqrt}[c]*x)/\text{Sqrt}[a + c*x^2]] - \text{Sqrt}[2*c*e^2 - 4*c*d*f + 4*a* \\
&f^2 - 2*c*e*\text{Sqrt}[e^2 - 4*d*f]]*\text{ArcTanh}[(2*a*f + c*(-e + \text{Sqrt}[e^2 - 4*d*f]) \\
&*x)/(\text{Sqrt}[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f])] * \text{Sqrt}[a + c*x^2 \\
&])))/f^2))/(8*f) - (15*(-e^2 + d*f + (e*(e^2 - 3*d*f)))/\text{Sqrt}[e^2 - 4*d*f])* \\
&((2*\text{Sqrt}[c]*(e + \text{Sqrt}[e^2 - 4*d*f])*\text{Sqrt}[a + c*x^2]*(\text{Sqrt}[c]*x*\text{Sqrt}[1 + (c* \\
&x^2)/a] + \text{Sqrt}[a]*\text{ArcSinh}[(\text{Sqrt}[c]*x)/\text{Sqrt}[a]]))/\text{Sqrt}[1 + (c*x^2)/a] + (2*(\\
&2*a*f^2 + c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]))*(-2*f*\text{Sqrt}[a + c*x^2] + \text{Sq} \\
&\text{rt}[c]*(e + \text{Sqrt}[e^2 - 4*d*f])*\text{ArcTanh}[(\text{Sqrt}[c]*x)/\text{Sqrt}[a + c*x^2]] + \text{Sqrt}[4 \\
&*a*f^2 + 2*c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]])*\text{ArcTanh}[(2*a*f - c*(e + \text{S} \\
&\text{qrt}[e^2 - 4*d*f])*x)/(\text{Sqrt}[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f] \\
&))] * \text{Sqrt}[a + c*x^2])))/f^2))/(8*f) + 10*(e^2 - d*f)*(\text{Sqrt}[a + c*x^2]*(4*a + \\
&c*x^2) - 3*a^{3/2}*\text{ArcTanh}[\text{Sqrt}[a + c*x^2]/\text{Sqrt}[a]]) + (30*a*d*e*\text{Sqrt}[a +
\end{aligned}$$

$c*x^2]*\text{Hypergeometric2F1}[-3/2, -1/2, 1/2, -((c*x^2)/a)]/(x*\text{Sqrt}[1 + (c*x^2)/a]) + (6*c*d^2*(a + c*x^2)^{(5/2)}*\text{Hypergeometric2F1}[2, 5/2, 7/2, 1 + (c*x^2)/a])/a^2)/(30*d^3)$

IntegrateAlgebraic [C] time = 0.95, size = 624, normalized size = 0.93

RootSum[...]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + c*x^2)^(3/2)/(x^3*(d + e*x + f*x^2)),x]
[Out] -1/2*(a*(d - 2*e*x)*Sqrt[a + c*x^2])/(d^2*x^2) + (2*a^(3/2)*(-e^2 + d*f)*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + c*x^2])/Sqrt[a]])/d^3 + (3*Sqrt[a]*c*ArcTanh[(Sqrt[c]*x)/Sqrt[a] - Sqrt[a + c*x^2]/Sqrt[a]])/d - RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (a*c^2*d^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - 2*a^2*c*d^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - a^3*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + a^3*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - 4*a*c^(3/2)*d^2*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 2*a^2*Sqrt[c]*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + 4*a^2*Sqrt[c]*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - c^2*d^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 + 2*a*c*d^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 + a^2*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - a^2*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]/d^3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x^3/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage2

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x^3/(f*x^2+e*x+d),x, algorithm="giac")
[Out] sage2
```

maple [B] time = 0.02, size = 10298, normalized size = 15.42

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)/x^3/(f*x^2+e*x+d),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2 + a)^{\frac{3}{2}}}{(fx^2 + ex + d)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(3/2)/x^3/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] integrate((c*x^2 + a)^(3/2)/((f*x^2 + e*x + d)*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^2 + a)^{3/2}}{x^3 (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c*x^2)^(3/2)/(x^3*(d + e*x + f*x^2)),x)

[Out] int((a + c*x^2)^(3/2)/(x^3*(d + e*x + f*x^2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+a)**(3/2)/x**3/(f*x**2+e*x+d),x)

[Out] Timed out

$$3.64 \quad \int \frac{x^3}{\sqrt{a+cx^2}(d+ex+fx^2)} dx$$

Optimal. Leaf size=380

$$\frac{(2def - (e^2 - df)(e - \sqrt{e^2 - 4df})) \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2}f^2\sqrt{e^2 - 4df}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} + \frac{(2def - (e^2 - df)(\sqrt{e^2 - 4df} + e)) \tanh^{-1}\left(\frac{2af - cx(\sqrt{e^2 - 4df} + e)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2}f^2\sqrt{e^2 - 4df}\sqrt{2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)}} - \frac{e \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f^2} + \frac{\sqrt{a+cx^2}}{cf}$$

Rubi [A] time = 1.17, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 27, number of rules / integrand size = 0.222, Rules used = {6728, 217, 206, 261, 1034, 725}

$$\frac{(2def - (e^2 - df)(e - \sqrt{e^2 - 4df})) \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2}f^2\sqrt{e^2 - 4df}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} + \frac{(2def - (e^2 - df)(\sqrt{e^2 - 4df} + e)) \tanh^{-1}\left(\frac{2af - cx(\sqrt{e^2 - 4df} + e)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2}f^2\sqrt{e^2 - 4df}\sqrt{2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)}} - \frac{e \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f^2} + \frac{\sqrt{a+cx^2}}{cf}$$

Antiderivative was successfully verified.

[In] Int[x^3/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]

[Out] Sqrt[a + c*x^2]/(c*f) - (e*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/(Sqrt[c]*f^2) - ((2*d*e*f - (e^2 - d*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]])]*Sqrt[a + c*x^2]))/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((2*d*e*f - (e^2 - d*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])]*Sqrt[a + c*x^2]))/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 725

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 1034

Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,

b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rule 6728

Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; SumQ[v] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^3}{\sqrt{a+cx^2}(d+ex+fx^2)} dx &= \int \left(-\frac{e}{f^2\sqrt{a+cx^2}} + \frac{x}{f\sqrt{a+cx^2}} + \frac{de+(e^2-df)x}{f^2\sqrt{a+cx^2}(d+ex+fx^2)} \right) dx \\ &= \frac{\int \frac{de+(e^2-df)x}{\sqrt{a+cx^2}(d+ex+fx^2)} dx}{f^2} - \frac{e \int \frac{1}{\sqrt{a+cx^2}} dx}{f^2} + \frac{\int \frac{x}{\sqrt{a+cx^2}} dx}{f} \\ &= \frac{\sqrt{a+cx^2}}{cf} - \frac{e \operatorname{Subst}\left(\int \frac{1}{1-cx^2} dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{f^2} + \frac{(2def - (e^2 - df)(e - \sqrt{e^2 - 4df})) \operatorname{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}} dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{f^2\sqrt{e^2 - 4df}} \\ &= \frac{\sqrt{a+cx^2}}{cf} - \frac{e \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{\sqrt{c} f^2} - \frac{(2def - (e^2 - df)(e - \sqrt{e^2 - 4df})) \operatorname{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}} dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{f^2\sqrt{e^2 - 4df}} \\ &= \frac{\sqrt{a+cx^2}}{cf} - \frac{e \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{\sqrt{c} f^2} - \frac{(2def - (e^2 - df)(e - \sqrt{e^2 - 4df})) \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{\sqrt{2} f^2 \sqrt{e^2 - 4df} \sqrt{2af^2 + c}} \end{aligned}$$

Mathematica [A] time = 1.33, size = 378, normalized size = 0.99

$$\frac{\sqrt{2}((e^2-df)(\sqrt{e^2-4df}-e)+2def) \tanh^{-1}\left(\frac{2af+cx(\sqrt{e^2-4df}-e)}{\sqrt{a+cx^2}\sqrt{4af^2-2c(e\sqrt{e^2-4df}+2df+e^2)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}} + \frac{\sqrt{2}(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3) \tanh^{-1}\left(\frac{2af-cx(\sqrt{e^2-4df}+e)}{\sqrt{a+cx^2}\sqrt{4af^2+2c(e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}} + \frac{2e \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{\sqrt{c}} - \frac{2f\sqrt{a+cx^2}}{c}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)), x]

[Out] -1/2*((-2*f*Sqrt[a + c*x^2])/c + (2*e*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/Sqrt[c] + (Sqrt[2]*(2*d*e*f + (e^2 - d*f)*(-e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (Sqrt[2]*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]))/f^2

IntegrateAlgebraic [C] time = 0.49, size = 315, normalized size = 0.83

$$\frac{\operatorname{RootSum}\left[\#1^4 f - 2\#1^3 \sqrt{c} e - 2\#1^2 a f + 4\#1^2 c d + 2\#1 a \sqrt{c} e + a^2 f \&, \frac{\#1^2 d f \log(-\#1 + \sqrt{a+c\#1^2} - \sqrt{c} x) + \#1^2 d^2 (-\log(-\#1 + \sqrt{a+c\#1^2} - \sqrt{c} x)) + 2\#1 \sqrt{c} d e \log(-\#1 + \sqrt{a+c\#1^2} - \sqrt{c} x) - a d f \log(-\#1 + \sqrt{a+c\#1^2} - \sqrt{c} x) + a^2 \log(-\#1 + \sqrt{a+c\#1^2} - \sqrt{c} x)}{2\#1^3 f - 2\#1^2 \sqrt{c} e - 2\#1 a \sqrt{c} e + a^2 f \&}\right]}{f^2} + \frac{e \log\left(\frac{\sqrt{a+cx^2} - \sqrt{cx}}{\sqrt{a+cx^2}}\right)}{\sqrt{c} f^2} + \frac{\sqrt{a+cx^2}}{c f}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)), x]

```
[Out] Sqrt[a + c*x^2]/(c*f) + (e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(Sqrt[c]*f^2) - RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (a*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - a*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*Sqrt[c]*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - e^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 + d*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]/f^2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 11.34sym2poly/r2sym(const g
en & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [B] time = 0.03, size = 2397, normalized size = 6.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] (c*x^2+a)^(1/2)/c/f-1/f^2*e*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)+1/2/f^2*2^(1/2)/((( -4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((( -4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((( -4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*d-1/2/f^3*2^(1/2)/((( -4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((( -4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((( -4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*d*e-1/2/f^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/((( -4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((( -4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((( -4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c
```

$$\begin{aligned}
& e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)})/(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})))/f)) * e^3 + \\
& 1/2/f^2*2^{(1/2)})/((-(-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)} \\
& * \ln(((-(-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^{(1/2)})/f \\
& *(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})))/f)+1/2*2^{(1/2)}*((-(-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)} \\
& *(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})))/f)^2*c-4*c*(e-(-4*d*f+e^2)^{(1/2)})/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})))/f)+2*((-(-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)})/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})))/f)) * d-1/2/f^3*2^{(1/2)})/((-(-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)} \\
& * \ln(((-(-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^{(1/2)})/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})))/f)+1/2*2^{(1/2)}*((-(-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)} \\
& *(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})))/f)^2*c-4*c*(e-(-4*d*f+e^2)^{(1/2)})/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})))/f)+2*((-(-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)})/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})))/f)) * e^2-3/2/f^2/(-4*d*f+e^2)^{(1/2)}*2^{(1/2)})/((-(-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)} \\
& * \ln(((-(-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^{(1/2)})/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})))/f)+1/2*2^{(1/2)}*((-(-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)} \\
& *(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})))/f)^2*c-4*c*(e-(-4*d*f+e^2)^{(1/2)})/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})))/f)+2*((-(-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)})/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})))/f)) * d * \\
& e+1/2/f^3/(-4*d*f+e^2)^{(1/2)}*2^{(1/2)})/((-(-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)} \\
& * \ln(((-(-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^{(1/2)})/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})))/f)+1/2*2^{(1/2)}*((-(-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)} \\
& *(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})))/f)^2*c-4*c*(e-(-4*d*f+e^2)^{(1/2)})/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})))/f)+2*((-(-4*d*f+e^2)^{(1/2)})*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^{(1/2)})/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})))/f)) * e^3
\end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{cx^2+a} (fx^2+ex+d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)

[Out] int(x^3/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a+cx^2} (d+ex+fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)

[Out] Integral(x**3/(sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)

$$3.65 \quad \int \frac{x^2}{\sqrt{a+cx^2}(d+ex+fx^2)} dx$$

Optimal. Leaf size=344

$$\frac{(-e\sqrt{e^2-4df}-2df+e^2)\tanh^{-1}\left(\frac{2af-cx(e-\sqrt{e^2-4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}} - \frac{(2df-e(\sqrt{e^2-4df}+e))\tanh^{-1}\left(\frac{2af-cx(\sqrt{e^2-4df}+e)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}}$$

Rubi [A] time = 0.54, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, number of rules / integrand size = 0.185, Rules used = {1081, 217, 206, 1034, 725}

$$\frac{(-e\sqrt{e^2-4df}-2df+e^2)\tanh^{-1}\left(\frac{2af-cx(e-\sqrt{e^2-4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}} - \frac{(2df-e(\sqrt{e^2-4df}+e))\tanh^{-1}\left(\frac{2af-cx(\sqrt{e^2-4df}+e)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f}$$

Antiderivative was successfully verified.

[In] Int[x^2/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]

[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/(Sqrt[c]*f) - ((e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]]) - ((2*d*f - e*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 725

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 1034

Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rule 1081

Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C - b*C*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]), x], x]

/; FreeQ[{a, b, c, d, f, A, C}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{\sqrt{a+cx^2}(d+ex+fx^2)} dx &= \frac{\int \frac{1}{\sqrt{a+cx^2}} dx}{f} + \frac{\int \frac{-d-ex}{\sqrt{a+cx^2}(d+ex+fx^2)} dx}{f} \\ &= \frac{\text{Subst}\left(\int \frac{1}{1-cx^2} dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{f} + \frac{(e^2 - 2df - e\sqrt{e^2 - 4df}) \int \frac{1}{(e - \sqrt{e^2 - 4df} + 2fx)\sqrt{e^2 - 4df}} dx}{f\sqrt{e^2 - 4df}} \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f} - \frac{(e^2 - 2df - e\sqrt{e^2 - 4df}) \text{Subst}\left(\int \frac{1}{4af^2 + c(e - \sqrt{e^2 - 4df})^2 - x^2} dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{f\sqrt{e^2 - 4df}} \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f} - \frac{(e^2 - 2df - e\sqrt{e^2 - 4df}) \tanh^{-1}\left(\frac{2af - c(e - \sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{2af^2 + c(e^2 - 2df - e\sqrt{e^2 - 4df})}}\right)}{\sqrt{2}f\sqrt{e^2 - 4df}\sqrt{2af^2 + c(e^2 - 2df - e\sqrt{e^2 - 4df})}} \end{aligned}$$

Mathematica [A] time = 0.70, size = 334, normalized size = 0.97

$$\frac{\sqrt{2}(e\sqrt{e^2-4df}+2df-e^2)\tanh^{-1}\left(\frac{2af+cx(\sqrt{e^2-4df}-e)}{\sqrt{a+cx^2}\sqrt{4af^2-2c(e\sqrt{e^2-4df}+2df-e^2)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}} + \frac{\sqrt{2}(e\sqrt{e^2-4df}-2df+e^2)\tanh^{-1}\left(\frac{2af-cx(\sqrt{e^2-4df}+e)}{\sqrt{a+cx^2}\sqrt{4af^2+2c(e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}}$$

2f

Antiderivative was successfully verified.

[In] Integrate[x^2/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)), x]

[Out] ((2*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/Sqrt[c] + (Sqrt[2]*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x]/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2])))/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (Sqrt[2]*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x]/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2])))/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]))/(2*f)

IntegrateAlgebraic [C] time = 0.43, size = 229, normalized size = 0.67

$$\frac{\text{RootSum}\left[\#1^4 f - 2\#1^3 \sqrt{c} e - 2\#1^2 a f + 4\#1^2 c d + 2\#1 a \sqrt{c} e + a^2 f \&, \frac{\#1^2 (-e) \log(-\#1 + \sqrt{a+cx^2} - \sqrt{c}x) + 2\#1 \sqrt{c} d \log(-\#1 + \sqrt{a+cx^2} - \sqrt{c}x) + a e \log(-\#1 + \sqrt{a+cx^2} - \sqrt{c}x)}{2\#1^3 f - 3\#1^2 \sqrt{c} e - 2\#1 a f + 4\#1 c d + a \sqrt{c} e} \& \right]}{f} - \frac{\log(\sqrt{a+cx^2} - \sqrt{c}x)}{\sqrt{c}f}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)), x]

[Out] -(Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]]/(Sqrt[c]*f)) + RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 &, (a*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*Sqrt[c]*d*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) &]/f

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

```
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Evaluation time: 6.03Error index.cc index_gc
d Error: Bad Argument Value
```

```
maple [B] time = 0.02, size = 1796, normalized size = 5.22
```

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/f*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)+1/2/f^2*2^(1/2)/((2*a*f^2-2*c*d*f
+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-e+(-4*d*f+e^2)^(1/2))*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c
*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1
/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x+1/2
*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*
c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*e-1/(-4*d*f+e^2)^(1/2)/f
*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-e
+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+
c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d
*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+
(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f
+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)
)*d+1/2/(-4*d*f+e^2)^(1/2)/f^2*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)
^(1/2)*c*e)/f^2)^(1/2)*ln((-e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(
1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)
*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-
4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1
/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*e^2+1/2/f^2*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2
-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-e+(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+
(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f
^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*
(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e
)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e+1/(-4*d*f+e^2)^(1/2)/f*2
^(1/2)/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-e+(-
4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c
*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*
f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+
(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*
f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/
f))*d-1/2/(-4*d*f+e^2)^(1/2)/f^2*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^
2)^(1/2)*c*e)/f^2)^(1/2)*ln((-e+(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)
^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1
/2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^

```

$2)^{(1/2)})/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)})/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f))*e^2$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{cx^2+a} (fx^2+ex+d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)

[Out] int(x^2/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+cx^2} (d+ex+fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)

[Out] Integral(x**2/(sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)

$$3.66 \quad \int \frac{x}{\sqrt{a+cx^2}(d+ex+fx^2)} dx$$

Optimal. Leaf size=294

$$\frac{(e - \sqrt{e^2 - 4df}) \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2} \sqrt{a+cx^2} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2} \sqrt{e^2 - 4df} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} - \frac{(\sqrt{e^2 - 4df} + e) \tanh^{-1}\left(\frac{2af - cx(\sqrt{e^2 - 4df} + e)}{\sqrt{2} \sqrt{a+cx^2} \sqrt{2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2} \sqrt{e^2 - 4df} \sqrt{2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)}}$$

Rubi [A] time = 0.24, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1034, 725, 206}

$$\frac{(e - \sqrt{e^2 - 4df}) \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2} \sqrt{a+cx^2} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2} \sqrt{e^2 - 4df} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} - \frac{(\sqrt{e^2 - 4df} + e) \tanh^{-1}\left(\frac{2af - cx(\sqrt{e^2 - 4df} + e)}{\sqrt{2} \sqrt{a+cx^2} \sqrt{2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2} \sqrt{e^2 - 4df} \sqrt{2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)}}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]

[Out] ((e - Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - ((e + Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 725

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 1034

Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rubi steps

$$\int \frac{x}{\sqrt{a+cx^2}(d+ex+fx^2)} dx = -\left(\left(-1 - \frac{e}{\sqrt{e^2-4df}}\right) \int \frac{1}{(e + \sqrt{e^2-4df} + 2fx)\sqrt{a+cx^2}} dx\right) + \left(1 - \frac{e}{\sqrt{e^2-4df}}\right) \int \frac{1}{(e - \sqrt{e^2-4df} + 2fx)\sqrt{a+cx^2}} dx$$

$$= \left(-1 + \frac{e}{\sqrt{e^2-4df}}\right) \text{Subst}\left(\int \frac{1}{4af^2 + c(e - \sqrt{e^2-4df})^2 - x^2} dx, x, \frac{2af - c}{\sqrt{e^2-4df}}\right)$$

$$= \frac{\left(1 - \frac{e}{\sqrt{e^2-4df}}\right) \tanh^{-1}\left(\frac{2af - c(e - \sqrt{e^2-4df})x}{\sqrt{2}\sqrt{2af^2 + c(e^2 - 2df - e\sqrt{e^2-4df})}\sqrt{a+cx^2}}\right)}{\sqrt{2}\sqrt{2af^2 + c(e^2 - 2df - e\sqrt{e^2-4df})}} - \frac{\left(1 + \frac{e}{\sqrt{e^2-4df}}\right) \tanh^{-1}\left(\frac{2af + c(e + \sqrt{e^2-4df})x}{\sqrt{2}\sqrt{2af^2 + c(e^2 - 2df + e\sqrt{e^2-4df})}\sqrt{a+cx^2}}\right)}{\sqrt{2}\sqrt{2af^2 + c(e^2 - 2df + e\sqrt{e^2-4df})}}$$

Mathematica [A] time = 0.32, size = 275, normalized size = 0.94

$$\frac{\sqrt{2} \left[\frac{(\sqrt{e^2-4df}-e) \tanh^{-1}\left(\frac{2af+cx(\sqrt{e^2-4df}-e)}{\sqrt{a+cx^2}\sqrt{4af^2-2c(e\sqrt{e^2-4df}+2df-e^2)}}\right)}{2\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}} - \frac{(\sqrt{e^2-4df}+e) \tanh^{-1}\left(\frac{2af-cx(\sqrt{e^2-4df}+e)}{\sqrt{a+cx^2}\sqrt{4af^2+2c(e\sqrt{e^2-4df}-2df+e^2)}}\right)}{2\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}} \right]}{\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]

[Out] (Sqrt[2]*(-1/2*((-e + Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x]/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]))*Sqrt[a + c*x^2]])/Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]]) - ((e + Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x]/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*Sqrt[a + c*x^2]])/(2*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))]))/Sqrt[e^2 - 4*d*f]

IntegrateAlgebraic [C] time = 0.38, size = 156, normalized size = 0.53

$$\text{RootSum}\left[\#1^4 f - 2\#1^3 \sqrt{ce} - 2\#1^2 af + 4\#1^2 cd + 2\#1 a \sqrt{ce} + a^2 f \&, \frac{\#1^2 \log(-\#1 + \sqrt{a+cx^2} - \sqrt{cx}) - a \log(-\#1 + \sqrt{a+cx^2} - \sqrt{cx})}{2\#1^3 f - 3\#1^2 \sqrt{ce} - 2\#1 af + 4\#1 cd + a \sqrt{ce}} \& \right]$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]

[Out] RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (- (a*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]) + Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1])*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) &]

fricas [B] time = 1.65, size = 5085, normalized size = 17.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")

[Out] -1/4*sqrt(2)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*

$$\begin{aligned}
& + a^3 c e^4) f^2 - 4(c^4 d^5 + 3 a^3 c^3 d^3 e^2 + 2 a^2 c^2 d^2 e^4) f)) \sqrt{c x^2 + a} \sqrt{(2 c^2 d^2 + a e^2 - 2 a d f - (c^2 d^2 e^2 + a c e^4 - 4 a^2 d f^3 + (8 a^3 c d^2 + a^2 e^2) f^2 - 2(2 c^2 d^3 + 3 a^3 c d e^2) f) \sqrt{a^2 e^2 / (c^4 d^4 e^2 + 2 a^3 c^3 d^2 e^4 + a^2 c^2 e^6 - 4 a^4 d f^5 + (16 a^3 c d^2 + a^4 e^2) f^4 - 12(2 a^2 c^2 d^3 + a^3 c d e^2) f^3 + 2(8 a^3 c^3 d^4 + 11 a^2 c^2 d^2 e^2 + a^3 c e^4) f^2 - 4(c^4 d^5 + 3 a^3 c^3 d^3 e^2 + 2 a^2 c^2 d^2 e^4) f)) / (c^2 d^2 e^2 + a c e^4 - 4 a^2 d f^3 + (8 a^3 c d^2 + a^2 e^2) f^2 - 2(2 c^2 d^3 + 3 a^3 c d e^2) f) - 2(a c^2 d^3 e^2 + a^2 c d e^4 - 4 a^3 d^2 f^3 + (8 a^2 c d^3 + a^3 d e^2) f^2 - 2(2 a^3 c^2 d^4 + 3 a^2 c d^2 e^2) f) \sqrt{a^2 e^2 / (c^4 d^4 e^2 + 2 a^3 c^3 d^2 e^4 + a^2 c^2 e^6 - 4 a^4 d f^5 + (16 a^3 c d^2 + a^4 e^2) f^4 - 12(2 a^2 c^2 d^3 + a^3 c d e^2) f^3 + 2(8 a^3 c^3 d^4 + 11 a^2 c^2 d^2 e^2 + a^3 c e^4) f^2 - 4(c^4 d^5 + 3 a^3 c^3 d^3 e^2 + 2 a^2 c^2 d^2 e^4) f)) / x} + 1/4 \sqrt{2} \sqrt{(2 c^2 d^2 + a e^2 - 2 a d f - (c^2 d^2 e^2 + a c e^4 - 4 a^2 d f^3 + (8 a^3 c d^2 + a^2 e^2) f^2 - 2(2 c^2 d^3 + 3 a^3 c d e^2) f) \sqrt{a^2 e^2 / (c^4 d^4 e^2 + 2 a^3 c^3 d^2 e^4 + a^2 c^2 e^6 - 4 a^4 d f^5 + (16 a^3 c d^2 + a^4 e^2) f^4 - 12(2 a^2 c^2 d^3 + a^3 c d e^2) f^3 + 2(8 a^3 c^3 d^4 + 11 a^2 c^2 d^2 e^2 + a^3 c e^4) f^2 - 4(c^4 d^5 + 3 a^3 c^3 d^3 e^2 + 2 a^2 c^2 d^2 e^4) f)) / (c^2 d^2 e^2 + a c e^4 - 4 a^2 d f^3 + (8 a^3 c d^2 + a^2 e^2) f^2 - 2(2 c^2 d^3 + 3 a^3 c d e^2) f) \log((4 a^3 c d^2 e^2 x - 2 a^2 d e^2 - \sqrt{2})(a^2 e^4 - 4 a^2 d e^2 f + (2 c^3 d^4 e^2 + 3 a^3 c^2 d^2 e^4 + a^2 c e^6 + 8 a^3 d^2 f^4 - 6(4 a^2 c d^3 + a^3 d e^2) f^3 + (24 a^3 c^2 d^4 + 22 a^2 c d^2 e^2 + a^3 e^4) f^2 - 2(4 c^3 d^5 + 9 a^3 c^2 d^3 e^2 + 4 a^2 c d e^4) f) \sqrt{a^2 e^2 / (c^4 d^4 e^2 + 2 a^3 c^3 d^2 e^4 + a^2 c^2 e^6 - 4 a^4 d f^5 + (16 a^3 c d^2 + a^4 e^2) f^4 - 12(2 a^2 c^2 d^3 + a^3 c d e^2) f^3 + 2(8 a^3 c^3 d^4 + 11 a^2 c^2 d^2 e^2 + a^3 c e^4) f^2 - 4(c^4 d^5 + 3 a^3 c^3 d^3 e^2 + 2 a^2 c^2 d^2 e^4) f)) \sqrt{c x^2 + a} \sqrt{(2 c^2 d^2 + a e^2 - 2 a d f - (c^2 d^2 e^2 + a c e^4 - 4 a^2 d f^3 + (8 a^3 c d^2 + a^2 e^2) f^2 - 2(2 c^2 d^3 + 3 a^3 c d e^2) f) \sqrt{a^2 e^2 / (c^4 d^4 e^2 + 2 a^3 c^3 d^2 e^4 + a^2 c^2 e^6 - 4 a^4 d f^5 + (16 a^3 c d^2 + a^4 e^2) f^4 - 12(2 a^2 c^2 d^3 + a^3 c d e^2) f^3 + 2(8 a^3 c^3 d^4 + 11 a^2 c^2 d^2 e^2 + a^3 c e^4) f^2 - 4(c^4 d^5 + 3 a^3 c^3 d^3 e^2 + 2 a^2 c^2 d^2 e^4) f)) / (c^2 d^2 e^2 + a c e^4 - 4 a^2 d f^3 + (8 a^3 c d^2 + a^2 e^2) f^2 - 2(2 c^2 d^3 + 3 a^3 c d e^2) f) - 2(a c^2 d^3 e^2 + a^2 c d e^4 - 4 a^3 d^2 f^3 + (8 a^2 c d^3 + a^3 d e^2) f^2 - 2(2 a^3 c^2 d^4 + 3 a^2 c d^2 e^2) f) \sqrt{a^2 e^2 / (c^4 d^4 e^2 + 2 a^3 c^3 d^2 e^4 + a^2 c^2 e^6 - 4 a^4 d f^5 + (16 a^3 c d^2 + a^4 e^2) f^4 - 12(2 a^2 c^2 d^3 + a^3 c d e^2) f^3 + 2(8 a^3 c^3 d^4 + 11 a^2 c^2 d^2 e^2 + a^3 c e^4) f^2 - 4(c^4 d^5 + 3 a^3 c^3 d^3 e^2 + 2 a^2 c^2 d^2 e^4) f)) / x)
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INP UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 1.77index.cc index_m operat or + Error: Bad Argument Value

maple [B] time = 0.01, size = 1172, normalized size = 3.99

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)

[Out]
$$-1/2/(-4df+e^2)^{(1/2)}/f^{2^{(1/2)}}/((2af^2-2cdf+ce^2+(-4df+e^2)^{(1/2)})^2 * ce)/f^2)^{(1/2)} * \ln((-e+(-4df+e^2)^{(1/2)}) * (x+1/2 * (e+(-4df+e^2)^{(1/2)})))$$

```

/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*
a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f
+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2)))/
f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)/(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f))*e-1/2/f*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+
e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^
2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(
1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(
e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2
)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)
)/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))-1/2/f*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2-(
-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-
4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2
+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*
(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-
4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/
f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))+1/2/(-4*d*f+e^2)^(1/2)/f*2^(
1/2)/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-e-(-4
*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e
^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+
e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-(-
4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+
c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)
)*e

```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{cx^2+a} (fx^2+ex+d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)

[Out] int(x/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+cx^2} (d+ex+fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)

[Out] Integral(x/(sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)

$$3.67 \quad \int \frac{1}{\sqrt{a+cx^2} (d+ex+fx^2)} dx$$

Optimal. Leaf size=266

$$\frac{\sqrt{2} f \tanh^{-1} \left(\frac{2af-cx(\sqrt{e^2-4df}+e)}{\sqrt{2} \sqrt{a+cx^2} \sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}} \right)}{\sqrt{e^2-4df} \sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}} - \frac{\sqrt{2} f \tanh^{-1} \left(\frac{2af-cx(e-\sqrt{e^2-4df})}{\sqrt{2} \sqrt{a+cx^2} \sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}} \right)}{\sqrt{e^2-4df} \sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}$$

Rubi [A] time = 0.15, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {985, 725, 206}

$$\frac{\sqrt{2} f \tanh^{-1} \left(\frac{2af-cx(\sqrt{e^2-4df}+e)}{\sqrt{2} \sqrt{a+cx^2} \sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}} \right)}{\sqrt{e^2-4df} \sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}} - \frac{\sqrt{2} f \tanh^{-1} \left(\frac{2af-cx(e-\sqrt{e^2-4df})}{\sqrt{2} \sqrt{a+cx^2} \sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}} \right)}{\sqrt{e^2-4df} \sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]

[Out] -((Sqrt[2]*f*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f]))*x]/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2])))/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (Sqrt[2]*f*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f]))*x]/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2])))/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 725

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 985

Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c)/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rubi steps

$$\int \frac{1}{\sqrt{a+cx^2}(d+ex+fx^2)} dx = \frac{(2f) \int \frac{1}{(e-\sqrt{e^2-4df}+2fx)\sqrt{a+cx^2}} dx}{\sqrt{e^2-4df}} - \frac{(2f) \int \frac{1}{(e+\sqrt{e^2-4df}+2fx)\sqrt{a+cx^2}} dx}{\sqrt{e^2-4df}}$$

$$= \frac{(2f) \text{Subst} \left(\int \frac{1}{4af^2+c(e-\sqrt{e^2-4df})^2-x^2} dx, x, \frac{2af-c(e-\sqrt{e^2-4df})x}{\sqrt{a+cx^2}} \right)}{\sqrt{e^2-4df}} + \frac{(2f) \text{Subst} \left(\int \frac{1}{4af^2+c(e+\sqrt{e^2-4df})^2-x^2} dx, x, \frac{2af-c(e+\sqrt{e^2-4df})x}{\sqrt{a+cx^2}} \right)}{\sqrt{e^2-4df}}$$

$$= \frac{\sqrt{2} f \tanh^{-1} \left(\frac{2af-c(e-\sqrt{e^2-4df})x}{\sqrt{2} \sqrt{2af^2+c(e^2-2df-e\sqrt{e^2-4df})} \sqrt{a+cx^2}} \right)}{\sqrt{e^2-4df} \sqrt{2af^2+c(e^2-2df-e\sqrt{e^2-4df})}} + \frac{\sqrt{2} f \tanh^{-1} \left(\frac{2af-c(e+\sqrt{e^2-4df})x}{\sqrt{2} \sqrt{2af^2+c(e^2-2df+e\sqrt{e^2-4df})} \sqrt{a+cx^2}} \right)}{\sqrt{e^2-4df} \sqrt{2af^2+c(e^2-2df+e\sqrt{e^2-4df})}}$$

Mathematica [A] time = 0.29, size = 247, normalized size = 0.93

$$\frac{2\sqrt{2} f \left(\frac{\tanh^{-1} \left(\frac{2af-cx(\sqrt{e^2-4df}+e)}{\sqrt{a+cx^2} \sqrt{4af^2+2c(e\sqrt{e^2-4df}-2df+e^2)}} \right)}{2\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}} - \frac{\tanh^{-1} \left(\frac{2af+cx(\sqrt{e^2-4df}-e)}{\sqrt{a+cx^2} \sqrt{4af^2-2c(e\sqrt{e^2-4df}+2df-e^2)}} \right)}{2\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}} \right)}{\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
[Out] (2*Sqrt[2]*f*(-1/2*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])] + ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(2*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]))/Sqrt[e^2 - 4*d*f]
```

IntegrateAlgebraic [C] time = 0.36, size = 131, normalized size = 0.49

$$-2\sqrt{c} \text{RootSum} \left[\#1^4 f - 2\#1^3 \sqrt{c} e - 2\#1^2 a f + 4\#1^2 c d + 2\#1 a \sqrt{c} e + a^2 f \&, \frac{\#1 \log(-\#1 + \sqrt{a + c x^2} - \sqrt{c} x)}{2\#1^3 f - 3\#1^2 \sqrt{c} e - 2\#1 a f + 4\#1 c d + a \sqrt{c} e} \& \right]$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
[Out] -2*Sqrt[c]*RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 &, (Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]
```

fricas [B] time = 1.62, size = 5073, normalized size = 19.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
[Out] -1/4*sqrt(2)*sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 + (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^
```



```

^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*
(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))*sqrt(c*x^2 + a)*sqrt((c*
e^2 - 2*c*d*f + 2*a*f^2 - (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2
+ a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2
+ 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^
4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*
e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/
(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^
2*d^3 + 3*a*c*d*e^2)*f)) - 2*(4*a^3*d*f^4 - (8*a^2*c*d^2 + a^3*e^2)*f^3 + 2
*(2*a*c^2*d^3 + 3*a^2*c*d*e^2)*f^2 - (a*c^2*d^2*e^2 + a^2*c*e^4)*f)*sqrt(c^
2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*
c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^
4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*
a^2*c^2*d*e^4)*f)))/x) + 1/4*sqrt(2)*sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 - (c^2
*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3
+ 3*a*c*d*e^2)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^
6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*
d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*
d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^
2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f))*log((
4*c^2*d*e*f*x - 2*a*c*e^2*f - sqrt(2)*(c^2*d*e^3 + 4*a*c*d*e*f^2 - (4*c^2*d
^2*e + a*c*e^3)*f + (c^3*d^3*e^3 + a*c^2*d*e^5 - 4*a^3*d*e*f^4 + (4*a^2*c*d
^2*e + a^3*e^3)*f^3 + (4*a*c^2*d^3*e - 5*a^2*c*d*e^3)*f^2 - (4*c^3*d^4*e +
5*a*c^2*d^2*e^3 - a^2*c*e^5)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4
+ a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2
*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*
f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))*sqrt(c*x^2 + a)*
sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 - (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8
*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(c^2*e^2/(c^4*
d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4
*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*
c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*
e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 -
2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)) - 2*(4*a^3*d*f^4 - (8*a^2*c*d^2 + a^3*e^2)
*f^3 + 2*(2*a*c^2*d^3 + 3*a^2*c*d*e^2)*f^2 - (a*c^2*d^2*e^2 + a^2*c*e^4)*f)
)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 +
(16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*
a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*
e^2 + 2*a^2*c^2*d*e^4)*f)))/x)

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x);OUTPUT:Evaluation time: 1.45Error index.cc index_gc
 d Error: Bad Argument Value

maple [B] time = 0.01, size = 589, normalized size = 2.21

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)

```
[Out] 1/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e
)/f^2)^(1/2)*ln((-e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c
/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2
-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)
^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/
f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/2*(e+(-
4*d*f+e^2)^(1/2))/f))-1/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2-
(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-
4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^
2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4
*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+
(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)
/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{cx^2+a} (fx^2+ex+d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a+c*x^2)^(1/2)*(d+e*x+f*x^2)),x)
```

```
[Out] int(1/((a+c*x^2)^(1/2)*(d+e*x+f*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+cx^2} (d+ex+fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a+c*x**2)*(d+e*x+f*x**2)), x)
```

$$3.68 \quad \int \frac{1}{x\sqrt{a+cx^2}(d+ex+fx^2)} dx$$

Optimal. Leaf size=330

$$\frac{f(\sqrt{e^2-4df}+e)\tanh^{-1}\left(\frac{2af-cx(e-\sqrt{e^2-4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}} - \frac{f(e-\sqrt{e^2-4df})\tanh^{-1}\left(\frac{2af-cx(\sqrt{e^2-4df}+e)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{ad}}$$

Rubi [A] time = 0.82, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, number of rules / integrand size = 0.259, Rules used = {6728, 266, 63, 208, 1034, 725, 206}

$$\frac{f(\sqrt{e^2-4df}+e)\tanh^{-1}\left(\frac{2af-cx(e-\sqrt{e^2-4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}} - \frac{f(e-\sqrt{e^2-4df})\tanh^{-1}\left(\frac{2af-cx(\sqrt{e^2-4df}+e)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{ad}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]

[Out] (f*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]]) - (f*(e - Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]]) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(Sqrt[a]*d)

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 725

Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ

{a, c, d, e}, x]

Rule 1034

Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rule 6728

Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{x\sqrt{a+cx^2}(d+ex+fx^2)} dx &= \int \left(\frac{1}{dx\sqrt{a+cx^2}} + \frac{-e-fx}{d\sqrt{a+cx^2}(d+ex+fx^2)} \right) dx \\ &= \frac{\int \frac{1}{x\sqrt{a+cx^2}} dx}{d} + \frac{\int \frac{-e-fx}{\sqrt{a+cx^2}(d+ex+fx^2)} dx}{d} \\ &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx^2}} dx, x, x^2\right)}{2d} - \frac{\left(f\left(1 - \frac{e}{\sqrt{e^2-4df}}\right)\right) \int \frac{1}{(e+\sqrt{e^2-4df}+2fx)\sqrt{a+cx^2}} dx}{d} \\ &= \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c} + \frac{x^2}{c}} dx, x, \sqrt{a+cx^2}\right)}{cd} + \frac{\left(f\left(1 - \frac{e}{\sqrt{e^2-4df}}\right)\right) \text{Subst}\left(\int \frac{1}{4af^2+c(e+\sqrt{e^2-4df}+2fx)} dx, x, \sqrt{a+cx^2}\right)}{d} \\ &= \frac{f\left(1 + \frac{e}{\sqrt{e^2-4df}}\right) \tanh^{-1}\left(\frac{2af-c(e-\sqrt{e^2-4df})x}{\sqrt{2}\sqrt{2af^2+c(e^2-2df-e\sqrt{e^2-4df})}\sqrt{a+cx^2}}\right)}{\sqrt{2}d\sqrt{2af^2+c(e^2-2df-e\sqrt{e^2-4df})}} + \frac{f\left(1 - \frac{e}{\sqrt{e^2-4df}}\right) \tanh^{-1}\left(\frac{2af-cx(\sqrt{e^2-4df}+e)}{\sqrt{a+cx^2}\sqrt{4af^2+2c(e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}d\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}} \end{aligned}$$

Mathematica [A] time = 0.74, size = 319, normalized size = 0.97

$$\frac{\frac{\sqrt{2}f(\sqrt{e^2-4df}+e) \tanh^{-1}\left(\frac{2af+cx(\sqrt{e^2-4df}-e)}{\sqrt{a+cx^2}\sqrt{4af^2-2c(e\sqrt{e^2-4df}+2df+e^2)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}} + \frac{\sqrt{2}f(\sqrt{e^2-4df}-e) \tanh^{-1}\left(\frac{2af-cx(\sqrt{e^2-4df}+e)}{\sqrt{a+cx^2}\sqrt{4af^2+2c(e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}}}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[a + c*x^2]*(d + e*x + f*x^2)), x]

[Out] ((Sqrt[2]*f*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2])))/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (Sqrt[2]*f*(-e + Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2])))/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) - (2*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/Sqrt[a])/(2*d)

IntegrateAlgebraic [C] time = 0.45, size = 242, normalized size = 0.73

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a}} - \frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{ad}} - \frac{\text{RootSum}\left[\#1^4 f - 2\#1^3 \sqrt{ce} - 2\#1^2 af + 4\#1^2 cd + 2\#1a \sqrt{ce} + a^2 f \&, \frac{\#1^2(-f) \log(-\#1 + \sqrt{a+cx^2} - \sqrt{cx}) + 2\#1 \sqrt{ce} \log(-\#1 + \sqrt{a+cx^2} - \sqrt{cx}) + af \log(-\#1 + \sqrt{a+cx^2} - \sqrt{cx})}{-2\#1^3 f + 3\#1^2 \sqrt{ce} + 2\#1af - 4\#1cd - a \sqrt{ce}} \&\right]}{d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
[Out] (2*ArcTanh[(sqrt[c]*x)/sqrt[a] - sqrt[a + c*x^2]/sqrt[a]])/(sqrt[a]*d) - Ro
otSum[a^2*f + 2*a*sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*sqrt[c]*e*#1^3
+ f*#1^4 & , (a*f*Log[-(sqrt[c]*x) + sqrt[a + c*x^2] - #1] + 2*sqrt[c]*e*L
og[-(sqrt[c]*x) + sqrt[a + c*x^2] - #1]*#1 - f*Log[-(sqrt[c]*x) + sqrt[a +
c*x^2] - #1]*#1^2)/(-(a*sqrt[c]*e) - 4*c*d*#1 + 2*a*f*#1 + 3*sqrt[c]*e*#1^2
- 2*f*#1^3) & ]/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.02, size = 681, normalized size = 2.06

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
[Out] -2*f/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-2*c*d*f+c*
e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-e+(-4*d*f+e^2)^(1/2))*(x+1/2*(
e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)
/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)
*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e
+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)
/f^2)^(1/2)/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))-2*f/(-e+(-4*d*f+e^2)^(1/2))
/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)
/f^2)^(1/2)*ln((-e+(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c
/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2
-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)
)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*
c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)/(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f))+4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))
/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)
```


maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + a}(fx^2 + ex + d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\sqrt{cx^2 + a}(fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)

[Out] int(1/(x*(a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a + cx^2}(d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)

[Out] Integral(1/(x*sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)

$$3.69 \quad \int \frac{1}{x^2 \sqrt{a+cx^2} (d+ex+fx^2)} dx$$

Optimal. Leaf size=367

$$\frac{f(e\sqrt{e^2-4df}-2df+e^2) \tanh^{-1}\left(\frac{2af-cx(e-\sqrt{e^2-4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}} + \frac{f(-e\sqrt{e^2-4df}-2df+e^2) \tanh^{-1}\left(\frac{2af-cx(e+\sqrt{e^2-4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}} + \frac{e \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^2} - \frac{\sqrt{a+cx^2}}{adx}$$

Rubi [A] time = 1.20, antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, number of rules / integrand size = 0.296, Rules used = {6728, 264, 266, 63, 208, 1034, 725, 206}

$$\frac{f(e\sqrt{e^2-4df}-2df+e^2) \tanh^{-1}\left(\frac{2af-cx(e-\sqrt{e^2-4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}} + \frac{f(-e\sqrt{e^2-4df}-2df+e^2) \tanh^{-1}\left(\frac{2af-cx(e+\sqrt{e^2-4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}} + \frac{e \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^2} - \frac{\sqrt{a+cx^2}}{adx}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]

[Out] -(Sqrt[a + c*x^2]/(a*d*x)) - (f*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]]) + (f*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]]) + (e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^2)

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 264

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b

, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 725

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]

Rule 1034

Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rule 6728

Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{x^2 \sqrt{a+cx^2} (d+ex+fx^2)} dx &= \int \left(\frac{1}{dx^2 \sqrt{a+cx^2}} - \frac{e}{d^2 x \sqrt{a+cx^2}} + \frac{e^2 - df + efx}{d^2 \sqrt{a+cx^2} (d+ex+fx^2)} \right) dx \\ &= \frac{\int \frac{e^2 - df + efx}{\sqrt{a+cx^2} (d+ex+fx^2)} dx}{d^2} + \frac{\int \frac{1}{x^2 \sqrt{a+cx^2}} dx}{d} - \frac{e \int \frac{1}{x \sqrt{a+cx^2}} dx}{d^2} \\ &= -\frac{\sqrt{a+cx^2}}{adx} - \frac{e \operatorname{Subst}\left(\int \frac{1}{x \sqrt{a+cx}} dx, x, x^2\right)}{2d^2} - \frac{f(e^2 - 2df - e\sqrt{e^2 - 4df})}{d^2 \sqrt{e^2 - 4df}} \\ &= -\frac{\sqrt{a+cx^2}}{adx} - \frac{e \operatorname{Subst}\left(\int \frac{1}{\frac{a}{c} + \frac{x^2}{c}} dx, x, \sqrt{a+cx^2}\right)}{cd^2} + \frac{f(e^2 - 2df - e\sqrt{e^2 - 4df})}{cd^2} \\ &= -\frac{\sqrt{a+cx^2}}{adx} - \frac{f(e^2 - 2df + e\sqrt{e^2 - 4df}) \tanh^{-1}\left(\frac{2af - c(e - \sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{2af^2 + c(e^2 - 2df - e\sqrt{e^2 - 4df})}}\right)}{\sqrt{2} d^2 \sqrt{e^2 - 4df} \sqrt{2af^2 + c(e^2 - 2df - e\sqrt{e^2 - 4df})}} \end{aligned}$$

Mathematica [A] time = 0.84, size = 356, normalized size = 0.97

$$\frac{\sqrt{2} f (e \sqrt{e^2 - 4df} - 2df + e^2) \tanh^{-1}\left(\frac{2af + cx \sqrt{e^2 - 4df} - e}{\sqrt{a+cx^2} \sqrt{4af^2 - 2c(e \sqrt{e^2 - 4df} + 2df + e^2)}}\right)}{\sqrt{e^2 - 4df} \sqrt{2af^2 + c(e \sqrt{e^2 - 4df} - 2df + e^2)}} + \frac{\sqrt{2} f (e \sqrt{e^2 - 4df} + 2df - e^2) \tanh^{-1}\left(\frac{2af - cx \sqrt{e^2 - 4df} + e}{\sqrt{a+cx^2} \sqrt{4af^2 + 2c(e \sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{e^2 - 4df} \sqrt{2af^2 + c(e \sqrt{e^2 - 4df} - 2df + e^2)}} + \frac{2d \sqrt{a+cx^2}}{ax} - \frac{2e \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]

[Out] -1/2*((2*d*Sqrt[a + c*x^2])/(a*x) + (Sqrt[2]*f*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f])

$$\begin{aligned} & \sqrt{2af^2 + c(e^2 - 2df - e\sqrt{e^2 - 4df})} + (\sqrt{2}f(-e^2 + 2df + e\sqrt{e^2 - 4df})\operatorname{ArcTanh}((2af - c(e + \sqrt{e^2 - 4df}))x) / (\sqrt{4af^2 + 2c(e^2 - 2df + e\sqrt{e^2 - 4df})}\sqrt{a + cx^2})) / (\sqrt{e^2 - 4df}\sqrt{2af^2 + c(e^2 - 2df + e\sqrt{e^2 - 4df})}) - (2e\operatorname{ArcTanh}(\sqrt{a + cx^2}/\sqrt{a})/\sqrt{a})/d^2 \end{aligned}$$

IntegrateAlgebraic [C] time = 0.55, size = 304, normalized size = 0.83

$$\operatorname{RootSum}\left[\#1^4 f - 2\#1^3 \sqrt{c} e - 2\#1^2 a f + 4\#1^2 c d + 2\#1 a \sqrt{c} e + a^2 f \& e, \frac{-\#1^2 e \log(-\#1 + \sqrt{a + c x^2} - \sqrt{c} x) - 2\#1 \sqrt{c} d f \log(-\#1 + \sqrt{a + c x^2} - \sqrt{c} x) + 2\#1 \sqrt{c} e^2 \log(-\#1 + \sqrt{a + c x^2} - \sqrt{c} x) + a e f \log(-\#1 + \sqrt{a + c x^2} - \sqrt{c} x)}{d^2}\right] - \frac{2e \operatorname{tanh}^{-1}\left(\frac{\sqrt{c} x}{\sqrt{a}} - \frac{\sqrt{a + c x^2}}{\sqrt{a}}\right)}{\sqrt{a} d^2} - \frac{\sqrt{a + c x^2}}{a d x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
[Out] -(sqrt[a + c*x^2]/(a*d*x)) - (2*e*ArcTanh[(sqrt[c]*x)/sqrt[a] - sqrt[a + c*x^2]/sqrt[a]])/(sqrt[a]*d^2) - RootSum[a^2*f + 2*a*sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*sqrt[c]*e*#1^3 + f*#1^4 & , (a*e*f*Log[-(sqrt[c]*x) + sqrt[a + c*x^2] - #1] + 2*sqrt[c]*e^2*Log[-(sqrt[c]*x) + sqrt[a + c*x^2] - #1]*#1 - 2*sqrt[c]*d*f*Log[-(sqrt[c]*x) + sqrt[a + c*x^2] - #1]*#1 - e*f*Log[-(sqrt[c]*x) + sqrt[a + c*x^2] - #1]*#1^2)/(a*sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*sqrt[c]*e*#1^2 + 2*f*#1^3) & ]/d^2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 9.45Error index.cc index_gcd Error: Bad Argument Value
```

maple [B] time = 0.02, size = 736, normalized size = 2.01

$$\frac{4\sqrt{f} \ln\left(\frac{(x + \sqrt{a + c x^2}) \sqrt{c x^2 + a} + \sqrt{a + c x^2} \sqrt{c x^2 + a}}{(x + \sqrt{a + c x^2}) \sqrt{c x^2 + a} - \sqrt{a + c x^2} \sqrt{c x^2 + a}}\right)}{(c + \sqrt{4d^2 + e^2}) \sqrt{c x^2 + a}} - \frac{4\sqrt{f} \ln\left(\frac{(x + \sqrt{a + c x^2}) \sqrt{c x^2 + a} + \sqrt{a + c x^2} \sqrt{c x^2 + a}}{(x + \sqrt{a + c x^2}) \sqrt{c x^2 + a} - \sqrt{a + c x^2} \sqrt{c x^2 + a}}\right)}{(c + \sqrt{4d^2 + e^2}) \sqrt{c x^2 + a}} + \frac{4\sqrt{c} f}{(c + \sqrt{4d^2 + e^2}) (c + \sqrt{4d^2 + e^2}) x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
[Out] 4*f^2/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))+4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a/x*(c*x^2+a)^(1/2)-4*f^2/(-e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c
```

$\frac{e}{f^2} \ln\left(\frac{-\left(e - \sqrt{-4df + e^2}\right) \left(x - \frac{1}{2} \frac{-e + \sqrt{-4df + e^2}}{f}\right)}{c/f + \left(2af^2 - 2c*df + ce^2 - \sqrt{-4df + e^2}\right) \frac{c}{f^2} + \frac{1}{2} \frac{2af^2 - 2c*df + ce^2 - \sqrt{-4df + e^2}}{f^2} \left(4\left(x - \frac{1}{2} \frac{-e + \sqrt{-4df + e^2}}{f}\right)\right)^2} \right) + \frac{4\left(x - \frac{1}{2} \frac{-e + \sqrt{-4df + e^2}}{f}\right)}{f^2} c - 4\left(e - \sqrt{-4df + e^2}\right) \frac{c}{f} + 2\left(2af^2 - 2c*df + ce^2 - \sqrt{-4df + e^2}\right) \frac{c}{f^2} \left(x - \frac{1}{2} \frac{-e + \sqrt{-4df + e^2}}{f}\right) + 16f^2 e \left(-e + \sqrt{-4df + e^2}\right)^2 \left(e + \sqrt{-4df + e^2}\right)^2} a^{1/2} \ln\left(\frac{2a + 2\left(cx^2 + a\right)^{1/2} a^{1/2}}{x}\right)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + a} (fx^2 + ex + d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2 \sqrt{cx^2 + a} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)

[Out] int(1/(x^2*(a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{a + cx^2} (d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)

[Out] Integral(1/(x**2*sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)

$$3.70 \quad \int \frac{1}{x^3 \sqrt{a+cx^2} (d+ex+fx^2)} dx$$

Optimal. Leaf size=457

$$\frac{c \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right) (e^2 - df) \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right) f(- (e^2 - df)(e - \sqrt{e^2 - 4df}) - 4def + 2e^3) \tanh^{-1}\left(\frac{2}{\sqrt{2} \sqrt{a+cx^2}}\right)}{2a^{3/2}d} - \frac{(e^2 - df) \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a} d^3} + \frac{f(- (e^2 - df)(e - \sqrt{e^2 - 4df}) - 4def + 2e^3) \tanh^{-1}\left(\frac{2}{\sqrt{2} \sqrt{a+cx^2}}\right)}{\sqrt{2} d^3 \sqrt{e^2 - 4df} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2af + e^2)}}$$

Rubi [A] time = 1.86, antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 27, number of rules / integrand size = 0.333, Rules used = {6728, 266, 51, 63, 208, 264, 1034, 725, 206}

$$\frac{c \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d} + \frac{f(- (e^2 - df)(e - \sqrt{e^2 - 4df}) - 4def + 2e^3) \tanh^{-1}\left(\frac{2af - c(\sqrt{e^2 - 4df})}{\sqrt{2} \sqrt{a+cx^2} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2af + e^2)}}\right)}{\sqrt{2} d^3 \sqrt{e^2 - 4df} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2af + e^2)}} - \frac{f(- (e^2 - df)(\sqrt{e^2 - 4df} + e) - 4def + 2e^3) \tanh^{-1}\left(\frac{2af - c(\sqrt{e^2 - 4df} + e)}{\sqrt{2} \sqrt{a+cx^2} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2af + e^2)}}\right)}{\sqrt{2} d^3 \sqrt{e^2 - 4df} \sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2af + e^2)}} - \frac{(e^2 - df) \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a} d^3} + \frac{c\sqrt{a+cx^2}}{ad^3x} - \frac{\sqrt{a+cx^2}}{2ad^3x^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]

[Out] -Sqrt[a + c*x^2]/(2*a*d*x^2) + (e*Sqrt[a + c*x^2])/(a*d^2*x) + (f*(2*e^3 - 4*d*e*f - (e^2 - d*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f]))*x]/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))]*Sqrt[a + c*x^2])]/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))] - (f*(2*e^3 - 4*d*e*f - (e^2 - d*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f]))*x]/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))]*Sqrt[a + c*x^2])]/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))] + (c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*a^(3/2)*d) - ((e^2 - d*f)*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^3)

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 264

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a+b*x^n)^(p+1))/(a*c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n+p+1, 0] && NeQ[m, -1]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m+1)/n]-1)*(a+b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m+1)/n]]

Rule 725

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 1034

Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rule 6728

Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^3 \sqrt{a+cx^2} (d+ex+fx^2)} dx &= \int \left(\frac{1}{dx^3 \sqrt{a+cx^2}} - \frac{e}{d^2 x^2 \sqrt{a+cx^2}} + \frac{e^2-df}{d^3 x \sqrt{a+cx^2}} + \frac{-e(e^2-2df)-f(e^2-df)}{d^3 \sqrt{a+cx^2} (d+ex+fx^2)} \right) dx \\
 &= \frac{\int \frac{-e(e^2-2df)-f(e^2-df)x}{\sqrt{a+cx^2} (d+ex+fx^2)} dx}{d^3} + \frac{\int \frac{1}{x^3 \sqrt{a+cx^2}} dx}{d} - \frac{e \int \frac{1}{x^2 \sqrt{a+cx^2}} dx}{d^2} + \frac{(e^2-df) \int \frac{1}{\sqrt{a+cx^2} (d+ex+fx^2)} dx}{d^3} \\
 &= \frac{e\sqrt{a+cx^2}}{ad^2x} + \frac{\text{Subst}\left(\int \frac{1}{x^2 \sqrt{a+cx}} dx, x, x^2\right)}{2d} + \frac{(e^2-df) \text{Subst}\left(\int \frac{1}{x \sqrt{a+cx}} dx, x, x^2\right)}{2d^3} \\
 &= -\frac{\sqrt{a+cx^2}}{2adx^2} + \frac{e\sqrt{a+cx^2}}{ad^2x} - \frac{c \text{Subst}\left(\int \frac{1}{x \sqrt{a+cx}} dx, x, x^2\right)}{4ad} + \frac{(e^2-df) \text{Subst}\left(\int \frac{1}{\sqrt{a+cx}} dx, x, x^2\right)}{2d^3} \\
 &= -\frac{\sqrt{a+cx^2}}{2adx^2} + \frac{e\sqrt{a+cx^2}}{ad^2x} + \frac{f(2e^3-4def-(e^2-df)(e-\sqrt{e^2-4df}))}{\sqrt{2} d^3 \sqrt{e^2-4df} \sqrt{2af^2+c}} \\
 &= -\frac{\sqrt{a+cx^2}}{2adx^2} + \frac{e\sqrt{a+cx^2}}{ad^2x} + \frac{f(2e^3-4def-(e^2-df)(e-\sqrt{e^2-4df}))}{\sqrt{2} d^3 \sqrt{e^2-4df} \sqrt{2af^2+c}}
 \end{aligned}$$

Mathematica [A] time = 1.57, size = 460, normalized size = 1.01

$$\frac{cd^2\sqrt{a+cx^2}\left(\frac{a}{c^2}\frac{\tanh^{-1}\left(\sqrt{\frac{a+cx^2}}{a}}\right)}{\sqrt{a+cx^2}}\right) + \frac{2(e^2-df)\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{2}f(\sqrt{e^2-4df}-df\sqrt{e^2-4df-3def+e^2})\tanh^{-1}\left(\frac{2df+cd\sqrt{e^2-4df-d}}{\sqrt{a+cx^2}\sqrt{4f^2-2(c\sqrt{e^2-4df}+2df-d^2)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c(c\sqrt{e^2-4df}-2df+d^2)}} + \frac{\sqrt{2}f(-e^2\sqrt{e^2-4df}+df\sqrt{e^2-4df-3def+e^2})\tanh^{-1}\left(\frac{2df-cd\sqrt{e^2-4df+d}}{\sqrt{a+cx^2}\sqrt{4f^2+2(c\sqrt{e^2-4df}-2df+d^2)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c(c\sqrt{e^2-4df}-2df+d^2)}} - \frac{2dc\sqrt{a+cx^2}}{ax}}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
[Out] -1/2*((-2*d*e*sqrt[a + c*x^2])/(a*x) - (sqrt[2]*f*(e^3 - 3*d*e*f + e^2*sqrt[e^2 - 4*d*f] - d*f*sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f + c*(-e + sqrt[e^2 - 4*d*f])*x)/(sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*sqrt[e^2 - 4*d*f]])*sqrt[a + c*x^2]]))/(sqrt[e^2 - 4*d*f]*sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*sqrt[e^2 - 4*d*f])]) + (sqrt[2]*f*(e^3 - 3*d*e*f - e^2*sqrt[e^2 - 4*d*f] + d*f*sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + sqrt[e^2 - 4*d*f])*x)/(sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*sqrt[e^2 - 4*d*f]])*sqrt[a + c*x^2]]))/(sqrt[e^2 - 4*d*f]*sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*sqrt[e^2 - 4*d*f])]) + (2*(e^2 - d*f)*ArcTanh[sqrt[a + c*x^2]/sqrt[a]])/sqrt[a] + (c*d^2*sqrt[a + c*x^2]*(a/(c*x^2) - ArcTanh[sqrt[1 + (c*x^2)/a]]/sqrt[1 + (c*x^2)/a]))/a^2)/d^3
```

IntegrateAlgebraic [C] time = 0.84, size = 430, normalized size = 0.94

$$\frac{\text{RootSum}\left[\#1^4 f^2 - 2\#1^3 \sqrt{c} e - 2\#1^2 a f + 4\#1 c d + 2\#1 a \sqrt{c} + a^2 f^2, \frac{a^{3/2} f^2 \log(-\#1 + \sqrt{a+cx^2} - \sqrt{c}) - \#1^2 d \log(-\#1 + \sqrt{a+cx^2} - \sqrt{c}) - \#1 \sqrt{c} d \log(-\#1 + \sqrt{a+cx^2} - \sqrt{c}) - a d f^2 \log(-\#1 + \sqrt{a+cx^2} - \sqrt{c}) + 2\#1 \sqrt{c} \log(-\#1 + \sqrt{a+cx^2} - \sqrt{c}) + a^2 \log(-\#1 + \sqrt{a+cx^2} - \sqrt{c})}{2\#1^3 f^2 \sqrt{c} - 2\#1^2 a f + 4\#1 c d + 4\#1 a \sqrt{c}}\right] c \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}} - \frac{\sqrt{a+cx^2}}{\sqrt{a}}\right) + \frac{2(df - e^2)\tanh^{-1}\left(\frac{\sqrt{a+cx^2} - \sqrt{a}}{\sqrt{a}}\right) + \sqrt{a+cx^2}(2ex - d)}{2ad^2c^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
[Out] ((-d + 2*e*x)*sqrt[a + c*x^2])/(2*a*d^2*x^2) + (2*(-e^2 + d*f)*ArcTanh[(-(sqrt[c]*x) + sqrt[a + c*x^2])/sqrt[a]])/(sqrt[a]*d^3) - (c*ArcTanh[(sqrt[c]*x)/sqrt[a] - sqrt[a + c*x^2]/sqrt[a]])/(a^(3/2)*d) + RootSum[a^2*f + 2*a*sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*sqrt[c]*e*#1^3 + f*#1^4 &, (a*e^2*f*Log[-(sqrt[c]*x) + sqrt[a + c*x^2] - #1] - a*d*f^2*Log[-(sqrt[c]*x) + sqrt[a + c*x^2] - #1] + 2*sqrt[c]*e^3*Log[-(sqrt[c]*x) + sqrt[a + c*x^2] - #1]*#1 - 4*sqrt[c]*d*e*f*Log[-(sqrt[c]*x) + sqrt[a + c*x^2] - #1]*#1 - e^2*f*Log[-(sqrt[c]*x) + sqrt[a + c*x^2] - #1]*#1^2 + d*f^2*Log[-(sqrt[c]*x) + sqrt[a + c*x^2] - #1]*#1^2)/(a*sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*sqrt[c]*e*#1^2 + 2*f*#1^3) & ]/d^3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.02, size = 911, normalized size = 1.99

$$\frac{\frac{cd^2\sqrt{a+cx^2}\left(\frac{a}{c^2}\frac{\tanh^{-1}\left(\sqrt{\frac{a+cx^2}}{a}}\right)}{\sqrt{a+cx^2}}\right) + \frac{2(e^2-df)\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{2}f(\sqrt{e^2-4df}-df\sqrt{e^2-4df-3def+e^2})\tanh^{-1}\left(\frac{2df+cd\sqrt{e^2-4df-d}}{\sqrt{a+cx^2}\sqrt{4f^2-2(c\sqrt{e^2-4df}+2df-d^2)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c(c\sqrt{e^2-4df}-2df+d^2)}} + \frac{\sqrt{2}f(-e^2\sqrt{e^2-4df}+df\sqrt{e^2-4df-3def+e^2})\tanh^{-1}\left(\frac{2df-cd\sqrt{e^2-4df+d}}{\sqrt{a+cx^2}\sqrt{4f^2+2(c\sqrt{e^2-4df}-2df+d^2)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c(c\sqrt{e^2-4df}-2df+d^2)}} - \frac{2dc\sqrt{a+cx^2}}{ax}}{2d^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)`

[Out]
$$\begin{aligned} & -8*f^3/(e+(-4*d*f+e^2)^{(1/2)})^3/(-4*d*f+e^2)^{(1/2)}*2^{(1/2)}/((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*\ln((-e+(-4*d*f+e^2)^{(1/2)})*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c-4*(e+(-4*d*f+e^2)^{(1/2)})*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}/(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)+16*f^2*e/(-e+(-4*d*f+e^2)^{(1/2)})^2/(e+(-4*d*f+e^2)^{(1/2)})^2/a/x*(c*x^2+a)^{(1/2)}+2*f/(-e+(-4*d*f+e^2)^{(1/2)})/(e+(-4*d*f+e^2)^{(1/2)})/a/x^2*(c*x^2+a)^{(1/2)}-2*f/(-e+(-4*d*f+e^2)^{(1/2)})/(e+(-4*d*f+e^2)^{(1/2)})*c/a^(3/2)*\ln((2*a+2*(c*x^2+a)^{(1/2)}*a^(1/2))/x)-8*f^3/(-e+(-4*d*f+e^2)^{(1/2)})^3/(-4*d*f+e^2)^{(1/2)}*2^{(1/2)}/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*\ln((-e+(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c-4*(e+(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)-64*f^4/(-e+(-4*d*f+e^2)^{(1/2)})^3/(e+(-4*d*f+e^2)^{(1/2)})^3/a^(1/2)*\ln((2*a+2*(c*x^2+a)^{(1/2)}*a^(1/2))/x)*d+64*f^3/(-e+(-4*d*f+e^2)^{(1/2)})^3/(e+(-4*d*f+e^2)^{(1/2)})^3/a^(1/2)*\ln((2*a+2*(c*x^2+a)^{(1/2)}*a^(1/2))/x)*e^2 \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+a}(fx^2+ex+d)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(c*x^2+a)*(f*x^2+e*x+d)*x^3),x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3 \sqrt{cx^2+a}(fx^2+ex+d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^3*(a+c*x^2)^(1/2)*(d+e*x+f*x^2)),x)`

[Out] `int(1/(x^3*(a+c*x^2)^(1/2)*(d+e*x+f*x^2)),x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \sqrt{a+cx^2}(d+ex+fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)`

[Out] `Integral(1/(x**3*sqrt(a+c*x**2)*(d+e*x+f*x**2)),x)`

$$3.71 \quad \int \frac{x^3}{(a+cx^2)^{3/2}(d+ex+fx^2)} dx$$

Optimal. Leaf size=499

$$\frac{cex(a(e^2-2df)+cd^2)+af(a(e^2-df)+cd^2)}{af^2\sqrt{a+cx^2}((cd-af)^2+ace^2)} \frac{(2adef-(e-\sqrt{e^2-4df})(a(e^2-df)+cd^2))\tanh^{-1}\left(\frac{2af-cx}{\sqrt{2}\sqrt{a+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}((cd-af)^2+ace^2)\sqrt{2af^2+c(-e\sqrt{e^2-4df})}}$$

Rubi [A] time = 2.11, antiderivative size = 499, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, number of rules / integrand size = 0.259, Rules used = {6728, 191, 261, 1017, 1034, 725, 206}

$$\frac{cex(a(e^2-2df)+cd^2)+af(a(e^2-df)+cd^2)}{af^2\sqrt{a+cx^2}((cd-af)^2+ace^2)} \frac{(2adef-(e-\sqrt{e^2-4df})(a(e^2-df)+cd^2))\tanh^{-1}\left(\frac{2af-cx}{\sqrt{2}\sqrt{a+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}((cd-af)^2+ace^2)\sqrt{2af^2+c(-e\sqrt{e^2-4df})}} + \frac{(2adef-(e+\sqrt{e^2-4df})(a(e^2-df)+cd^2))\tanh^{-1}\left(\frac{2af-cx}{\sqrt{2}\sqrt{a+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}((cd-af)^2+ace^2)\sqrt{2af^2+c(e\sqrt{e^2-4df})}} - \frac{ex}{af^2\sqrt{a+cx^2}} - \frac{1}{cf\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

[In] Int[x^3/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

[Out] -(1/(c*f*Sqrt[a + c*x^2])) - (e*x)/(a*f^2*Sqrt[a + c*x^2]) + (a*f*(c*d^2 + a*(e^2 - d*f)) + c*e*(c*d^2 + a*(e^2 - 2*d*f))*x)/(a*f^2*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2]) - ((2*a*d*e*f - (e - Sqrt[e^2 - 4*d*f])*(c*d^2 + a*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]]) + ((2*a*d*e*f - (e + Sqrt[e^2 - 4*d*f])*(c*d^2 + a*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])

Rule 191

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 725

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 1017

Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q_))

```
(q + 1)*(g*c*(2*a*c*e) + (-a*h))*(2*c^2*d - c*(2*a*f)) + c*(g*(2*c^2*d - c
*(2*a*f)) - h*(-2*a*c*e))*x)/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)),
x] + Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^
(p + 1)*(d + e*x + f*x^2)^q* Simp[(-2*g*c)*((c*d - a*f)^2 - (-a*e)*(c*e))
*(p + 1) + (2*(g*c*(c*d - a*f) - a*(-h*c*e)))*(a*f*(p + 1) - c*d*(p + 2))
- e*((g*c)*(2*a*c*e) + (-a*h))*(2*c^2*d - c*(Plus[2]*a*f)))*(p + q + 2)
- (2*f*((g*c)*(2*a*c*e) + (-a*h))*(2*c^2*d - c*(Plus[2]*a*f)))*(p + q +
2) - (2*(g*c*(c*d - a*f) - a*(-h*c*e)))*(-c*e*(2*p + q + 4)))]*x - c*f*(
2*(g*c*(c*d - a*f) - a*(-h*c*e)))*(2*p + 2*q + 5)*x^2, x], x] /; Free
Q[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[
a*c*e^2 + (c*d - a*f)^2, 0] && !(IntegerQ[p] && ILtQ[q, -1])
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (f
_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^3}{(a+cx^2)^{3/2}(d+ex+fx^2)} dx = \int \left(-\frac{e}{f^2(a+cx^2)^{3/2}} + \frac{x}{f(a+cx^2)^{3/2}} + \frac{de+(e^2-df)x}{f^2(a+cx^2)^{3/2}(d+ex+fx^2)} \right) dx$$

$$= \frac{\int \frac{de+(e^2-df)x}{(a+cx^2)^{3/2}(d+ex+fx^2)} dx}{f^2} - \frac{e \int \frac{1}{(a+cx^2)^{3/2}} dx}{f^2} + \frac{\int \frac{x}{(a+cx^2)^{3/2}} dx}{f}$$

$$= -\frac{1}{cf\sqrt{a+cx^2}} - \frac{ex}{af^2\sqrt{a+cx^2}} + \frac{af(cd^2+a(e^2-df))+ce(cd^2+a(e^2-df))}{af^2(ace^2+(cd-af)^2)\sqrt{a+cx^2}}$$

$$= -\frac{1}{cf\sqrt{a+cx^2}} - \frac{ex}{af^2\sqrt{a+cx^2}} + \frac{af(cd^2+a(e^2-df))+ce(cd^2+a(e^2-df))}{af^2(ace^2+(cd-af)^2)\sqrt{a+cx^2}}$$

$$= -\frac{1}{cf\sqrt{a+cx^2}} - \frac{ex}{af^2\sqrt{a+cx^2}} + \frac{af(cd^2+a(e^2-df))+ce(cd^2+a(e^2-df))}{af^2(ace^2+(cd-af)^2)\sqrt{a+cx^2}}$$

$$= -\frac{1}{cf\sqrt{a+cx^2}} - \frac{ex}{af^2\sqrt{a+cx^2}} + \frac{af(cd^2+a(e^2-df))+ce(cd^2+a(e^2-df))}{af^2(ace^2+(cd-af)^2)\sqrt{a+cx^2}}$$

Mathematica [A] time = 2.70, size = 577, normalized size = 1.16

$$\frac{\left(\frac{d^2-3df}{\sqrt{a+cx^2}}-df+e\right)(2df+cx(e-\sqrt{a+cx^2}))}{af^2\sqrt{a+cx^2}(4f^2+c(e-\sqrt{a+cx^2}))} + \frac{\left(\frac{d^2-3df}{\sqrt{a+cx^2}}-df+e\right)(2df+cx(\sqrt{a+cx^2}+e))}{af^2\sqrt{a+cx^2}(4f^2+c(\sqrt{a+cx^2}+e))} - \frac{\sqrt{2}(-e^2\sqrt{a+cx^2}+df\sqrt{a+cx^2}-3def+e)\tanh^{-1}\left(\frac{2df+cx(\sqrt{a+cx^2}+e)}{\sqrt{a+cx^2}(4f^2+c(\sqrt{a+cx^2}+e))}\right)}{\sqrt{a+cx^2}(2df+cx(-e\sqrt{a+cx^2}-2df+e))^2} - \frac{\sqrt{2}(e^2\sqrt{a+cx^2}-df\sqrt{a+cx^2}-3def+e)\tanh^{-1}\left(\frac{2df+cx(\sqrt{a+cx^2}-e)}{\sqrt{a+cx^2}(4f^2+c(\sqrt{a+cx^2}-e))}\right)}{\sqrt{a+cx^2}(2df+cx(e\sqrt{a+cx^2}-2df+e))^2} - \frac{ex}{af^2\sqrt{a+cx^2}} - \frac{1}{cf\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

[Out] $-(1/(c*f*\text{Sqrt}[a + c*x^2])) - (e*x)/(a*f^2*\text{Sqrt}[a + c*x^2]) + ((e^2 - d*f - (e*(e^2 - 3*d*f))/\text{Sqrt}[e^2 - 4*d*f])*(2*a*f + c*(e - \text{Sqrt}[e^2 - 4*d*f])*x))/ (a*f^2*(4*a*f^2 + c*(e - \text{Sqrt}[e^2 - 4*d*f])^2)*\text{Sqrt}[a + c*x^2]) + ((e^2 - d*f + (e*(e^2 - 3*d*f))/\text{Sqrt}[e^2 - 4*d*f])*(2*a*f + c*(e + \text{Sqrt}[e^2 - 4*d*f])*x))/ (a*f^2*(4*a*f^2 + c*(e + \text{Sqrt}[e^2 - 4*d*f])^2)*\text{Sqrt}[a + c*x^2]) + (\text{Sqrt}[2]*(e^3 - 3*d*e*f - e^2*\text{Sqrt}[e^2 - 4*d*f] + d*f*\text{Sqrt}[e^2 - 4*d*f])* \text{ArcTanh}[(2*a*f + c*(-e + \text{Sqrt}[e^2 - 4*d*f])*x)/(\text{Sqrt}[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f])]*\text{Sqrt}[a + c*x^2])]) / (\text{Sqrt}[e^2 - 4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f - e*\text{Sqrt}[e^2 - 4*d*f]))^(3/2)) - (\text{Sqrt}[2]*(e^3 - 3*d*e*f + e^2*\text{Sqrt}[e^2 - 4*d*f] - d*f*\text{Sqrt}[e^2 - 4*d*f])* \text{ArcTanh}[(2*a*f - c*(e + \text{Sqrt}[e^2 - 4*d*f])*x)/(\text{Sqrt}[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f])]*\text{Sqrt}[a + c*x^2])]) / (\text{Sqrt}[e^2 - 4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]))^(3/2))$

IntegrateAlgebraic [C] time = 0.76, size = 425, normalized size = 0.85

$$\text{RootSum}\left[\frac{\#1^4 f^3 - 2\#1^3 \sqrt{c} e - 2\#1^2 a f + 4\#1^2 c d + 2\#1 a \sqrt{c} e + a^2 f^2}{a^2 f^2 - 2a c d f + a c^2 + c^2 d^2}, \frac{\#1^2 a f \log(-\#1 + \sqrt{a c x^2 - c}) - \#1^2 a f \log(-\#1 + \sqrt{a c x^2 - c}) + \#1^2 a^2 \log(-\#1 + \sqrt{a c x^2 - c}) + \#1^2 a^2 \log(-\#1 + \sqrt{a c x^2 - c}) + \#1^2 a^2 \log(-\#1 + \sqrt{a c x^2 - c}) + \#1^2 a^2 \log(-\#1 + \sqrt{a c x^2 - c})}{2\#1^3 \sqrt{c} e - 2\#1 a f + 4\#1^2 c d + 2\#1 a \sqrt{c} e + a^2 f^2}\right] + \frac{a^2(-f) + a c d - a c e x}{c \sqrt{a + c x^2} (a^2 f^2 - 2a c d f + a c^2 + c^2 d^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

[Out] $(a*c*d - a^2*f - a*c*e*x)/(c*(c^2*d^2 + a*c*e^2 - 2*a*c*d*f + a^2*f^2)*\text{Sqrt}[a + c*x^2]) + \text{RootSum}[a^2*f + 2*a*\text{Sqrt}[c]*e*\#1 + 4*c*d*\#1^2 - 2*a*f*\#1^2 - 2*\text{Sqrt}[c]*e*\#1^3 + f*\#1^4 \& , (-a*c*d^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + c*x^2] - \#1]) - a^2*e^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + c*x^2] - \#1] + a^2*d*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + c*x^2] - \#1] - 2*a*\text{Sqrt}[c]*d*e*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + c*x^2] - \#1]*\#1 + c*d^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + c*x^2] - \#1]*\#1^2 + a*e^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + c*x^2] - \#1]*\#1^2 - a*d*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + c*x^2] - \#1]*\#1^2)/(a*\text{Sqrt}[c]*e + 4*c*d*\#1 - 2*a*f*\#1 - 3*\text{Sqrt}[c]*e*\#1^2 + 2*f*\#1^3) \&]/(c^2*d^2 + a*c*e^2 - 2*a*c*d*f + a^2*f^2)$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage2

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] sage2

maple [B] time = 0.03, size = 6124, normalized size = 12.27

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)

[Out] result too large to display

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{(cx^2 + a)^{3/2} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)

[Out] int(x^3/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a + cx^2)^{\frac{3}{2}} (d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)

[Out] Integral(x**3/((a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)

$$3.72 \quad \int \frac{x^2}{(a+cx^2)^{3/2}(d+ex+fx^2)} dx$$

Optimal. Leaf size=410

$$\frac{f(2d(cd-af) + ae(e - \sqrt{e^2 - 4df})) \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2}\sqrt{e^2 - 4df}((cd-af)^2 + ace^2)\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} + \frac{f(2d(cd-af) + ae(\sqrt{e^2 - 4df})) \tanh^{-1}\left(\frac{2af - cx(\sqrt{e^2 - 4df} + e)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2}\sqrt{e^2 - 4df}((cd-af)^2 + ace^2)\sqrt{2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)}} - \frac{x(cd-af) + ae}{\sqrt{a+cx^2}((cd-af)^2 + ace^2)}$$

Rubi [A] time = 0.71, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1063, 1034, 725, 206}

$$\frac{f(2d(cd-af) + ae(e - \sqrt{e^2 - 4df})) \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2}\sqrt{e^2 - 4df}((cd-af)^2 + ace^2)\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} + \frac{f(2d(cd-af) + ae(\sqrt{e^2 - 4df} + e)) \tanh^{-1}\left(\frac{2af - cx(\sqrt{e^2 - 4df} + e)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2}\sqrt{e^2 - 4df}((cd-af)^2 + ace^2)\sqrt{2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)}} - \frac{x(cd-af) + ae}{\sqrt{a+cx^2}((cd-af)^2 + ace^2)}$$

Antiderivative was successfully verified.

[In] Int[x^2/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x]

[Out] -((a*e + (c*d - a*f)*x)/((a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2])) - (f*(2*d*(c*d - a*f) + a*e*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]])]*Sqrt[a + c*x^2]))/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]])] + (f*(2*d*(c*d - a*f) + a*e*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])]*Sqrt[a + c*x^2]))/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 725

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 1034

Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rule 1063

Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + c*x^2)^(p+1)*(d + e*x + f*x^2)^(q+1)*((A*c - a*C)*(2*a*c*e) + c*(A*(2*c^2*d - c*(2*a*f)) + C*(-2*a*(c*d - a*f))))*x)/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p+1)), x] + Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p+1)), Int[(a + c*x^2)^(p+1)*(d + e*x + f*x^2)^q*Simp[(-2*A*c - 2*a*C)*((c*d - a*f)^2 - (-a*e)*(c*e))*(p+1) + (2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(a*f*(p+1) - c*d*(p+2)) -

```
e*((A*c - a*C)*(2*a*c*e))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e))*(p + q
+ 2) - (2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(-(c*e*(2*p + q + 4))))*x
- c*f*(2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(2*p + 2*q + 5)*x^2, x
], x] /; FreeQ[{a, c, d, e, f, A, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p
, -1] && NeQ[a*c*e^2 + (c*d - a*f)^2, 0] && !( !IntegerQ[p] && ILtQ[q, -1]
) && !IGtQ[q, 0]
```

Rubi steps

$$\int \frac{x^2}{(a + cx^2)^{3/2} (d + ex + fx^2)} dx = -\frac{ae + (cd - af)x}{(ace^2 + (cd - af)^2) \sqrt{a + cx^2}} + \frac{\int \frac{2acd(cd-af)-2a^2cef x}{\sqrt{a+cx^2}(d+ex+fx^2)} dx}{2ac(ace^2 + (cd - af)^2)}$$

$$= -\frac{ae + (cd - af)x}{(ace^2 + (cd - af)^2) \sqrt{a + cx^2}} + \frac{(f(2d(cd - af) + ae(e - \sqrt{e^2 - 4df})))}{\sqrt{e^2 - 4df} (ace^2 + (cd - af)^2)}$$

$$= -\frac{ae + (cd - af)x}{(ace^2 + (cd - af)^2) \sqrt{a + cx^2}} - \frac{(f(2d(cd - af) + ae(e - \sqrt{e^2 - 4df})))}{\sqrt{e^2 - 4df} (ace^2 + (cd - af)^2)}$$

$$= -\frac{ae + (cd - af)x}{(ace^2 + (cd - af)^2) \sqrt{a + cx^2}} - \frac{f(2d(cd - af) + ae(e - \sqrt{e^2 - 4df})) \operatorname{tanh}^{-1}\left(\frac{\sqrt{2} \sqrt{e^2 - 4df}}{\sqrt{a + cx^2}}\right)}{\sqrt{2} \sqrt{e^2 - 4df} (ace^2 + (cd - af)^2)}$$

Mathematica [A] time = 2.07, size = 509, normalized size = 1.24

$$\frac{\left(\frac{2df - e^2}{\sqrt{e^2 - 4df}} + c\right)(2af + cx(e - \sqrt{e^2 - 4df}))}{a\sqrt{a + cx^2}(4af^2 + c(e - \sqrt{e^2 - 4df})^2)} - \frac{\left(\frac{e^2 - 2df}{\sqrt{e^2 - 4df}} + c\right)(2af + cx(\sqrt{e^2 - 4df} + e))}{a\sqrt{a + cx^2}(4af^2 + c(\sqrt{e^2 - 4df} + e)^2)} + \frac{\sqrt{2} f^2 (e\sqrt{e^2 - 4df} + 2df - e^2) \operatorname{tanh}^{-1}\left(\frac{2df + cx(\sqrt{e^2 - 4df} - e)}{\sqrt{a + cx^2} \sqrt{4af^2 - 2c(e\sqrt{e^2 - 4df} + 2df - e^2)}}\right)}{\sqrt{e^2 - 4df} (2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2))^2} + \frac{\sqrt{2} f^2 (e\sqrt{e^2 - 4df} - 2df + e^2) \operatorname{tanh}^{-1}\left(\frac{2df - cx(\sqrt{e^2 - 4df} + e)}{\sqrt{a + cx^2} \sqrt{4af^2 + 2c(e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{e^2 - 4df} (2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2))^2} + \frac{x}{a\sqrt{a + cx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

[Out] (x/(a*Sqrt[a + c*x^2]) - ((e + (-e^2 + 2*d*f)/Sqrt[e^2 - 4*d*f])*(2*a*f + c*(e - Sqrt[e^2 - 4*d*f])*x))/(a*(4*a*f^2 + c*(e - Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^2]) - ((e + (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*(2*a*f + c*(e + Sqrt[e^2 - 4*d*f])*x))/(a*(4*a*f^2 + c*(e + Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^2]) + (Sqrt[2]*f^2*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))^(3/2)) + (Sqrt[2]*f^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))^(3/2)))/f

IntegrateAlgebraic [C] time = 0.63, size = 323, normalized size = 0.79

$$\frac{\operatorname{RootSum}\left[\#1^4 f - 2\#1^3 \sqrt{c} e - 2\#1^2 a f + 4\#1^2 c d + 2\#1 a \sqrt{c} e + a^2 f \&, \frac{-\#1^2 a e f \log(-\#1 + \sqrt{a + c x^2} - \sqrt{c} x) + a^2 e f \log(-\#1 + \sqrt{a + c x^2} - \sqrt{c} x) - 2\#1 c^{3/2} d^2 \log(-\#1 + \sqrt{a + c x^2} - \sqrt{c} x) + 2\#1 a \sqrt{c} d f \log(-\#1 + \sqrt{a + c x^2} - \sqrt{c} x)}{2\#1^3 f - 3\#1^2 \sqrt{c} e - 2\#1 a f + 4\#1 c d + a^2 \sqrt{c}}\right] \&}{a^2 f^2 - 2 a c d f + a c e^2 + c^2 d^2} + \frac{-a e + a f x - c d x}{\sqrt{a + c x^2} (a^2 f^2 - 2 a c d f + a c e^2 + c^2 d^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

[Out] (-a*e) - c*d*x + a*f*x)/((c^2*d^2 + a*c*e^2 - 2*a*c*d*f + a^2*f^2)*Sqrt[a + c*x^2]) + RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*

$$\frac{\sqrt{c}e^{\sqrt{c}x^3 + f^{\sqrt{c}x^4}} \left(a^2 e^f \log\left[-\sqrt{c}x\right] + \sqrt{a + cx^2} - \sqrt{c}x - 2c^{3/2}d^2 \log\left[-\sqrt{c}x\right] + \sqrt{a + cx^2} - \sqrt{c}x + 2a\sqrt{c} \log\left[-\sqrt{c}x\right] + \sqrt{a + cx^2} - \sqrt{c}x - a e^f \log\left[-\sqrt{c}x\right] + \sqrt{a + cx^2} - \sqrt{c}x \right) \left(a\sqrt{c}e + 4c^2d^2 - 2a^2f - 3\sqrt{c}e^2 + 2f^2 \right)}{(c^2d^2 + a^2c^2e^2 - 2a^2c^2d^2f + a^2f^2)}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage2

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] sage2

maple [B] time = 0.02, size = 4752, normalized size = 11.59

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)

[Out]
$$\frac{1/f*x/a/(c*x^2+a)^{1/2} - 1/(2*a*f^2 - 2*c*d*f + c*e^2 + (-4*d*f + e^2)^{1/2}*c*e)}{\left(\left(x + 1/2*(e + (-4*d*f + e^2)^{1/2})/f \right)^2 * c - (e + (-4*d*f + e^2)^{1/2}) * (x + 1/2*(e + (-4*d*f + e^2)^{1/2})/f) * c/f + 1/2*(2*a*f^2 - 2*c*d*f + c*e^2 + (-4*d*f + e^2)^{1/2}*c*e)/f^2 \right)^{1/2} * e + 2/(-4*d*f + e^2)^{1/2} * f / (2*a*f^2 - 2*c*d*f + c*e^2 + (-4*d*f + e^2)^{1/2}*c*e)}{\left(\left(x + 1/2*(e + (-4*d*f + e^2)^{1/2})/f \right)^2 * c - (e + (-4*d*f + e^2)^{1/2}) * (x + 1/2*(e + (-4*d*f + e^2)^{1/2})/f) * c/f + 1/2*(2*a*f^2 - 2*c*d*f + c*e^2 + (-4*d*f + e^2)^{1/2}*c*e)/f^2 \right)^{1/2} * d - 1/(-4*d*f + e^2)^{1/2} / (2*a*f^2 - 2*c*d*f + c*e^2 + (-4*d*f + e^2)^{1/2}*c*e)}{\left(\left(x + 1/2*(e + (-4*d*f + e^2)^{1/2})/f \right)^2 * c - (e + (-4*d*f + e^2)^{1/2}) * (x + 1/2*(e + (-4*d*f + e^2)^{1/2})/f) * c/f + 1/2*(2*a*f^2 - 2*c*d*f + c*e^2 + (-4*d*f + e^2)^{1/2}*c*e)/f^2 \right)^{1/2} * e^2 - 2*(-4*d*f + e^2)^{1/2} / f * c^2 / (2*a*f^2 - 2*c*d*f + c*e^2 + (-4*d*f + e^2)^{1/2}*c*e)}{\left(4*a*c - 4*c^2/f*d + c^2/f^2 * e^2 - c^2/f^2 * (-4*d*f + e^2) \right) / \left(\left(x + 1/2*(e + (-4*d*f + e^2)^{1/2})/f \right)^2 * c - (e + (-4*d*f + e^2)^{1/2}) * (x + 1/2*(e + (-4*d*f + e^2)^{1/2})/f) * c/f + 1/2*(2*a*f^2 - 2*c*d*f + c*e^2 + (-4*d*f + e^2)^{1/2}*c*e)/f^2 \right)^{1/2} * c - (e + (-4*d*f + e^2)^{1/2}) * (x + 1/2*(e + (-4*d*f + e^2)^{1/2})/f) * c/f + 1/2*(2*a*f^2 - 2*c*d*f + c*e^2 + (-4*d*f + e^2)^{1/2}*c*e)/f^2 \right)^{1/2} * x * e + 4/c^2 / (2*a*f^2 - 2*c*d*f + c*e^2 + (-4*d*f + e^2)^{1/2}*c*e)}{\left(4*a*c - 4*c^2/f*d + c^2/f^2 * e^2 - c^2/f^2 * (-4*d*f + e^2) \right) / \left(\left(x + 1/2*(e + (-4*d*f + e^2)^{1/2})/f \right)^2 * c - (e + (-4*d*f + e^2)^{1/2}) * (x + 1/2*(e + (-4*d*f + e^2)^{1/2})/f) * c/f + 1/2*(2*a*f^2 - 2*c*d*f + c*e^2 + (-4*d*f + e^2)^{1/2}*c*e)/f^2 \right)^{1/2} * x * d - 4/f * c^2 / (2*a*f^2 - 2*c*d*f + c*e^2 + (-4*d*f + e^2)^{1/2}*c*e)}{\left(4*a*c - 4*c^2/f*d + c^2/f^2 * e^2 - c^2/f^2 * (-4*d*f + e^2) \right) / \left(\left(x + 1/2*(e + (-4*d*f + e^2)^{1/2})/f \right)^2 * c - (e + (-4*d*f + e^2)^{1/2}) * (x + 1/2*(e + (-4*d*f + e^2)^{1/2})/f) * c/f + 1/2*(2*a*f^2 - 2*c*d*f + c*e^2 + (-4*d*f + e^2)^{1/2}*c*e)/f^2 \right)^{1/2} * e * x * d - 2/(-4*d*f + e^2)^{1/2} / f * c^2 / (2*a*f^2 - 2*c*d*f + c*e^2 + (-4*d*f + e^2)^{1/2}*c*e)}{\left(4*a*c - 4*c^2/f*d + c^2/f^2 * e^2 - c^2/f^2 * (-4*d*f + e^2) \right) / \left(\left(x + 1/2*(e + (-4*d*f + e^2)^{1/2})/f \right)^2 * c - (e + (-4*d*f + e^2)^{1/2}) * (x + 1/2*(e + (-4*d*f + e^2)^{1/2})/f) * c/f + 1/2*(2*a*f^2 - 2*c*d*f + c*e^2 + (-4*d*f + e^2)^{1/2}*c*e)/f^2 \right)^{1/2}}$$


```

2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)
)/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*d-1/(-4*d*f+e^2)^(1/2)/(2*a*f^2-2*c*d*
f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2
)^(1/2)*c*e)/f^2)^(1/2)*ln((-e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/
2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)
)/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e^2

```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{(cx^2 + a)^{3/2} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(x^2/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + cx^2)^{\frac{3}{2}} (d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x**2/((a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

$$3.73 \quad \int \frac{x}{(a+cx^2)^{3/2}(d+ex+fx^2)} dx$$

Optimal. Leaf size=411

$$\frac{f(2cde - (e - \sqrt{e^2 - 4df})(cd - af)) \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}} - \frac{f(2cde - (\sqrt{e^2-4df}+e)(cd-af)) \tanh^{-1}\left(\frac{2af - cx(\sqrt{e^2-4df}+e)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}} - \frac{-af+cd-cex}{\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}$$

Rubi [A] time = 0.83, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, number of rules / integrand size = 0.160, Rules used = {1017, 1034, 725, 206}

$$\frac{f(2cde - (e - \sqrt{e^2 - 4df})(cd - af)) \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c(-e\sqrt{e^2-4df}-2df+e^2)}} - \frac{f(2cde - (\sqrt{e^2-4df}+e)(cd-af)) \tanh^{-1}\left(\frac{2af - cx(\sqrt{e^2-4df}+e)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c(e\sqrt{e^2-4df}-2df+e^2)}} - \frac{-af+cd-cex}{\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}$$

Antiderivative was successfully verified.

[In] Int[x/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

[Out] -((c*d - a*f - c*e*x)/((a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2])) + (f*(2*c*d*e - (c*d - a*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]]) - (f*(2*c*d*e - (c*d - a*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 725

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 1017

Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + c*x^2)^(p+1)*(d + e*x + f*x^2)^(q+1)*(g*c*(2*a*c*e) + (-a*h)*(2*c^2*d - c*(2*a*f)) + c*(g*(2*c^2*d - c*(2*a*f)) - h*(-2*a*c*e))*x)/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p+1)), x] + Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p+1)), Int[(a + c*x^2)^(p+1)*(d + e*x + f*x^2)^q*Simp[(-2*g*c)*((c*d - a*f)^2 - (-a*e)*(c*e))^(p+1) + (2*(g*c*(c*d - a*f) - a*(-h*c*e)))*(a*f*(p+1) - c*d*(p+2)) - e*((g*c)*(2*a*c*e) + (-a*h)*(2*c^2*d - c*(Plus[2]*a*f)))*(p+q+2) - (2*f*((g*c)*(2*a*c*e) + (-a*h)*(2*c^2*d - c*(Plus[2]*a*f)))*(p+q+2) - (2*(g*c*(c*d - a*f) - a*(-h*c*e)))*(-c*e*(2*p+q+4)))]*x - c*f*(2*(g*c*(c*d - a*f) - a*(-h*c*e)))*(2*p+2*q+5)*x^2, x], x] /; FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[a*c*e^2 + (c*d - a*f)^2, 0] && !(IntegerQ[p] && ILtQ[q, -1])

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x}{(a + cx^2)^{3/2} (d + ex + fx^2)} dx = -\frac{cd - af - cex}{(ace^2 + (cd - af)^2) \sqrt{a + cx^2}} + \frac{\int \frac{-2ac^2de - 2acf(cd - af)x}{\sqrt{a + cx^2} (d + ex + fx^2)} dx}{2ac (ace^2 + (cd - af)^2)}$$

$$= -\frac{cd - af - cex}{(ace^2 + (cd - af)^2) \sqrt{a + cx^2}} - \frac{(f(2cde - (cd - af)(e - \sqrt{e^2 - 4df}))) \int \frac{1}{\sqrt{e^2 - 4df} (ace^2 + (cd - af)^2)} dx}{\sqrt{e^2 - 4df} (ace^2 + (cd - af)^2)}$$

$$= -\frac{cd - af - cex}{(ace^2 + (cd - af)^2) \sqrt{a + cx^2}} + \frac{(f(2cde - (cd - af)(e - \sqrt{e^2 - 4df}))) \operatorname{Subst}\left(\int \frac{1}{\sqrt{e^2 - 4df} (ace^2 + (cd - af)^2)} dx, x, \frac{e - \sqrt{e^2 - 4df}}{2}\right)}{\sqrt{e^2 - 4df}}$$

$$= -\frac{cd - af - cex}{(ace^2 + (cd - af)^2) \sqrt{a + cx^2}} + \frac{f(2cde - (cd - af)(e - \sqrt{e^2 - 4df})) \operatorname{tanh}^{-1}\left(\frac{e - \sqrt{e^2 - 4df}}{2}\right)}{\sqrt{2} \sqrt{e^2 - 4df} (ace^2 + (cd - af)^2) \sqrt{2}}$$

Mathematica [A] time = 0.81, size = 457, normalized size = 1.11

$$\frac{\left(1 - \frac{e}{\sqrt{e^2 - 4df}}\right) (2af + cx(e - \sqrt{e^2 - 4df})) + \left(\frac{e}{\sqrt{e^2 - 4df}} + 1\right) (2af + cx(\sqrt{e^2 - 4df} + e))}{a\sqrt{a + cx^2} (4af^2 + c(e - \sqrt{e^2 - 4df})^2)} + \frac{\sqrt{2} f^2 (e - \sqrt{e^2 - 4df}) \operatorname{tanh}^{-1}\left(\frac{2af + cx(\sqrt{e^2 - 4df} + e)}{\sqrt{a + cx^2} \sqrt{4af^2 - 2c(e\sqrt{e^2 - 4df} + 2df + e^2)}}\right)}{\sqrt{e^2 - 4df} (2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2))^{3/2}} - \frac{\sqrt{2} f^2 (\sqrt{e^2 - 4df} + e) \operatorname{tanh}^{-1}\left(\frac{2af - cx(\sqrt{e^2 - 4df} + e)}{\sqrt{a + cx^2} \sqrt{4af^2 + 2c(e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{e^2 - 4df} (2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x]
[Out] ((1 - e/Sqrt[e^2 - 4*d*f])*(2*a*f + c*(e - Sqrt[e^2 - 4*d*f])*x))/(a*(4*a*f^2 + c*(e - Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^2]) + ((1 + e/Sqrt[e^2 - 4*d*f])*(2*a*f + c*(e + Sqrt[e^2 - 4*d*f])*x))/(a*(4*a*f^2 + c*(e + Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^2]) + (Sqrt[2]*f^2*(e - Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[e^2 - 4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))^(3/2)) - (Sqrt[2]*f^2*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[e^2 - 4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))^(3/2))
```

IntegrateAlgebraic [C] time = 0.64, size = 352, normalized size = 0.86

$$\frac{\operatorname{RootSum}\left[\#1^4 f - 2\#1^3 \sqrt{c} e - 2\#1^2 af + 4\#1^2 cd + 2\#1 a \sqrt{c} e + a^2 f \&e, \frac{\#1^2 cd \log(-\#1 + \sqrt{a + cx^2} - \sqrt{c} x) + \#1^2 (-d)^2 \log(-\#1 + \sqrt{a + cx^2} - \sqrt{c} x) + a^2 f^2 \log(-\#1 + \sqrt{a + cx^2} - \sqrt{c} x) - 2\#1 c^2 d \log(-\#1 + \sqrt{a + cx^2} - \sqrt{c} x) - acd f \log(-\#1 + \sqrt{a + cx^2} - \sqrt{c} x)}{2\#1^3 f - 3\#1^2 \sqrt{c} e - 2\#1 af + 4\#1 cd + a \sqrt{c} e}\right] \&e}{-a^2 f^2 + 2acd f - ace^2 - c^2 d^2} + \frac{af - cd + cex}{\sqrt{a + cx^2} (a^2 f^2 - 2acd f + ace^2 + c^2 d^2)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x]
[Out] -(c*d) + a*f + c*e*x/((c^2*d^2 + a*c*e^2 - 2*a*c*d*f + a^2*f^2)*Sqrt[a + c*x^2]) + RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (-a*c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]) + a^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - 2*c^(3/2)*d*e*Log[
```

$$-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + c*x^2] - \#1*\#1 + c*d*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + c*x^2] - \#1]*\#1^2 - a*f^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + c*x^2] - \#1]*\#1^2)/(a*\text{Sqrt}[c]*e + 4*c*d*\#1 - 2*a*f*\#1 - 3*\text{Sqrt}[c]*e*\#1^2 + 2*f*\#1^3) \&]/(-(c^2*d^2) - a*c*e^2 + 2*a*c*d*f - a^2*f^2)$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage2

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] sage2

maple [B] time = 0.02, size = 3000, normalized size = 7.30

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)

[Out]
$$\frac{1/(-4*d*f+e^2)^{(1/2)}*f*e/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/((x+1/2*(e+(-4*d*f+e^2)^{(1/2}))/f)^2*c-(e+(-4*d*f+e^2)^{(1/2}))*x+1/2*(e+(-4*d*f+e^2)^{(1/2}))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)+4*e*c^2/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2)))/((x+1/2*(e+(-4*d*f+e^2)^{(1/2}))/f)^2*c-(e+(-4*d*f+e^2)^{(1/2}))*x+1/2*(e+(-4*d*f+e^2)^{(1/2}))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)*x+2/(-4*d*f+e^2)^{(1/2)}*c^2/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2)))/((x+1/2*(e+(-4*d*f+e^2)^{(1/2}))/f)^2*c-(e+(-4*d*f+e^2)^{(1/2}))*x+1/2*(e+(-4*d*f+e^2)^{(1/2}))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)*e^2*x-1/(-4*d*f+e^2)^{(1/2)}*f*e/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)*2^{(1/2)}/((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)*\ln((-e+(-4*d*f+e^2)^{(1/2}))*x+1/2*(e+(-4*d*f+e^2)^{(1/2}))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2}))/f)^2*c-4*(e+(-4*d*f+e^2)^{(1/2}))*x+1/2*(e+(-4*d*f+e^2)^{(1/2}))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}}/(x+1/2*(e+(-4*d*f+e^2)^{(1/2}))/f)+f/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/((x+1/2*(e+(-4*d*f+e^2)^{(1/2}))/f)^2*c-(e+(-4*d*f+e^2)^{(1/2}))*x+1/2*(e+(-4*d*f+e^2)^{(1/2}))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)+2*(-4*d*f+e^2)^{(1/2)}*c^2/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2)))/((x+1/2*(e+(-4*d*f+e^2)^{(1/2}))/f)^2*c-(e+(-4*d*f+e^2)^{(1/2}))*x+1/2*(e+(-4*d*f+e^2)^{(1/2}))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)*x-f/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)*2^{(1/2)}/((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)*\ln((-e+(-4*d*f+e^2)^{(1/2}))*x+1/2*(e+(-4*d*f+e^2)^{(1/2}))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2}))/f)^2*c-4*(e+(-4*d*f+e^2)^{(1/2}))*x+1/2*(e+(-4*d*f+e^2)^{(1/2}))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}$$

$$x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}/(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f))+f/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f))^2*c-(e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}-2*(-4*d*f+e^2)^{(1/2)}*c^2/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f))^2*c-(e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*x+4*c^2/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f))^2*c-(e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*e*x-f/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)*2^{(1/2)}/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*ln((-e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f))^2*c-4*(e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f))-1/(-4*d*f+e^2)^{(1/2)}*f*e/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f))^2*c-(e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}-2/(-4*d*f+e^2)^{(1/2)}*c^2/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f))^2*c-(e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*e^2*x+1/(-4*d*f+e^2)^{(1/2)}*f*e/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)*2^{(1/2)}/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*ln((-e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f))^2*c-4*(e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f))$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{(cx^2+a)^{3/2}(fx^2+ex+d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((a+c*x^2)^(3/2)*(d+e*x+f*x^2)),x)

[Out] int(x/((a+c*x^2)^(3/2)*(d+e*x+f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+cx^2)^{\frac{3}{2}}(d+ex+fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x/((a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

$$3.74 \quad \int \frac{1}{(a+cx^2)^{3/2}(d+ex+fx^2)} dx$$

Optimal. Leaf size=416

$$\frac{f(2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)) \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2}\sqrt{e^2 - 4df}((cd - af)^2 + ace^2)\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} + \frac{f(2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)) \tanh^{-1}\left(\frac{2af - cx(e + \sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2}\sqrt{e^2 - 4df}((cd - af)^2 + ace^2)\sqrt{2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)}} + \frac{c(x(cd - af) + ae)}{a\sqrt{a+cx^2}((cd - af)^2 + ace^2)}$$

Rubi [A] time = 0.62, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, number of rules / integrand size = 0.167, Rules used = {976, 1034, 725, 206}

$$\frac{f(2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)) \tanh^{-1}\left(\frac{2af - cx(e - \sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2}\sqrt{e^2 - 4df}((cd - af)^2 + ace^2)\sqrt{2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}} + \frac{f(2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2)) \tanh^{-1}\left(\frac{2af - cx(e + \sqrt{e^2 - 4df})}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{2}\sqrt{e^2 - 4df}((cd - af)^2 + ace^2)\sqrt{2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2)}} + \frac{c(x(cd - af) + ae)}{a\sqrt{a+cx^2}((cd - af)^2 + ace^2)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

[Out] (c*(a*e + (c*d - a*f)*x))/(a*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2]) - (f*(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]])]*Sqrt[a + c*x^2])]/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (f*(2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])]*Sqrt[a + c*x^2])]/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 725

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 976

Int[((a_.) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((2*a*c^2*e + c*(2*c^2*d - c*(2*a*f))*x)*(a + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1))/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), x] - Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - (-a*e)*(c*e))*(p + 1) - (2*c^2*d - c*(2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(-2*a*c^2*e)*(p + q + 2) + (2*f*(2*a*c^2*e)*(p + q + 2) - (2*c^2*d - c*(2*a*f))*(-(c*e*(2*p + q + 4)))]*x + c*f*(2*c^2*d - c*(2*a*f))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, c, d, e, f, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[a*c*e^2 + (c*d - a*f)^2, 0] && (!IntegerQ[p] && !LtQ[q, -1]) && !IGtQ[q, 0]

Rule 1034

Int[((g_.) + (h_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(

$(b - q)/q, \text{Int}[1/((b - q + 2*c*x)*\text{Sqrt}[d + f*x^2]), x], x] - \text{Dist}[(2*c*g - h*(b + q))/q, \text{Int}[1/((b + q + 2*c*x)*\text{Sqrt}[d + f*x^2]), x], x]] /; \text{FreeQ}[\{a, b, c, d, f, g, h\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{PosQ}[b^2 - 4*a*c]$

Rubi steps

$$\begin{aligned} \int \frac{1}{(a + cx^2)^{3/2} (d + ex + fx^2)} dx &= \frac{c(ae + (cd - af)x)}{a(ace^2 + (cd - af)^2) \sqrt{a + cx^2}} - \frac{\int \frac{-2ac(af^2 + c(e^2 - df)) - 2ac^2efx}{\sqrt{a + cx^2} (d + ex + fx^2)} dx}{2ac(ace^2 + (cd - af)^2)} \\ &= \frac{c(ae + (cd - af)x)}{a(ace^2 + (cd - af)^2) \sqrt{a + cx^2}} - \frac{(f(2af^2 + c(e^2 - 2df - e\sqrt{e^2 - 4df})))}{\sqrt{e^2 - 4df} (ace^2 + (cd - af)^2)} \\ &= \frac{c(ae + (cd - af)x)}{a(ace^2 + (cd - af)^2) \sqrt{a + cx^2}} + \frac{(f(2af^2 + c(e^2 - 2df - e\sqrt{e^2 - 4df})))}{\sqrt{e^2 - 4df} (ace^2 + (cd - af)^2)} \\ &= \frac{c(ae + (cd - af)x)}{a(ace^2 + (cd - af)^2) \sqrt{a + cx^2}} - \frac{f(2af^2 + c(e^2 - 2df + e\sqrt{e^2 - 4df}))}{\sqrt{2} \sqrt{e^2 - 4df} (ace^2 + (cd - af)^2)} \end{aligned}$$

Mathematica [A] time = 2.06, size = 320, normalized size = 0.77

$$\frac{c(a(e - fx) + cdx)}{a\sqrt{a + cx^2} (a^2f^2 + ac(e^2 - 2df) + c^2d^2)} - \frac{2\sqrt{2}f^3 \tanh^{-1}\left(\frac{2af + cx(\sqrt{e^2 - 4df} - e)}{\sqrt{a + cx^2} \sqrt{4af^2 - 2c(e\sqrt{e^2 - 4df} + 2df - e^2)}}\right)}{\sqrt{e^2 - 4df} (2af^2 + c(-e\sqrt{e^2 - 4df} - 2df + e^2))^{3/2}} + \frac{2\sqrt{2}f^3 \tanh^{-1}\left(\frac{2af - cx(\sqrt{e^2 - 4df} + e)}{\sqrt{a + cx^2} \sqrt{4af^2 + 2c(e\sqrt{e^2 - 4df} - 2df + e^2)}}\right)}{\sqrt{e^2 - 4df} (2af^2 + c(e\sqrt{e^2 - 4df} - 2df + e^2))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x]

[Out] $(c*(c*d*x + a*(e - f*x)))/(a*(c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))*\text{Sqrt}[a + c*x^2] - (2*\text{Sqrt}[2]*f^3*\text{ArcTanh}[(2*a*f + c*(-e + \text{Sqrt}[e^2 - 4*d*f]))*x]/(\text{Sqrt}[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f])]*\text{Sqrt}[a + c*x^2]))/(\text{Sqrt}[e^2 - 4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f - e*\text{Sqrt}[e^2 - 4*d*f]))^{3/2}) + (2*\text{Sqrt}[2]*f^3*\text{ArcTanh}[(2*a*f - c*(e + \text{Sqrt}[e^2 - 4*d*f]))*x]/(\text{Sqrt}[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f])]*\text{Sqrt}[a + c*x^2]))/(\text{Sqrt}[e^2 - 4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]))^{3/2})$

IntegrateAlgebraic [C] time = 0.64, size = 368, normalized size = 0.88

$$\frac{\text{RootSum}\left[\frac{\#1^4 f - 2\#1^3 \sqrt{c} e - 2\#1^2 a f + 4\#1^2 c d + 2\#1 a \sqrt{c} e + a^2 f \&e, -\#1^2 c f \log(-\#1 + \sqrt{a + c x^2} - \sqrt{c}) - 2\#1 a^2 d f \log(-\#1 + \sqrt{a + c x^2} - \sqrt{c}) + 2\#1 c^2 d^2 \log(-\#1 + \sqrt{a + c x^2} - \sqrt{c}) + a c f \log(-\#1 + \sqrt{a + c x^2} - \sqrt{c}) + 2\#1 a \sqrt{c} f^2 \log(-\#1 + \sqrt{a + c x^2} - \sqrt{c}) \&e}{-a^2 f^2 + 2a c d f - a c^2 - c^2 d^2}\right] + \frac{a c e - a c f x + c^2 d x}{a \sqrt{a + c x^2} (a^2 f^2 - 2a c d f + a c^2 + c^2 d^2)}}{a \sqrt{a + c x^2} (a^2 f^2 + a c (e^2 - 2 d f) + c^2 d^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x]

[Out] $(a*c*e + c^2*d*x - a*c*f*x)/(a*(c^2*d^2 + a*c*e^2 - 2*a*c*d*f + a^2*f^2))*\text{Sqrt}[a + c*x^2] + \text{RootSum}[a^2*f + 2*a*\text{Sqrt}[c]*e*\#1 + 4*c*d*\#1^2 - 2*a*f*\#1^2 - 2*\text{Sqrt}[c]*e*\#1^3 + f*\#1^4 \& , (a*c*e*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + c*x^2] - \#1] + 2*c^(3/2)*e^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + c*x^2] - \#1]*\#1 - 2*c^(3/2)*d*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + c*x^2] - \#1]*\#1 + 2*a*\text{Sqrt}[c]*f^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + c*x^2] - \#1]*\#1 - c*e*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + c*x^2] - \#1]*\#1^2)/(a*\text{Sqrt}[c]*e + 4*c*d*\#1 - 2*a*f*\#1 - 3*\text{Sqrt}[c]*e*\#1^2 + 2*f*\#1^3) \&]/(-(c^2*d^2) - a*c*e^2 + 2*a*c*d*f - a^2*f^2)$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage₂

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] sage2

maple [B] time = 0.02, size = 1713, normalized size = 4.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)

[Out]
$$\begin{aligned} & -2/(-4*d*f+e^2)^{(1/2)}/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)*f^2/((\\ & x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-(e+(-4*d*f+e^2)^{(1/2)})*(x+1/2*(e+(-4*d*f \\ & +e^2)^{(1/2)})/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2 \\ &)^{(1/2)}-4*c^2*f/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/(4*a*c-4*c^2 \\ & /f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)^2* \\ & c-(e+(-4*d*f+e^2)^{(1/2)})*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+1/2*(2*a*f^2- \\ & 2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*x-4/(-4*d*f+e^2)^{(1/2)}*c^2 \\ & *f/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2* \\ & e^2-c^2/f^2*(-4*d*f+e^2))/((x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-(e+(-4*d*f+ \\ & e^2)^{(1/2)})*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2 \\ & +(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*e*x+2/(-4*d*f+e^2)^{(1/2)}/(2*a*f^2-2*c*d \\ & *f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)*f^2*2^{(1/2)}/((2*a*f^2-2*c*d*f+c*e^2+(-4*d* \\ & f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*\ln((-e+(-4*d*f+e^2)^{(1/2)})*(x+1/2*(e+(-4*d*f+ \\ & e^2)^{(1/2)})/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2 \\ & ^{(1/2)}*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x+1/2 \\ & *(e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-4*(e+(-4*d*f+e^2)^{(1/2)})*(x+1/2*(e+(-4*d*f+e \\ & ^2)^{(1/2)})/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/ \\ & 2)}/(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f))+2/(-4*d*f+e^2)^{(1/2)}/(2*a*f^2-2*c*d*f \\ & +c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)*f^2/((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-(\\ & e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+1/2*(2*a*f^2-2* \\ & c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}-4*c^2*f/(2*a*f^2-2*c*d*f+c*e \\ & ^2-(-4*d*f+e^2)^{(1/2)}*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2 \\ &))/((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-(e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e \\ & +(-4*d*f+e^2)^{(1/2)})/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c \\ & *e)/f^2)^{(1/2)}*x+4/(-4*d*f+e^2)^{(1/2)}*c^2*f/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+ \\ & e^2)^{(1/2)}*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x-1/2* \\ & (-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c-(e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e \\ & ^2)^{(1/2)})/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1 \\ & /2)}*e*x-2/(-4*d*f+e^2)^{(1/2)}/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e) \\ & *f^2*2^{(1/2)}/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*\ln(\\ & (-e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+(2*a*f^2-2*c \\ & *d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-2*c*d*f+c*e^2- \\ & (-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c \\ & -4*(e-(-4*d*f+e^2)^{(1/2)})*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)*c/f+2*(2*a*f^2- \end{aligned}$$

$2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)*c*e}/f^2)^{(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2))/f))$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(cx^2 + a)^{3/2} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)

[Out] int(1/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + cx^2)^{\frac{3}{2}} (d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)

[Out] Integral(1/((a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)

$$3.75 \quad \int \frac{1}{x(a+cx^2)^{3/2}(d+ex+fx^2)} dx$$

Optimal. Leaf size=526

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{a(af^2+c(e^2-df))+c^2dex}{ad\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)} + \frac{f\left(2e(af^2+c(e^2-2df))-(e-\sqrt{e^2-4df})(af^2+c(e^2-2df))\right)}{\sqrt{2}d\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2a}}$$

Rubi [A] time = 2.18, antiderivative size = 526, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 27, number of rules / integrand size = 0.333, Rules used = {6728, 266, 51, 63, 208, 1017, 1034, 725, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{a(af^2+c(e^2-df))+c^2dex}{ad\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)} + \frac{f\left(2e(af^2+c(e^2-2df))-(e-\sqrt{e^2-4df})(af^2+c(e^2-df))\right)}{\sqrt{2}d\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2a}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

[Out] 1/(a*d*Sqrt[a + c*x^2]) - (a*(a*f^2 + c*(e^2 - d*f)) + c^2*d*e*x)/(a*d*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2]) + (f*(2*e*(a*f^2 + c*(e^2 - 2*d*f)) - (e - Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]]) - (f*(2*e*(a*f^2 + c*(e^2 - 2*d*f)) - (e + Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2]])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]]) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(a^(3/2)*d)

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 206

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (c_)*(x_)^2]), x_Symbol] := -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1017

```
Int[((g_) + (h_)*(x_))*((a_) + (c_)*(x_)^2)^(p_)*((d_) + (e_)*(x_) + (f
_)*(x_)^2)^(q_), x_Symbol] := Simp[((a + c*x^2)^(p + 1)*(d + e*x + f*x^2)^
(q + 1)*(g*c*(2*a*c*e) + (-a*h))*(2*c^2*d - c*(2*a*f)) + c*(g*(2*c^2*d - c
*(2*a*f)) - h*(-2*a*c*e))*x)/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)),
x] + Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^
(p + 1)*(d + e*x + f*x^2)^q*Simp[(-2*g*c)*((c*d - a*f)^2 - (-a*e))*(c*e))*
(p + 1) + (2*(g*c*(c*d - a*f) - a*(-h*c*e)))*(a*f*(p + 1) - c*d*(p + 2))
- e*((g*c)*(2*a*c*e) + (-a*h))*(2*c^2*d - c*(Plus[2]*a*f))*(p + q + 2)
- (2*f*((g*c)*(2*a*c*e) + (-a*h))*(2*c^2*d - c*(Plus[2]*a*f))*(p + q +
2) - (2*(g*c*(c*d - a*f) - a*(-h*c*e)))*(-c*e*(2*p + q + 4)))*x - c*f*(
2*(g*c*(c*d - a*f) - a*(-h*c*e)))*(2*p + 2*q + 5)*x^2, x], x] /; Free
Q[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[
a*c*e^2 + (c*d - a*f)^2, 0] && !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1034

```
Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (f
_)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x_Symbol] := With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x(a+cx^2)^{3/2}(d+ex+fx^2)} dx = \int \left(\frac{1}{dx(a+cx^2)^{3/2}} + \frac{-e-fx}{d(a+cx^2)^{3/2}(d+ex+fx^2)} \right) dx$$

$$= \frac{\int \frac{1}{x(a+cx^2)^{3/2}} dx}{d} + \frac{\int \frac{-e-fx}{(a+cx^2)^{3/2}(d+ex+fx^2)} dx}{d}$$

$$= \frac{a(af^2+c(e^2-df))+c^2dex}{ad(ace^2+(cd-af)^2)\sqrt{a+cx^2}} + \frac{\text{Subst}\left(\int \frac{1}{x(a+cx)^{3/2}} dx, x, x^2\right)}{2d} + \frac{\int \frac{-2ace}{(a+cx)^{3/2}} dx}{2d}$$

$$= \frac{1}{ad\sqrt{a+cx^2}} - \frac{a(af^2+c(e^2-df))+c^2dex}{ad(ace^2+(cd-af)^2)\sqrt{a+cx^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}} dx, x, x^2\right)}{2ad}$$

$$= \frac{1}{ad\sqrt{a+cx^2}} - \frac{a(af^2+c(e^2-df))+c^2dex}{ad(ace^2+(cd-af)^2)\sqrt{a+cx^2}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}} dx, x, \sqrt{a+cx}\right)}{acd}$$

$$= \frac{1}{ad\sqrt{a+cx^2}} - \frac{a(af^2+c(e^2-df))+c^2dex}{ad(ace^2+(cd-af)^2)\sqrt{a+cx^2}} + \frac{f(2e(af^2+c(e^2-2df)))}{\sqrt{2}d\sqrt{a+cx^2}}$$

Mathematica [C] time = 3.50, size = 497, normalized size = 0.94

$$\frac{f\left(\frac{e}{\sqrt{2-4df}}+1\right)(2af+cx(e-\sqrt{2-4df}))}{a\sqrt{a+cx^2}(4af^2+c(e-\sqrt{2-4df})^2)} - \frac{f\left(1-\frac{e}{\sqrt{2-4df}}\right)(2af+cx(\sqrt{2-4df}+e))}{a\sqrt{a+cx^2}(4af^2+c(\sqrt{2-4df}+e)^2)} + \frac{\sqrt{2}f^3(\sqrt{2-4df}+e)\tanh^{-1}\left(\frac{2af+cx(\sqrt{2-4df}-e)}{\sqrt{a+cx^2}\sqrt{4af^2-2c(\sqrt{2-4df}+2df-2)}}\right)}{\sqrt{2-4df}(2af^2+c(-e\sqrt{2-4df}-2df+e^2))^2} + \frac{\sqrt{2}f^3(\sqrt{2-4df}-e)\tanh^{-1}\left(\frac{2af-cx(\sqrt{2-4df}+e)}{\sqrt{a+cx^2}\sqrt{4af^2+2c(\sqrt{2-4df}-2df+e^2)}}\right)}{\sqrt{2-4df}(2af^2+c(e\sqrt{2-4df}-2df+e^2))^2} + \frac{{}_2F_1\left(-\frac{1}{2}, 1, \frac{1}{2}, \frac{cx^2}{a}+1\right)}{a\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x]

[Out] $(-(f*(1 + e/\text{Sqrt}[e^2 - 4*d*f]))*(2*a*f + c*(e - \text{Sqrt}[e^2 - 4*d*f]))*x)/(a*(4*a*f^2 + c*(e - \text{Sqrt}[e^2 - 4*d*f])^2)*\text{Sqrt}[a + c*x^2]) - (f*(1 - e/\text{Sqrt}[e^2 - 4*d*f]))*(2*a*f + c*(e + \text{Sqrt}[e^2 - 4*d*f]))*x)/(a*(4*a*f^2 + c*(e + \text{Sqrt}[e^2 - 4*d*f])^2)*\text{Sqrt}[a + c*x^2]) + (\text{Sqrt}[2]*f^3*(e + \text{Sqrt}[e^2 - 4*d*f]))*\text{ArcTanh}[(2*a*f + c*(-e + \text{Sqrt}[e^2 - 4*d*f]))*x]/(\text{Sqrt}[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]))*\text{Sqrt}[a + c*x^2])]/(\text{Sqrt}[e^2 - 4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f - e*\text{Sqrt}[e^2 - 4*d*f]))^(3/2)) + (\text{Sqrt}[2]*f^3*(-e + \text{Sqrt}[e^2 - 4*d*f]))*\text{ArcTanh}[(2*a*f - c*(e + \text{Sqrt}[e^2 - 4*d*f]))*x]/(\text{Sqrt}[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]))*\text{Sqrt}[a + c*x^2])]/(\text{Sqrt}[e^2 - 4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]))^(3/2)) + \text{Hypergeometric2F1}[-1/2, 1, 1/2, 1 + (c*x^2)/a]/(a*\text{Sqrt}[a + c*x^2])/d$

IntegrateAlgebraic [C] time = 0.95, size = 552, normalized size = 1.05

$$\frac{\text{RootSum}\left[\frac{e^2 f^2 - 2af^2 + ac^2 + c^2 d}{d(e^2 - 2adf + ac^2 + c^2 d)}\right] \sqrt{\frac{e^2 f^2 - 2af^2 + ac^2 + c^2 d}{d(e^2 - 2adf + ac^2 + c^2 d)}}}{2 \text{tanh}^{-1}\left(\frac{\sqrt{c} x}{\sqrt{a}}\right)} + \frac{-def + c^2 d - e^2 c}{a\sqrt{a+cx^2}(e^2 - 2adf + ac^2 + c^2 d)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x]

[Out] $(c^2*d - a*c*f - c^2*e*x)/(a*(c^2*d^2 + a*c*e^2 - 2*a*c*d*f + a^2*f^2)*\text{Sqrt}[a + c*x^2]) + (2*\text{ArcTanh}[(\text{Sqrt}[c]*x)/\text{Sqrt}[a] - \text{Sqrt}[a + c*x^2]/\text{Sqrt}[a]])/(a^(3/2)*d) + \text{RootSum}[a^2*f + 2*a*\text{Sqrt}[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*\text{Sqrt}[c]*e*#1^3 + f*#1^4 \& , (a*c*e^2*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + c*x^2])]$

- #1] - a*c*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + a^2*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*c^(3/2)*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 4*c^(3/2)*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + 2*a*Sqrt[c]*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - c*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 + c*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - a*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) &]/(d*(c^2*d^2 + a*c*e^2 - 2*a*c*d*f + a^2*f^2))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.02, size = 1945, normalized size = 3.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)

[Out]
$$\frac{4f^3(e+(-4df+e^2)^{1/2})/(-4df+e^2)^{1/2}/(2af^2-2cdf+ce^2+(-4df+e^2)^{1/2}ce)/((x+1/2(e+(-4df+e^2)^{1/2})/f)^2c-(e+(-4df+e^2)^{1/2})(x+1/2(e+(-4df+e^2)^{1/2})/f)cf+1/2(2af^2-2cdf+ce^2+(-4df+e^2)^{1/2}ce)/f^2)^{1/2}+8f^2/(e+(-4df+e^2)^{1/2})c^2/(2af^2-2cdf+ce^2+(-4df+e^2)^{1/2}ce)/(4ac-4c^2d/f+c^2e^2/f^2-(-4df+e^2)c^2/f^2)/((x+1/2(e+(-4df+e^2)^{1/2})/f)^2c-(e+(-4df+e^2)^{1/2})(x+1/2(e+(-4df+e^2)^{1/2})/f)cf+1/2(2af^2-2cdf+ce^2+(-4df+e^2)^{1/2}ce)/f^2)^{1/2}+8f^2/(e+(-4df+e^2)^{1/2})/(-4df+e^2)^{1/2}c^2/(2af^2-2cdf+ce^2+(-4df+e^2)^{1/2}ce)/(4ac-4c^2d/f+c^2e^2/f^2-(-4df+e^2)c^2/f^2)/((x+1/2(e+(-4df+e^2)^{1/2})/f)^2c-(e+(-4df+e^2)^{1/2})(x+1/2(e+(-4df+e^2)^{1/2})/f)cf+1/2(2af^2-2cdf+ce^2+(-4df+e^2)^{1/2}ce)/f^2)^{1/2}+e^2x-4f^3/(e+(-4df+e^2)^{1/2})/(-4df+e^2)^{1/2}/(2af^2-2cdf+ce^2+(-4df+e^2)^{1/2}ce)*2^{1/2}/((2af^2-2cdf+ce^2+(-4df+e^2)^{1/2}ce)/f^2)^{1/2}*\ln((-e+(-4df+e^2)^{1/2})(x+1/2(e+(-4df+e^2)^{1/2})/f)cf+(2af^2-2cdf+ce^2+(-4df+e^2)^{1/2}ce)/f^2+1/2*2^{1/2}*((2af^2-2cdf+ce^2+(-4df+e^2)^{1/2}ce)/f^2)^{1/2}*(4*(x+1/2(e+(-4df+e^2)^{1/2})/f)^2c-4*(e+(-4df+e^2)^{1/2})*(x+1/2(e+(-4df+e^2)^{1/2})/f)cf+2*(2af^2-2cdf+ce^2+(-4df+e^2)^{1/2}ce)/f^2)^{1/2})/(x+1/2(e+(-4df+e^2)^{1/2})/f)+4f^3/(-e+(-4df+e^2)^{1/2})/(-4df+e^2)^{1/2}/(2af^2-2cdf+ce^2-(-4df+e^2)^{1/2}ce)/((x-1/2(-e+(-4df+e^2)^{1/2})/f)^2c-(e+(-4df+e^2)^{1/2})*(x-1/2(-e+(-4df+e^2)^{1/2})/f)cf+1/2(2af^2-2cdf+ce^2-(-4df+e^2)^{1/2}ce)/f^2)^{1/2}-8f^2/(-e+(-4df+e^2)^{1/2})c^2/(2af^2-2cdf+ce^2-(-4df+e^2)^{1/2}ce)/(4ac-4c^2d/f+c^2e^2/f^2-(-4df+e^2)c^2/f^2)/((x-1/2(-e+(-4df+e^2)^{1/2})/f)^2c-(e+(-4df+e^2)^{1/2})*(x-1/2(-e+(-4df+e^2)^{1/2})/f)cf+1/2(2af^2-2cdf+ce^2-(-4df+e^2)^{1/2}ce)/f^2)^{1/2}}$$

$$\begin{aligned} & \left(-2 \right)^{\frac{1}{2}} * x + 8 * f^2 / \left(-e + \left(-4 * d * f + e^2 \right)^{\frac{1}{2}} \right) / \left(-4 * d * f + e^2 \right)^{\frac{1}{2}} * c^2 / \left(2 * a * f^2 - 2 * c * d * f + c * e^2 - \left(-4 * d * f + e^2 \right)^{\frac{1}{2}} * c * e \right) / \left(4 * a * c - 4 * c^2 * d / f + c^2 * e^2 / f^2 - \left(-4 * d * f + e^2 \right)^{\frac{1}{2}} * c^2 / f^2 \right) / \left(\left(x - 1/2 * \left(-e + \left(-4 * d * f + e^2 \right)^{\frac{1}{2}} \right) / f \right)^2 * c - \left(e - \left(-4 * d * f + e^2 \right)^{\frac{1}{2}} \right) * \left(x - 1/2 * \left(-e + \left(-4 * d * f + e^2 \right)^{\frac{1}{2}} \right) / f \right) * c / f + 1/2 * \left(2 * a * f^2 - 2 * c * d * f + c * e^2 - \left(-4 * d * f + e^2 \right)^{\frac{1}{2}} * c * e \right) / f^2 \right)^{\frac{1}{2}} * e * x - 4 * f^3 / \left(-e + \left(-4 * d * f + e^2 \right)^{\frac{1}{2}} \right) / \left(-4 * d * f + e^2 \right)^{\frac{1}{2}} / \left(2 * a * f^2 - 2 * c * d * f + c * e^2 - \left(-4 * d * f + e^2 \right)^{\frac{1}{2}} * c * e \right) * 2^{\frac{1}{2}} / \left(\left(2 * a * f^2 - 2 * c * d * f + c * e^2 - \left(-4 * d * f + e^2 \right)^{\frac{1}{2}} * c * e \right) / f^2 \right)^{\frac{1}{2}} * \ln \left(\left(-e - \left(-4 * d * f + e^2 \right)^{\frac{1}{2}} \right) * \left(x - 1/2 * \left(-e + \left(-4 * d * f + e^2 \right)^{\frac{1}{2}} \right) / f \right) * c / f + \left(2 * a * f^2 - 2 * c * d * f + c * e^2 - \left(-4 * d * f + e^2 \right)^{\frac{1}{2}} * c * e \right) / f^2 + 1/2 * 2^{\frac{1}{2}} * \left(\left(2 * a * f^2 - 2 * c * d * f + c * e^2 - \left(-4 * d * f + e^2 \right)^{\frac{1}{2}} * c * e \right) / f^2 \right)^{\frac{1}{2}} * \left(4 * \left(x - 1/2 * \left(-e + \left(-4 * d * f + e^2 \right)^{\frac{1}{2}} \right) / f \right)^2 * c - 4 * \left(e - \left(-4 * d * f + e^2 \right)^{\frac{1}{2}} \right) * \left(x - 1/2 * \left(-e + \left(-4 * d * f + e^2 \right)^{\frac{1}{2}} \right) / f \right) * c / f + 2 * \left(2 * a * f^2 - 2 * c * d * f + c * e^2 - \left(-4 * d * f + e^2 \right)^{\frac{1}{2}} * c * e \right) / f^2 \right)^{\frac{1}{2}} \right) / \left(x - 1/2 * \left(-e + \left(-4 * d * f + e^2 \right)^{\frac{1}{2}} \right) / f \right) - 4 * f / \left(-e + \left(-4 * d * f + e^2 \right)^{\frac{1}{2}} \right) / \left(e + \left(-4 * d * f + e^2 \right)^{\frac{1}{2}} \right) / a / \left(c * x^2 + a \right)^{\frac{1}{2}} + 4 * f / \left(-e + \left(-4 * d * f + e^2 \right)^{\frac{1}{2}} \right) / \left(e + \left(-4 * d * f + e^2 \right)^{\frac{1}{2}} \right) / a^{\frac{3}{2}} * \ln \left(\left(2 * a + 2 * \left(c * x^2 + a \right)^{\frac{1}{2}} * a^{\frac{1}{2}} \right) / x \right) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + a)^{\frac{3}{2}} (fx^2 + ex + d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] integrate(1/((c*x^2 + a)^(3/2)*(f*x^2 + e*x + d)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x (cx^2 + a)^{\frac{3}{2}} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)

[Out] int(1/(x*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x (a + cx^2)^{\frac{3}{2}} (d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)

[Out] Integral(1/(x*(a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)

$$3.76 \quad \int \frac{1}{x^2(a+cx^2)^{3/2}(d+ex+fx^2)} dx$$

Optimal. Leaf size=618

$$\frac{e \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^2} - \frac{2cx}{a^2d\sqrt{a+cx^2}} + \frac{cdx(af^2+c(e^2-df))+ae(af^2+c(e^2-2df))}{ad^2\sqrt{a+cx^2}((cd-af)^2+ace^2)} + \frac{f(e(e-\sqrt{e^2-4df}))(a$$

Rubi [A] time = 2.28, antiderivative size = 618, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 27, number of rules / integrand size = 0.407, Rules used = {6728, 271, 191, 266, 51, 63, 208, 1017, 1034, 725, 206}

$$\frac{e \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^2} - \frac{2cx}{a^2d\sqrt{a+cx^2}} + \frac{cdx(af^2+c(e^2-df))+ae(af^2+c(e^2-2df))}{ad^2\sqrt{a+cx^2}((cd-af)^2+ace^2)} + \frac{f(e(e-\sqrt{e^2-4df}))(a$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

[Out] -(e/(a*d^2*Sqrt[a + c*x^2])) - 1/(a*d*x*Sqrt[a + c*x^2]) - (2*c*x)/(a^2*d*Sqrt[a + c*x^2]) + (a*e*(a*f^2 + c*(e^2 - 2*d*f)) + c*d*(a*f^2 + c*(e^2 - d*f))*x)/(a*d^2*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2]) + (f*(e*(e - Sqrt[e^2 - 4*d*f]))*(a*f^2 + c*(e^2 - 2*d*f)) - 2*(a*f^2*(e^2 - d*f) + c*(e^4 - 3*d*e^2*f + d^2*f^2)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f]))*x]/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2])]/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (f*(e*(e + Sqrt[e^2 - 4*d*f]))*(a*f^2 + c*(e^2 - 2*d*f)) - 2*(a*f^2*(e^2 - d*f) + c*(e^4 - 3*d*e^2*f + d^2*f^2)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f]))*x]/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]])*Sqrt[a + c*x^2])]/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) + (e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(a^(3/2)*d^2)

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 191

Int[((a_.) + (b_.)*(x_))^(n_)]^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 206

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

$Q[a, 0] \parallel LtQ[b, 0]$

Rule 208

$Int[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[\{a, b\}, x] \&\& NegQ[a/b]$

Rule 266

$Int[(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}), x_Symbol] \rightarrow Dist[1/n, Subst[Int[x^{(Simplify[(m + 1)/n] - 1)*(a + b*x)^p}, x], x, x^n], x] /; FreeQ[\{a, b, m, n, p\}, x] \&\& IntegerQ[Simplify[(m + 1)/n]]$

Rule 271

$Int[(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}), x_Symbol] \rightarrow Simp[(x^{(m + 1)}*(a + b*x^n)^{(p + 1)})/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m + 1)), Int[x^{(m + n)}*(a + b*x^n)^p, x], x] /; FreeQ[\{a, b, m, n, p\}, x] \&\& ILtQ[Simplify[(m + 1)/n + p + 1], 0] \&\& NeQ[m, -1]$

Rule 725

$Int[1/(((d_ + (e_)*(x_))*Sqrt[(a_ + (c_)*(x_)^2])), x_Symbol] \rightarrow -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[\{a, c, d, e\}, x]$

Rule 1017

$Int[((g_ + (h_)*(x_))*((a_ + (c_)*(x_)^2)^{(p_)}*((d_ + (e_)*(x_ + (f_)*(x_)^2)^{(q_)}), x_Symbol] \rightarrow Simp[((a + c*x^2)^{(p + 1)}*(d + e*x + f*x^2)^{(q + 1)}*(g*c*(2*a*c*e) + (-a*h)*(2*c^2*d - c*(2*a*f)) + c*(g*(2*c^2*d - c*(2*a*f)) - h*(-2*a*c*e))*x)/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^{(p + 1)}*(d + e*x + f*x^2)^q*Simp[(-2*g*c)*((c*d - a*f)^2 - (-a*e)*(c*e))^(p + 1) + (2*(g*c*(c*d - a*f) - a*(-h*c*e)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e) + (-a*h)*(2*c^2*d - c*(Plus[2]*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*a*c*e) + (-a*h)*(2*c^2*d - c*(Plus[2]*a*f)))*(p + q + 2) - (2*(g*c*(c*d - a*f) - a*(-h*c*e)))*(-c*e*(2*p + q + 4)))*x - c*f*(2*(g*c*(c*d - a*f) - a*(-h*c*e)))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[\{a, c, d, e, f, g, h, q\}, x] \&\& NeQ[e^2 - 4*d*f, 0] \&\& LtQ[p, -1] \&\& NeQ[a*c*e^2 + (c*d - a*f)^2, 0] \&\& !(IntegerQ[p] \&\& ILtQ[q, -1])$

Rule 1034

$Int[((g_ + (h_)*(x_))/((a_ + (b_)*(x_ + (c_)*(x_)^2))*Sqrt[(d_ + (f_)*(x_)^2])), x_Symbol] \rightarrow With[\{q = Rt[b^2 - 4*a*c, 2]\}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[\{a, b, c, d, f, g, h\}, x] \&\& NeQ[b^2 - 4*a*c, 0] \&\& PosQ[b^2 - 4*a*c]$

Rule 6728

$Int[(u_)/((a_ + (b_)*(x_)^{(n_)} + (c_)*(x_)^{(2*n_)}), x_Symbol] \rightarrow With[\{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]\}, Int[v, x] /; SumQ[v]] /; FreeQ[\{a, b, c\}, x] \&\& EqQ[n2, 2*n] \&\& IGtQ[n, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 (a + cx^2)^{3/2} (d + ex + fx^2)} dx &= \int \left(\frac{1}{dx^2 (a + cx^2)^{3/2}} - \frac{e}{d^2 x (a + cx^2)^{3/2}} + \frac{e^2 - df + efx}{d^2 (a + cx^2)^{3/2} (d + ex + fx^2)} \right) dx \\
&= \frac{\int \frac{e^2 - df + efx}{(a + cx^2)^{3/2} (d + ex + fx^2)} dx}{d^2} + \frac{\int \frac{1}{x^2 (a + cx^2)^{3/2}} dx}{d} - \frac{e \int \frac{1}{x (a + cx^2)^{3/2}} dx}{d^2} \\
&= -\frac{1}{adx\sqrt{a + cx^2}} + \frac{ae (af^2 + c (e^2 - 2df)) + cd (af^2 + c (e^2 - df)) x}{ad^2 (ace^2 + (cd - af)^2) \sqrt{a + cx^2}} \\
&= -\frac{e}{ad^2\sqrt{a + cx^2}} - \frac{1}{adx\sqrt{a + cx^2}} - \frac{2cx}{a^2 d\sqrt{a + cx^2}} + \frac{ae (af^2 + c (e^2 - 2df))}{ad^2 (ace^2 + (cd - af)^2)} \\
&= -\frac{e}{ad^2\sqrt{a + cx^2}} - \frac{1}{adx\sqrt{a + cx^2}} - \frac{2cx}{a^2 d\sqrt{a + cx^2}} + \frac{ae (af^2 + c (e^2 - 2df))}{ad^2 (ace^2 + (cd - af)^2)} \\
&= -\frac{e}{ad^2\sqrt{a + cx^2}} - \frac{1}{adx\sqrt{a + cx^2}} - \frac{2cx}{a^2 d\sqrt{a + cx^2}} + \frac{ae (af^2 + c (e^2 - 2df))}{ad^2 (ace^2 + (cd - af)^2)}
\end{aligned}$$

Mathematica [C] time = 3.60, size = 557, normalized size = 0.90

$$\frac{d(e+2cx^2)}{a^2x\sqrt{a+cx^2}} - \frac{f\left(\frac{2-2df}{\sqrt{2-4df}}+e\right)(2af+cx(e-\sqrt{2-4df}))}{a\sqrt{a+cx^2}\left(4af^2+c(e-\sqrt{2-4df})^2\right)} - \frac{f\left(\frac{2df-c^2}{\sqrt{2-4df}}+e\right)(2af+cx(\sqrt{2-4df}+e))}{a\sqrt{a+cx^2}\left(4af^2+c(\sqrt{2-4df}+e)^2\right)} + \frac{\sqrt{2}f^3(e\sqrt{2-4df}-2df+c^2)\tanh^{-1}\left(\frac{2af+cx(\sqrt{2-4df}+e)}{\sqrt{a+cx^2}\sqrt{4af^2-2c(\sqrt{2-4df}+2df-c^2)}}\right)}{\sqrt{2-4df}\left(2af^2+c(-c\sqrt{2-4df}-2df+c^2)\right)^{3/2}} + \frac{\sqrt{2}f^3(c\sqrt{2-4df}+2df-c^2)\tanh^{-1}\left(\frac{2af-cx(\sqrt{2-4df}+e)}{\sqrt{a+cx^2}\sqrt{4af^2+2c(\sqrt{2-4df}-2df+c^2)}}\right)}{\sqrt{2-4df}\left(2af^2+c(e\sqrt{2-4df}-2df+c^2)\right)^{3/2}} + \frac{e^2F_1\left(-\frac{1}{2}, 1, \frac{1}{2}, 1 + \frac{cx^2}{a}\right)}{a\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

[Out]
$$\begin{aligned}
& -\left(\left(-\left(\frac{f(e + (e^2 - 2df)/\sqrt{e^2 - 4df})}{\sqrt{e^2 - 4df}}\right)(2af + c(e - \sqrt{e^2 - 4df}))\right)*x\right) / \left(a(4af^2 + c(e - \sqrt{e^2 - 4df})^2)\sqrt{a + cx^2}\right) - \left(\frac{f(e + (-e^2 + 2df)/\sqrt{e^2 - 4df})}{\sqrt{e^2 - 4df}}\right)(2af + c(e + \sqrt{e^2 - 4df}))x \\
& / \left(a(4af^2 + c(e + \sqrt{e^2 - 4df})^2)\sqrt{a + cx^2}\right) + \left(\frac{d(a + 2cx^2)}{a^2x\sqrt{a + cx^2}}\right) + \left(\frac{\sqrt{2}f^3(e^2 - 2df + e\sqrt{e^2 - 4df})}{\sqrt{2-4df}}\right) \text{ArcTanh}\left[\frac{(2af + c(-e + \sqrt{e^2 - 4df}))x}{(\sqrt{4af^2 - 2c(-e^2 + 2df + e\sqrt{e^2 - 4df})})\sqrt{a + cx^2}}\right] \\
& / \left(\sqrt{e^2 - 4df}(2af^2 + c(e^2 - 2df - e\sqrt{e^2 - 4df}))^{3/2}\right) + \left(\frac{\sqrt{2}f^3(-e^2 + 2df + e\sqrt{e^2 - 4df})}{\sqrt{2-4df}}\right) \text{ArcTanh}\left[\frac{(2af - c(e + \sqrt{e^2 - 4df}))x}{(\sqrt{4af^2 + 2c(e^2 - 2df + e\sqrt{e^2 - 4df})})\sqrt{a + cx^2}}\right] \\
& / \left(\sqrt{e^2 - 4df}(2af^2 + c(e^2 - 2df + e\sqrt{e^2 - 4df}))^{3/2}\right) + \left(\frac{e^2 \text{Hypergeometric2F1}\left[-\frac{1}{2}, 1, \frac{1}{2}, 1 + \frac{cx^2}{a}\right]}{a\sqrt{a + cx^2}}\right) / d^2
\end{aligned}$$

IntegrateAlgebraic [C] time = 1.29, size = 720, normalized size = 1.17

$$\frac{d(e+2cx^2)}{a^2x\sqrt{a+cx^2}} - \frac{f\left(\frac{2-2df}{\sqrt{2-4df}}+e\right)(2af+cx(e-\sqrt{2-4df}))}{a\sqrt{a+cx^2}\left(4af^2+c(e-\sqrt{2-4df})^2\right)} - \frac{f\left(\frac{2df-c^2}{\sqrt{2-4df}}+e\right)(2af+cx(\sqrt{2-4df}+e))}{a\sqrt{a+cx^2}\left(4af^2+c(\sqrt{2-4df}+e)^2\right)} + \frac{\sqrt{2}f^3(e\sqrt{2-4df}-2df+c^2)\tanh^{-1}\left(\frac{2af+cx(\sqrt{2-4df}+e)}{\sqrt{a+cx^2}\sqrt{4af^2-2c(\sqrt{2-4df}+2df-c^2)}}\right)}{\sqrt{2-4df}\left(2af^2+c(-c\sqrt{2-4df}-2df+c^2)\right)^{3/2}} + \frac{\sqrt{2}f^3(c\sqrt{2-4df}+2df-c^2)\tanh^{-1}\left(\frac{2af-cx(\sqrt{2-4df}+e)}{\sqrt{a+cx^2}\sqrt{4af^2+2c(\sqrt{2-4df}-2df+c^2)}}\right)}{\sqrt{2-4df}\left(2af^2+c(e\sqrt{2-4df}-2df+c^2)\right)^{3/2}} + \frac{e^2F_1\left(-\frac{1}{2}, 1, \frac{1}{2}, 1 + \frac{cx^2}{a}\right)}{a\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

[Out]
$$(-a^2c^2d^2 - a^2c^2e^2 + 2a^2c^2d^2f - a^2c^2f^2 - a^2c^2d^2e^2x - 2c^3d^2x^2 - a^2c^2e^2x^2 + 3a^2c^2d^2fx^2 - a^2c^2f^2x^2) / (a^2d(c^2d^2 + a$$

```
*c*e^2 - 2*a*c*d*f + a^2*f^2)*x*Sqrt[a + c*x^2]) - (2*e*ArcTanh[(Sqrt[c]*x)
/Sqrt[a] - Sqrt[a + c*x^2]/Sqrt[a]])/(a^(3/2)*d^2) - RootSum[a^2*f + 2*a*Sq
rt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (a*c*e
^3*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - 2*a*c*d*e*f^2*Log[-(Sqrt[c]
*x) + Sqrt[a + c*x^2] - #1] + a^2*e*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]
- #1] + 2*c^(3/2)*e^4*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 6*c^(3/
2)*d*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + 2*c^(3/2)*d^2*f^2*
Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + 2*a*Sqrt[c]*e^2*f^2*Log[-(Sqr
t[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 2*a*Sqrt[c]*d*f^3*Log[-(Sqrt[c]*x) + S
qrt[a + c*x^2] - #1]*#1 - c*e^3*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*
#1^2 + 2*c*d*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - a*e*f^3*
Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*
a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]/(d^2*(c^2*d^2 + a*c*e^2 - 2*a*c*d
*f + a^2*f^2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage2

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

[Out] sage2

maple [B] time = 0.02, size = 2046, normalized size = 3.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] -8*f^4/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)/(2*a*f^2-2*c*d*f+c*e^2+(-
4*d*f+e^2)^(1/2)*c*e)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2
)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-
4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)-16*f^3/(e+(-4*d*f+e^2)^(1/2))^2*c^2/(2*a*f
^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d
*f+e^2)*c^2/f^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2)^(1/2
))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+
e^2)^(1/2)*c*e)/f^2)^(1/2)*x-16*f^3/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(
1/2)*c^2/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^
2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-
4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f
+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*e*x+8*f^4/(e+(-4*d*f+e^2)^(1/2))^
2/(-4*d*f+e^2)^(1/2)/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)
/((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-e+(-4*d*f
+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-
4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(
1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+
e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-
4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))+4*f/(-
e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a/x/(c*x^2+a)^(1/2)+8*f/(-e+(-
```

$$\frac{4df+e^2)^{1/2}}{(e+(-4df+e^2)^{1/2})} \cdot \frac{c}{a^2x} \cdot \frac{1}{(cx^2+a)^{1/2}} + \frac{8f^4}{(-e+(-4df+e^2)^{1/2})^2} \cdot \frac{1}{(-4df+e^2)^{1/2}} \cdot \frac{1}{(2af^2-2cdf+ce^2-(-4df+e^2)^{1/2})^{1/2}} \cdot ce \cdot \frac{1}{((x-1/2)(-e+(-4df+e^2)^{1/2}))/f} \cdot c - (e+(-4df+e^2)^{1/2}) \cdot \frac{1}{(x-1/2)(-e+(-4df+e^2)^{1/2}))/f} \cdot \frac{c}{f} + \frac{1}{2} \cdot \frac{1}{(2af^2-2cdf+ce^2-(-4df+e^2)^{1/2})^{1/2}} \cdot ce \cdot \frac{1}{f^2} - \frac{16f^3}{(-e+(-4df+e^2)^{1/2})^2} \cdot \frac{1}{(2af^2-2cdf+ce^2-(-4df+e^2)^{1/2})^{1/2}} \cdot ce \cdot \frac{1}{(4ac-4c^2d/f+c^2e^2/f^2-(-4df+e^2)^{1/2})^{1/2}} \cdot \frac{1}{f^2} \cdot \frac{1}{((x-1/2)(-e+(-4df+e^2)^{1/2}))/f} \cdot c - (e+(-4df+e^2)^{1/2}) \cdot \frac{1}{(x-1/2)(-e+(-4df+e^2)^{1/2}))/f} \cdot \frac{c}{f} + \frac{1}{2} \cdot \frac{1}{(2af^2-2cdf+ce^2-(-4df+e^2)^{1/2})^{1/2}} \cdot ce \cdot \frac{1}{f^2} \cdot x + \frac{16f^3}{(-e+(-4df+e^2)^{1/2})^2} \cdot \frac{1}{(-4df+e^2)^{1/2}} \cdot \frac{1}{(2af^2-2cdf+ce^2-(-4df+e^2)^{1/2})^{1/2}} \cdot ce \cdot \frac{1}{(4ac-4c^2d/f+c^2e^2/f^2-(-4df+e^2)^{1/2})^{1/2}} \cdot \frac{1}{f^2} \cdot \frac{1}{((x-1/2)(-e+(-4df+e^2)^{1/2}))/f} \cdot c - (e+(-4df+e^2)^{1/2}) \cdot \frac{1}{(x-1/2)(-e+(-4df+e^2)^{1/2}))/f} \cdot \frac{c}{f} + \frac{1}{2} \cdot \frac{1}{(2af^2-2cdf+ce^2-(-4df+e^2)^{1/2})^{1/2}} \cdot ce \cdot \frac{1}{f^2} \cdot e \cdot x - \frac{8f^4}{(-e+(-4df+e^2)^{1/2})^2} \cdot \frac{1}{(-4df+e^2)^{1/2}} \cdot \frac{1}{(2af^2-2cdf+ce^2-(-4df+e^2)^{1/2})^{1/2}} \cdot ce \cdot \frac{1}{f^2} \cdot \frac{1}{((2af^2-2cdf+ce^2-(-4df+e^2)^{1/2})^{1/2})^{1/2}} \cdot \ln\left(\frac{(-e+(-4df+e^2)^{1/2}) \cdot (x-1/2)(-e+(-4df+e^2)^{1/2}))/f}{(2af^2-2cdf+ce^2-(-4df+e^2)^{1/2})^{1/2}}\right) + \frac{1}{2} \cdot \frac{1}{(2af^2-2cdf+ce^2-(-4df+e^2)^{1/2})^{1/2}} \cdot ce \cdot \frac{1}{f^2} \cdot \frac{1}{(4(x-1/2)(-e+(-4df+e^2)^{1/2}))/f} \cdot c - 4 \cdot \frac{1}{(e+(-4df+e^2)^{1/2})^{1/2}} \cdot \frac{1}{(x-1/2)(-e+(-4df+e^2)^{1/2}))/f} \cdot \frac{c}{f} + 2 \cdot \frac{1}{(2af^2-2cdf+ce^2-(-4df+e^2)^{1/2})^{1/2}} \cdot ce \cdot \frac{1}{f^2} \cdot \frac{1}{(x-1/2)(-e+(-4df+e^2)^{1/2}))/f} - \frac{16f^2e}{(-e+(-4df+e^2)^{1/2})^2} \cdot \frac{1}{(e+(-4df+e^2)^{1/2})^2} \cdot \frac{1}{a} \cdot \frac{1}{(cx^2+a)^{1/2}} + \frac{16f^2e}{(-e+(-4df+e^2)^{1/2})^2} \cdot \frac{1}{(e+(-4df+e^2)^{1/2})^2} \cdot \frac{1}{a^{3/2}} \cdot \ln\left(\frac{(2af+2(cx^2+a)^{1/2}) \cdot a^{1/2}}{x}\right)$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2+a)^{\frac{3}{2}}(fx^2+ex+d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] integrate(1/((c*x^2+a)^(3/2)*(f*x^2+e*x+d)*x^2),x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2(c x^2+a)^{3/2}(f x^2+e x+d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(a+c*x^2)^(3/2)*(d+e*x+f*x^2)),x)

[Out] int(1/(x^2*(a+c*x^2)^(3/2)*(d+e*x+f*x^2)),x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(a+cx^2)^{\frac{3}{2}}(d+ex+fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)

[Out] Integral(1/(x**2*(a+c*x**2)**(3/2)*(d+e*x+f*x**2)),x)

$$3.77 \quad \int \frac{x^3 \sqrt{a+bx+cx^2}}{d-fx^2} dx$$

Optimal. Leaf size=392

$$\frac{b(b^2 - 4ac) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}f} + \frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f} - \frac{d\sqrt{af+b(-\sqrt{d})}\sqrt{f}+cd \tanh^{-1}\left(\frac{-2a\sqrt{f}+x}{2\sqrt{a+bx+cx^2}}\right)}{2f^{5/2}}$$

Rubi [A] time = 0.95, antiderivative size = 392, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {6725, 640, 612, 621, 206, 1021, 1078, 1033, 724}

$$\frac{b(b^2 - 4ac) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}f} + \frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f} - \frac{d\sqrt{a+bx+cx^2}}{f^2} - \frac{bd \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2} - \frac{d\sqrt{af+b(-\sqrt{d})}\sqrt{f}+cd \tanh^{-1}\left(\frac{-2a\sqrt{f}+x}{2\sqrt{a+bx+cx^2}}\right)}{2f^{5/2}} + \frac{d\sqrt{af+b\sqrt{d}}\sqrt{f}+cd \tanh^{-1}\left(\frac{2a\sqrt{f}+x}{2\sqrt{a+bx+cx^2}}\right)}{2f^{5/2}} - \frac{(a+bx+cx^2)^{3/2}}{3cf}$$

Antiderivative was successfully verified.

[In] Int[(x^3*Sqrt[a + b*x + c*x^2])/(d - f*x^2), x]

[Out] -((d*Sqrt[a + b*x + c*x^2])/f^2) + (b*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(8*c^2*f) - (a + b*x + c*x^2)^(3/2)/(3*c*f) - (b*d*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*f^2) - (b*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(5/2)*f) - (d*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(5/2)) + (d*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(5/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,

d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1021

Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]

Rule 1033

Int[((g_.) + (h_.)*(x_))/((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]

Rule 1078

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f, 0]

Rule 6725

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^3 \sqrt{a + bx + cx^2}}{d - fx^2} dx &= \int \left(-\frac{x \sqrt{a + bx + cx^2}}{f} + \frac{dx \sqrt{a + bx + cx^2}}{f(d - fx^2)} \right) dx \\
 &= -\frac{\int x \sqrt{a + bx + cx^2} dx}{f} + \frac{d \int \frac{x \sqrt{a + bx + cx^2}}{d - fx^2} dx}{f} \\
 &= -\frac{d \sqrt{a + bx + cx^2}}{f^2} - \frac{(a + bx + cx^2)^{3/2}}{3cf} + \frac{d \int \frac{\frac{bd}{2} + (cd + af)x + \frac{1}{2}bfx^2}{\sqrt{a + bx + cx^2}(d - fx^2)} dx}{f^2} + \frac{b \int \sqrt{a + bx + cx^2}}{2cf} \\
 &= -\frac{d \sqrt{a + bx + cx^2}}{f^2} + \frac{b(b + 2cx)\sqrt{a + bx + cx^2}}{8c^2f} - \frac{(a + bx + cx^2)^{3/2}}{3cf} - \frac{d \int \frac{-bdf - f(cd + af)}{\sqrt{a + bx + cx^2}(d - fx^2)} dx}{f^3} \\
 &= -\frac{d \sqrt{a + bx + cx^2}}{f^2} + \frac{b(b + 2cx)\sqrt{a + bx + cx^2}}{8c^2f} - \frac{(a + bx + cx^2)^{3/2}}{3cf} - \frac{(bd) \text{Subst} \left(\int \frac{1}{4c - \dots} \right)}{f^3} \\
 &= -\frac{d \sqrt{a + bx + cx^2}}{f^2} + \frac{b(b + 2cx)\sqrt{a + bx + cx^2}}{8c^2f} - \frac{(a + bx + cx^2)^{3/2}}{3cf} - \frac{bd \tanh^{-1} \left(\frac{b + \dots}{2\sqrt{c} \sqrt{a + \dots}} \right)}{2\sqrt{c} f^2} \\
 &= -\frac{d \sqrt{a + bx + cx^2}}{f^2} + \frac{b(b + 2cx)\sqrt{a + bx + cx^2}}{8c^2f} - \frac{(a + bx + cx^2)^{3/2}}{3cf} - \frac{bd \tanh^{-1} \left(\frac{b + \dots}{2\sqrt{c} \sqrt{a + \dots}} \right)}{2\sqrt{c} f^2}
 \end{aligned}$$

Mathematica [A] time = 0.89, size = 327, normalized size = 0.83

$$\frac{2\sqrt{f}\sqrt{a+bx+cx^2}(2cf(4a+bx)-3b^2f+8c^2(3d+fx^2))}{c^2} - \frac{3b\sqrt{f}(-4acf+b^2f+8c^2d)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^2} + 24d\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right) - 24d\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+b(-\sqrt{d})+b(-\sqrt{d})x+2c\sqrt{d}x}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)$$

Antiderivative was successfully verified.

```

[In] Integrate[(x^3*Sqrt[a + b*x + c*x^2])/(d - f*x^2), x]
[Out] ((-2*Sqrt[f]*Sqrt[a + x*(b + c*x)]*(-3*b^2*f + 2*c*f*(4*a + b*x) + 8*c^2*(3*d + f*x^2)))/c^2 - (3*b*Sqrt[f]*(8*c^2*d + b^2*f - 4*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/c^(5/2) + 24*d*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] - 24*d*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/ (48*f^(5/2))
    
```

IntegrateAlgebraic [C] time = 1.05, size = 414, normalized size = 1.06

$$\frac{d\text{RootSum}\left[\left(1^2(-f) + 2d^2f + 4d^2cd - 4d^2\sqrt{c}d - d^2 + b^2d\right)\sqrt{a+bx+cx^2} + \dots\right]}{2f^2} \log\left(\frac{(-bdf + b^2f + 8bc^2d)\log\left(\frac{-2\sqrt{c}\sqrt{a+bx+cx^2} + b + 2cx}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right) + \dots}{16c^2f^2}\right) + \frac{\sqrt{a+bx+cx^2}(-3acf + 3b^2f - 2bc^2d - 8c^2f^2)}{24c^2f^2}$$

Antiderivative was successfully verified.

```

[In] IntegrateAlgebraic[(x^3*Sqrt[a + b*x + c*x^2])/(d - f*x^2), x]
[Out] (Sqrt[a + b*x + c*x^2]*(-24*c^2*d + 3*b^2*f - 8*a*c*f - 2*b*c*f*x - 8*c^2*f*x^2))/(24*c^2*f^2) + ((8*b*c^2*d + b^3*f - 4*a*b*c*f)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(16*c^(5/2)*f^2) - (d*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (b^2*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*b*Sqrt[c]*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + c*d*Log[-(S
    
```



```

qrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]**#1^2 + a*f*Log[-(Sqrt[c]*x) + Sqrt[
a + b*x + c*x^2] - #1]**#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ]
)/(2*f^2)

```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type

maple [B] time = 0.02, size = 1817, normalized size = 4.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/3*(c*x^2+b*x+a)^(3/2)/c/f+1/4/f*b/c*x*(c*x^2+b*x+a)^(1/2)+1/8/f*b^2/c^2*
(c*x^2+b*x+a)^(1/2)+1/4/f*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1
/2))*a-1/16/f*b^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/2/f
^2*d*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+
c*d-(d*f)^(1/2)*b)/f)^(1/2)+1/2/f^3*d*ln((1/2/f*(b*f-2*(d*f)^(1/2)*c)+c*(x+
(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*
f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)-1/4/f^2
*d*ln((1/2/f*(b*f-2*(d*f)^(1/2)*c)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(
1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*
b)/f)^(1/2))/c^(1/2)*b-1/2/f^3*d/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a
*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c
*d-(d*f)^(1/2)*b)/f)^(1/2))*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+
(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*(d*f)
^(1/2)*b+1/2/f^2*d/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(
1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)
*b)/f)^(1/2))*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)
/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*a+1/2/f^3*d^2/((a*f
+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(
1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*((x+(d*f)^(
1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*
b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*c-1/2/f^2*d*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(
d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)-1/2/f^3*
d*ln((1/2*(b*f+2*(d*f)^(1/2)*c)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1
/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b
)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)-1/4/f^2*d*ln((1/2*(b*f+2*(d*f)^(1/2)*c)/f+c
*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x
-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b+1/2/f^3*d/((a
*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)
^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*((x-(d*f)
^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)

```

```

)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*(d*f)^(1/2)*b+1/2/f^2*d/((a*f+c*d+(d*f)^(
1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(
d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+
(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))
/(x-(d*f)^(1/2)/f))*a+1/2/f^3*d^2/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(
a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+
c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x
-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c

```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2
-(c*((b*sqrt(4*d*f)))/(2*f) + (c*d)/f+a)
/f^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \sqrt{cx^2 + bx + a}}{d - fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x + c*x^2)^(1/2))/(d - f*x^2),x)
```

```
[Out] int((x^3*(a + b*x + c*x^2)^(1/2))/(d - f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3 \sqrt{a + bx + cx^2}}{-d + fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x**3*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)
```

$$3.78 \quad \int \frac{x^2 \sqrt{a+bx+cx^2}}{d-fx^2} dx$$

Optimal. Leaf size=316

$$\frac{(4acf + b^2(-f) + 8c^2d) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) \sqrt{d}\sqrt{af+b(-\sqrt{d})}\sqrt{f} + cd \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}}\right)}{8c^{3/2}f^2} + \frac{\sqrt{d}\sqrt{af+b(-\sqrt{d})}\sqrt{f} + cd \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}}\right)}{2f^2}$$

Rubi [A] time = 0.49, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 28, number of rules / integrand size = 0.214, Rules used = {1071, 1078, 621, 206, 1033, 724}

$$\frac{(4acf + b^2(-f) + 8c^2d) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) \sqrt{d}\sqrt{af+b(-\sqrt{d})}\sqrt{f} + cd \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}}\right)}{8c^{3/2}f^2} + \frac{\sqrt{d}\sqrt{af+b(-\sqrt{d})}\sqrt{f} + cd \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}}\right)}{2f^2} + \frac{\sqrt{d}\sqrt{af+b\sqrt{d}}\sqrt{f} + cd \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}}\sqrt{f}+cd}\right)}{2f^2} - \frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf}$$

Antiderivative was successfully verified.

[In] Int[(x^2*sqrt[a + b*x + c*x^2])/(d - f*x^2), x]

[Out] -((b + 2*c*x)*sqrt[a + b*x + c*x^2])/(4*c*f) - ((8*c^2*d - b^2*f + 4*a*c*f)*ArcTanh[(b + 2*c*x)/(2*sqrt[c]*sqrt[a + b*x + c*x^2])])/(8*c^(3/2)*f^2) + (sqrt[d]*sqrt[c*d - b*sqrt[d]*sqrt[f] + a*f)*ArcTanh[(b*sqrt[d] - 2*a*sqrt[f] + (2*c*sqrt[d] - b*sqrt[f])*x)/(2*sqrt[c*d - b*sqrt[d]*sqrt[f] + a*f]*sqrt[a + b*x + c*x^2])])/(2*f^2) + (sqrt[d]*sqrt[c*d + b*sqrt[d]*sqrt[f] + a*f)*ArcTanh[(b*sqrt[d] + 2*a*sqrt[f] + (2*c*sqrt[d] + b*sqrt[f])*x)/(2*sqrt[c*d + b*sqrt[d]*sqrt[f] + a*f]*sqrt[a + b*x + c*x^2])])/(2*f^2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1033

Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]

Rule 1071

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +

```
3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*(C*(-(b*f)))*(q + 1)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f) + f*(-2*A*f))*(2*p + 2*q + 3))] + (2*p*(c*d - a*f)*(C*(-(b*f)))*(q + 1)) + (p + q + 1)*(-(b*c*(C*(-4*d*f))*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))]*x + (p*(-(b*f))*(C*(-(b*f)))*(q + 1)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(-4*d*f))*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))*x^2, x], x] /; FreeQ[{a, b, c, d, f, A, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{x^2 \sqrt{a + bx + cx^2}}{d - fx^2} dx = -\frac{(b + 2cx)\sqrt{a + bx + cx^2}}{4cf} - \frac{\int \frac{-\frac{1}{4}(b^2 + 4ac)df - 2bcdfx - \frac{1}{4}f(8c^2d - b^2f + 4acf)x^2}{\sqrt{a + bx + cx^2}(d - fx^2)} dx}{2cf^2}$$

$$= -\frac{(b + 2cx)\sqrt{a + bx + cx^2}}{4cf} + \frac{\int \frac{\frac{1}{4}(b^2 + 4ac)df^2 + \frac{1}{4}df(8c^2d - b^2f + 4acf) + 2bcdf^2x}{\sqrt{a + bx + cx^2}(d - fx^2)} dx}{2cf^3} - \frac{(8c^2d - b^2f)}{2cf^3}$$

$$= -\frac{(b + 2cx)\sqrt{a + bx + cx^2}}{4cf} - \frac{(\sqrt{d}(cd - b\sqrt{d}\sqrt{f} + af)) \int \frac{1}{(-\sqrt{d}\sqrt{f} - fx)\sqrt{a + bx + cx^2}} dx}{2f^{3/2}} + \dots$$

$$= -\frac{(b + 2cx)\sqrt{a + bx + cx^2}}{4cf} - \frac{(8c^2d - b^2f + 4acf) \tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right)}{8c^{3/2}f^2} + \frac{(\sqrt{d}(cd - b\sqrt{d}\sqrt{f} + af)) \int \frac{1}{(-\sqrt{d}\sqrt{f} - fx)\sqrt{a + bx + cx^2}} dx}{2f^{3/2}} + \dots$$

$$= -\frac{(b + 2cx)\sqrt{a + bx + cx^2}}{4cf} - \frac{(8c^2d - b^2f + 4acf) \tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right)}{8c^{3/2}f^2} + \frac{\sqrt{d}\sqrt{cd - b^2f}}{4cf}$$

Mathematica [A] time = 0.49, size = 302, normalized size = 0.96

$$\frac{(-4acf + b^2f - 8c^2d) \tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right) - 2\sqrt{d}\left(-2c\sqrt{d}\sqrt{af + b\sqrt{d}\sqrt{f} + cd} \tanh^{-1}\left(\frac{2a\sqrt{f} + b\sqrt{d} + b\sqrt{fx + 2c\sqrt{d}x}}{2\sqrt{d + x(b + cx)}}\sqrt{af + b\sqrt{d}\sqrt{f} + cd}\right) - 2c\sqrt{d}\sqrt{af + b(-\sqrt{d})\sqrt{f} + cd} \tanh^{-1}\left(\frac{-2a\sqrt{f} + b(\sqrt{d} - \sqrt{fx}) + 2c\sqrt{d}x}{2\sqrt{d + x(b + cx)}}\sqrt{af + b(-\sqrt{d})\sqrt{f} + cd}\right) + f(b + 2cx)\sqrt{a + x(b + cx)}}{8c^{3/2}f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[a + b*x + c*x^2])/(d - f*x^2), x]
```

```
[Out] ((-8*c^2*d + b^2*f - 4*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - 2*Sqrt[c]*(f*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - 2*c*Sqrt[d]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] - 2*c*Sqrt[d]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]))/(8*c^(3/2)*f^2)
```

IntegrateAlgebraic [C] time = 0.83, size = 327, normalized size = 1.03

$$\frac{d\text{RootSum}\left[\frac{1}{2f^2} \left(11^4(-f) + 281^2af + 481^2cd - 481b\sqrt{c}d - a^2f + b^2d\sqrt{c}, \frac{81^2b^2\log(-1 + \sqrt{a + bx + cx^2} - \sqrt{c}x) - 281c^2d\log(-1 + \sqrt{a + bx + cx^2} - \sqrt{c}x) + bcd\log(-1 + \sqrt{a + bx + cx^2} - \sqrt{c}x) - 281a\sqrt{c}f\log(-1 + \sqrt{a + bx + cx^2} - \sqrt{c}x)}{81^2 - 81af - 281cd + \sqrt{c}d}\right) \right]}{2f^2} + \frac{(4acf + b^2(-f) + 8c^2d)\log\left(\frac{-2c\sqrt{d}\sqrt{a + bx + cx^2} + bc + 2c^2x}{8c^{3/2}f^2}\right)}{8c^{3/2}f^2} + \frac{(-b - 2cx)\sqrt{a + bx + cx^2}}{4cf}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
[Out] ((-b - 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c*f) + ((8*c^2*d - b^2*f + 4*a*c*f)
*Log[b*c + 2*c^2*x - 2*c^(3/2)*Sqrt[a + b*x + c*x^2]])/(8*c^(3/2)*f^2) - (d
*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^
4 & , (b*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*c^(3/2)*d*L
og[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*a*Sqrt[c]*f*Log[-(Sqrt
[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + b*f*Log[-(Sqrt[c]*x) + Sqrt[a + b
*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ])/(2*
f^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.02, size = 1810, normalized size = 5.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/2/f*x*(c*x^2+b*x+a)^(1/2)-1/4/f/c*(c*x^2+b*x+a)^(1/2)*b-1/2/f/c^(1/2)*ln
((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+1/8/f/c^(3/2)*ln((c*x+1/2*b)/c^
(1/2)+(c*x^2+b*x+a)^(1/2))*b^2+1/2*d/(d*f)^(1/2)/f*((x+(d*f)^(1/2)/f)^2*c+(
b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)-1
/2*d/f^2*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(
d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(
1/2)*b)/f)^(1/2))*c^(1/2)+1/4*d/(d*f)^(1/2)/f*ln(((x+(d*f)^(1/2)/f)*c+1/2*
(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*
c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b+1/2*d/f^
2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*
(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*((x+
(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)
^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*b-1/2*d/(d*f)^(1/2)/f/((a*f+c*d-(d*f)
)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*
(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*((x+(d*f)^(1/2)/f)^2
*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/
2))/(x+(d*f)^(1/2)/f))*a-1/2*d^2/(d*f)^(1/2)/f^2/((a*f+c*d-(d*f)^(1/2)*b)/f)
^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)
)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d
*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)
^(1/2)/f))*c-1/2*d/(d*f)^(1/2)/f*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*
c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)-1/2*d/f^2*ln(((x-(d
```

```

*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+
(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))
*c^(1/2)-1/4*d/(d*f)^(1/2)/f*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)
*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/
f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b+1/2*d/f^2/((a*f+c*d+(d*f)^(
1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-
(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*((x-(d*f)^(1/2)/f)^2*c
+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)
)/(x-(d*f)^(1/2)/f))*b+1/2*d/(d*f)^(1/2)/f/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)
)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f
+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1
/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)
/f))*a+1/2*d^2/(d*f)^(1/2)/f^2/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f
+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d
+(d*f)^(1/2)*b)/f)^(1/2))*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d
*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c

```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2 -c*((b*sqrt(4*d*f)) / (2*f) + (c*d)/f+a) /f^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \sqrt{cx^2 + bx + a}}{d - fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(a + b*x + c*x^2)^(1/2))/(d - f*x^2),x)

[Out] int((x^2*(a + b*x + c*x^2)^(1/2))/(d - f*x^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2 \sqrt{a + bx + cx^2}}{-d + fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)

[Out] -Integral(x**2*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)

$$3.79 \quad \int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2} dx$$

Optimal. Leaf size=282

$$\frac{\sqrt{af+b(-\sqrt{d})}\sqrt{f}+cd \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f}+cd}\right)}{2f^{3/2}} + \frac{\sqrt{af+b\sqrt{d}}\sqrt{f}+cd \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}}\sqrt{f}+cd}\right)}{2f^{3/2}}$$

Rubi [A] time = 0.29, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1021, 1078, 621, 206, 1033, 724}

$$\frac{\sqrt{af+b(-\sqrt{d})}\sqrt{f}+cd \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f}+cd}\right)}{2f^{3/2}} + \frac{\sqrt{af+b\sqrt{d}}\sqrt{f}+cd \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}}\sqrt{f}+cd}\right)}{2f^{3/2}} - \frac{\sqrt{a+bx+cx^2}}{f} - \frac{b \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f}$$

Antiderivative was successfully verified.

[In] Int[(x*Sqrt[a + b*x + c*x^2])/(d - f*x^2), x]

[Out] -(Sqrt[a + b*x + c*x^2]/f) - (b*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*f) - (Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(3/2)) + (Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(3/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1021

Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-b*f)) + c*(-2*g*f)*(p + q + 1))*x^2, x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2} dx = -\frac{\sqrt{a+bx+cx^2}}{f} + \frac{\int \frac{\frac{bd}{2}+(cd+af)x+\frac{1}{2}bfx^2}{\sqrt{a+bx+cx^2}(d-fx^2)} dx}{f}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{f} - \frac{\int \frac{-bdf-f(cd+af)x}{\sqrt{a+bx+cx^2}(d-fx^2)} dx}{f^2} - \frac{b \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{2f}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{f} - \frac{b \operatorname{Subst}\left(\int \frac{1}{4c-x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f} + \frac{(cd - b\sqrt{d}\sqrt{f} + af) \int \frac{1}{(-\sqrt{d}\sqrt{f} - \sqrt{a+bx+cx^2})}}{2f}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{f} - \frac{b \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f} - \frac{(cd - b\sqrt{d}\sqrt{f} + af) \operatorname{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}\sqrt{f}-x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{2f}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{f} - \frac{b \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f} - \frac{\sqrt{cd - b\sqrt{d}\sqrt{f} + af} \tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}}{2\sqrt{cd-b\sqrt{d}\sqrt{f} + af}}\right)}{2f^{3/2}}$$

Mathematica [A] time = 0.29, size = 272, normalized size = 0.96

$$\frac{\sqrt{af + b\sqrt{d}\sqrt{f} + cd} \tanh^{-1}\left(\frac{2a\sqrt{f} + b\sqrt{d} + b\sqrt{f}x + 2c\sqrt{d}x}{2\sqrt{a+bx+cx^2}\sqrt{af + b\sqrt{d}\sqrt{f} + cd}}\right) - \sqrt{af + b(-\sqrt{d})\sqrt{f} + cd} \tanh^{-1}\left(\frac{-2a\sqrt{f} + b(\sqrt{d} - \sqrt{f}x) + 2c\sqrt{d}x}{2\sqrt{a+bx+cx^2}\sqrt{af + b(-\sqrt{d})\sqrt{f} + cd}}\right) - 2\sqrt{f}\sqrt{a+bx+cx^2} - \frac{b\sqrt{f} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}}{2f^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[a + b*x + c*x^2])/(d - f*x^2), x]
```

```
[Out] (-2*Sqrt[f]*Sqrt[a + x*(b + c*x)] - (b*Sqrt[f]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] - Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(2*f^(3/2))
```

IntegrateAlgebraic [C] time = 0.58, size = 361, normalized size = 1.28

$$\frac{\operatorname{RootSum}\left[\#1^4(-f) + 2\#1^2af + 4\#1^2cd - 4\#1b\sqrt{c}d - a^2f + b^2d6e, \frac{\#1^2d \log\left(\frac{\#1 + \sqrt{a+bx+cx^2} - \sqrt{c}}{\#1 + \sqrt{a+bx+cx^2} + \sqrt{c}}\right) + \#1^2d \log\left(\frac{\#1 + \sqrt{a+bx+cx^2} - \sqrt{c}}{\#1 + \sqrt{a+bx+cx^2} + \sqrt{c}}\right) + \#1^2d \log\left(\frac{\#1 + \sqrt{a+bx+cx^2} - \sqrt{c}}{\#1 + \sqrt{a+bx+cx^2} + \sqrt{c}}\right) + \#1^2d \log\left(\frac{\#1 + \sqrt{a+bx+cx^2} - \sqrt{c}}{\#1 + \sqrt{a+bx+cx^2} + \sqrt{c}}\right) + \#1^2d \log\left(\frac{\#1 + \sqrt{a+bx+cx^2} - \sqrt{c}}{\#1 + \sqrt{a+bx+cx^2} + \sqrt{c}}\right)}{\#1^7 - \#1^5f - 2\#1^3d + b\sqrt{c}d}\right]}{f} + \frac{b \log\left(-2\sqrt{c}f\sqrt{a+bx+cx^2} + bf + 2cf\right)}{2\sqrt{c}f}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]

[Out] $-(\sqrt{a + b*x + c*x^2}/f) + (b*\log[b*f + 2*c*f*x - 2*\sqrt{c}*f*\sqrt{a + b*x + c*x^2}])/(2*\sqrt{c}*f) - \text{RootSum}[b^2*d - a^2*f - 4*b*\sqrt{c}*d*\#1 + 4*c*d*\#1^2 + 2*a*f*\#1^2 - f*\#1^4 \& , (b^2*d*\log[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1] - a*c*d*\log[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1] - a^2*f*\log[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1] - 2*b*\sqrt{c}*d*\log[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1]*\#1 + c*d*\log[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1]*\#1^2 + a*f*\log[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1]*\#1^2)/(b*\sqrt{c}*d - 2*c*d*\#1 - a*f*\#1 + f*\#1^3) \&]/(2*f)$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type

maple [B] time = 0.01, size = 1667, normalized size = 5.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)

[Out] $-1/2/f*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)+1/2/f^2*\ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)-1/4/f*\ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b-1/2/f^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*\ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/((x+(d*f)^(1/2)/f))*((d*f)^(1/2)*b+1/2/f/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*\ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/((x+(d*f)^(1/2)/f))*a+1/2/f^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*\ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/((x+(d*f)^(1/2)/f))*c*d-1/2/f*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)-1/2/f^2*\ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)-1/4/f*\ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b+1/2/f^2/((a*f+c*d+(d*f)^(1/2)*b)/f)$

$$\begin{aligned} & \int \frac{x \sqrt{(a+bx+cx^2)} (d-fx^2)^{3/2}}{(d-fx^2)^2} dx \\ &= \frac{1}{2} \int \frac{x \sqrt{(a+bx+cx^2)} (d-fx^2)^{1/2}}{(d-fx^2)^2} dx + \frac{1}{2} \int \frac{x \sqrt{(a+bx+cx^2)} (d-fx^2)^{3/2}}{(d-fx^2)^2} dx \\ &= \frac{1}{2} \int \frac{x \sqrt{(a+bx+cx^2)}}{(d-fx^2)^{3/2}} dx + \frac{1}{2} \int \frac{x \sqrt{(a+bx+cx^2)} (d-fx^2)^{1/2}}{(d-fx^2)^2} dx \\ &= \frac{1}{2} \int \frac{x \sqrt{(a+bx+cx^2)}}{(d-fx^2)^{3/2}} dx + \frac{1}{2} \int \frac{x \sqrt{(a+bx+cx^2)}}{(d-fx^2)^{3/2}} dx \\ &= \int \frac{x \sqrt{(a+bx+cx^2)}}{(d-fx^2)^{3/2}} dx \end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation may help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see 'assume?' for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2 -c*((b*sqrt(4*d*f)) / (2*f) + (c*d)/f+a) /f^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x \sqrt{cx^2 + bx + a}}{d - fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(a + b*x + c*x^2)^(1/2))/(d - f*x^2),x)

[Out] int((x*(a + b*x + c*x^2)^(1/2))/(d - f*x^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x \sqrt{a + bx + cx^2}}{-d + fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)

[Out] -Integral(x*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)

$$3.80 \quad \int \frac{\sqrt{a+bx+cx^2}}{d-fx^2} dx$$

Optimal. Leaf size=266

$$\frac{\sqrt{af+b(-\sqrt{d})}\sqrt{f}+cd \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f}+cd}\right)}{2\sqrt{d}f} + \frac{\sqrt{af+b\sqrt{d}}\sqrt{f}+cd \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}}\sqrt{f}+cd}\right)}{2\sqrt{d}f}$$

Rubi [A] time = 0.23, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {990, 621, 206, 1033, 724}

$$\frac{\sqrt{af+b(-\sqrt{d})}\sqrt{f}+cd \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f}+cd}\right)}{2\sqrt{d}f} + \frac{\sqrt{af+b\sqrt{d}}\sqrt{f}+cd \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}}\sqrt{f}+cd}\right)}{2\sqrt{d}f} - \frac{\sqrt{c} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x + c*x^2]/(d - f*x^2), x]

[Out] $-\left(\frac{\text{Sqrt}[c] \text{ArcTanh}\left[\frac{b + 2c x}{2 \text{Sqrt}[c] \text{Sqrt}[a + b x + c x^2]}\right]}{f}\right) + \left(\frac{\text{Sqrt}[c d - b \text{Sqrt}[d] \text{Sqrt}[f] + a f] \text{ArcTanh}\left[\frac{b \text{Sqrt}[d] - 2 a \text{Sqrt}[f] + (2 c \text{Sqrt}[d] - b \text{Sqrt}[f]) x}{2 \text{Sqrt}[c d - b \text{Sqrt}[d] \text{Sqrt}[f] + a f] \text{Sqrt}[a + b x + c x^2]}\right]}{2 \text{Sqrt}[d] f}\right) + \left(\frac{\text{Sqrt}[c d + b \text{Sqrt}[d] \text{Sqrt}[f] + a f] \text{ArcTanh}\left[\frac{b \text{Sqrt}[d] + 2 a \text{Sqrt}[f] + (2 c \text{Sqrt}[d] + b \text{Sqrt}[f]) x}{2 \text{Sqrt}[c d + b \text{Sqrt}[d] \text{Sqrt}[f] + a f] \text{Sqrt}[a + b x + c x^2]}\right]}{2 \text{Sqrt}[d] f}\right)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 990

Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (f_.)*(x_)^2), x_Symbol] := Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*f - b*f*x)/(Sqrt[a + b*x + c*x^2]*(d + f*x^2)), x], x] /; FreeQ[{a, b, c, d, f}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1033

Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/(q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f}

, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{d-fx^2} dx = \frac{\int \frac{cd+af+bf x}{\sqrt{a+bx+cx^2}(d-fx^2)} dx}{f} - \frac{c \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{f}$$

$$= -\frac{(2c) \text{Subst}\left(\int \frac{1}{4c-x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f} + \frac{1}{2} \left(b - \frac{cd+af}{\sqrt{d}\sqrt{f}}\right) \int \frac{1}{(-\sqrt{d}\sqrt{f}-fx)\sqrt{a+bx+cx^2}} dx$$

$$= -\frac{\sqrt{c} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f} + \left(-b - \frac{cd+af}{\sqrt{d}\sqrt{f}}\right) \text{Subst}\left(\int \frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)$$

$$= -\frac{\sqrt{c} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f} + \frac{\sqrt{cd-b\sqrt{d}\sqrt{f}+af} \tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+(2c\sqrt{d}-b\sqrt{f})x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}f}$$

Mathematica [A] time = 0.17, size = 253, normalized size = 0.95

$$\frac{\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd} \tanh^{-1}\left(\frac{-2a\sqrt{f}+b\sqrt{d}-b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right) + \sqrt{af+b\sqrt{d}\sqrt{f}+cd} \tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right) - 2\sqrt{c}\sqrt{d} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{2\sqrt{d}f}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*x + c*x^2]/(d - f*x^2), x]

[Out] (-2*Sqrt[c]*Sqrt[d]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + 2*c*Sqrt[d]*x - b*Sqrt[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(2*Sqrt[d]*f)

IntegrateAlgebraic [C] time = 0.42, size = 273, normalized size = 1.03

$$\frac{\sqrt{c} \log\left(-2\sqrt{c}\sqrt{a+bx+cx^2} + bf + 2cfx\right)}{f} - \frac{\text{RootSum}\left[\#1^4(-f) + 2\#1^2af + 4\#1^2cd - 4\#1b\sqrt{c}d - a^2f + b^2d\&, \frac{\#1^2bf \log\left(\frac{\#1 + \sqrt{a+b\sqrt{c}x^2} - \sqrt{c}x}{-1 + \sqrt{a+b\sqrt{c}x^2} - \sqrt{c}x}\right) + bcd \log\left(\frac{\#1 + \sqrt{a+b\sqrt{c}x^2} - \sqrt{c}x}{-1 + \sqrt{a+b\sqrt{c}x^2} - \sqrt{c}x}\right) - 2\#1a\sqrt{c}f \log\left(\frac{\#1 + \sqrt{a+b\sqrt{c}x^2} - \sqrt{c}x}{-1 + \sqrt{a+b\sqrt{c}x^2} - \sqrt{c}x}\right) \&}{\#1^3f - \#1bf - 2\#1cd + b\sqrt{c}d}\right]}{2f}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a + b*x + c*x^2]/(d - f*x^2), x]

[Out] (Sqrt[c]*Log[b*f + 2*c*f*x - 2*Sqrt[c]*f*Sqrt[a + b*x + c*x^2]])/f - RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 &, (b*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*c^(3/2)*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*a*Sqrt[c]*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + b*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) &]/(2*f)

fricas [B] time = 98.76, size = 1139, normalized size = 4.28



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(1/2)/(-f*x^2+d), x, algorithm="fricas")

```
[Out] [1/4*(f*sqrt((d*f^2*sqrt(b^2/(d*f^3)) + c*d + a*f)/(d*f^2))*log((2*b*c*x +
2*sqrt(c*x^2 + b*x + a)*b*f*sqrt((d*f^2*sqrt(b^2/(d*f^3)) + c*d + a*f)/(d*f
^2)) + b^2 + (b*f^2*x + 2*a*f^2)*sqrt(b^2/(d*f^3)))/x) - f*sqrt((d*f^2*sqrt
(b^2/(d*f^3)) + c*d + a*f)/(d*f^2))*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*
b*f*sqrt((d*f^2*sqrt(b^2/(d*f^3)) + c*d + a*f)/(d*f^2)) + b^2 + (b*f^2*x +
2*a*f^2)*sqrt(b^2/(d*f^3)))/x) + f*sqrt(-(d*f^2*sqrt(b^2/(d*f^3)) - c*d - a
*f)/(d*f^2))*log((2*b*c*x + 2*sqrt(c*x^2 + b*x + a)*b*f*sqrt(-(d*f^2*sqrt(b
^2/(d*f^3)) - c*d - a*f)/(d*f^2)) + b^2 - (b*f^2*x + 2*a*f^2)*sqrt(b^2/(d*f
^3)))/x) - f*sqrt(-(d*f^2*sqrt(b^2/(d*f^3)) - c*d - a*f)/(d*f^2))*log((2*b*
c*x - 2*sqrt(c*x^2 + b*x + a)*b*f*sqrt(-(d*f^2*sqrt(b^2/(d*f^3)) - c*d - a*
f)/(d*f^2)) + b^2 - (b*f^2*x + 2*a*f^2)*sqrt(b^2/(d*f^3)))/x) + 2*sqrt(c)*l
og(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c)
- 4*a*c))/f, 1/4*(f*sqrt((d*f^2*sqrt(b^2/(d*f^3)) + c*d + a*f)/(d*f^2))*lo
g((2*b*c*x + 2*sqrt(c*x^2 + b*x + a)*b*f*sqrt((d*f^2*sqrt(b^2/(d*f^3)) + c*
d + a*f)/(d*f^2)) + b^2 + (b*f^2*x + 2*a*f^2)*sqrt(b^2/(d*f^3)))/x) - f*sq
rt((d*f^2*sqrt(b^2/(d*f^3)) + c*d + a*f)/(d*f^2))*log((2*b*c*x - 2*sqrt(c*x^
2 + b*x + a)*b*f*sqrt((d*f^2*sqrt(b^2/(d*f^3)) + c*d + a*f)/(d*f^2)) + b^2
+ (b*f^2*x + 2*a*f^2)*sqrt(b^2/(d*f^3)))/x) + f*sqrt(-(d*f^2*sqrt(b^2/(d*f^
3)) - c*d - a*f)/(d*f^2))*log((2*b*c*x + 2*sqrt(c*x^2 + b*x + a)*b*f*sqrt(-
(d*f^2*sqrt(b^2/(d*f^3)) - c*d - a*f)/(d*f^2)) + b^2 - (b*f^2*x + 2*a*f^2)*
sqrt(b^2/(d*f^3)))/x) - f*sqrt(-(d*f^2*sqrt(b^2/(d*f^3)) - c*d - a*f)/(d*f^
2))*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*f*sqrt(-(d*f^2*sqrt(b^2/(d*f^3
)) - c*d - a*f)/(d*f^2)) + b^2 - (b*f^2*x + 2*a*f^2)*sqrt(b^2/(d*f^3)))/x)
+ 4*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2
+ b*c*x + a*c)))/f]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.01, size = 1669, normalized size = 6.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] 1/2/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)
/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)-1/2/f*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b
*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)
*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)+1/4/(d*f)^(1
/2)*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(
1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)
*b)/f)^(1/2))/c^(1/2)*b+1/2/f/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+
c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-
(d*f)^(1/2)*b)/f)^(1/2))*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*
f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/((x+(d*f)^(1/2)/f))*b-1/2/(d
*f)^(1/2)/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f
+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1
/2))*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c
*d-(d*f)^(1/2)*b)/f)^(1/2))/((x+(d*f)^(1/2)/f))*a-1/2/(d*f)^(1/2)/f/((a*f+c*
d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)
)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*((x+(d*f)^(1/2)
)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/
```

```
f)^(1/2))/(x+(d*f)^(1/2)/f))*c*d-1/2/(d*f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*
f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)-1/2
/f*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(
1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*
b)/f)^(1/2))*c^(1/2)-1/4/(d*f)^(1/2)*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*
f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f
)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b+1/2/f/((a*f+c*d+(d
*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)
*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)
^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(
1/2))/(x-(d*f)^(1/2)/f))*b+1/2/(d*f)^(1/2)/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)
)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f
+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1
/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)
/f))*a+1/2/(d*f)^(1/2)/f/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(
d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)
^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1
/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2 -c*((b*sqrt(4*d*f)) / (2*f) + (c*d)/f+a)) /f^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{d - fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x + c*x^2)^(1/2)/(d - f*x^2),x)

[Out] int((a + b*x + c*x^2)^(1/2)/(d - f*x^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{a + bx + cx^2}}{-d + fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)

[Out] -Integral(sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)

$$3.81 \quad \int \frac{\sqrt{a+bx+cx^2}}{x(d-fx^2)} dx$$

Optimal. Leaf size=267

$$\frac{\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd} \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2d\sqrt{f}} + \frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd} \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{d}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d\sqrt{f}}$$

Rubi [A] time = 0.78, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 28, number of rules / integrand size = 0.321, Rules used = {6725, 734, 843, 621, 206, 724, 1021, 1078, 1033}

$$\frac{\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd} \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2d\sqrt{f}} + \frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd} \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{d}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d\sqrt{f}} - \frac{\sqrt{a} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x + c*x^2]/(x*(d - f*x^2)), x]

[Out] -((Sqrt[a]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d) - (Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d*Sqrt[f]) + (Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d*Sqrt[f])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 734

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && (!RationalQ[m] || LtQ[m, 1]) && !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]

Rule 843

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +

$c*x^2)^p, x], x] + \text{Dist}[(e*f - d*g)/e, \text{Int}[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m, p\}, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& !\text{IGtQ}[m, 0]$

Rule 1021

$\text{Int}[(g_.) + (h_.)*(x_))*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^{(p_.)}*((d_.) + (f_.)*(x_.)^2)^{(q_.)}, x_Symbol] \rightarrow \text{Simp}[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^{(q + 1)})/(2*f*(p + q + 1)), x] - \text{Dist}[1/(2*f*(p + q + 1)), \text{Int}[(a + b*x + c*x^2)^{(p - 1)}*(d + f*x^2)^q*\text{Simp}[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q + 1))*x^2, x], x], x] /; \text{FreeQ}\{a, b, c, d, f, g, h, q\}, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{GtQ}[p, 0] \&\& \text{NeQ}[p + q + 1, 0]$

Rule 1033

$\text{Int}[(g_.) + (h_.)*(x_)]/(((a_.) + (c_.)*(x_.)^2)*\text{Sqrt}[(d_.) + (e_.)*(x_.) + (f_.)*(x_.)^2]), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[-(a*c), 2]\}, \text{Dist}[h/2 + (c*g)/(2*q), \text{Int}[1/((-q + c*x)*\text{Sqrt}[d + e*x + f*x^2]), x], x] + \text{Dist}[h/2 - (c*g)/(2*q), \text{Int}[1/((q + c*x)*\text{Sqrt}[d + e*x + f*x^2]), x], x]] /; \text{FreeQ}\{a, c, d, e, f, g, h\}, x\} \&\& \text{NeQ}[e^2 - 4*d*f, 0] \&\& \text{PosQ}[-(a*c)]$

Rule 1078

$\text{Int}[(A_.) + (B_.)*(x_.) + (C_.)*(x_.)^2)/(((a_.) + (c_.)*(x_.)^2)*\text{Sqrt}[(d_.) + (e_.)*(x_.) + (f_.)*(x_.)^2]), x_Symbol] \rightarrow \text{Dist}[C/c, \text{Int}[1/\text{Sqrt}[d + e*x + f*x^2], x], x] + \text{Dist}[1/c, \text{Int}[(A*c - a*C + B*c*x)/((a + c*x^2)*\text{Sqrt}[d + e*x + f*x^2]), x], x] /; \text{FreeQ}\{a, c, d, e, f, A, B, C\}, x\} \&\& \text{NeQ}[e^2 - 4*d*f, 0]$

Rule 6725

$\text{Int}[(u_)/((a_.) + (b_.)*(x_.)^n)], x_Symbol] \rightarrow \text{With}\{v = \text{RationalFunctionExpand}[u/(a + b*x^n), x]\}, \text{Int}[v, x] /; \text{SumQ}[v] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{IGtQ}[n, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx+cx^2}}{x(d-fx^2)} dx &= \int \left(\frac{\sqrt{a+bx+cx^2}}{dx} - \frac{fx\sqrt{a+bx+cx^2}}{d(-d+fx^2)} \right) dx \\
&= \frac{\int \frac{\sqrt{a+bx+cx^2}}{x} dx}{d} - \frac{f \int \frac{x\sqrt{a+bx+cx^2}}{-d+fx^2} dx}{d} \\
&= -\frac{\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}} dx}{2d} + \frac{\int \frac{-\frac{bd}{2}-(cd+af)x-\frac{1}{2}bfx^2}{\sqrt{a+bx+cx^2}(-d+fx^2)} dx}{d} \\
&= \frac{a \int \frac{1}{x\sqrt{a+bx+cx^2}} dx}{d} + \frac{\int \frac{-bdf+f(-cd-af)x}{\sqrt{a+bx+cx^2}(-d+fx^2)} dx}{df} \\
&= -\frac{(2a) \text{Subst} \left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}} \right)}{d} - \frac{(cd-b\sqrt{d}\sqrt{f}+af) \int \frac{1}{(\sqrt{d}\sqrt{f}+fx)\sqrt{a+bx+cx^2}} dx}{2d} \\
&= -\frac{\sqrt{a} \tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}} \right)}{d} + \frac{(cd-b\sqrt{d}\sqrt{f}+af) \text{Subst} \left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2} dx \right)}{d} \\
&= -\frac{\sqrt{a} \tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}} \right)}{d} - \frac{\sqrt{cd-b\sqrt{d}\sqrt{f}+af} \tanh^{-1} \left(\frac{b\sqrt{d}-2a\sqrt{f}+(2c\sqrt{d}-b\sqrt{f})x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}} \right)}{2d\sqrt{f}}
\end{aligned}$$

Mathematica [A] time = 0.30, size = 255, normalized size = 0.96

$$\frac{\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd} \tanh^{-1} \left(\frac{-2a\sqrt{f}+b\sqrt{d}-b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} \right) - \sqrt{af+b\sqrt{d}\sqrt{f}+cd} \tanh^{-1} \left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} \right) + 2\sqrt{a}\sqrt{f} \tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}} \right)}{2d\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*x + c*x^2]/(x*(d - f*x^2)), x]

[Out] $-1/2*(2*\text{Sqrt}[a]*\text{Sqrt}[f]*\text{ArcTanh}[(2*a + b*x)/(2*\text{Sqrt}[a]*\text{Sqrt}[a + x*(b + c*x)])] + \text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{ArcTanh}[(b*\text{Sqrt}[d] - 2*a*\text{Sqrt}[f] + 2*c*\text{Sqrt}[d]*x - b*\text{Sqrt}[f]*x)/(2*\text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{Sqrt}[a + x*(b + c*x)])] - \text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{ArcTanh}[(b*\text{Sqrt}[d] + 2*a*\text{Sqrt}[f] + 2*c*\text{Sqrt}[d]*x + b*\text{Sqrt}[f]*x)/(2*\text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{Sqrt}[a + x*(b + c*x)])])/(d*\text{Sqrt}[f])$

IntegrateAlgebraic [C] time = 0.39, size = 342, normalized size = 1.28

$$\frac{2\sqrt{a} \tanh^{-1} \left(\frac{\sqrt{c}}{\sqrt{d}} - \frac{\sqrt{af+bx+cx^2}}{\sqrt{d}} \right) - \text{RootSum} \left[\#1^4(-f) + 2\#1^2af + 4\#1^2cd - 4\#1b\sqrt{d} - a^2f + b^2d\&, \frac{\#1^2cd \log(-\#1 + \sqrt{af+bx+cx^2} - \sqrt{c}) + \#1^2af \log(-\#1 + \sqrt{af+bx+cx^2} - \sqrt{c}) + a^2(-f) \log(-\#1 + \sqrt{af+bx+cx^2} - \sqrt{c}) + b^2d \log(-\#1 + \sqrt{af+bx+cx^2} - \sqrt{c}) - a^2d \log(-\#1 + \sqrt{af+bx+cx^2} - \sqrt{c}) - 2\#1b\sqrt{d} \log(-\#1 + \sqrt{af+bx+cx^2} - \sqrt{c})}{\#1^4 - \#1af - 2\#1cd + b\sqrt{c}d} \right]}{2d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a + b*x + c*x^2]/(x*(d - f*x^2)), x]

[Out] $(2*\text{Sqrt}[a]*\text{ArcTanh}[(\text{Sqrt}[c]*x)/\text{Sqrt}[a] - \text{Sqrt}[a + b*x + c*x^2]/\text{Sqrt}[a]])/d - \text{RootSum}[b^2*d - a^2*f - 4*b*\text{Sqrt}[c]*d*\#1 + 4*c*d*\#1^2 + 2*a*f*\#1^2 - f*\#1^4 \&, (b^2*d*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] - a*c*d*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] - a^2*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] - 2*b*\text{Sqrt}[c]*d*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1 + c*d*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1^2 + a*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1^2)/(b*\text{Sqrt}[c]*d - 2*c*d*\#1 - a*f*\#1 + f*\#1^3) \&]/(2*d)$

fricas [B] time = 17.02, size = 1253, normalized size = 4.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/4*(d*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a*f)/(d^2*f))*log((2*sqrt(c*x^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a*f)/(d^2*f)) + 2*b*c*x + b^2 + (b*d*f*x + 2*a*d*f)*sqrt(b^2/(d^3*f)))/x) - d*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a*f)/(d^2*f))*log(-(2*sqrt(c*x^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a*f)/(d^2*f)) - 2*b*c*x - b^2 - (b*d*f*x + 2*a*d*f)*sqrt(b^2/(d^3*f)))/x) - d*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)/(d^2*f))*log((2*sqrt(c*x^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)/(d^2*f)) + 2*b*c*x + b^2 - (b*d*f*x + 2*a*d*f)*sqrt(b^2/(d^3*f)))/x) + d*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)/(d^2*f))*log(-(2*sqrt(c*x^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)/(d^2*f)) - 2*b*c*x - b^2 + (b*d*f*x + 2*a*d*f)*sqrt(b^2/(d^3*f)))/x) + 2*sqrt(a)*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2))/d, 1/4*(d*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a*f)/(d^2*f))*log((2*sqrt(c*x^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a*f)/(d^2*f)) + 2*b*c*x + b^2 + (b*d*f*x + 2*a*d*f)*sqrt(b^2/(d^3*f)))/x) - d*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a*f)/(d^2*f))*log(-(2*sqrt(c*x^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a*f)/(d^2*f)) - 2*b*c*x - b^2 - (b*d*f*x + 2*a*d*f)*sqrt(b^2/(d^3*f)))/x) - d*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)/(d^2*f))*log((2*sqrt(c*x^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)/(d^2*f)) + 2*b*c*x + b^2 - (b*d*f*x + 2*a*d*f)*sqrt(b^2/(d^3*f)))/x) + d*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)/(d^2*f))*log(-(2*sqrt(c*x^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)/(d^2*f)) - 2*b*c*x - b^2 + (b*d*f*x + 2*a*d*f)*sqrt(b^2/(d^3*f)))/x) + 4*sqrt(-a)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)))/d]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage2

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] sage2
```

maple [B] time = 0.02, size = 1764, normalized size = 6.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/x/(-f*x^2+d),x)
```

```
[Out] -1/2/d*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)+1/2/d/f*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)-1/4/d*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b-1/2/d/f/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f)*(d*f)^(1/2)*b+1/2/d/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+
```

$$(b*f-2*(d*f)^{(1/2)*c}*(x+(d*f)^{(1/2)/f})/f+2*((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)*((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c}*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2))}/(x+(d*f)^{(1/2)/f}))*a+1/2/f/((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)*\ln((2*(a*f+c*d-(d*f)^{(1/2)*b})/f+(b*f-2*(d*f)^{(1/2)*c}*(x+(d*f)^{(1/2)/f})/f+2*((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)*((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c}*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2))}/(x+(d*f)^{(1/2)/f}))*c-1/2/d*((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)-1/2}/d/f*\ln(((x-(d*f)^{(1/2)/f})^2*c+1/2*(b*f+2*(d*f)^{(1/2)*c})/f)/c^{(1/2)}+((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2))*c^{(1/2)}*(d*f)^{(1/2)-1/4}/d*\ln(((x-(d*f)^{(1/2)/f})^2*c+1/2*(b*f+2*(d*f)^{(1/2)*c})/f)/c^{(1/2)}+((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2))}/c^{(1/2)*b+1/2}/d/f/((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)*\ln((2*(a*f+c*d+(d*f)^{(1/2)*b})/f+(b*f+2*(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f})/f+2*((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)*((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2))}/(x-(d*f)^{(1/2)/f}))*c^{(1/2)*b+1/2}/d/((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)*\ln((2*(a*f+c*d+(d*f)^{(1/2)*b})/f+(b*f+2*(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f})/f+2*((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)*((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2))}/(x-(d*f)^{(1/2)/f}))*a+1/2/f/((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)*\ln((2*(a*f+c*d+(d*f)^{(1/2)*b})/f+(b*f+2*(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f})/f+2*((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)*((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2))}/(x-(d*f)^{(1/2)/f}))*c+1/d*(c*x^2+b*x+a)^{(1/2)+1/2}/d*b*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2))}/c^{(1/2)-1/d*a^{(1/2)*\ln((2*a+b*x+2*a^{(1/2)}*(c*x^2+b*x+a)^{(1/2))}/x)$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{cx^2 + bx + a}}{(fx^2 - d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(1/2)/x/(-f*x^2+d),x, algorithm="maxima")

[Out] -integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 - d)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{x(d - fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x + c*x^2)^(1/2)/(x*(d - f*x^2)),x)

[Out] int((a + b*x + c*x^2)^(1/2)/(x*(d - f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{a + bx + cx^2}}{-dx + fx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)**(1/2)/x/(-f*x**2+d),x)

[Out] -Integral(sqrt(a + b*x + c*x**2)/(-d*x + f*x**3), x)

$$3.82 \quad \int \frac{\sqrt{a+bx+cx^2}}{x^2(d-fx^2)} dx$$

Optimal. Leaf size=286

$$\frac{\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd} \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2d^{3/2}} + \frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd} \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{3/2}}$$

Rubi [A] time = 0.71, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6725, 732, 843, 621, 206, 724, 990, 1033}

$$\frac{\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd} \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2d^{3/2}} + \frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd} \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{3/2}} - \frac{\sqrt{a+bx+cx^2}}{dx} - \frac{b \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x + c*x^2]/(x^2*(d - f*x^2)),x]

[Out] -(Sqrt[a + b*x + c*x^2]/(d*x)) - (b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[a]*d) + (Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)) + (Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 732

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Dist[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1]) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]

Rule 843

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

Rule 990

Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (f_.)*(x_)^2), x_Symbol] := Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*f - b*f*x)/(Sqrt[a + b*x + c*x^2]*(d + f*x^2)), x], x] /; FreeQ[{a, b, c, d, f}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1033

Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/(q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]

Rule 6725

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a+bx+cx^2}}{x^2(d-fx^2)} dx &= \int \left(\frac{\sqrt{a+bx+cx^2}}{dx^2} + \frac{f\sqrt{a+bx+cx^2}}{d(d-fx^2)} \right) dx \\ &= \frac{\int \frac{\sqrt{a+bx+cx^2}}{x^2} dx}{d} + \frac{f \int \frac{\sqrt{a+bx+cx^2}}{d-fx^2} dx}{d} \\ &= -\frac{\sqrt{a+bx+cx^2}}{dx} + \frac{\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}} dx}{2d} + \frac{\int \frac{cd+af+bf x}{\sqrt{a+bx+cx^2}(d-fx^2)} dx}{d} - \frac{c \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{d} \\ &= -\frac{\sqrt{a+bx+cx^2}}{dx} + \frac{b \int \frac{1}{x\sqrt{a+bx+cx^2}} dx}{2d} + \frac{c \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{d} - \frac{(2c) \text{Subst} \left(\int \frac{1}{4c-x^2} dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}} \right)}{d} \\ &= -\frac{\sqrt{a+bx+cx^2}}{dx} - \frac{\sqrt{c} \tanh^{-1} \left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}} \right)}{d} - \frac{b \text{Subst} \left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}} \right)}{d} + \\ &= -\frac{\sqrt{a+bx+cx^2}}{dx} - \frac{b \tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}} \right)}{2\sqrt{a}d} + \frac{\sqrt{cd-b\sqrt{d}}\sqrt{f}+af \tanh^{-1} \left(\frac{b\sqrt{d}-2a}{2\sqrt{cd-b\sqrt{d}}} \right)}{2d^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.43, size = 275, normalized size = 0.96

$$\frac{\sqrt{af+b\sqrt{d}}\sqrt{f}+cd \tanh^{-1} \left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}}\sqrt{f}+cd} \right) + \sqrt{af+b(-\sqrt{d})}\sqrt{f}+cd \tanh^{-1} \left(\frac{-2a\sqrt{f}+b(\sqrt{d}-\sqrt{f}x)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b(-\sqrt{d})}\sqrt{f}+cd} \right) - \frac{2\sqrt{d}\sqrt{a+x(b+cx)}}{x} - \frac{b\sqrt{d} \tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}} \right)}{\sqrt{a}}}{2d^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*x + c*x^2]/(x^2*(d - f*x^2)),x]

```
[Out] ((-2*Sqrt[d]*Sqrt[a + x*(b + c*x))]/x - (b*Sqrt[d]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/Sqrt[a] + Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(2*d^(3/2))
```

IntegrateAlgebraic [C] time = 0.53, size = 299, normalized size = 1.05

$$\frac{\text{RootSum}\left[\#1^4(-f) + 2\#1^2af + 4\#1^2cd - 4\#1b\sqrt{cd} - a^2f + b^2d\&, \frac{\#1^2bf\log(-\#1 + \sqrt{a+bx+cx^2} - \sqrt{cx}) - 2\#1c^2d\log(-\#1 + \sqrt{a+bx+cx^2} - \sqrt{cx}) + bcd\log(-\#1 + \sqrt{a+bx+cx^2} - \sqrt{cx}) - 2\#1a\sqrt{cf}\log(-\#1 + \sqrt{a+bx+cx^2} - \sqrt{cx})}{\#1^3f - \#1af - 2\#1cd + b\sqrt{cd}}\&]{\frac{\sqrt{a+bx+cx^2}}{dx} + \frac{b \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a}} - \frac{\sqrt{a+bx+cx^2}}{\sqrt{a}}\right)}{\sqrt{ad}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x + c*x^2]/(x^2*(d - f*x^2)),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/(d*x)) + (b*ArcTanh[(Sqrt[c]*x)/Sqrt[a] - Sqrt[a + b*x + c*x^2]/Sqrt[a]])/(Sqrt[a]*d) - RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (b*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*c^(3/2)*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*a*Sqrt[c]*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + b*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ]/(2*d)
```

fricas [B] time = 24.59, size = 1094, normalized size = 3.83



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/4*(a*d*x*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3)*log((2*b*c*x + 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3) + b^2 + (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) - a*d*x*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3)*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3) + b^2 + (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) + a*d*x*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3)*log((2*b*c*x + 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3) + b^2 - (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) - a*d*x*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3)*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3) + b^2 - (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) + sqrt(a)*b*x*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*sqrt(c*x^2 + b*x + a)*a/(a*d*x) , 1/4*(a*d*x*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3)*log((2*b*c*x + 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3) + b^2 + (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) - a*d*x*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3)*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3) + b^2 + (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) + a*d*x*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3)*log((2*b*c*x + 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3) + b^2 - (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) - a*d*x*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3)*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3) + b^2 - (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) + 2*sqrt(-a)*b*x*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - 4*sqrt(c*x^2 + b*x + a)*a/(a*d*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.87sym2poly/r2sym(const ge
n & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.02, size = 1819, normalized size = 6.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/x^2/(-f*x^2+d),x)
```

```
[Out] 1/2*f/d/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(
1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)-1/2/d*ln(((x+(d*f)^(1/2)/f)*c+1/
2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)
)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)+1/4*f/d/
(d*f)^(1/2)*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((
x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*
f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b+1/2/d/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln(
(2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((
a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c
)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*
b-1/2*f/d/(d*f)^(1/2)/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)
)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1
/2)*b)/f)^(1/2))*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)
)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*a-1/2/(d*f)^(1/2
)/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*
(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*((x+
(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)
^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*c-1/d/a/x*(c*x^2+b*x+a)^(3/2)+1/d*b/
a*(c*x^2+b*x+a)^(1/2)-1/2/d*b/a^(1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2))*a^(
1/2)/x)+1/d*c/a*(c*x^2+b*x+a)^(1/2)*x+1/d*c^(1/2)*ln((c*x+1/2*b)/c^(1/2)+(
c*x^2+b*x+a)^(1/2))-1/2*f/d/(d*f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)
^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)-1/2/d*ln(((x
-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2
*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/
2))*c^(1/2)-1/4*f/d/(d*f)^(1/2)*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1
/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/
2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b+1/2/d/((a*f+c*d+(d*f)^(
1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(
d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*((x-(d*f)^(1/2)/f)^2*c+
(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))
/(x-(d*f)^(1/2)/f))*b+1/2*f/d/(d*f)^(1/2)/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)
*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+
2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/
2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/
f))*a+1/2/(d*f)^(1/2)/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)
)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1
/2)*b)/f)^(1/2))*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)
)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{cx^2 + bx + a}}{(fx^2 - d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(-f*x^2+d),x, algorithm="maxima")

[Out] -integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 - d)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{x^2 (d - fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x + c*x^2)^(1/2)/(x^2*(d - f*x^2)),x)

[Out] int((a + b*x + c*x^2)^(1/2)/(x^2*(d - f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{\sqrt{a + bx + cx^2}}{-dx^2 + fx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)**(1/2)/x**2/(-f*x**2+d),x)

[Out] -Integral(sqrt(a + b*x + c*x**2)/(-d*x**2 + f*x**4), x)

$$3.83 \quad \int \frac{\sqrt{a+bx+cx^2}}{x^3(d-fx^2)} dx$$

Optimal. Leaf size=353

$$\frac{(b^2 - 4ac) \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d} - \frac{\sqrt{a} f \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2} - \frac{\sqrt{f} \sqrt{af+b(-\sqrt{d})} \sqrt{f} + cd \tanh^{-1}\left(\frac{-2a}{2\sqrt{a+bx+cx^2}}\right)}{2d^2}$$

Rubi [A] time = 0.88, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 10, integrand size = 28, number of rules / integrand size = 0.357, Rules used = {6725, 720, 724, 206, 734, 843, 621, 1021, 1078, 1033}

$$\frac{(b^2 - 4ac) \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d} - \frac{\sqrt{a} f \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2} - \frac{\sqrt{f} \sqrt{af+b(-\sqrt{d})} \sqrt{f} + cd \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f}+cd}\right)}{2d^2} + \frac{\sqrt{f} \sqrt{af+b\sqrt{d}} \sqrt{f} + cd \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}}\sqrt{f}+cd}\right)}{2d^2} - \frac{(2a+bx)\sqrt{a+bx+cx^2}}{4ndx^2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x + c*x^2]/(x^3*(d - f*x^2)),x]

[Out] -((2*a + b*x)*Sqrt[a + b*x + c*x^2])/(4*a*d*x^2) + ((b^2 - 4*a*c)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(3/2)*d) - (Sqrt[a]*f*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d^2 - (Sqrt[f]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2) + (Sqrt[f]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 720

Int[((d_.) + (e_.)*(x_)^(m_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && (!RationalQ[m] || LtQ[m, 1]) && !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1021

```
Int[((g_.) + (h_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-b*f)) + c*(-2*g*f)*(p + q + 1))*x^2, x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2], x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2], x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f, 0]
```

Rule 6725

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx+cx^2}}{x^3(d-fx^2)} dx &= \int \left(\frac{\sqrt{a+bx+cx^2}}{dx^3} + \frac{f\sqrt{a+bx+cx^2}}{d^2x} + \frac{f^2x\sqrt{a+bx+cx^2}}{d^2(d-fx^2)} \right) dx \\
&= \frac{\int \frac{\sqrt{a+bx+cx^2}}{x^3} dx}{d} + \frac{f \int \frac{\sqrt{a+bx+cx^2}}{x} dx}{d^2} + \frac{f^2 \int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2} dx}{d^2} \\
&= -\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2} - \frac{(b^2-4ac) \int \frac{1}{x\sqrt{a+bx+cx^2}} dx}{8ad} - \frac{f \int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}} dx}{2d^2} + \frac{f \int \frac{\frac{bd}{2\sqrt{a+bx+cx^2}}}{d-fx^2} dx}{d^2} \\
&= -\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2} - \frac{\int \frac{-bdf-f(cd+af)x}{\sqrt{a+bx+cx^2}(d-fx^2)} dx}{d^2} + \frac{(b^2-4ac) \text{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{bd}{2\sqrt{a+bx+cx^2}}\right)}{4ad} \\
&= -\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2} + \frac{(b^2-4ac) \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d} - \frac{(2af) \text{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{bd}{2\sqrt{a+bx+cx^2}}\right)}{d^2} \\
&= -\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2} + \frac{(b^2-4ac) \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d} - \frac{\sqrt{a} f \tanh^{-1}\left(\frac{2a}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2} \\
&= -\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2} + \frac{(b^2-4ac) \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d} - \frac{\sqrt{a} f \tanh^{-1}\left(\frac{2a}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2}
\end{aligned}$$

Mathematica [A] time = 0.52, size = 316, normalized size = 0.90

$$\frac{x^2 (b^2 d - 4a(2af + cd)) \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right) - 2\sqrt{a} \left(-2a\sqrt{f} x^2 \sqrt{af+b\sqrt{d}\sqrt{f}+cd} \tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}\sqrt{f}+2c\sqrt{d}x}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right) + 2a\sqrt{f} x^2 \sqrt{af+b(-\sqrt{d})\sqrt{f}+cd} \tanh^{-1}\left(\frac{-2a\sqrt{f}+b(\sqrt{d}-\sqrt{f})x+2c\sqrt{d}x}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right) + d(2a+bx)\sqrt{a+bx+cx^2} \right)}{8a^{3/2}d^2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*x + c*x^2]/(x^3*(d - f*x^2)), x]

[Out] ((b^2*d - 4*a*(c*d + 2*a*f))*x^2*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])] - 2*Sqrt[a]*(d*(2*a + b*x)*Sqrt[a + x*(b + c*x)] - 2*a*Sqrt[f]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*x^2*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + 2*a*Sqrt[f]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*x^2*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(8*a^(3/2)*d^2*x^2)

IntegrateAlgebraic [C] time = 0.86, size = 392, normalized size = 1.11

$$\frac{f\text{RootSum}\left[\#1^3(-f) + 2\#1^2af + 4\#1cd - 4\#1b\sqrt{cd - a^2f} + b^2dk, \frac{a^2\sqrt{af}\log\left(\frac{d+\sqrt{a+bx+cx^2}-\sqrt{c}}{d-\sqrt{a+bx+cx^2}-\sqrt{c}}\right)+a^2f\log\left(\frac{d+\sqrt{a+bx+cx^2}-\sqrt{c}}{d-\sqrt{a+bx+cx^2}-\sqrt{c}}\right)+a^2f\log\left(\frac{d+\sqrt{a+bx+cx^2}-\sqrt{c}}{d-\sqrt{a+bx+cx^2}-\sqrt{c}}\right)+2ab\sqrt{cd}\log\left(\frac{d+\sqrt{a+bx+cx^2}-\sqrt{c}}{d-\sqrt{a+bx+cx^2}-\sqrt{c}}\right)}{2d^2}\right] + \frac{(-8d^2f - 4acd + b^2d)\tanh^{-1}\left(\frac{\sqrt{a+bx+cx^2}-\sqrt{c}}{\sqrt{d}}\right) + (-2d-b)\sqrt{d}\sqrt{a+bx+cx^2}}{4a^{3/2}d^2}}{4a^{3/2}d^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a + b*x + c*x^2]/(x^3*(d - f*x^2)), x]

[Out] ((-2*a - b*x)*Sqrt[a + b*x + c*x^2])/(4*a*d*x^2) + ((b^2*d - 4*a*c*d - 8*a^2*f)*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2])/Sqrt[a]])/(4*a^(3/2)*d^2) - (f*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (b^2*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*b*Sqrt[c]*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + a*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) &])/(2*d^2)

fricas [B] time = 133.83, size = 1485, normalized size = 4.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(1/2)/x^3/(-f*x^2+d),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/16*(4*a^2*d^2*x^2*\sqrt{(d^4*\sqrt{b^2*f^3/d^7} + c*d*f + a*f^2)/d^4})*\log(\\ & (2*\sqrt{c*x^2 + b*x + a}*d^5*\sqrt{b^2*f^3/d^7})*\sqrt{(d^4*\sqrt{b^2*f^3/d^7} \\ & + c*d*f + a*f^2)/d^4} + 2*b*c*f^2*x + b^2*f^2 + (b*d^3*f*x + 2*a*d^3*f)*\sqrt{ \\ & (b^2*f^3/d^7)}/x) - 4*a^2*d^2*x^2*\sqrt{(d^4*\sqrt{b^2*f^3/d^7} + c*d*f + a \\ & f^2)/d^4})*\log(-(2*\sqrt{c*x^2 + b*x + a}*d^5*\sqrt{b^2*f^3/d^7})*\sqrt{(d^4*\sqrt{ \\ & (b^2*f^3/d^7) + c*d*f + a*f^2)/d^4} - 2*b*c*f^2*x - b^2*f^2 - (b*d^3*f*x + \\ & 2*a*d^3*f)*\sqrt{b^2*f^3/d^7)}/x) - 4*a^2*d^2*x^2*\sqrt{-(d^4*\sqrt{b^2*f^3/d \\ & ^7) - c*d*f - a*f^2)/d^4})*\log((2*\sqrt{c*x^2 + b*x + a}*d^5*\sqrt{b^2*f^3/d^7} \\ &)*\sqrt{-(d^4*\sqrt{b^2*f^3/d^7} - c*d*f - a*f^2)/d^4} + 2*b*c*f^2*x + b^2*f^2 \\ & - (b*d^3*f*x + 2*a*d^3*f)*\sqrt{b^2*f^3/d^7)}/x) + 4*a^2*d^2*x^2*\sqrt{-(d^4 \\ & *\sqrt{b^2*f^3/d^7} - c*d*f - a*f^2)/d^4})*\log(-(2*\sqrt{c*x^2 + b*x + a}*d^5 \\ & *\sqrt{b^2*f^3/d^7})*\sqrt{-(d^4*\sqrt{b^2*f^3/d^7} - c*d*f - a*f^2)/d^4} - 2*b \\ & *c*f^2*x - b^2*f^2 + (b*d^3*f*x + 2*a*d^3*f)*\sqrt{b^2*f^3/d^7)}/x) + (8*a^2 \\ & *f - (b^2 - 4*a*c)*d)*\sqrt{a}*x^2*\log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*\sqrt{ \\ & (c*x^2 + b*x + a)*(b*x + 2*a)*\sqrt{a} + 8*a^2)/x^2) - 4*(a*b*d*x + 2*a^2*d \\ &)*\sqrt{c*x^2 + b*x + a})/(a^2*d^2*x^2), 1/8*(2*a^2*d^2*x^2*\sqrt{(d^4*\sqrt{b^2*f^3/d^7} \\ & + c*d*f + a*f^2)/d^4})*\log((2*\sqrt{c*x^2 + b*x + a}*d^5*\sqrt{b^2*f^3/d^7} \\ &)*\sqrt{(d^4*\sqrt{b^2*f^3/d^7} + c*d*f + a*f^2)/d^4} + 2*b*c*f^2*x + \\ & b^2*f^2 + (b*d^3*f*x + 2*a*d^3*f)*\sqrt{b^2*f^3/d^7)}/x) - 2*a^2*d^2*x^2*\sqrt{ \\ & (d^4*\sqrt{b^2*f^3/d^7} + c*d*f + a*f^2)/d^4})*\log(-(2*\sqrt{c*x^2 + b*x + \\ & a}*d^5*\sqrt{b^2*f^3/d^7})*\sqrt{(d^4*\sqrt{b^2*f^3/d^7} + c*d*f + a*f^2)/d^4} \\ & - 2*b*c*f^2*x - b^2*f^2 - (b*d^3*f*x + 2*a*d^3*f)*\sqrt{b^2*f^3/d^7)}/x) - 2 \\ & *a^2*d^2*x^2*\sqrt{-(d^4*\sqrt{b^2*f^3/d^7} - c*d*f - a*f^2)/d^4})*\log((2*\sqrt{ \\ & (c*x^2 + b*x + a)*d^5*\sqrt{b^2*f^3/d^7})*\sqrt{-(d^4*\sqrt{b^2*f^3/d^7} - c*d* \\ & f - a*f^2)/d^4} + 2*b*c*f^2*x + b^2*f^2 - (b*d^3*f*x + 2*a*d^3*f)*\sqrt{b^2* \\ & f^3/d^7)}/x) + 2*a^2*d^2*x^2*\sqrt{-(d^4*\sqrt{b^2*f^3/d^7} - c*d*f - a*f^2)/ \\ & d^4})*\log(-(2*\sqrt{c*x^2 + b*x + a}*d^5*\sqrt{b^2*f^3/d^7})*\sqrt{-(d^4*\sqrt{b^2 \\ & *f^3/d^7} - c*d*f - a*f^2)/d^4} - 2*b*c*f^2*x - b^2*f^2 + (b*d^3*f*x + 2*a \\ & *d^3*f)*\sqrt{b^2*f^3/d^7)}/x) + (8*a^2*f - (b^2 - 4*a*c)*d)*\sqrt{-a}*x^2*ar \\ & ctan(1/2*\sqrt{c*x^2 + b*x + a)*(b*x + 2*a)*\sqrt{-a})/(a*c*x^2 + a*b*x + a^2) \\ &) - 2*(a*b*d*x + 2*a^2*d)*\sqrt{c*x^2 + b*x + a})/(a^2*d^2*x^2)] \end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(1/2)/x^3/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:Evaluation time: 1.36sym2poly/r2sym(const ge
n & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [B] time = 0.02, size = 1953, normalized size = 5.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x+a)^(1/2)/x^3/(-f*x^2+d),x)

[Out]
$$\begin{aligned} & -1/2*f/d^2*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f \\ & +(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)+1/2/d^2*\ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f- \\ & 2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x \\ & +(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)-1/4 \end{aligned}$$

```

*f/d^2*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*
f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1
/2)*b)/f)^(1/2))/c^(1/2)*b-1/2/d^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*
(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f
+c*d-(d*f)^(1/2)*b)/f)^(1/2))*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(
x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*(d*
f)^(1/2)*b+1/2*f/d^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)
^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/
2)*b)/f)^(1/2))*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/
f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*a+1/2/d/((a*f+c*d
-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)
*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*((x+(d*f)^(1/2)
/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f
)^(1/2))/(x+(d*f)^(1/2)/f))*c-1/2*f/d^2*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)
^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)-1/2/d^2*ln((
(x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)
^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(
1/2))*c^(1/2)*(d*f)^(1/2)-1/4*f/d^2*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)
^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)
^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b+1/2/d^2/((a*f+c*d+(
d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c
)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*((x-(d*f)^(1/2)/f
)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(
1/2))/(x-(d*f)^(1/2)/f))*(d*f)^(1/2)*b+1/2*f/d^2/((a*f+c*d+(d*f)^(1/2)*b)/
f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/
2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*
(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f
)^(1/2)/f))*a+1/2/d/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(
1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)
*b)/f)^(1/2))*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f
)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c-1/2/d/a/x^2*(c*x
^2+b*x+a)^(3/2)+1/4/d*b/a^2/x*(c*x^2+b*x+a)^(3/2)-1/4/d*b^2/a^2*(c*x^2+b*x+a
)^(1/2)+1/8/d*b^2/a^(3/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-1/
4/d*b/a^2*c*(c*x^2+b*x+a)^(1/2)*x+1/2/d*c/a*(c*x^2+b*x+a)^(1/2)-1/2/d*c/a^(
1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)+f/d^2*(c*x^2+b*x+a)^(1/2
)+1/2*f/d^2*b*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-f/d^2*a^(
1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{cx^2 + bx + a}}{(fx^2 - d)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(1/2)/x^3/(-f*x^2+d),x, algorithm="maxima")

[Out] -integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 - d)*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{x^3 (d - fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x + c*x^2)^(1/2)/(x^3*(d - f*x^2)),x)

[Out] int((a + b*x + c*x^2)^(1/2)/(x^3*(d - f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{a + bx + cx^2}}{-dx^3 + fx^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)**(1/2)/x**3/(-f*x**2+d), x)

[Out] -Integral(sqrt(a + b*x + c*x**2)/(-d*x**3 + f*x**5), x)

$$3.84 \quad \int \frac{x^3(a+bx+cx^2)^{3/2}}{d-fx^2} dx$$

Optimal. Leaf size=501

$$\frac{3b(b^2 - 4ac)^2 \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{7/2}f} - \frac{3b(b^2 - 4ac)(b + 2cx)\sqrt{a + bx + cx^2}}{128c^3f} - \frac{d\sqrt{a + bx + cx^2}(8acf + b^2f + \dots)}{8cf^3}$$

Rubi [A] time = 1.41, antiderivative size = 501, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 28, number of rules / integrand size = 0.357, Rules used = {6725, 640, 612, 621, 206, 1021, 1070, 1078, 1033, 724}

$$\frac{d\sqrt{a+bx+cx^2}(8acf+b^2f+8c^2d)}{8c^3f^2} - \frac{3b(2acf+b^2f+24c^2d)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}f} - \frac{3b(b^2-4ac)(b+2cx)\sqrt{a+bx+cx^2}}{128c^3f} - \frac{3b(b^2-4ac)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{7/2}f} - \frac{3b(b+2cx)(a+bx+cx^2)^{3/2}}{16c^3f} - \frac{d(a+bx+cx^2)^{3/2}}{3f^2} - \frac{d(ej+b(-\sqrt{d})\sqrt{f+cd})^{3/2}\tanh^{-1}\left(\frac{2c\sqrt{a+bx+cx^2}+d\sqrt{f+cd}}{2\sqrt{c}\sqrt{a+bx+cx^2}+d\sqrt{f+cd}}\right)}{2f^{3/2}} - \frac{d(ej+b\sqrt{d}\sqrt{f+cd})^{3/2}\tanh^{-1}\left(\frac{2c\sqrt{a+bx+cx^2}+d\sqrt{f+cd}}{2\sqrt{c}\sqrt{a+bx+cx^2}+d\sqrt{f+cd}}\right)}{2f^{3/2}} - \frac{(a+bx+cx^2)^{5/2}}{8cf}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(a + b*x + c*x^2)^(3/2))/(d - f*x^2), x]

[Out] (-3*b*(b^2 - 4*a*c)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(128*c^3*f) - (d*(8*c^2*d + b^2*f + 8*a*c*f + 2*b*c*f*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^3) - (d*(a + b*x + c*x^2)^(3/2))/(3*f^2) + (b*(b + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(16*c^2*f) - (a + b*x + c*x^2)^(5/2)/(5*c*f) + (3*b*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*c^(7/2)*f) - (b*d*(24*c^2*d - b^2*f + 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*f^3) - (d*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + b*x + c*x^2]])/(2*f^(7/2)) + (d*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + b*x + c*x^2]])/(2*f^(7/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1021

```
Int[((g_.) + (h_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-b*f)) + c*(-2*g*f)*(p + q + 1))*x^2, x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1070

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_.) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((B*c*f*(2*p + 2*q + 3) + C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*(C*(-b*f))*(q + 1) - c*(-B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(-b*f))*(q + 1) - c*(-B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(-(b*c*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3))))*x + (p*(-b*f))*(C*(-b*f))*(q + 1) - c*(-B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))]*x^2, x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f, 0]
```

Rule 6725

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^3 (a + bx + cx^2)^{3/2}}{d - fx^2} dx = \int \left(-\frac{x (a + bx + cx^2)^{3/2}}{f} + \frac{dx (a + bx + cx^2)^{3/2}}{f(d - fx^2)} \right) dx$$

$$= -\frac{\int x (a + bx + cx^2)^{3/2} dx}{f} + \frac{d \int \frac{x(a+bx+cx^2)^{3/2}}{d-fx^2} dx}{f}$$

$$= -\frac{d (a + bx + cx^2)^{3/2}}{3f^2} - \frac{(a + bx + cx^2)^{5/2}}{5cf} + \frac{d \int \frac{\sqrt{a+bx+cx^2} \left(\frac{3bd}{2} + 3(cd+af)x + \frac{3}{2}bfx^2 \right)}{d-fx^2} dx}{3f^2}$$

$$= -\frac{d(8c^2d + b^2f + 8acf + 2bcfx) \sqrt{a + bx + cx^2}}{8cf^3} - \frac{d(a + bx + cx^2)^{3/2}}{3f^2} + \frac{b(b + 2cx) \sqrt{a + bx + cx^2}}{128c^3f}$$

$$= -\frac{3b(b^2 - 4ac)(b + 2cx) \sqrt{a + bx + cx^2}}{128c^3f} - \frac{d(8c^2d + b^2f + 8acf + 2bcfx) \sqrt{a + bx + cx^2}}{8cf^3}$$

$$= -\frac{3b(b^2 - 4ac)(b + 2cx) \sqrt{a + bx + cx^2}}{128c^3f} - \frac{d(8c^2d + b^2f + 8acf + 2bcfx) \sqrt{a + bx + cx^2}}{8cf^3}$$

$$= -\frac{3b(b^2 - 4ac)(b + 2cx) \sqrt{a + bx + cx^2}}{128c^3f} - \frac{d(8c^2d + b^2f + 8acf + 2bcfx) \sqrt{a + bx + cx^2}}{8cf^3}$$

$$= -\frac{3b(b^2 - 4ac)(b + 2cx) \sqrt{a + bx + cx^2}}{128c^3f} - \frac{d(8c^2d + b^2f + 8acf + 2bcfx) \sqrt{a + bx + cx^2}}{8cf^3}$$

Mathematica [A] time = 1.49, size = 447, normalized size = 0.89

$$\frac{\sqrt{a+bx+cx^2} \left(2b^2 f^2 c^2 d^2 + 2b^2 f^2 c^2 d^2 + 2b^2 f^2 c^2 d^2 + 2b^2 f^2 c^2 d^2 + 2b^2 f^2 c^2 d^2 + 2b^2 f^2 c^2 d^2 + 2b^2 f^2 c^2 d^2 + 2b^2 f^2 c^2 d^2 + 2b^2 f^2 c^2 d^2 + 2b^2 f^2 c^2 d^2 \right)}{1920 f^6} + \frac{960 d (af + b(-\sqrt{d})\sqrt{f} + cd)^{3/2} \operatorname{tanh}^{-1} \left(\frac{2a\sqrt{d} + \sqrt{d}\sqrt{f} + 2a\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{d-fx^2}} \right) - 960 d (af + b\sqrt{d}\sqrt{f} + cd)^{3/2} \operatorname{tanh}^{-1} \left(\frac{2a\sqrt{d} - \sqrt{d}\sqrt{f} + 2a\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{d-fx^2}} \right)}{256c^3 f^3} + \frac{b(16c^2 f (3a^2 f + b^2 d) - 24a^2 f^2 - 192ac^2 f + 3b^2 f^2 - 384c^2 d^2) \operatorname{tanh}^{-1} \left(\frac{b+2cx}{2\sqrt{a+bx+cx^2}} \right)}{256c^3 f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x + c*x^2)^(3/2))/(d - f*x^2), x]
```

```
[Out] (b*(-384*c^4*d^2 - 192*a*c^3*d*f + 3*b^4*f^2 - 24*a*b^2*c*f^2 + 16*c^2*f*(b^2*d + 3*a^2*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(256*c^(7/2)*f^3) + (-(Sqrt[f]*Sqrt[a + x*(b + c*x)]*(45*b^4*f^2 - 30*b^2*c*f^2*(10*a + b*x) + 16*c^3*f*(160*a*d + 70*b*d*x + 48*a*f*x^2 + 33*b*f*x^3) + 128*c^4*(15*d^2 + 5*d*f*x^2 + 3*f^2*x^4) + 24*c^2*f*(16*a^2*f + 7*a*b*f*x + b^2*(10*d + f*x^2))))/c^3) + 960*d*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + x*(b + c*x)])] - 960*d*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + x*(b + c*x)])])/(1920*f^(7/2))
```

IntegrateAlgebraic [C] time = 3.07, size = 774, normalized size = 1.54

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(a + b*x + c*x^2)^(3/2))/(d - f*x^2), x]
```

```
[Out] (Sqrt[a + b*x + c*x^2]*(-1920*c^4*d^2 - 240*b^2*c^2*d*f - 2560*a*c^3*d*f -
45*b^4*f^2 + 300*a*b^2*c*f^2 - 384*a^2*c^2*f^2 - 1120*b*c^3*d*f*x + 30*b^3*
c*f^2*x - 168*a*b*c^2*f^2*x - 640*c^4*d*f*x^2 - 24*b^2*c^2*f^2*x^2 - 768*a*
c^3*f^2*x^2 - 528*b*c^3*f^2*x^3 - 384*c^4*f^2*x^4))/(1920*c^3*f^3) + ((384*
b*c^4*d^2 - 16*b^3*c^2*d*f + 192*a*b*c^3*d*f - 3*b^5*f^2 + 24*a*b^3*c*f^2 -
48*a^2*b*c^2*f^2)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(256*c
^(7/2)*f^3) - (d*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*
a*f*#1^2 - f*#1^4 & , (2*b^2*c*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2]
- #1] - a*c^2*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a*b^2*d
*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*a^2*c*d*f*Log[-(Sqrt[
c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^3*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b
*x + c*x^2] - #1] - 4*b*c^(3/2)*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2
] - #1]*#1 - 4*a*b*Sqrt[c]*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #
1]*#1 + c^2*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + b^2*d
*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + 2*a*c*d*f*Log[-(Sq
rt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + a^2*f^2*Log[-(Sqrt[c]*x) + Sq
rt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3)
& ])/(2*f^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.03, size = 4884, normalized size = 9.75

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] -1/5*(c*x^2+b*x+a)^(5/2)/c/f+1/16/f*b^2/c^2*(c*x^2+b*x+a)^(3/2)-3/128/f*b^4
/c^3*(c*x^2+b*x+a)^(1/2)+3/256/f*b^5/c^(7/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))-1/2/f^2*d*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*
f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*a-1/2/f^3*d^2*((x-(d*f)^(1/2
)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/
f)^(1/2)*c-1/2/f^2*d*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(
1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*a-1/2/f^3*d^2*((x+(d*f)^(1/2)/f
)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(
1/2)*c+1/2/f^4*d^3/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(
1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2
)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f
)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f)*c^2-1/8/f^2*d*((x+
(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)
^(1/2)*b)/f)^(1/2)*x*b+5/8/f^3*d*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*
```

$$\begin{aligned}
& c) * (x + (d*f)^{(1/2)} / f) / f + (a*f + c*d - (d*f)^{(1/2)} * b) / f)^{(1/2)} * b * (d*f)^{(1/2)} - 1/16 / \\
& f^2 * d / c * ((x + (d*f)^{(1/2)} / f)^2 * c + (b*f - 2 * (d*f)^{(1/2)} * c) * (x + (d*f)^{(1/2)} / f) / f + (a \\
& * f + c*d - (d*f)^{(1/2)} * b) / f)^{(1/2)} * b^2 + 1/32 / f^2 * d / c^{(3/2)} * \ln(((x + (d*f)^{(1/2)} / f) \\
& * c + 1/2 * (b*f - 2 * (d*f)^{(1/2)} * c) / f) / c^{(1/2)} + ((x + (d*f)^{(1/2)} / f)^2 * c + (b*f - 2 * (d*f) \\
& ^{(1/2)} * c) * (x + (d*f)^{(1/2)} / f) / f + (a*f + c*d - (d*f)^{(1/2)} * b) / f)^{(1/2)} * b^3 - 3/4 / f^3 \\
& * d^2 * \ln(((x + (d*f)^{(1/2)} / f) * c + 1/2 * (b*f - 2 * (d*f)^{(1/2)} * c) / f) / c^{(1/2)} + ((x + (d*f) \\
& ^{(1/2)} / f)^2 * c + (b*f - 2 * (d*f)^{(1/2)} * c) * (x + (d*f)^{(1/2)} / f) / f + (a*f + c*d - (d*f)^{(1/2)} \\
&) * b) / f)^{(1/2)} * c^{(1/2)} * b + 1/2 / f^4 * d^2 * \ln(((x + (d*f)^{(1/2)} / f) * c + 1/2 * (b*f - 2 * (d* \\
& f)^{(1/2)} * c) / f) / c^{(1/2)} + ((x + (d*f)^{(1/2)} / f)^2 * c + (b*f - 2 * (d*f)^{(1/2)} * c) * (x + (d*f) \\
&)^{(1/2)} / f) / f + (a*f + c*d - (d*f)^{(1/2)} * b) / f)^{(1/2)} * c^{(3/2)} * (d*f)^{(1/2)} + 1/2 / f^3 * \\
& d^2 / ((a*f + c*d - (d*f)^{(1/2)} * b) / f)^{(1/2)} * \ln((2 * (a*f + c*d - (d*f)^{(1/2)} * b) / f + (b*f - \\
& 2 * (d*f)^{(1/2)} * c) * (x + (d*f)^{(1/2)} / f) / f + 2 * ((a*f + c*d - (d*f)^{(1/2)} * b) / f)^{(1/2)} * ((\\
& x + (d*f)^{(1/2)} / f)^2 * c + (b*f - 2 * (d*f)^{(1/2)} * c) * (x + (d*f)^{(1/2)} / f) / f + (a*f + c*d - (d* \\
& f)^{(1/2)} * b) / f)^{(1/2)})) / (x + (d*f)^{(1/2)} / f) * b^2 + 1/2 / f^2 * d / ((a*f + c*d - (d*f)^{(1/2)} \\
&) * b) / f)^{(1/2)} * \ln((2 * (a*f + c*d - (d*f)^{(1/2)} * b) / f + (b*f - 2 * (d*f)^{(1/2)} * c) * (x + (d*f) \\
&)^{(1/2)} / f) / f + 2 * ((a*f + c*d - (d*f)^{(1/2)} * b) / f)^{(1/2)} * ((x + (d*f)^{(1/2)} / f)^2 * c + (b* \\
& f - 2 * (d*f)^{(1/2)} * c) * (x + (d*f)^{(1/2)} / f) / f + (a*f + c*d - (d*f)^{(1/2)} * b) / f)^{(1/2)})) / (x \\
& + (d*f)^{(1/2)} / f) * a^2 + 1/2 / f^4 * d^3 / ((a*f + c*d - (d*f)^{(1/2)} * b) / f)^{(1/2)} * \ln((2 * (a \\
& * f + c*d - (d*f)^{(1/2)} * b) / f + (b*f - 2 * (d*f)^{(1/2)} * c) * (x + (d*f)^{(1/2)} / f) / f + 2 * ((a*f + c \\
& * d - (d*f)^{(1/2)} * b) / f)^{(1/2)} * ((x + (d*f)^{(1/2)} / f)^2 * c + (b*f - 2 * (d*f)^{(1/2)} * c) * (x + \\
& (d*f)^{(1/2)} / f) / f + (a*f + c*d - (d*f)^{(1/2)} * b) / f)^{(1/2)})) / (x + (d*f)^{(1/2)} / f) * c^2 + 1 \\
& / 8 / f * b / c * x * (c * x^2 + b * x + a)^{(3/2)} - 3/64 / f * b^3 / c^2 * (c * x^2 + b * x + a)^{(1/2)} * x + 3/32 / f * \\
& b^2 / c^2 * (c * x^2 + b * x + a)^{(1/2)} * a + 3/16 / f * b / c^{(3/2)} * \ln((c * x + 1/2 * b) / c^{(1/2)} + (c * x^2 \\
& + b * x + a)^{(1/2)} * a^2 - 3/32 / f * b^3 / c^{(5/2)} * \ln((c * x + 1/2 * b) / c^{(1/2)} + (c * x^2 + b * x + a) \\
& ^{(1/2)} * a - 1/4 / f^3 * d * ((x - (d*f)^{(1/2)} / f)^2 * c + (b*f + 2 * (d*f)^{(1/2)} * c) * (x - (d*f)^{(\\
& 1/2)} / f) / f + (a*f + c*d + (d*f)^{(1/2)} * b) / f)^{(1/2)} * x * c * (d*f)^{(1/2)} - 3/4 / f^3 * d * \ln(((x \\
& - (d*f)^{(1/2)} / f) * c + 1/2 * (b*f + 2 * (d*f)^{(1/2)} * c) / f) / c^{(1/2)} + ((x - (d*f)^{(1/2)} / f)^2 \\
& * c + (b*f + 2 * (d*f)^{(1/2)} * c) * (x - (d*f)^{(1/2)} / f) / f + (a*f + c*d + (d*f)^{(1/2)} * b) / f)^{(1/ \\
& 2)) * c^{(1/2)} * (d*f)^{(1/2)} * a - 3/8 / f^2 * d / c^{(1/2)} * \ln(((x - (d*f)^{(1/2)} / f) * c + 1/2 * (b* \\
& f + 2 * (d*f)^{(1/2)} * c) / f) / c^{(1/2)} + ((x - (d*f)^{(1/2)} / f)^2 * c + (b*f + 2 * (d*f)^{(1/2)} * c) * \\
& (x - (d*f)^{(1/2)} / f) / f + (a*f + c*d + (d*f)^{(1/2)} * b) / f)^{(1/2)} * a * b - 3/16 / f^3 * d * \ln(((x \\
& - (d*f)^{(1/2)} / f) * c + 1/2 * (b*f + 2 * (d*f)^{(1/2)} * c) / f) / c^{(1/2)} + ((x - (d*f)^{(1/2)} / f)^2 \\
& * c + (b*f + 2 * (d*f)^{(1/2)} * c) * (x - (d*f)^{(1/2)} / f) / f + (a*f + c*d + (d*f)^{(1/2)} * b) / f)^{(1/ \\
& 2)) / c^{(1/2)} * b^2 * (d*f)^{(1/2)} + 1 / f^3 * d^2 / ((a*f + c*d + (d*f)^{(1/2)} * b) / f)^{(1/2)} * \ln(\\
& (2 * (a*f + c*d + (d*f)^{(1/2)} * b) / f + (b*f + 2 * (d*f)^{(1/2)} * c) * (x - (d*f)^{(1/2)} / f) / f + 2 * ((\\
& a*f + c*d + (d*f)^{(1/2)} * b) / f)^{(1/2)} * ((x - (d*f)^{(1/2)} / f)^2 * c + (b*f + 2 * (d*f)^{(1/2)} * c) \\
&) * (x - (d*f)^{(1/2)} / f) / f + (a*f + c*d + (d*f)^{(1/2)} * b) / f)^{(1/2)})) / (x - (d*f)^{(1/2)} / f) * \\
& a * c + 3/16 / f^3 * d * \ln(((x + (d*f)^{(1/2)} / f) * c + 1/2 * (b*f - 2 * (d*f)^{(1/2)} * c) / f) / c^{(1/2)} \\
& + ((x + (d*f)^{(1/2)} / f)^2 * c + (b*f - 2 * (d*f)^{(1/2)} * c) * (x + (d*f)^{(1/2)} / f) / f + (a*f + c*d - \\
& (d*f)^{(1/2)} * b) / f)^{(1/2)})) / c^{(1/2)} * b^2 * (d*f)^{(1/2)} + 1 / f^3 * d^2 / ((a*f + c*d - (d*f)^{(1/2)} * b) / f)^{(1/2)} * \ln((2 * (a*f + c*d - (d*f)^{(1/2)} * b) / f + (b*f - 2 * (d*f)^{(1/2)} * c) * (x + \\
& (d*f)^{(1/2)} / f) / f + 2 * ((a*f + c*d - (d*f)^{(1/2)} * b) / f)^{(1/2)} * ((x + (d*f)^{(1/2)} / f)^2 * c \\
& + (b*f - 2 * (d*f)^{(1/2)} * c) * (x + (d*f)^{(1/2)} / f) / f + (a*f + c*d - (d*f)^{(1/2)} * b) / f)^{(1/2)} \\
&) / (x + (d*f)^{(1/2)} / f) * a * c + 3/16 / f * b / c * (c * x^2 + b * x + a)^{(1/2)} * x * a + 1/4 / f^3 * d * ((x + \\
& (d*f)^{(1/2)} / f)^2 * c + (b*f - 2 * (d*f)^{(1/2)} * c) * (x + (d*f)^{(1/2)} / f) / f + (a*f + c*d - (d*f)^{(\\
& 1/2)} * b) / f)^{(1/2)} * x * c * (d*f)^{(1/2)} + 3/4 / f^3 * d * \ln(((x + (d*f)^{(1/2)} / f) * c + 1/2 * (b* \\
& f - 2 * (d*f)^{(1/2)} * c) / f) / c^{(1/2)} + ((x + (d*f)^{(1/2)} / f)^2 * c + (b*f - 2 * (d*f)^{(1/2)} * c) * \\
& (x + (d*f)^{(1/2)} / f) / f + (a*f + c*d - (d*f)^{(1/2)} * b) / f)^{(1/2)} * c^{(1/2)} * (d*f)^{(1/2)} * a \\
& - 3/8 / f^2 * d / c^{(1/2)} * \ln(((x + (d*f)^{(1/2)} / f) * c + 1/2 * (b*f - 2 * (d*f)^{(1/2)} * c) / f) / c^{(\\
& 1/2)} + ((x + (d*f)^{(1/2)} / f)^2 * c + (b*f - 2 * (d*f)^{(1/2)} * c) * (x + (d*f)^{(1/2)} / f) / f + (a*f + \\
& c*d - (d*f)^{(1/2)} * b) / f)^{(1/2)} * a * b + 1/2 / f^3 * d^2 / ((a*f + c*d + (d*f)^{(1/2)} * b) / f)^{(1 \\
& / 2)} * \ln((2 * (a*f + c*d + (d*f)^{(1/2)} * b) / f + (b*f + 2 * (d*f)^{(1/2)} * c) * (x - (d*f)^{(1/2)} / f) \\
& / f + 2 * ((a*f + c*d + (d*f)^{(1/2)} * b) / f)^{(1/2)} * ((x - (d*f)^{(1/2)} / f)^2 * c + (b*f + 2 * (d*f)^{(\\
& 1/2)} * c) * (x - (d*f)^{(1/2)} / f) / f + (a*f + c*d + (d*f)^{(1/2)} * b) / f)^{(1/2)})) / (x - (d*f)^{(1/ \\
& 2)} / f) * b^2 + 1/2 / f^2 * d / ((a*f + c*d + (d*f)^{(1/2)} * b) / f)^{(1/2)} * \ln((2 * (a*f + c*d + (d*f) \\
& ^{(1/2)} * b) / f + (b*f + 2 * (d*f)^{(1/2)} * c) * (x - (d*f)^{(1/2)} / f) / f + 2 * ((a*f + c*d + (d*f)^{(1/ \\
& 2)} * b) / f)^{(1/2)} * ((x - (d*f)^{(1/2)} / f)^2 * c + (b*f + 2 * (d*f)^{(1/2)} * c) * (x - (d*f)^{(1/2)} / \\
& f) / f + (a*f + c*d + (d*f)^{(1/2)} * b) / f)^{(1/2)})) / (x - (d*f)^{(1/2)} / f) * a^2 + 1/32 / f^2 * d / c^{ \\
& (3/2)} * \ln(((x - (d*f)^{(1/2)} / f) * c + 1/2 * (b*f + 2 * (d*f)^{(1/2)} * c) / f) / c^{(1/2)} + ((x - (d*f)
\end{aligned}$$

$$\begin{aligned} &)^{(1/2)/f)^2*c+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2))*b^3-3/4/f^3*d^2*\ln(((x-(d*f)^{(1/2)/f)*c+1/2*(b*f+2*(d*f)^{(1/2)*c)/f)/c^{(1/2)+((x-(d*f)^{(1/2)/f)^2*c+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2))*c^{(1/2)*b-1/2/f^4*d^2*\ln(((x-(d*f)^{(1/2)/f)*c+1/2*(b*f+2*(d*f)^{(1/2)*c)/f)/c^{(1/2)+((x-(d*f)^{(1/2)/f)^2*c+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2))*c^{(3/2)*(d*f)^{(1/2)-1/16/f^2*d/c*((x-(d*f)^{(1/2)/f)^2*c+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2))*b^2-1/6/f^2*d*((x-(d*f)^{(1/2)/f)^2*c+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(3/2)-1/6/f^2*d*((x+(d*f)^{(1/2)/f)^2*c+(b*f-2*(d*f)^{(1/2)*c)*(x+(d*f)^{(1/2)/f)/f+(a*f+c*d-(d*f)^{(1/2)*b)/f)^{(3/2)+1/f^3*d/((a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*\ln((2*(a*f+c*d+(d*f)^{(1/2)*b)/f+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f)/f+2*((a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2))*((x-(d*f)^{(1/2)/f)^2*c+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)))/(x-(d*f)^{(1/2)/f))*b*(d*f)^{(1/2)*a+1/f^4*d^2/((a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*\ln((2*(a*f+c*d+(d*f)^{(1/2)*b)/f+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f)/f+2*((a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2))*((x-(d*f)^{(1/2)/f)^2*c+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)))/(x-(d*f)^{(1/2)/f))*b*(d*f)^{(1/2)*c-1/f^3*d/((a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2)*\ln((2*(a*f+c*d-(d*f)^{(1/2)*b)/f+(b*f-2*(d*f)^{(1/2)*c)*(x+(d*f)^{(1/2)/f)/f+2*((a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2))*((x+(d*f)^{(1/2)/f)^2*c+(b*f-2*(d*f)^{(1/2)*c)*(x+(d*f)^{(1/2)/f)/f+(a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2)))/(x+(d*f)^{(1/2)/f))*b*(d*f)^{(1/2)*a-1/f^4*d^2/((a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2)*\ln((2*(a*f+c*d-(d*f)^{(1/2)*b)/f+(b*f-2*(d*f)^{(1/2)*c)*(x+(d*f)^{(1/2)/f)/f+2*((a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2))*((x+(d*f)^{(1/2)/f)^2*c+(b*f-2*(d*f)^{(1/2)*c)*(x+(d*f)^{(1/2)/f)/f+(a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2)))/(x+(d*f)^{(1/2)/f))*b*(d*f)^{(1/2)*c-1/8/f^2*d*((x-(d*f)^{(1/2)/f)^2*c+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*x*b-5/8/f^3*d*((x-(d*f)^{(1/2)/f)^2*c+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*b*(d*f)^{(1/2)}}} \end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2 -c*((b*sqrt(4*d*f)) / (2*f) + (c*d)/f+a) /f^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 (c x^2 + b x + a)^{3/2}}{d - f x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x)

[Out] int((x^3*(a + b*x + c*x^2)^(3/2))/(d - f*x^2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

$$3.85 \quad \int \frac{x^2(a+bx+cx^2)^{3/2}}{d-fx^2} dx$$

Optimal. Leaf size=417

$$\frac{(48c^2f(a^2f + b^2d) - 24ab^2cf^2 + 192ac^3df + 3b^4f^2 + 128c^4d^2) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) \sqrt{a+bx+cx^2} (2cx(128c^{5/2}f^3) - \dots)}{128c^{5/2}f^3}$$

Rubi [A] time = 1.02, antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1071, 1070, 1078, 621, 206, 1033, 724}

$$\frac{(48c^2f(a^2f + b^2d) - 24ab^2cf^2 + 192ac^3df + 3b^4f^2 + 128c^4d^2) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) \sqrt{a+bx+cx^2} (2cx(128c^{5/2}f^3) - \dots)}{128c^{5/2}f^3}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(a + b*x + c*x^2)^(3/2))/(d - f*x^2), x]

[Out] -((b*(80*c^2*d - 3*b^2*f + 12*a*c*f) + 2*c*(16*c^2*d - 3*b^2*f + 12*a*c*f)*x)*Sqrt[a + b*x + c*x^2])/(64*c^2*f^2) - ((b + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(8*c*f) - (((128*c^4*d^2 + 192*a*c^3*d*f + 3*b^4*f^2 - 24*a*b^2*c*f^2 + 48*c^2*f*(b^2*d + a^2*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(5/2)*f^3) + (Sqrt[d]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^3) + (Sqrt[d]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^3)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1033

Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]

Rule 1070

```

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x_)^2)*((d_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((B*c*f*(2*p + 2*q + 3) + C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*(C*(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(-(b*c*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))))*x + (p*(-(b*f))*(C*(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]

```

Rule 1071

```

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (C_)*(x_)^2)*((d_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*(C*(-(b*f))*(q + 1)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(-(b*f))*(q + 1)) + (p + q + 1)*(-(b*c*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))))*x + (p*(-(b*f))*(C*(-(b*f))*(q + 1)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x] /; FreeQ[{a, b, c, d, f, A, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]

```

Rule 1078

```

Int[((A_) + (B_)*(x_) + (C_)*(x_)^2)/(((a_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f, 0]

```

Rubi steps

$$\int \frac{x^2 (a + bx + cx^2)^{3/2}}{d - fx^2} dx = -\frac{(b + 2cx)(a + bx + cx^2)^{3/2}}{8cf} - \frac{\int \frac{\sqrt{a+bx+cx^2} \left(-\frac{3}{4}(3b^2+4ac)df - 12bcdfx - \frac{3}{4}f(16c^2d - 3(b^2-4ac)f)x^2\right)}{d-fx^2}}{12cf^2}$$

$$= -\frac{(b(80c^2d - 3b^2f + 12acf) + 2c(16c^2d - 3b^2f + 12acf)x)\sqrt{a + bx + cx^2}}{64c^2f^2} - \frac{(b + 2cx)(a + bx + cx^2)^{3/2}}{8cf}$$

$$= -\frac{(b(80c^2d - 3b^2f + 12acf) + 2c(16c^2d - 3b^2f + 12acf)x)\sqrt{a + bx + cx^2}}{64c^2f^2} - \frac{(b + 2cx)(a + bx + cx^2)^{3/2}}{8cf}$$

$$= -\frac{(b(80c^2d - 3b^2f + 12acf) + 2c(16c^2d - 3b^2f + 12acf)x)\sqrt{a + bx + cx^2}}{64c^2f^2} - \frac{(b + 2cx)(a + bx + cx^2)^{3/2}}{8cf}$$

$$= -\frac{(b(80c^2d - 3b^2f + 12acf) + 2c(16c^2d - 3b^2f + 12acf)x)\sqrt{a + bx + cx^2}}{64c^2f^2} - \frac{(b + 2cx)(a + bx + cx^2)^{3/2}}{8cf}$$

$$= -\frac{(b(80c^2d - 3b^2f + 12acf) + 2c(16c^2d - 3b^2f + 12acf)x)\sqrt{a + bx + cx^2}}{64c^2f^2} - \frac{(b + 2cx)(a + bx + cx^2)^{3/2}}{8cf}$$

Mathematica [A] time = 1.02, size = 395, normalized size = 0.95

$$\frac{-((48c^2f^2 + b^2d) - 24ab^2cf^2 + 192ac^3df + 3b^4f^2 + 128c^4d^2) \operatorname{tanh}^{-1}\left(\frac{2b\sqrt{a+bx+cx^2}}{2\sqrt{a+bx+cx^2} + \sqrt{d-fx^2}}\right) - 2\sqrt{a+bx+cx^2} \left(4bc(5af + 2bd + 6cf^2) + 8c^2(5af + 4bd + 2cf^2) - 3b^2f + 2b^2cf + 32c^2\sqrt{d}(af + b(-\sqrt{d})\sqrt{f} + cd)\right) \operatorname{tanh}^{-1}\left(\frac{2b\sqrt{a+bx+cx^2} - 2\sqrt{d}}{2\sqrt{a+bx+cx^2} + \sqrt{d-fx^2}}\right) + 32c^2\sqrt{d}(af + b\sqrt{d}\sqrt{f} + cd) \operatorname{tanh}^{-1}\left(\frac{2b\sqrt{a+bx+cx^2} + 2\sqrt{d}}{2\sqrt{a+bx+cx^2} + \sqrt{d-fx^2}}\right)}{128c^{5/2}f^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(a + b*x + c*x^2)^(3/2))/(d - f*x^2), x]

[Out] (-((128*c^4*d^2 + 192*a*c^3*d*f + 3*b^4*f^2 - 24*a*b^2*c*f^2 + 48*c^2*f*(b^2*d + a^2*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]) - 2*Sqrt[c]*(f*Sqrt[a + x*(b + c*x)]*(-3*b^3*f + 2*b^2*c*f*x + 8*c^2*x*(4*c*d + 5*a*f + 2*c*f*x^2)) + 4*b*c*(20*c*d + 5*a*f + 6*c*f*x^2)) + 32*c^2*Sqrt[d]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + x*(b + c*x)])] + 32*c^2*Sqrt[d]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + x*(b + c*x)])])/(128*c^(5/2)*f^3)

IntegrateAlgebraic [C] time = 2.56, size = 620, normalized size = 1.49

$$\frac{(128c^2f^2 + b^2d) \operatorname{tanh}^{-1}\left(\frac{2b\sqrt{a+bx+cx^2}}{2\sqrt{a+bx+cx^2} + \sqrt{d-fx^2}}\right) - 2\sqrt{a+bx+cx^2} \left(4bc(5af + 2bd + 6cf^2) + 8c^2(5af + 4bd + 2cf^2) - 3b^2f + 2b^2cf + 32c^2\sqrt{d}(af + b(-\sqrt{d})\sqrt{f} + cd)\right) \operatorname{tanh}^{-1}\left(\frac{2b\sqrt{a+bx+cx^2} - 2\sqrt{d}}{2\sqrt{a+bx+cx^2} + \sqrt{d-fx^2}}\right) + 32c^2\sqrt{d}(af + b\sqrt{d}\sqrt{f} + cd) \operatorname{tanh}^{-1}\left(\frac{2b\sqrt{a+bx+cx^2} + 2\sqrt{d}}{2\sqrt{a+bx+cx^2} + \sqrt{d-fx^2}}\right)}{128c^{5/2}f^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^2*(a + b*x + c*x^2)^(3/2))/(d - f*x^2), x]

[Out] (Sqrt[a + b*x + c*x^2]*(-80*b*c^2*d + 3*b^3*f - 20*a*b*c*f - 32*c^3*d*x - 2*b^2*c*f*x - 40*a*c^2*f*x - 24*b*c^2*f*x^2 - 16*c^3*f*x^3)/(64*c^2*f^2) + ((128*c^4*d^2 + 48*b^2*c^2*d*f + 192*a*c^3*d*f + 3*b^4*f^2 - 24*a*b^2*c*f^2 + 48*a^2*c^2*f^2)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2])/(128*c^(5/2)*f^3) - (d*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (b*c^2*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + b^3*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^2*b*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*c^(5/2)*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*b^2*Sqrt[c]*d*f*Log[-(Sqrt[c]*x) +

$$\frac{\sqrt{ax^2+bx+c} - \sqrt{4ac^3} \operatorname{Log}\left[-\sqrt{c}x + \sqrt{ax^2+bx+c} - \sqrt{4ac^3}\right] - 2a^2\sqrt{c} \operatorname{Log}\left[-\sqrt{c}x + \sqrt{ax^2+bx+c} - \sqrt{4ac^3}\right] + 2b^2\sqrt{c} \operatorname{Log}\left[-\sqrt{c}x + \sqrt{ax^2+bx+c} - \sqrt{4ac^3}\right] - \sqrt{4ac^3} \operatorname{Log}\left[-\sqrt{c}x + \sqrt{ax^2+bx+c} - \sqrt{4ac^3}\right]}{2f^3}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type

maple [B] time = 0.02, size = 4900, normalized size = 11.75

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)

[Out]
$$\begin{aligned} & -\frac{3}{8} \frac{f}{c^{1/2}} \ln\left(\frac{c^2 x + \frac{1}{2} b c}{c^{1/2} + (c^2 x^2 + b^2 x + a)^{1/2}}\right) a^2 - \frac{3}{128} \frac{f}{c^{5/2}} \ln\left(\frac{c^2 x + \frac{1}{2} b c}{c^{1/2} + (c^2 x^2 + b^2 x + a)^{1/2}}\right) b^4 - \frac{1}{6} \frac{d}{f} \frac{1}{(d^2 x^2 + b^2 x + a)^{1/2}} \frac{1}{f} \left(x - \frac{(d^2 x^2 + b^2 x + a)^{1/2}}{f} \right)^2 c + (b^2 x + 2(d^2 x^2 + b^2 x + a)^{1/2} c) \left(x - \frac{(d^2 x^2 + b^2 x + a)^{1/2}}{f} \right) / f + (a^2 x + c^2 d + (d^2 x^2 + b^2 x + a)^{1/2} b) / f^{3/2} - \frac{5}{8} \frac{d}{f^2} \left(x - \frac{(d^2 x^2 + b^2 x + a)^{1/2}}{f} \right)^2 c + (b^2 x + 2(d^2 x^2 + b^2 x + a)^{1/2} c) \left(x - \frac{(d^2 x^2 + b^2 x + a)^{1/2}}{f} \right) / f + (a^2 x + c^2 d + (d^2 x^2 + b^2 x + a)^{1/2} b) / f^{3/2} \ln\left(\frac{c^2 x + \frac{1}{2} b c}{c^{1/2} + (c^2 x^2 + b^2 x + a)^{1/2}}\right) \\ & + \frac{1}{2} \frac{d^2}{f^3} \ln\left(\frac{c^2 x + \frac{1}{2} b c}{c^{1/2} + (c^2 x^2 + b^2 x + a)^{1/2}}\right) c + \frac{1}{6} \frac{d}{f} \frac{1}{(d^2 x^2 + b^2 x + a)^{1/2}} \frac{1}{f} \left(x + \frac{(d^2 x^2 + b^2 x + a)^{1/2}}{f} \right)^2 c + (b^2 x - 2(d^2 x^2 + b^2 x + a)^{1/2} c) \left(x + \frac{(d^2 x^2 + b^2 x + a)^{1/2}}{f} \right) / f + (a^2 x + c^2 d - (d^2 x^2 + b^2 x + a)^{1/2} b) / f^{3/2} - \frac{5}{8} \frac{d}{f^2} \left(x + \frac{(d^2 x^2 + b^2 x + a)^{1/2}}{f} \right)^2 c + (b^2 x - 2(d^2 x^2 + b^2 x + a)^{1/2} c) \left(x + \frac{(d^2 x^2 + b^2 x + a)^{1/2}}{f} \right) / f + (a^2 x + c^2 d - (d^2 x^2 + b^2 x + a)^{1/2} b) / f^{3/2} \ln\left(\frac{c^2 x + \frac{1}{2} b c}{c^{1/2} + (c^2 x^2 + b^2 x + a)^{1/2}}\right) \\ & + \frac{1}{2} \frac{d^2}{f^3} \ln\left(\frac{c^2 x + \frac{1}{2} b c}{c^{1/2} + (c^2 x^2 + b^2 x + a)^{1/2}}\right) c + \frac{1}{8} \frac{f}{c} (c^2 x^2 + b^2 x + a)^{3/2} b^3 - \frac{3}{8} \frac{f}{c} (c^2 x^2 + b^2 x + a)^{1/2} x a + \frac{3}{64} \frac{f}{c^2} (c^2 x^2 + b^2 x + a)^{1/2} b^3 - \frac{d^2}{(d^2 x^2 + b^2 x + a)^{1/2}} \frac{1}{f^2} \left(\frac{a^2 x + c^2 d - (d^2 x^2 + b^2 x + a)^{1/2} b}{f} \right)^2 \ln\left(\frac{2(a^2 x + c^2 d - (d^2 x^2 + b^2 x + a)^{1/2} b)}{f + (b^2 x - 2(d^2 x^2 + b^2 x + a)^{1/2} c) \left(x + \frac{(d^2 x^2 + b^2 x + a)^{1/2}}{f} \right) / f + 2 \left(\frac{a^2 x + c^2 d - (d^2 x^2 + b^2 x + a)^{1/2} b}{f} \right)^2} \right) \\ & + \frac{1}{2} \frac{d}{f} \frac{1}{(d^2 x^2 + b^2 x + a)^{1/2}} \frac{1}{f} \left(x + \frac{(d^2 x^2 + b^2 x + a)^{1/2}}{f} \right)^2 c + (b^2 x - 2(d^2 x^2 + b^2 x + a)^{1/2} c) \left(x + \frac{(d^2 x^2 + b^2 x + a)^{1/2}}{f} \right) / f + (a^2 x + c^2 d - (d^2 x^2 + b^2 x + a)^{1/2} b) / f^{3/2} \ln\left(\frac{c^2 x + \frac{1}{2} b c}{c^{1/2} + (c^2 x^2 + b^2 x + a)^{1/2}}\right) \\ & + \frac{1}{2} \frac{d^2}{f^3} \ln\left(\frac{c^2 x + \frac{1}{2} b c}{c^{1/2} + (c^2 x^2 + b^2 x + a)^{1/2}}\right) c + \frac{1}{8} \frac{f}{c} (c^2 x^2 + b^2 x + a)^{3/2} b^3 - \frac{3}{8} \frac{f}{c} (c^2 x^2 + b^2 x + a)^{1/2} x a + \frac{3}{64} \frac{f}{c^2} (c^2 x^2 + b^2 x + a)^{1/2} b^3 - \frac{d^2}{(d^2 x^2 + b^2 x + a)^{1/2}} \frac{1}{f^2} \left(\frac{a^2 x + c^2 d - (d^2 x^2 + b^2 x + a)^{1/2} b}{f} \right)^2 \ln\left(\frac{2(a^2 x + c^2 d - (d^2 x^2 + b^2 x + a)^{1/2} b)}{f + (b^2 x - 2(d^2 x^2 + b^2 x + a)^{1/2} c) \left(x + \frac{(d^2 x^2 + b^2 x + a)^{1/2}}{f} \right) / f + 2 \left(\frac{a^2 x + c^2 d - (d^2 x^2 + b^2 x + a)^{1/2} b}{f} \right)^2} \right) \\ & + \frac{1}{2} \frac{d}{f} \frac{1}{(d^2 x^2 + b^2 x + a)^{1/2}} \frac{1}{f} \left(x + \frac{(d^2 x^2 + b^2 x + a)^{1/2}}{f} \right)^2 c + (b^2 x - 2(d^2 x^2 + b^2 x + a)^{1/2} c) \left(x + \frac{(d^2 x^2 + b^2 x + a)^{1/2}}{f} \right) / f + (a^2 x + c^2 d - (d^2 x^2 + b^2 x + a)^{1/2} b) / f^{3/2} \ln\left(\frac{c^2 x + \frac{1}{2} b c}{c^{1/2} + (c^2 x^2 + b^2 x + a)^{1/2}}\right) \\ & + \frac{1}{2} \frac{d^2}{f^3} \ln\left(\frac{c^2 x + \frac{1}{2} b c}{c^{1/2} + (c^2 x^2 + b^2 x + a)^{1/2}}\right) c + \frac{1}{8} \frac{f}{c} (c^2 x^2 + b^2 x + a)^{3/2} b^3 - \frac{3}{8} \frac{f}{c} (c^2 x^2 + b^2 x + a)^{1/2} x a + \frac{3}{64} \frac{f}{c^2} (c^2 x^2 + b^2 x + a)^{1/2} b^3 - \frac{d^2}{(d^2 x^2 + b^2 x + a)^{1/2}} \frac{1}{f^2} \left(\frac{a^2 x + c^2 d - (d^2 x^2 + b^2 x + a)^{1/2} b}{f} \right)^2 \ln\left(\frac{2(a^2 x + c^2 d - (d^2 x^2 + b^2 x + a)^{1/2} b)}{f + (b^2 x - 2(d^2 x^2 + b^2 x + a)^{1/2} c) \left(x + \frac{(d^2 x^2 + b^2 x + a)^{1/2}}{f} \right) / f + 2 \left(\frac{a^2 x + c^2 d - (d^2 x^2 + b^2 x + a)^{1/2} b}{f} \right)^2} \right) \\ & + \frac{1}{2} \frac{d}{f} \frac{1}{(d^2 x^2 + b^2 x + a)^{1/2}} \frac{1}{f} \left(x + \frac{(d^2 x^2 + b^2 x + a)^{1/2}}{f} \right)^2 c + (b^2 x - 2(d^2 x^2 + b^2 x + a)^{1/2} c) \left(x + \frac{(d^2 x^2 + b^2 x + a)^{1/2}}{f} \right) / f + (a^2 x + c^2 d - (d^2 x^2 + b^2 x + a)^{1/2} b) / f^{3/2} \ln\left(\frac{c^2 x + \frac{1}{2} b c}{c^{1/2} + (c^2 x^2 + b^2 x + a)^{1/2}}\right) \\ & + \frac{1}{2} \frac{d^2}{f^3} \ln\left(\frac{c^2 x + \frac{1}{2} b c}{c^{1/2} + (c^2 x^2 + b^2 x + a)^{1/2}}\right) c + \frac{1}{8} \frac{f}{c} (c^2 x^2 + b^2 x + a)^{3/2} b^3 - \frac{3}{8} \frac{f}{c} (c^2 x^2 + b^2 x + a)^{1/2} x a + \frac{3}{64} \frac{f}{c^2} (c^2 x^2 + b^2 x + a)^{1/2} b^3 - \frac{d^2}{(d^2 x^2 + b^2 x + a)^{1/2}} \frac{1}{f^2} \left(\frac{a^2 x + c^2 d - (d^2 x^2 + b^2 x + a)^{1/2} b}{f} \right)^2 \ln\left(\frac{2(a^2 x + c^2 d - (d^2 x^2 + b^2 x + a)^{1/2} b)}{f + (b^2 x - 2(d^2 x^2 + b^2 x + a)^{1/2} c) \left(x + \frac{(d^2 x^2 + b^2 x + a)^{1/2}}{f} \right) / f + 2 \left(\frac{a^2 x + c^2 d - (d^2 x^2 + b^2 x + a)^{1/2} b}{f} \right)^2} \right) \end{aligned}$$

$$3.86 \quad \int \frac{x(a+bx+cx^2)^{3/2}}{d-fx^2} dx$$

Optimal. Leaf size=349

$$\frac{\sqrt{a+bx+cx^2} (8acf + b^2f + 2bcfx + 8c^2d)}{8cf^2} - \frac{b(12acf + b^2(-f) + 24c^2d) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}f^2} - \frac{(af + b(-\sqrt{d}\sqrt{f+cd}) \tanh^{-1}\left(\frac{-2a\sqrt{f} + (2c\sqrt{d}-b)\sqrt{f+cd}}{2\sqrt{a+bx+cx^2}\sqrt{f+(-\sqrt{d})\sqrt{f+cd}}}\right) + (af + b\sqrt{d}\sqrt{f+cd})^{3/2} \tanh^{-1}\left(\frac{2a\sqrt{f} + (b\sqrt{f} + 2c\sqrt{d})\sqrt{f+cd}}{2\sqrt{a+bx+cx^2}\sqrt{f+(-\sqrt{d})\sqrt{f+cd}}}\right) - (a+bx+cx^2)^{3/2}}{2f^{5/2}}$$

Rubi [A] time = 0.52, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1021, 1070, 1078, 621, 206, 1033, 724}

$$\frac{\sqrt{a+bx+cx^2} (8acf + b^2f + 2bcfx + 8c^2d)}{8cf^2} - \frac{b(12acf + b^2(-f) + 24c^2d) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}f^2} - \frac{(af + b(-\sqrt{d}\sqrt{f+cd}) \tanh^{-1}\left(\frac{-2a\sqrt{f} + (2c\sqrt{d}-b)\sqrt{f+cd}}{2\sqrt{a+bx+cx^2}\sqrt{f+(-\sqrt{d})\sqrt{f+cd}}}\right) + (af + b\sqrt{d}\sqrt{f+cd})^{3/2} \tanh^{-1}\left(\frac{2a\sqrt{f} + (b\sqrt{f} + 2c\sqrt{d})\sqrt{f+cd}}{2\sqrt{a+bx+cx^2}\sqrt{f+(-\sqrt{d})\sqrt{f+cd}}}\right) - (a+bx+cx^2)^{3/2}}{2f^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(x*(a + b*x + c*x^2)^(3/2))/(d - f*x^2), x]

[Out] -((8*c^2*d + b^2*f + 8*a*c*f + 2*b*c*f*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^2) - (a + b*x + c*x^2)^(3/2)/(3*f) - (b*(24*c^2*d - b^2*f + 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*f^2) - ((c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + b*x + c*x^2]])/(2*f^(5/2)) + ((c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + b*x + c*x^2]])/(2*f^(5/2))

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1021

Int[((g_) + (h_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-b*f)) + c*(-2*g*f)*(p + q + 1))*x^2, x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]

Rule 1033

Int[((g_) + (h_)*(x_))/(((a_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]

Rule 1070

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x_)^2)*((d_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((B*c*f*(2*p + 2*q + 3) + C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*(C*(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(-(b*c*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))))*x + (p*(-(b*f))*(C*(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))))*x^2, x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]

Rule 1078

Int[((A_) + (B_)*(x_) + (C_)*(x_)^2)/(((a_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f, 0]

Rubi steps

$$\begin{aligned} \int \frac{x(a+bx+cx^2)^{3/2}}{d-fx^2} dx &= -\frac{(a+bx+cx^2)^{3/2}}{3f} + \frac{\int \frac{\sqrt{a+bx+cx^2} \left(\frac{3bd}{2} + 3(cd+af)x + \frac{3}{2}bfx^2 \right)}{d-fx^2} dx}{3f} \\ &= -\frac{(8c^2d+b^2f+8acf+2bcfx)\sqrt{a+bx+cx^2}}{8cf^2} - \frac{(a+bx+cx^2)^{3/2}}{3f} - \frac{\int \frac{-\frac{3}{8}bdf(8c^2d+)}{d-fx^2} dx}{3f} \\ &= -\frac{(8c^2d+b^2f+8acf+2bcfx)\sqrt{a+bx+cx^2}}{8cf^2} - \frac{(a+bx+cx^2)^{3/2}}{3f} + \frac{\int \frac{\frac{3}{8}bdf^2(24c^2d+)}{d-fx^2} dx}{3f} \\ &= -\frac{(8c^2d+b^2f+8acf+2bcfx)\sqrt{a+bx+cx^2}}{8cf^2} - \frac{(a+bx+cx^2)^{3/2}}{3f} + \frac{(cd-b\sqrt{d}\sqrt{a+bx+cx^2})}{3f} \\ &= -\frac{(8c^2d+b^2f+8acf+2bcfx)\sqrt{a+bx+cx^2}}{8cf^2} - \frac{(a+bx+cx^2)^{3/2}}{3f} - \frac{b(24c^2d-b^2)}{3f} \\ &= -\frac{(8c^2d+b^2f+8acf+2bcfx)\sqrt{a+bx+cx^2}}{8cf^2} - \frac{(a+bx+cx^2)^{3/2}}{3f} - \frac{b(24c^2d-b^2)}{3f} \end{aligned}$$

Mathematica [A] time = 0.80, size = 330, normalized size = 0.95

$$\frac{b(-12acf + b^2f - 24c^2d) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{-b+3d+cx}}\right) - \sqrt{f}\sqrt{a+x(b+cx)}(2cf(16a+7bx) + 3b^2f + 8c^2(3d+fx^2)) - 12c(af + b(-\sqrt{d})\sqrt{f} + cd)^{3/2} \tanh^{-1}\left(\frac{2a\sqrt{f} + b\sqrt{d} + b\sqrt{f} - 2c\sqrt{d}x}{2\sqrt{-b+3d+cx}}\sqrt{af + (-\sqrt{d})\sqrt{f} + cd}\right) + 12c(af + b\sqrt{d}\sqrt{f} + cd)^{3/2} \tanh^{-1}\left(\frac{-2(a\sqrt{f} + c\sqrt{d}x) - (b\sqrt{d} + \sqrt{f}x)}{2\sqrt{-b+3d+cx}}\sqrt{af + b\sqrt{d}\sqrt{f} + cd}\right)}{16c^{3/2}f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x + c*x^2)^(3/2))/(d - f*x^2), x]
```

```
[Out] (b*(-24*c^2*d + b^2*f - 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]/(16*c^(3/2)*f^2) - (Sqrt[f]*Sqrt[a + x*(b + c*x)]*(3*b^2*f + 2*c*f*(16*a + 7*b*x) + 8*c^2*(3*d + f*x^2)) - 12*c*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(-b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)]) + 12*c*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/(24*c*f^(5/2))
```

IntegrateAlgebraic [C] time = 1.29, size = 626, normalized size = 1.79

$$\frac{b(-12acf + b^2f - 24c^2d) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{-b+3d+cx}}\right) - \sqrt{f}\sqrt{a+x(b+cx)}(2cf(16a+7bx) + 3b^2f + 8c^2(3d+fx^2)) - 12c(af + b(-\sqrt{d})\sqrt{f} + cd)^{3/2} \tanh^{-1}\left(\frac{2a\sqrt{f} + b\sqrt{d} + b\sqrt{f} - 2c\sqrt{d}x}{2\sqrt{-b+3d+cx}}\sqrt{af + (-\sqrt{d})\sqrt{f} + cd}\right) + 12c(af + b\sqrt{d}\sqrt{f} + cd)^{3/2} \tanh^{-1}\left(\frac{-2(a\sqrt{f} + c\sqrt{d}x) - (b\sqrt{d} + \sqrt{f}x)}{2\sqrt{-b+3d+cx}}\sqrt{af + b\sqrt{d}\sqrt{f} + cd}\right)}{16c^{3/2}f^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(a + b*x + c*x^2)^(3/2))/(d - f*x^2), x]
```

```
[Out] (Sqrt[a + b*x + c*x^2]*(-24*c^2*d - 3*b^2*f - 32*a*c*f - 14*b*c*f*x - 8*c^2*f*x^2))/(24*c*f^2) + ((24*b*c^2*d - b^3*f + 12*a*b*c*f)*Log[b*c + 2*c^2*x - 2*c^(3/2)*Sqrt[a + b*x + c*x^2]]/(16*c^(3/2)*f^2) - RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (2*b^2*c*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*c^2*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a*b^2*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*a^2*c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^3*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 4*b*c^(3/2)*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 4*a*b*Sqrt[c]*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + c^2*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + b^2*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + 2*a*c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + a^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ]/(2*f^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d), x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d), x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.02, size = 4567, normalized size = 13.09

output too large to display


```

^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*c^(3/2)*(d*f)^(1/2)*d+1/2/f^2
/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(
d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(
d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(
1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*d*b^2+1/2/f^3/((a*f+c*d+(d*f)^(1/2)*b
)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(
1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2
*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d
*f)^(1/2)/f))*c^2*d^2+1/4/f^2*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*
(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*(d*f)^(1/2)*c+3/4/f^
2*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1
/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b
)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)*a-3/8/f/c^(1/2)*ln(((x+(d*f)^(1/2)/f)*c+1/2
*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)
*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*a*b+3/16/f^2*ln(((
x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)
^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(
1/2))/c^(1/2)*b^2*(d*f)^(1/2)-3/4/f^2*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d
*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*
f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*d*b+1/2/f^3*ln(((x+
(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*
c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2
))*c^(3/2)*(d*f)^(1/2)*d+1/2/f^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a
*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c
*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+
(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*d*b^2
+1/2/f^3/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+
(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/
2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*
d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*c^2*d^2-1/4/f^2*((x-(d*f)^(1/
2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b
)/f)^(1/2)*x*(d*f)^(1/2)*c+1/f^3/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a
*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*
d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(
d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*((d*f)^(
1/2)*b*c*d-1/f^3/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/
2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*
b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/
f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*((d*f)^(1/2)*b*c*d

```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see 'assume?' for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2 - (c*((b*sqrt(4*d*f)) / (2*f) + (c*d)/f+a)) /f^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x(c x^2 + b x + a)^{3/2}}{d - f x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] int((x*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x)
```

```
[Out] int((x*(a + b*x + c*x^2)^(3/2))/(d - f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax\sqrt{a+bx+cx^2}}{-d+fx^2} dx - \int \frac{bx^2\sqrt{a+bx+cx^2}}{-d+fx^2} dx - \int \frac{cx^3\sqrt{a+bx+cx^2}}{-d+fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(a*x*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(b*x**2*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(c*x**3*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)
```

$$3.87 \quad \int \frac{(a+bx+cx^2)^{3/2}}{d-fx^2} dx$$

Optimal. Leaf size=315

$$\frac{(12acf + 3b^2f + 8c^2d) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (af + b(-\sqrt{d})\sqrt{f} + cd)^{3/2} \tanh^{-1}\left(\frac{-2a\sqrt{f} + x(2c\sqrt{d}-b\sqrt{f}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{8\sqrt{c}f^2} + \frac{(af + b(-\sqrt{d})\sqrt{f} + cd)^{3/2} \tanh^{-1}\left(\frac{-2a\sqrt{f} + x(2c\sqrt{d}-b\sqrt{f}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2\sqrt{d}f^2}$$

Rubi [A] time = 0.52, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, number of rules / integrand size = 0.240, Rules used = {978, 1078, 621, 206, 1033, 724}

$$\frac{(12acf + 3b^2f + 8c^2d) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (af + b(-\sqrt{d})\sqrt{f} + cd)^{3/2} \tanh^{-1}\left(\frac{-2a\sqrt{f} + x(2c\sqrt{d}-b\sqrt{f}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{8\sqrt{c}f^2} + \frac{(af + b(-\sqrt{d})\sqrt{f} + cd)^{3/2} \tanh^{-1}\left(\frac{-2a\sqrt{f} + x(2c\sqrt{d}-b\sqrt{f}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2\sqrt{d}f^2} + \frac{(af + b\sqrt{d}\sqrt{f} + cd)^{3/2} \tanh^{-1}\left(\frac{2a\sqrt{f} + x(b\sqrt{f} + 2c\sqrt{d}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}f^2} - \frac{(5b + 2cx)\sqrt{a + bx + cx^2}}{4f}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + c*x^2)^(3/2)/(d - f*x^2), x]

[Out] -((5*b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*f) - ((8*c^2*d + 3*b^2*f + 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*f^2) + ((c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2]))/(2*Sqrt[d]*f^2) + ((c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2]))/(2*Sqrt[d]*f^2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 978

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((b*(3*p + 2*q) + 2*c*(p + q)*x)*(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^(q + 1))/(2*f*(p + q)*(2*p + 2*q + 1)), x] - Dist[1/(2*f*(p + q)*(2*p + 2*q + 1)), Int[(a + b*x + c*x^2)^(p - 2)*(d + f*x^2)^q*Simp[b^2*d*(p - 1)*(2*p + q) - (p + q)*(b^2*d*(1 - p) - 2*a*(c*d - a*f*(2*p + 2*q + 1))) - (2*b*(c*d - a*f)*(1 - p)*(2*p + q) - 2*(p + q)*b*(2*c*d*(2*p + q) - (c*d + a*f)*(2*p + 2*q + 1)))*x + (b^2*f*p*(1 - p) + 2*c*(p + q)*(c*d*(2*p - 1) - a*f*(4*p + 2*q - 1)))*x^2, x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*q + 1, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]

Rule 1033

Int[((g_.) + (h_.)*(x_))/(((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]

Rule 1078

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{(a + bx + cx^2)^{3/2}}{d - fx^2} dx &= -\frac{(5b + 2cx)\sqrt{a + bx + cx^2}}{4f} + \frac{\int \frac{\frac{1}{4}(5b^2d + 4a(cd + 2af)) + 4b(cd + af)x + \frac{1}{4}(8c^2d + 3b^2f + 12acf)x^2}{\sqrt{a + bx + cx^2}(d - fx^2)} dx}{2f} \\
 &= -\frac{(5b + 2cx)\sqrt{a + bx + cx^2}}{4f} - \frac{\int \frac{-\frac{1}{4}d(8c^2d + 3b^2f + 12acf) - \frac{1}{4}f(5b^2d + 4a(cd + 2af)) - 4bf(cd + af)x}{\sqrt{a + bx + cx^2}(d - fx^2)} dx}{2f^2} \\
 &= -\frac{(5b + 2cx)\sqrt{a + bx + cx^2}}{4f} - \frac{(cd - b\sqrt{d}\sqrt{f} + af)^2 \int \frac{1}{(-\sqrt{d}\sqrt{f} - fx)\sqrt{a + bx + cx^2}} dx}{2\sqrt{d}f^{3/2}} + \frac{(cd - b\sqrt{d}\sqrt{f} + af)^2}{2\sqrt{d}f^{3/2}} \\
 &= -\frac{(5b + 2cx)\sqrt{a + bx + cx^2}}{4f} - \frac{(8c^2d + 3b^2f + 12acf) \tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right)}{8\sqrt{c}f^2} + \frac{(cd - b\sqrt{d}\sqrt{f} + af)^2}{2\sqrt{d}f^{3/2}} \\
 &= -\frac{(5b + 2cx)\sqrt{a + bx + cx^2}}{4f} - \frac{(8c^2d + 3b^2f + 12acf) \tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right)}{8\sqrt{c}f^2} + \frac{(cd - b\sqrt{d}\sqrt{f} + af)^2}{2\sqrt{d}f^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.81, size = 298, normalized size = 0.95

$$\frac{(12acf + 3b^2f + 8c^2d) \tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right) + \frac{4(af + b(-\sqrt{d})\sqrt{f} + cd)^{3/2} \tanh^{-1}\left(\frac{2c\sqrt{f} - b\sqrt{d} + b\sqrt{f}x - 2c\sqrt{d}x}{2\sqrt{a + bx + cx^2}\sqrt{af + b(-\sqrt{d})\sqrt{f} + cd}}\right) + \frac{4(af + b\sqrt{d}\sqrt{f} + cd)^{3/2} \tanh^{-1}\left(\frac{-2(a\sqrt{f} + c\sqrt{d}x) - b(\sqrt{d} + \sqrt{f}x)}{2\sqrt{a + bx + cx^2}\sqrt{af + b\sqrt{d}\sqrt{f} + cd}}\right)}{\sqrt{d}} + 2f(5b + 2cx)\sqrt{a + x(b + cx)}}{8f^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d - f*x^2), x]

[Out] -1/8*(2*f*(5*b + 2*c*x)*Sqrt[a + x*(b + c*x)] + ((8*c^2*d + 3*b^2*f + 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + (4*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(-b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)]))/Sqrt[d] + (4*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/Sqrt[d])/f^2

IntegrateAlgebraic [C] time = 1.00, size = 524, normalized size = 1.66

$$\frac{(12acf + 3b^2f + 8c^2d) \tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right) + \frac{4(af + b(-\sqrt{d})\sqrt{f} + cd)^{3/2} \tanh^{-1}\left(\frac{2c\sqrt{f} - b\sqrt{d} + b\sqrt{f}x - 2c\sqrt{d}x}{2\sqrt{a + bx + cx^2}\sqrt{af + b(-\sqrt{d})\sqrt{f} + cd}}\right) + \frac{4(af + b\sqrt{d}\sqrt{f} + cd)^{3/2} \tanh^{-1}\left(\frac{-2(a\sqrt{f} + c\sqrt{d}x) - b(\sqrt{d} + \sqrt{f}x)}{2\sqrt{a + bx + cx^2}\sqrt{af + b\sqrt{d}\sqrt{f} + cd}}\right)}{\sqrt{d}} + 2f(5b + 2cx)\sqrt{a + x(b + cx)}}{8f^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)^(3/2)/(d - f*x^2),x]
```

```
[Out] ((-5*b - 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*f) + ((8*c^2*d + 3*b^2*f + 12*a*c*f)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(8*Sqrt[c]*f^2) - RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (b*c^2*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + b^3*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^2*b*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*c^(5/2)*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*b^2*Sqrt[c]*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 4*a*c^(3/2)*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*a^2*Sqrt[c]*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*b*c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + 2*a*b*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ]/(2*f^2)
```

```
fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

```
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);OUTPUT:Error: Bad Argument Type
```

```
maple [B] time = 0.02, size = 4574, normalized size = 14.52
```

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] -1/6/(d*f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(3/2)+1/6/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(3/2)-1/2/f^2*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(3/2)*d-1/2/(d*f)^(1/2)/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f)*a^2+1/32/(d*f)^(1/2)/c^(3/2)*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*b^3-1/2/f^2*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*c^(3/2)*d+1/2/(d*f)^(1/2)/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*c^(3/2)*d-1/2/(d*f)^(1/2)/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))
```

$$\begin{aligned}
& /f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)})/(x-(d*f)^{(1/2)/f})) * a^2-1/4/f*((x+(d \\
& *f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f)/f+(a*f+c*d-(d*f)^{(\\
& 1/2)*b)/f)^{(1/2)*x*c+1/8/(d*f)^{(1/2)*((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1 \\
& /2)*c)*(x+(d*f)^{(1/2)/f)/f+(a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2)*x*b+1/16/(d*f)^{(\\
& 1/2)/c*((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f)/f+(a \\
& *f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2)*b^2-3/4/f*\ln(((x+(d*f)^{(1/2)/f)*c+1/2*(b*f-2 \\
& *(d*f)^{(1/2)*c)/f)/c^{(1/2)}+((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+ \\
& (d*f)^{(1/2)/f)/f+(a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2))*c^{(1/2)*a-3/16/f*\ln(((x+ \\
& (d*f)^{(1/2)/f)*c+1/2*(b*f-2*(d*f)^{(1/2)*c)/f)/c^{(1/2)}+((x+(d*f)^{(1/2)/f})^2* \\
& c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f)/f+(a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2 \\
&))/c^{(1/2)*b^2-1/32/(d*f)^{(1/2)/c^{(3/2)*\ln(((x+(d*f)^{(1/2)/f)*c+1/2*(b*f-2* \\
& (d*f)^{(1/2)*c)/f)/c^{(1/2)}+((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(\\
& d*f)^{(1/2)/f)/f+(a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2))*b^3-1/4/f*((x-(d*f)^{(1/2) \\
& /f})^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f \\
&)^2*c-1/8/(d*f)^{(1/2)*((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x \\
& -(d*f)^{(1/2)/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*x*b-1/16/(d*f)^{(1/2)/c*((\\
& x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f)/f+(a*f+c*d+(d \\
& *f)^{(1/2)*b)/f)^{(1/2)*b^2-3/4/f*\ln(((x-(d*f)^{(1/2)/f)*c+1/2*(b*f+2*(d*f)^{(1 \\
& /2)*c)/f)/c^{(1/2)}+((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/ \\
& 2)/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2))*c^{(1/2)*a-3/16/f*\ln(((x-(d*f)^{(1/ \\
& 2)/f)*c+1/2*(b*f+2*(d*f)^{(1/2)*c)/f)/c^{(1/2)}+((x-(d*f)^{(1/2)/f})^2*c+(b*f+2* \\
& (d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2))/c^{(1/2 \\
&) * b^2+1/(d*f)^{(1/2)/f)/((a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*\ln((2*(a*f+c*d+(d*f \\
&)^2*c+(d*f)^{(1/2)*b)/f+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f)/f+2*((a*f+c*d+(d*f)^{(1 \\
& /2)*b)/f)^{(1/2)*((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2) \\
& /f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)))/(x-(d*f)^{(1/2)/f))*a*c*d+1/f^2/((a* \\
& f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*\ln((2*(a*f+c*d+(d*f)^{(1/2)*b)/f+(b*f+2*(d*f)^{(\\
& 1/2)*c)*(x-(d*f)^{(1/2)/f)/f+2*((a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*((x-(d*f)^{(\\
& 1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f)/f+(a*f+c*d+(d*f)^{(1/2) \\
& *b)/f)^{(1/2)))/(x-(d*f)^{(1/2)/f))*b*c*d+1/2/(d*f)^{(1/2)/f^2/((a*f+c*d+(d*f)^{(\\
& 1/2)*b)/f)^{(1/2)*\ln((2*(a*f+c*d+(d*f)^{(1/2)*b)/f+(b*f+2*(d*f)^{(1/2)*c})*(x- \\
& (d*f)^{(1/2)/f)/f+2*((a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*((x-(d*f)^{(1/2)/f})^2*c \\
& +(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2) \\
&))/(x-(d*f)^{(1/2)/f))*c^2*d^2-3/4/(d*f)^{(1/2)/f*\ln(((x-(d*f)^{(1/2)/f)*c+1/2* \\
& (b*f+2*(d*f)^{(1/2)*c)/f)/c^{(1/2)}+((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)* \\
& c)*(x-(d*f)^{(1/2)/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2))*c^{(1/2)*d*b+1/2/(d \\
& *f)^{(1/2)/f)/((a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*\ln((2*(a*f+c*d+(d*f)^{(1/2)*b) \\
& /f+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f)/f+2*((a*f+c*d+(d*f)^{(1/2)*b)/f)^{(\\
& 1/2)*((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f)/f+(a*f \\
& +c*d+(d*f)^{(1/2)*b)/f)^{(1/2)))/(x-(d*f)^{(1/2)/f))*d*b^2+3/4/(d*f)^{(1/2)/f*\ln \\
& (((x+(d*f)^{(1/2)/f)*c+1/2*(b*f-2*(d*f)^{(1/2)*c)/f)/c^{(1/2)}+((x+(d*f)^{(1/2)/ \\
& f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f)/f+(a*f+c*d-(d*f)^{(1/2)*b)/f) \\
&)^2*c^{(1/2)*d*b-1/2/(d*f)^{(1/2)/f)/((a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2)*\ln(\\
& (2*(a*f+c*d-(d*f)^{(1/2)*b)/f+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f)/f+2*((\\
& a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2)*((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c \\
&)*(x+(d*f)^{(1/2)/f)/f+(a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2)))/(x+(d*f)^{(1/2)/f))* \\
& d*b^2+1/f^2/((a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2)*\ln((2*(a*f+c*d-(d*f)^{(1/2)*b) \\
& /f+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f)/f+2*((a*f+c*d-(d*f)^{(1/2)*b)/f)^{(\\
& 1/2)*((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f)/f+(a*f \\
& +c*d-(d*f)^{(1/2)*b)/f)^{(1/2)))/(x+(d*f)^{(1/2)/f))*b*c*d-1/2/(d*f)^{(1/2)/f^2/ \\
& ((a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2)*\ln((2*(a*f+c*d-(d*f)^{(1/2)*b)/f+(b*f-2*(d \\
& *f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f)/f+2*((a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2)*((x+(d \\
& *f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f)/f+(a*f+c*d-(d*f)^{(\\
& 1/2)*b)/f)^{(1/2)))/(x+(d*f)^{(1/2)/f))*c^2*d^2-5/8/f*((x-(d*f)^{(1/2)/f})^2*c+(\\
& b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*b \\
& -1/2/(d*f)^{(1/2)*((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2) \\
&)/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)*a-5/8/f*((x+(d*f)^{(1/2)/f})^2*c+(b*f \\
& -2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f)/f+(a*f+c*d-(d*f)^{(1/2)*b)/f)^{(1/2)*b+1/ \\
& 2/(d*f)^{(1/2)*((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f}
\end{aligned}$$

```

)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*a+3/8/(d*f)^(1/2)/c^(1/2)*ln(((x+(d*f)
^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*
f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*a*
b+1/2/(d*f)^(1/2)/f*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(
1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*c*d+1/f/((a*f+c*d-(d*f)^(1/2)*b)
/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1
/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*
(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*
f)^(1/2)/f))*b*a-3/8/(d*f)^(1/2)/c^(1/2)*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2
*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-
(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*a*b-1/2/(d*f)^(1/2)/f*((
x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*
f)^(1/2)*b)/f)^(1/2)*c*d+1/f/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c
*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(
d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f
)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b*a-1/(d*
f)^(1/2)/f/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/
f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(
1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+
c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*a*c*d

```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2
-(c*((b*sqrt(4*d*f)) / (2*f) + (c*d)/f+a))
/f^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^2 + bx + a)^{3/2}}{d - fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)/(d - f*x^2),x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)/(d - f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a\sqrt{a+bx+cx^2}}{-d+fx^2} dx - \int \frac{bx\sqrt{a+bx+cx^2}}{-d+fx^2} dx - \int \frac{cx^2\sqrt{a+bx+cx^2}}{-d+fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(a*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(b*x*sqrt(a
+ b*x + c*x**2)/(-d + f*x**2), x) - Integral(c*x**2*sqrt(a + b*x + c*x**2)/
(-d + f*x**2), x)
```

$$3.88 \quad \int \frac{(a+bx+cx^2)^{3/2}}{x(d-fx^2)} dx$$

Optimal. Leaf size=469

$$\frac{a^{3/2} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right) - b(b^2-12ac) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) - \sqrt{a+bx+cx^2} (8acf + b^2f + 2bcfx + 8cd^2)}{d \cdot 16c^{3/2}d \cdot 8cdf}$$

Rubi [A] time = 1.27, antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, integrand size = 28, number of rules used = 0.393, Rules used = {6725, 734, 814, 843, 621, 206, 724, 1021, 1070, 1078, 1033}

$$\frac{a^{3/2} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right) - \sqrt{a+bx+cx^2} (8acf + b^2f + 2bcfx + 8cd^2) - \frac{b(12acf + b^2(-f) + 24c^2d) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) - b(b^2-12ac) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) - (8ac + b^2 + 2bcx) \sqrt{a+bx+cx^2} - (af + b(-\sqrt{d})\sqrt{f} + cd)^{3/2} \tanh^{-1}\left(\frac{-2a\sqrt{f} + b(2a\sqrt{f} + \sqrt{d}) + a\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{d+fx^2}}\right) - (af + b\sqrt{d}\sqrt{f} + cd)^{3/2} \tanh^{-1}\left(\frac{2a\sqrt{f} + b(2a\sqrt{f} + \sqrt{d}) + a\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{d+fx^2}}\right)}{d \cdot 16c^{3/2}d \cdot 8cdf}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + c*x^2)^(3/2)/(x*(d - f*x^2)), x]

[Out] ((b^2 + 8*a*c + 2*b*c*x)*Sqrt[a + b*x + c*x^2])/(8*c*d) - ((8*c^2*d + b^2*f + 8*a*c*f + 2*b*c*f*x)*Sqrt[a + b*x + c*x^2])/(8*c*d*f) - (a^(3/2)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d - (b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*d) - (b*(24*c^2*d - b^2*f + 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*d*f) - ((c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + b*x + c*x^2])])/(2*d*f^(3/2)) + ((c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + b*x + c*x^2])])/(2*d*f^(3/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 734

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && (!RationalQ[m] || LtQ[m, 1]) &&

& !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x*(a + b*x + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1021

```
Int[((g_.) + (h_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-b*f)) + c*(-2*g*f)*(p + q + 1))*x^2, x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2], x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1070

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_.) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((B*c*f*(2*p + 2*q + 3) + C*(b*f*p) + 2*c*C*f*(p + q + 1)*x*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*(C*(-b*f))*(q + 1) - c*(-B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(-b*f))*(q + 1) - c*(-B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(-b*c*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))]*x + (p*(-b*f))*(C*(-b*f))*(q + 1) - c*(-B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))]*x^2, x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]
```


Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
+ f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx+cx^2)^{3/2}}{x(d-fx^2)} dx &= \int \left(\frac{(a+bx+cx^2)^{3/2}}{dx} - \frac{fx(a+bx+cx^2)^{3/2}}{d(-d+fx^2)} \right) dx \\ &= \frac{\int \frac{(a+bx+cx^2)^{3/2}}{x} dx}{d} - \frac{f \int \frac{x(a+bx+cx^2)^{3/2}}{-d+fx^2} dx}{d} \\ &= \frac{\int \frac{\sqrt{a+bx+cx^2} \left(-\frac{3bd}{2} - 3(cd+af)x - \frac{3}{2}bfx^2 \right)}{-d+fx^2} dx}{3d} - \frac{\int \frac{(-2a-bx)\sqrt{a+bx+cx^2}}{x} dx}{2d} \\ &= \frac{(b^2 + 8ac + 2bcx) \sqrt{a+bx+cx^2}}{8cd} - \frac{(8c^2d + b^2f + 8acf + 2bcfx) \sqrt{a+bx+cx^2}}{8cdf} + \\ &= \frac{(b^2 + 8ac + 2bcx) \sqrt{a+bx+cx^2}}{8cd} - \frac{(8c^2d + b^2f + 8acf + 2bcfx) \sqrt{a+bx+cx^2}}{8cdf} + \\ &= \frac{(b^2 + 8ac + 2bcx) \sqrt{a+bx+cx^2}}{8cd} - \frac{(8c^2d + b^2f + 8acf + 2bcfx) \sqrt{a+bx+cx^2}}{8cdf} - \\ &= \frac{(b^2 + 8ac + 2bcx) \sqrt{a+bx+cx^2}}{8cd} - \frac{(8c^2d + b^2f + 8acf + 2bcfx) \sqrt{a+bx+cx^2}}{8cdf} - \\ &= \frac{(b^2 + 8ac + 2bcx) \sqrt{a+bx+cx^2}}{8cd} - \frac{(8c^2d + b^2f + 8acf + 2bcfx) \sqrt{a+bx+cx^2}}{8cdf} - \end{aligned}$$

Mathematica [A] time = 0.50, size = 755, normalized size = 1.61

Integrate[(a + b*x + c*x^2)^(3/2)/(x*(d - f*x^2)), x] // FullSimplify

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + c*x^2)^(3/2)/(x*(d - f*x^2)), x]

[Out] $-\frac{1}{2} \frac{(2cd\sqrt{f}\sqrt{a+bx+cx^2})}{\sqrt{a}\sqrt{a+bx+cx^2}} + 2a^{3/2}f^{3/2}\text{ArcTanh}\left[\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right] + 3b\sqrt{c}d\sqrt{f}\text{ArcTanh}\left[\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right] - c\sqrt{cd-b}\sqrt{d}\sqrt{a+bx+cx^2}$

$$\begin{aligned} & \text{rt}[f] + a*f]*\text{ArcTanh}[(-(b*\text{Sqrt}[d]) + 2*a*\text{Sqrt}[f] - 2*c*\text{Sqrt}[d]*x + b*\text{Sqrt}[f] \\ &]*x)/(2*\text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{Sqrt}[a + x*(b + c*x)])] + b*\text{Sqr} \\ & \text{t}[d]*\text{Sqrt}[f]*\text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{ArcTanh}[(-(b*\text{Sqrt}[d]) + 2* \\ & a*\text{Sqrt}[f] - 2*c*\text{Sqrt}[d]*x + b*\text{Sqrt}[f]*x)/(2*\text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + \\ & a*f]*\text{Sqrt}[a + x*(b + c*x)])] - a*f*\text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{ArcT} \\ & \text{anh}[(-(b*\text{Sqrt}[d]) + 2*a*\text{Sqrt}[f] - 2*c*\text{Sqrt}[d]*x + b*\text{Sqrt}[f]*x)/(2*\text{Sqrt}[c*d \\ & - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{Sqrt}[a + x*(b + c*x)])] + c*d*\text{Sqrt}[c*d + b*\text{Sqrt}[\\ & d]*\text{Sqrt}[f] + a*f]*\text{ArcTanh}[(-2*(a*\text{Sqrt}[f] + c*\text{Sqrt}[d]*x) - b*(\text{Sqrt}[d] + \text{Sqrt} \\ & [f]*x))/(2*\text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{Sqrt}[a + x*(b + c*x)])] + b* \\ & \text{Sqrt}[d]*\text{Sqrt}[f]*\text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{ArcTanh}[(-2*(a*\text{Sqrt}[f] \\ & + c*\text{Sqrt}[d]*x) - b*(\text{Sqrt}[d] + \text{Sqrt}[f]*x))/(2*\text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + \\ & a*f]*\text{Sqrt}[a + x*(b + c*x)])] + a*f*\text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{Arc} \\ & \text{Tanh}[(-2*(a*\text{Sqrt}[f] + c*\text{Sqrt}[d]*x) - b*(\text{Sqrt}[d] + \text{Sqrt}[f]*x))/(2*\text{Sqrt}[c*d + \\ & b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{Sqrt}[a + x*(b + c*x)])]/(d*f^(3/2)) \end{aligned}$$

IntegrateAlgebraic [C] time = 1.02, size = 618, normalized size = 1.32

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x + c*x^2)^(3/2)/(x*(d - f*x^2)),x]

[Out] $-\left(\frac{c*\text{Sqrt}[a + b*x + c*x^2]}{f}\right) + \frac{(2*a^{(3/2)}*\text{ArcTanh}[\frac{\text{Sqrt}[c]*x}{\text{Sqrt}[a]} - \text{Sqrt}[a + b*x + c*x^2]/\text{Sqrt}[a]])}{d} + \frac{(3*b*\text{Sqrt}[c]*\text{Log}[b*f + 2*c*f*x - 2*\text{Sqrt}[c]*f*\text{Sqrt}[a + b*x + c*x^2]])}{(2*f)} - \frac{\text{RootSum}[b^2*d - a^2*f - 4*b*\text{Sqrt}[c]*d*\#1 + 4*c*d*\#1^2 + 2*a*f*\#1^2 - f*\#1^4 \& , (2*b^2*c*d^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] - a*c^2*d^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] + a*b^2*d*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] - 2*a^2*c*d*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] - a^3*f^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] - 4*b*c^{(3/2)}*d^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1 - 4*a*b*\text{Sqrt}[c]*d*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1 + c^2*d^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1^2 + b^2*d*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1^2 + 2*a*c*d*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1^2 + a^2*f^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1^2)/(b*\text{Sqrt}[c]*d - 2*c*d*\#1 - a*f*\#1 + f*\#1^3) \&]}{(2*d*f)}$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(3/2)/x/(-f*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(3/2)/x/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x);OUTPUT:index.cc index_m operator + Error: Bad Argument Value

maple [B] time = 0.02, size = 4765, normalized size = 10.16

output too large to display

$$\frac{1}{(x+(d*f)^{(1/2)}/f)} * (d*f)^{(1/2)} * b*c + 1/4/d/f * ((x+(d*f)^{(1/2)}/f)^{2*c} + (b*f - 2*(d*f)^{(1/2)}*c) * (x+(d*f)^{(1/2)}/f) / f + (a*f + c*d - (d*f)^{(1/2)}*b) / f)^{(1/2)} * x * (d*f)^{(1/2)} * c + 3/4/d/f * \ln(((x+(d*f)^{(1/2)}/f)^{2*c} + (b*f - 2*(d*f)^{(1/2)}*c) / f) / c^{(1/2)} + ((x+(d*f)^{(1/2)}/f)^{2*c} + (b*f - 2*(d*f)^{(1/2)}*c) * (x+(d*f)^{(1/2)}/f) / f + (a*f + c*d - (d*f)^{(1/2)}*b) / f)^{(1/2)} * c^{(1/2)} * (d*f)^{(1/2)} * a + 1/f / ((a*f + c*d + (d*f)^{(1/2)}*b) / f)^{(1/2)} * \ln((2*(a*f + c*d + (d*f)^{(1/2)}*b) / f + (b*f + 2*(d*f)^{(1/2)}*c) * (x - (d*f)^{(1/2)}/f) / f + 2*((a*f + c*d + (d*f)^{(1/2)}*b) / f)^{(1/2)} * ((x - (d*f)^{(1/2)}/f)^{2*c} + (b*f + 2*(d*f)^{(1/2)}*c) * (x - (d*f)^{(1/2)}/f) / f + (a*f + c*d + (d*f)^{(1/2)}*b) / f)^{(1/2)}) / (x - (d*f)^{(1/2)}/f) * a * c + 1/2*d/f^2 / ((a*f + c*d + (d*f)^{(1/2)}*b) / f)^{(1/2)} * \ln((2*(a*f + c*d + (d*f)^{(1/2)}*b) / f + (b*f + 2*(d*f)^{(1/2)}*c) * (x - (d*f)^{(1/2)}/f) / f + 2*((a*f + c*d + (d*f)^{(1/2)}*b) / f)^{(1/2)} * ((x - (d*f)^{(1/2)}/f)^{2*c} + (b*f + 2*(d*f)^{(1/2)}*c) * (x - (d*f)^{(1/2)}/f) / f + (a*f + c*d + (d*f)^{(1/2)}*b) / f)^{(1/2)}) / (x - (d*f)^{(1/2)}/f) * c^{2+3/4} / d * b / c^{(1/2)} * \ln((c*x + 1/2*b) / c^{(1/2)} + (c*x^2 + b*x + a)^{(1/2)}) * a + 1/f / ((a*f + c*d - (d*f)^{(1/2)}*b) / f)^{(1/2)} * \ln((2*(a*f + c*d - (d*f)^{(1/2)}*b) / f + (b*f - 2*(d*f)^{(1/2)}*c) * (x + (d*f)^{(1/2)}/f) / f + 2*((a*f + c*d - (d*f)^{(1/2)}*b) / f)^{(1/2)} * ((x + (d*f)^{(1/2)}/f)^{2*c} + (b*f - 2*(d*f)^{(1/2)}*c) * (x + (d*f)^{(1/2)}/f) / f + (a*f + c*d - (d*f)^{(1/2)}*b) / f)^{(1/2)}) / (x + (d*f)^{(1/2)}/f) * a * c + 1/2*d/f^2 / ((a*f + c*d - (d*f)^{(1/2)}*b) / f)^{(1/2)} * \ln((2*(a*f + c*d - (d*f)^{(1/2)}*b) / f + (b*f - 2*(d*f)^{(1/2)}*c) * (x + (d*f)^{(1/2)}/f) / f + 2*((a*f + c*d - (d*f)^{(1/2)}*b) / f)^{(1/2)} * ((x + (d*f)^{(1/2)}/f)^{2*c} + (b*f - 2*(d*f)^{(1/2)}*c) * (x + (d*f)^{(1/2)}/f) / f + (a*f + c*d - (d*f)^{(1/2)}*b) / f)^{(1/2)}) / (x + (d*f)^{(1/2)}/f) * c^{2+5/8} / d / f * ((x + (d*f)^{(1/2)}/f)^{2*c} + (b*f - 2*(d*f)^{(1/2)}*c) * (x + (d*f)^{(1/2)}/f) / f + (a*f + c*d - (d*f)^{(1/2)}*b) / f)^{(1/2)} * b * (d*f)^{(1/2)} - 3/8 / d / c^{(1/2)} * \ln(((x + (d*f)^{(1/2)}/f)^{2*c} + 1/2*(b*f - 2*(d*f)^{(1/2)}*c) / f) / c^{(1/2)} + ((x + (d*f)^{(1/2)}/f)^{2*c} + (b*f - 2*(d*f)^{(1/2)}*c) * (x + (d*f)^{(1/2)}/f) / f + (a*f + c*d - (d*f)^{(1/2)}*b) / f)^{(1/2)}) * a * b + 1/d / f / ((a*f + c*d + (d*f)^{(1/2)}*b) / f)^{(1/2)} * \ln((2*(a*f + c*d + (d*f)^{(1/2)}*b) / f + (b*f + 2*(d*f)^{(1/2)}*c) * (x - (d*f)^{(1/2)}/f) / f + 2*((a*f + c*d + (d*f)^{(1/2)}*b) / f)^{(1/2)} * ((x - (d*f)^{(1/2)}/f)^{2*c} + (b*f + 2*(d*f)^{(1/2)}*c) * (x - (d*f)^{(1/2)}/f) / f + (a*f + c*d + (d*f)^{(1/2)}*b) / f)^{(1/2)}) / (x - (d*f)^{(1/2)}/f) * (d*f)^{(1/2)} * b * a - 1/d / f / ((a*f + c*d - (d*f)^{(1/2)}*b) / f)^{(1/2)} * \ln((2*(a*f + c*d - (d*f)^{(1/2)}*b) / f + (b*f - 2*(d*f)^{(1/2)}*c) * (x + (d*f)^{(1/2)}/f) / f + 2*((a*f + c*d - (d*f)^{(1/2)}*b) / f)^{(1/2)} * ((x + (d*f)^{(1/2)}/f)^{2*c} + (b*f - 2*(d*f)^{(1/2)}*c) * (x + (d*f)^{(1/2)}/f) / f + (a*f + c*d - (d*f)^{(1/2)}*b) / f)^{(1/2)}) / (x + (d*f)^{(1/2)}/f) * (d*f)^{(1/2)} * b * a - 1/2/d * ((x + (d*f)^{(1/2)}/f)^{2*c} + (b*f - 2*(d*f)^{(1/2)}*c) * (x + (d*f)^{(1/2)}/f) / f + (a*f + c*d - (d*f)^{(1/2)}*b) / f)^{(1/2)} * a - 1/2/f * ((x + (d*f)^{(1/2)}/f)^{2*c} + (b*f - 2*(d*f)^{(1/2)}*c) * (x + (d*f)^{(1/2)}/f) / f + (a*f + c*d - (d*f)^{(1/2)}*b) / f)^{(1/2)} * c + 1/d * a * (c*x^2 + b*x + a)^{(1/2)} - 1/d * a^{(3/2)} * \ln((b*x + 2*a + 2*(c*x^2 + b*x + a)^{(1/2)} * a^{(1/2)}) / x) - 1/2/d * ((x - (d*f)^{(1/2)}/f)^{2*c} + (b*f + 2*(d*f)^{(1/2)}*c) * (x - (d*f)^{(1/2)}/f) / f + (a*f + c*d + (d*f)^{(1/2)}*b) / f)^{(1/2)} * a - 1/2/f * ((x - (d*f)^{(1/2)}/f)^{2*c} + (b*f + 2*(d*f)^{(1/2)}*c) * (x - (d*f)^{(1/2)}/f) / f + (a*f + c*d + (d*f)^{(1/2)}*b) / f)^{(1/2)} * c - 5/8 / d / f * ((x - (d*f)^{(1/2)}/f)^{2*c} + (b*f + 2*(d*f)^{(1/2)}*c) * (x - (d*f)^{(1/2)}/f) / f + (a*f + c*d + (d*f)^{(1/2)}*b) / f)^{(1/2)} * b * (d*f)^{(1/2)} - 3/8 / d / c^{(1/2)} * \ln(((x - (d*f)^{(1/2)}/f)^{2*c} + 1/2*(b*f + 2*(d*f)^{(1/2)}*c) / f) / c^{(1/2)} + ((x - (d*f)^{(1/2)}/f)^{2*c} + (b*f + 2*(d*f)^{(1/2)}*c) * (x - (d*f)^{(1/2)}/f) / f + (a*f + c*d + (d*f)^{(1/2)}*b) / f)^{(1/2)}) * a * b$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(cx^2 + bx + a)^{\frac{3}{2}}}{(fx^2 - d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(3/2)/x/(-f*x^2+d),x, algorithm="maxima")

[Out] -integrate((c*x^2 + b*x + a)^(3/2)/((f*x^2 - d)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^2 + bx + a)^{\frac{3}{2}}}{x(d - fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x + c*x^2)^(3/2)/(x*(d - f*x^2)), x)`

[Out] `int((a + b*x + c*x^2)^(3/2)/(x*(d - f*x^2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a\sqrt{a+bx+cx^2}}{-dx+fx^3} dx - \int \frac{bx\sqrt{a+bx+cx^2}}{-dx+fx^3} dx - \int \frac{cx^2\sqrt{a+bx+cx^2}}{-dx+fx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+b*x+a)**(3/2)/x/(-f*x**2+d), x)`

[Out] `-Integral(a*sqrt(a + b*x + c*x**2)/(-d*x + f*x**3), x) - Integral(b*x*sqrt(a + b*x + c*x**2)/(-d*x + f*x**3), x) - Integral(c*x**2*sqrt(a + b*x + c*x**2)/(-d*x + f*x**3), x)`

$$3.89 \quad \int \frac{(a+bx+cx^2)^{3/2}}{x^2(d-fx^2)} dx$$

Optimal. Leaf size=463

$$\frac{(12acf + 3b^2f + 8c^2d) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) + 3(4ac + b^2) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (af + b(-\sqrt{d})\sqrt{f} + cd)}{8\sqrt{c}df} + \frac{3(4ac + b^2) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (af + b(-\sqrt{d})\sqrt{f} + cd)}{8\sqrt{c}d} + \frac{(af + b(-\sqrt{d})\sqrt{f} + cd)}{8\sqrt{c}df}$$

Rubi [A] time = 1.20, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 28, number of rules / integrand size = 0.357, Rules used = {6725, 732, 814, 843, 621, 206, 724, 978, 1078, 1033}

$$\frac{(12cf + 3b^2f + 8c^2d) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) + 3(4ac + b^2) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (af + b(-\sqrt{d})\sqrt{f} + cd)}{8\sqrt{c}df} + \frac{3(4ac + b^2) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (af + b(-\sqrt{d})\sqrt{f} + cd)}{8\sqrt{c}d} + \frac{(af + b(-\sqrt{d})\sqrt{f} + cd)}{8\sqrt{c}df}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + c*x^2)^(3/2)/(x^2*(d - f*x^2)), x]

[Out] (3*(3*b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*d) - ((5*b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*d) - (a + b*x + c*x^2)^(3/2)/(d*x) - (3*Sqrt[a]*b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*d) + (3*(b^2 + 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*d) - ((8*c^2*d + 3*b^2*f + 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*d*f) + ((c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)*f) + ((c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)*f)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 732

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Dist[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1]) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 978

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((b*(3*p + 2*q) + 2*c*(p + q)*x)*(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^(q + 1))/(2*f*(p + q)*(2*p + 2*q + 1)), x] - Dist[1/(2*f*(p + q)*(2*p + 2*q + 1)), Int[(a + b*x + c*x^2)^(p - 2)*(d + f*x^2)^q*Simp[b^2*d*(p - 1)*(2*p + q) - (p + q)*(b^2*d*(1 - p) - 2*a*(c*d - a*f*(2*p + 2*q + 1))) - (2*b*(c*d - a*f)*(1 - p)*(2*p + q) - 2*(p + q)*b*(2*c*d*(2*p + q) - (c*d + a*f)*(2*p + 2*q + 1)))*x + (b^2*f*p*(1 - p) + 2*c*(p + q)*(c*d*(2*p - 1) - a*f*(4*p + 2*q - 1)))*x^2, x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*q + 1, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/(a + c*x^2)*Sqrt[d + e*x + f*x^2], x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f, 0]
```

Rule 6725

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Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(a + bx + cx^2)^{3/2}}{x^2(d - fx^2)} dx = \int \left(\frac{(a + bx + cx^2)^{3/2}}{dx^2} + \frac{f(a + bx + cx^2)^{3/2}}{d(d - fx^2)} \right) dx$$

$$= \frac{\int \frac{(a+bx+cx^2)^{3/2}}{x^2} dx}{d} + \frac{f \int \frac{(a+bx+cx^2)^{3/2}}{d-fx^2} dx}{d}$$

$$= -\frac{(5b + 2cx)\sqrt{a + bx + cx^2}}{4d} - \frac{(a + bx + cx^2)^{3/2}}{dx} + \frac{\int \frac{\frac{1}{4}(5b^2d+4a(cd+2af))+4b(cd+af)x+\frac{1}{4}(8c^2d+3b^2)}{\sqrt{a+bx+cx^2}(d-fx^2)} dx}{2d}$$

$$= \frac{3(3b + 2cx)\sqrt{a + bx + cx^2}}{4d} - \frac{(5b + 2cx)\sqrt{a + bx + cx^2}}{4d} - \frac{(a + bx + cx^2)^{3/2}}{dx} - \frac{3 \int \frac{-4abc}{x\sqrt{a+bx+cx^2}} dx}{4d}$$

$$= \frac{3(3b + 2cx)\sqrt{a + bx + cx^2}}{4d} - \frac{(5b + 2cx)\sqrt{a + bx + cx^2}}{4d} - \frac{(a + bx + cx^2)^{3/2}}{dx} + \frac{(3ab) \int \frac{1}{x} dx}{4d}$$

$$= \frac{3(3b + 2cx)\sqrt{a + bx + cx^2}}{4d} - \frac{(5b + 2cx)\sqrt{a + bx + cx^2}}{4d} - \frac{(a + bx + cx^2)^{3/2}}{dx} - \frac{(8c^2d + 3b^2) \arctan\left(\frac{2cx + b}{\sqrt{a + bx + cx^2}}\right)}{4d}$$

$$= \frac{3(3b + 2cx)\sqrt{a + bx + cx^2}}{4d} - \frac{(5b + 2cx)\sqrt{a + bx + cx^2}}{4d} - \frac{(a + bx + cx^2)^{3/2}}{dx} - \frac{3\sqrt{a} b \arctan\left(\frac{2cx + b}{\sqrt{a + bx + cx^2}}\right)}{4d}$$

Mathematica [A] time = 0.64, size = 765, normalized size = 1.65

Integrate[(a + b*x + c*x^2)^(3/2)/(x^2*(d - f*x^2)), x]

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + c*x^2)^(3/2)/(x^2*(d - f*x^2)), x]

[Out]
$$-1/2*(2*a*\text{Sqrt}[d]*f*\text{Sqrt}[a + x*(b + c*x)] + 3*\text{Sqrt}[a]*b*\text{Sqrt}[d]*f*x*\text{ArcTanh}[(2*a + b*x)/(2*\text{Sqrt}[a]*\text{Sqrt}[a + x*(b + c*x)])] + 2*c^(3/2)*d^(3/2)*x*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + x*(b + c*x)])] + c*d*\text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*x*\text{ArcTanh}[(-b*\text{Sqrt}[d]) + 2*a*\text{Sqrt}[f] - 2*c*\text{Sqrt}[d]*x + b*\text{Sqrt}[f]*x)/(2*\text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{Sqrt}[a + x*(b + c*x)])] - b*\text{Sqrt}[d]*\text{Sqrt}[f]*\text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*x*\text{ArcTanh}[(-b*\text{Sqrt}[d]) + 2*a*\text{Sqrt}[f] - 2*c*\text{Sqrt}[d]*x + b*\text{Sqrt}[f]*x)/(2*\text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{Sqrt}[a + x*(b + c*x)])] + a*f*\text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*x*\text{ArcTanh}[(-b*\text{Sqrt}[d]) + 2*a*\text{Sqrt}[f] - 2*c*\text{Sqrt}[d]*x + b*\text{Sqrt}[f]*x)/(2*\text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{Sqrt}[a + x*(b + c*x)])] + c*d*\text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*x*\text{ArcTanh}[(-2*(a*\text{Sqrt}[f] + c*\text{Sqrt}[d]*x) - b*(\text{Sqrt}[d] + \text{Sqrt}[f]*x))/(2*\text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{Sqrt}[a + x*(b + c*x)])] + b*\text{Sqrt}[d]*\text{Sqrt}[f]*\text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*x*\text{ArcTanh}[(-2*(a*\text{Sqrt}[f] + c*\text{Sqrt}[d]*x) - b*(\text{Sqrt}[d] + \text{Sqrt}[f]*x))/(2*\text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{Sqrt}[a + x*(b + c*x)])] + a*f*\text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*x*\text{ArcTanh}[(-2*(a*\text{Sqrt}[f] + c*\text{Sqrt}[d]*x) - b*(\text{Sqrt}[d] + \text{Sqrt}[f]*x))/(2*\text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{Sqrt}[a + x*(b + c*x)])])]/(d^(3/2)*f*x)$$

IntegrateAlgebraic [C] time = 0.89, size = 550, normalized size = 1.19

Integrate[(a + b*x + c*x^2)^(3/2)/(x^2*(d - f*x^2)), x]

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x + c*x^2)^(3/2)/(x^2*(d - f*x^2)),x]

[Out] $-\frac{(a\sqrt{a+bx+cx^2})/(dx) + (3\sqrt{a}b\operatorname{ArcTanh}(\frac{\sqrt{c}x}{\sqrt{a}-\sqrt{a+bx+cx^2}})/d + (c^{3/2}\operatorname{Log}[bf+2cfx-2\sqrt{c}f\sqrt{a+bx+cx^2}])/f - \operatorname{RootSum}[b^2d-a^2f-4b\sqrt{c}d\#1+4c*d\#1^2+2a*f\#1^2-f\#1^4 \& , (b*c^2*d^2*\operatorname{Log}[-(\sqrt{c}x) + \sqrt{a+bx+cx^2}] - \#1) + b^3*d*f*\operatorname{Log}[-(\sqrt{c}x) + \sqrt{a+bx+cx^2}] - \#1) - a^2*b*f^2*\operatorname{Log}[-(\sqrt{c}x) + \sqrt{a+bx+cx^2}] - \#1) - 2*c^{5/2}*d^2*\operatorname{Log}[-(\sqrt{c}x) + \sqrt{a+bx+cx^2}] - \#1)*\#1 - 2*b^2*\sqrt{c}*d*f*\operatorname{Log}[-(\sqrt{c}x) + \sqrt{a+bx+cx^2}] - \#1)*\#1 - 4*a*c^{3/2}*d*f*\operatorname{Log}[-(\sqrt{c}x) + \sqrt{a+bx+cx^2}] - \#1)*\#1 - 2*a^2*\sqrt{c}*f^2*\operatorname{Log}[-(\sqrt{c}x) + \sqrt{a+bx+cx^2}] - \#1)*\#1 + 2*b*c*d*f*\operatorname{Log}[-(\sqrt{c}x) + \sqrt{a+bx+cx^2}] - \#1)*\#1^2 + 2*a*b*f^2*\operatorname{Log}[-(\sqrt{c}x) + \sqrt{a+bx+cx^2}] - \#1)*\#1^2)/(b*\sqrt{c}*d - 2*c*d*\#1 - a*f*\#1 + f*\#1^3) \&]/(2*d*f)$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(3/2)/x^2/(-f*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(3/2)/x^2/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type

maple [B] time = 0.02, size = 4799, normalized size = 10.37

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x+a)^(3/2)/x^2/(-f*x^2+d),x)

[Out] $-\frac{1}{6}f/d/(df)^{1/2}*((x-(df)^{1/2}/f)^2*c+(b*f+2*(df)^{1/2}*c)*(x-(df)^{1/2}/f)/f+(a*f+c*d+(df)^{1/2}*b)/f)^{3/2}-\frac{1}{4}d*((x-(df)^{1/2}/f)^2*c+(b*f+2*(df)^{1/2}*c)*(x-(df)^{1/2}/f)/f+(a*f+c*d+(df)^{1/2}*b)/f)^{1/2}*x*c-\frac{3}{4}d*\ln(((x-(df)^{1/2}/f)*c+1/2*(b*f+2*(df)^{1/2}*c)/f)/c^{1/2}+((x-(df)^{1/2}/f)^2*c+(b*f+2*(df)^{1/2}*c)*(x-(df)^{1/2}/f)/f+(a*f+c*d+(df)^{1/2}*b)/f)^{1/2})*c^{1/2}*a-\frac{3}{16}d*\ln(((x-(df)^{1/2}/f)*c+1/2*(b*f+2*(df)^{1/2}*c)/f)/c^{1/2}+((x-(df)^{1/2}/f)^2*c+(b*f+2*(df)^{1/2}*c)*(x-(df)^{1/2}/f)/f+(a*f+c*d+(df)^{1/2}*b)/f)^{1/2}))/c^{1/2}*b^2-\frac{3}{4}d*\ln(((x-(df)^{1/2}/f)*c+1/2*(b*f+2*(df)^{1/2}*c)/f)/c^{1/2}+((x-(df)^{1/2}/f)^2*c+(b*f+2*(df)^{1/2}*c)*(x-(df)^{1/2}/f)/f+(a*f+c*d+(df)^{1/2}*b)/f)^{1/2})*c^{1/2}*b+1/2/(df)^{1/2}/((a*f+c*d+(df)^{1/2}*b)/f)^{1/2})*\ln((2*(a*f+c*d+(df)^{1/2}*b)/f+(b*f+2*(df)^{1/2}*c)*(x-(df)^{1/2}/f)/f+2*(a*f+c*d+(df)^{1/2}*b)/f)^{1/2})*((x-(df)^{1/2}/f)^2*c+(b*f+2*(df)^{1/2}*c)*(x-(df)^{1/2}/f)/f+(a*f+c*d+(df)^{1/2}*b)/f)^{1/2}))/((x-(df)^{1/2}/f))^b^2+\frac{3}{8}d*b^2/c^{1/2}*\ln((c*x+1/2*b)/c^{1/2}+(c*x^2+b*x+a)^{1/2}))+1/d*b/a*(c*x^2+b*x+a)^{3/2}+\frac{1}{6}f/d/(df)^{1/2}*((x+(df)^{1/2}/f)^2*c+(b*f-2*(df)^{1/2}*c)*(x+(df)^{1/2}/f)/f+(a*f+c*d-(df)^{1/2}*b)/f)^{3/2}-\frac{1}{4}d*((x+(df)^{1/2}/f)^2*c+(b*f-2*(df)^{1/2}*c)*(x+(df)^{1/2}/f)/f+(a*f+c*d-(df)^{1/2}*b)/f)^{1/2})*x*c-\frac{3}{4}d*\ln(((x+(df)^{1/2}/f)*c+1/2*(b*f-2*(df)^{1/2}*c)/f)/$

$$\begin{aligned}
& c^{(1/2)} + ((x+(d*f)^{(1/2)}/f)^2*c + (b*f - 2*(d*f)^{(1/2)*c}) * (x+(d*f)^{(1/2)}/f) / f + (a \\
& *f + c*d - (d*f)^{(1/2)*b}) / f)^{(1/2)} * c^{(1/2)} * a - 3/16/d * \ln(((x+(d*f)^{(1/2)}/f)^2*c + 1/ \\
& 2*(b*f - 2*(d*f)^{(1/2)*c}) / f) / c^{(1/2)} + ((x+(d*f)^{(1/2)}/f)^2*c + (b*f - 2*(d*f)^{(1/2) \\
&) * c) * (x+(d*f)^{(1/2)}/f) / f + (a*f + c*d - (d*f)^{(1/2)*b}) / f)^{(1/2)} / c^{(1/2)} * b^2 + 3/4/ \\
& (d*f)^{(1/2)} * \ln(((x+(d*f)^{(1/2)}/f)^2*c + 1/2*(b*f - 2*(d*f)^{(1/2)*c}) / f) / c^{(1/2)} + ((\\
& x+(d*f)^{(1/2)}/f)^2*c + (b*f - 2*(d*f)^{(1/2)*c}) * (x+(d*f)^{(1/2)}/f) / f + (a*f + c*d - (d* \\
& f)^{(1/2)*b}) / f)^{(1/2)} * c^{(1/2)} * b - 1/2 / (d*f)^{(1/2)} / ((a*f + c*d - (d*f)^{(1/2)*b}) / f) \\
& ^{(1/2)} * \ln((2*(a*f + c*d - (d*f)^{(1/2)*b}) / f + (b*f - 2*(d*f)^{(1/2)*c}) * (x+(d*f)^{(1/2) \\
&) / f) / f + 2*((a*f + c*d - (d*f)^{(1/2)*b}) / f)^{(1/2)} * ((x+(d*f)^{(1/2)}/f)^2*c + (b*f - 2*(d* \\
& f)^{(1/2)*c}) * (x+(d*f)^{(1/2)}/f) / f + (a*f + c*d - (d*f)^{(1/2)*b}) / f)^{(1/2)}) / (x+(d*f)^{(1/2) \\
&) / f) * b^2 - 1/8*f/d / (d*f)^{(1/2)} * ((x - (d*f)^{(1/2)}/f)^2*c + (b*f + 2*(d*f)^{(1/2) \\
&) * c) * (x - (d*f)^{(1/2)}/f) / f + (a*f + c*d + (d*f)^{(1/2)*b}) / f)^{(1/2)} * x * b - 1/16*f/d / (d*f) \\
& ^{(1/2)} / c * ((x - (d*f)^{(1/2)}/f)^2*c + (b*f + 2*(d*f)^{(1/2)*c}) * (x - (d*f)^{(1/2)}/f) / f + (\\
& a*f + c*d + (d*f)^{(1/2)*b}) / f)^{(1/2)} * b^2 + 1/32*f/d / (d*f)^{(1/2)} / c^{(3/2)} * \ln(((x - (d* \\
& f)^{(1/2)}/f)^2*c + 1/2*(b*f + 2*(d*f)^{(1/2)*c}) / f) / c^{(1/2)} + ((x - (d*f)^{(1/2)}/f)^2*c + (\\
& b*f + 2*(d*f)^{(1/2)*c}) * (x - (d*f)^{(1/2)}/f) / f + (a*f + c*d + (d*f)^{(1/2)*b}) / f)^{(1/2)} * \\
& b^3 + 1/2*f/d / (d*f)^{(1/2)} / ((a*f + c*d + (d*f)^{(1/2)*b}) / f)^{(1/2)} * \ln((2*(a*f + c*d + (d \\
& *f)^{(1/2)*b}) / f + (b*f + 2*(d*f)^{(1/2)*c}) * (x - (d*f)^{(1/2)}/f) / f + 2*((a*f + c*d + (d*f) \\
& ^{(1/2)*b}) / f)^{(1/2)} * ((x - (d*f)^{(1/2)}/f)^2*c + (b*f + 2*(d*f)^{(1/2)*c}) * (x - (d*f)^{(1/2) \\
&) / f) / f + (a*f + c*d + (d*f)^{(1/2)*b}) / f)^{(1/2)}) / (x - (d*f)^{(1/2)}/f) * a^2 + 1/2/f*d / (d \\
& *f)^{(1/2)} / ((a*f + c*d + (d*f)^{(1/2)*b}) / f)^{(1/2)} * \ln((2*(a*f + c*d + (d*f)^{(1/2)*b}) / f \\
& + (b*f + 2*(d*f)^{(1/2)*c}) * (x - (d*f)^{(1/2)}/f) / f + 2*((a*f + c*d + (d*f)^{(1/2)*b}) / f)^{(1 \\
& / 2)} * ((x - (d*f)^{(1/2)}/f)^2*c + (b*f + 2*(d*f)^{(1/2)*c}) * (x - (d*f)^{(1/2)}/f) / f + (a*f + c \\
& *d + (d*f)^{(1/2)*b}) / f)^{(1/2)}) / (x - (d*f)^{(1/2)}/f) * c^2 + 1/8*f/d / (d*f)^{(1/2)} * ((x + \\
& (d*f)^{(1/2)}/f)^2*c + (b*f - 2*(d*f)^{(1/2)*c}) * (x+(d*f)^{(1/2)}/f) / f + (a*f + c*d - (d*f) \\
& ^{(1/2)*b}) / f)^{(1/2)} * x * b + 1/16*f/d / (d*f)^{(1/2)} / c * ((x+(d*f)^{(1/2)}/f)^2*c + (b*f - 2 \\
& * (d*f)^{(1/2)*c}) * (x+(d*f)^{(1/2)}/f) / f + (a*f + c*d - (d*f)^{(1/2)*b}) / f)^{(1/2)} * b^2 - 1/ \\
& 32*f/d / (d*f)^{(1/2)} / c^{(3/2)} * \ln(((x+(d*f)^{(1/2)}/f)^2*c + 1/2*(b*f - 2*(d*f)^{(1/2)*c} \\
&) / f) / c^{(1/2)} + ((x+(d*f)^{(1/2)}/f)^2*c + (b*f - 2*(d*f)^{(1/2)*c}) * (x+(d*f)^{(1/2)}/f) \\
& / f + (a*f + c*d - (d*f)^{(1/2)*b}) / f)^{(1/2)} * b^3 - 1/2*f/d / (d*f)^{(1/2)} / ((a*f + c*d - (d*f) \\
&) ^{(1/2)*b}) / f)^{(1/2)} * \ln((2*(a*f + c*d - (d*f)^{(1/2)*b}) / f + (b*f - 2*(d*f)^{(1/2)*c}) * (\\
& x+(d*f)^{(1/2)}/f) / f + 2*((a*f + c*d - (d*f)^{(1/2)*b}) / f)^{(1/2)} * ((x+(d*f)^{(1/2)}/f)^2 \\
& * c + (b*f - 2*(d*f)^{(1/2)*c}) * (x+(d*f)^{(1/2)}/f) / f + (a*f + c*d - (d*f)^{(1/2)*b}) / f)^{(1/ \\
& 2)}) / (x+(d*f)^{(1/2)}/f) * a^2 - 1/2/f*d / (d*f)^{(1/2)} / ((a*f + c*d - (d*f)^{(1/2)*b}) / f) \\
& ^{(1/2)} * \ln((2*(a*f + c*d - (d*f)^{(1/2)*b}) / f + (b*f - 2*(d*f)^{(1/2)*c}) * (x+(d*f)^{(1/2) \\
&) / f) / f + 2*((a*f + c*d - (d*f)^{(1/2)*b}) / f)^{(1/2)} * ((x+(d*f)^{(1/2)}/f)^2*c + (b*f - 2*(d* \\
& f)^{(1/2)*c}) * (x+(d*f)^{(1/2)}/f) / f + (a*f + c*d - (d*f)^{(1/2)*b}) / f)^{(1/2)}) / (x+(d*f)^{(1/2) \\
&) / f) * c^2 + 1/d / ((a*f + c*d + (d*f)^{(1/2)*b}) / f)^{(1/2)} * \ln((2*(a*f + c*d + (d*f)^{(1/2) \\
&) * b) / f + (b*f + 2*(d*f)^{(1/2)*c}) * (x - (d*f)^{(1/2)}/f) / f + 2*((a*f + c*d + (d*f)^{(1/2)*b} \\
&) / f)^{(1/2)} * ((x - (d*f)^{(1/2)}/f)^2*c + (b*f + 2*(d*f)^{(1/2)*c}) * (x - (d*f)^{(1/2)}/f) / f \\
& + (a*f + c*d + (d*f)^{(1/2)*b}) / f)^{(1/2)}) / (x - (d*f)^{(1/2)}/f) * b * a + 1/f / ((a*f + c*d + (d* \\
& f)^{(1/2)*b}) / f)^{(1/2)} * \ln((2*(a*f + c*d + (d*f)^{(1/2)*b}) / f + (b*f + 2*(d*f)^{(1/2)*c}) * \\
& (x - (d*f)^{(1/2)}/f) / f + 2*((a*f + c*d + (d*f)^{(1/2)*b}) / f)^{(1/2)} * ((x - (d*f)^{(1/2)}/f) \\
& ^2*c + (b*f + 2*(d*f)^{(1/2)*c}) * (x - (d*f)^{(1/2)}/f) / f + (a*f + c*d + (d*f)^{(1/2)*b}) / f)^{(1 \\
& / 2)}) / (x - (d*f)^{(1/2)}/f) * b * c + 1/2*f/d / (d*f)^{(1/2)} * ((x+(d*f)^{(1/2)}/f)^2*c + (b*f \\
& - 2*(d*f)^{(1/2)*c}) * (x+(d*f)^{(1/2)}/f) / f + (a*f + c*d - (d*f)^{(1/2)*b}) / f)^{(1/2)} * a + 1/ \\
& d / ((a*f + c*d - (d*f)^{(1/2)*b}) / f)^{(1/2)} * \ln((2*(a*f + c*d - (d*f)^{(1/2)*b}) / f + (b*f - 2* \\
& (d*f)^{(1/2)*c}) * (x+(d*f)^{(1/2)}/f) / f + 2*((a*f + c*d - (d*f)^{(1/2)*b}) / f)^{(1/2)} * ((x + \\
& (d*f)^{(1/2)}/f)^2*c + (b*f - 2*(d*f)^{(1/2)*c}) * (x+(d*f)^{(1/2)}/f) / f + (a*f + c*d - (d*f) \\
& ^{(1/2)*b}) / f)^{(1/2)}) / (x+(d*f)^{(1/2)}/f) * b * a + 1/f / ((a*f + c*d - (d*f)^{(1/2)*b}) / f) \\
& ^{(1/2)} * \ln((2*(a*f + c*d - (d*f)^{(1/2)*b}) / f + (b*f - 2*(d*f)^{(1/2)*c}) * (x+(d*f)^{(1/2) \\
&) / f) / f + 2*((a*f + c*d - (d*f)^{(1/2)*b}) / f)^{(1/2)} * ((x+(d*f)^{(1/2)}/f)^2*c + (b*f - 2*(d* \\
& f)^{(1/2)*c}) * (x+(d*f)^{(1/2)}/f) / f + (a*f + c*d - (d*f)^{(1/2)*b}) / f)^{(1/2)}) / (x+(d*f)^{(1/2) \\
&) / f) * b * c - 1 / (d*f)^{(1/2)} / ((a*f + c*d - (d*f)^{(1/2)*b}) / f)^{(1/2)} * \ln((2*(a*f + c*d \\
& - (d*f)^{(1/2)*b}) / f + (b*f - 2*(d*f)^{(1/2)*c}) * (x+(d*f)^{(1/2)}/f) / f + 2*((a*f + c*d - (d* \\
& f)^{(1/2)*b}) / f)^{(1/2)} * ((x+(d*f)^{(1/2)}/f)^2*c + (b*f - 2*(d*f)^{(1/2)*c}) * (x+(d*f)^{(1/2) \\
&) / f) / f + (a*f + c*d - (d*f)^{(1/2)*b}) / f)^{(1/2)}) / (x+(d*f)^{(1/2)}/f) * a * c + 1/d * c/a \\
& * (c * x^2 + b * x + a)^{(3/2)} * x - 5/8/d * ((x - (d*f)^{(1/2)}/f)^2*c + (b*f + 2*(d*f)^{(1/2)*c}) * (\\
& x - (d*f)^{(1/2)}/f) / f + (a*f + c*d + (d*f)^{(1/2)*b}) / f)^{(1/2)} * b - 1/2 / (d*f)^{(1/2)} * ((x - (
\end{aligned}$$

$$d*f)^{(1/2)}/f)^{2*c}+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)*c-1/2}/f*\ln(((x-(d*f)^{(1/2)}/f)*c+1/2*(b*f+2*(d*f)^{(1/2)*c})/f)/c^{(1/2)}+((x-(d*f)^{(1/2)}/f)^{2*c}+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)})^{c^{(3/2)}+9/4/d*b*(c*x^2+b*x+a)^{(1/2)}-5/8/d*((x+(d*f)^{(1/2)}/f)^{2*c}+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)*b+1/2}/(d*f)^{(1/2)*c}*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)*c-1/2}/f*\ln(((x+(d*f)^{(1/2)}/f)*c+1/2*(b*f-2*(d*f)^{(1/2)*c})/f)/c^{(1/2)}+((x+(d*f)^{(1/2)}/f)^{2*c}+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)})^{c^{(3/2)}+3/2/d*c*(c*x^2+b*x+a)^{(1/2)*x+3/2}/d*c^{(1/2)*a*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-1/d/a/x*(c*x^2+b*x+a)^{(5/2)}+3/8*f/d/(d*f)^{(1/2)}/c^{(1/2)*\ln(((x+(d*f)^{(1/2)}/f)*c+1/2*(b*f-2*(d*f)^{(1/2)*c})/f)/c^{(1/2)}+((x+(d*f)^{(1/2)}/f)^{2*c}+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)})^{a*b-3/8*f/d/(d*f)^{(1/2)}/c^{(1/2)*\ln(((x-(d*f)^{(1/2)}/f)*c+1/2*(b*f+2*(d*f)^{(1/2)*c})/f)/c^{(1/2)}+((x-(d*f)^{(1/2)}/f)^{2*c}+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)})^{a*b+1/(d*f)^{(1/2)}/((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)*\ln((2*(a*f+c*d+(d*f)^{(1/2)*b})/f+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+2*((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2))*((x-(d*f)^{(1/2)}/f)^{2*c}+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)))/(x-(d*f)^{(1/2)}/f)}^{a*c-1/2*f/d/(d*f)^{(1/2)*((x-(d*f)^{(1/2)}/f)^{2*c}+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)*a-3/2/d*b*a^{(1/2)*\ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)*a^{(1/2)})/x}}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(cx^2 + bx + a)^{\frac{3}{2}}}{(fx^2 - d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(3/2)/x^2/(-f*x^2+d),x, algorithm="maxima")

[Out] -integrate((c*x^2 + b*x + a)^(3/2)/((f*x^2 - d)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^2 + bx + a)^{\frac{3}{2}}}{x^2 (d - fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x + c*x^2)^(3/2)/(x^2*(d - f*x^2)),x)

[Out] int((a + b*x + c*x^2)^(3/2)/(x^2*(d - f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a\sqrt{a+bx+cx^2}}{-dx^2+fx^4} dx - \int \frac{bx\sqrt{a+bx+cx^2}}{-dx^2+fx^4} dx - \int \frac{cx^2\sqrt{a+bx+cx^2}}{-dx^2+fx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)**(3/2)/x**2/(-f*x**2+d),x)

[Out] -Integral(a*sqrt(a + b*x + c*x**2)/(-d*x**2 + f*x**4), x) - Integral(b*x*sqrt(a + b*x + c*x**2)/(-d*x**2 + f*x**4), x) - Integral(c*x**2*sqrt(a + b*x + c*x**2)/(-d*x**2 + f*x**4), x)

3.90 $\int \frac{(a+bx+cx^2)^{3/2}}{x^3(d-fx^2)} dx$

Optimal. Leaf size=614

$$\frac{a^{3/2} f \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right) - bf(b^2-12ac) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) - \sqrt{a+bx+cx^2} (8acf + b^2f + 2bcfx + 8cd^2)}{d^2 - 16c^{3/2}d^2 - 8cd^2}$$

Rubi [A] time = 1.44, antiderivative size = 614, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {6725, 732, 812, 843, 621, 206, 724, 734, 814, 1021, 1070, 1078, 1033}

$$\frac{a^{3/2} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right) - \sqrt{a+bx+cx^2} (8acf + b^2f + 2bcfx + 8cd^2) - bf(b^2-12ac) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) - (8acf + b^2f + 2bcfx + 8cd^2) \sqrt{a+bx+cx^2}}{d^2 - 16c^{3/2}d^2 - 8cd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(x^3*(d - f*x^2)), x]
[Out] (-3*(b - 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*d*x) + (f*(b^2 + 8*a*c + 2*b*c*x)
*Sqrt[a + b*x + c*x^2])/(8*c*d^2) - ((8*c^2*d + b^2*f + 8*a*c*f + 2*b*c*f*x)
*Sqrt[a + b*x + c*x^2])/(8*c*d^2) - (a + b*x + c*x^2)^(3/2)/(2*d*x^2) - (3
*(b^2 + 4*a*c)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*S
qrt[a]*d) - (a^(3/2)*f*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2]
)))/d^2 + (3*b*Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2]
)))/(2*d) - (b*(b^2 - 12*a*c)*f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x
+ c*x^2])])/(16*c^(3/2)*d^2) - (b*(24*c^2*d - b^2*f + 12*a*c*f)*ArcTanh[(b
+ 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*d^2) - ((c*d - b*
Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d]
- b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x
^2])])/(2*d^2*Sqrt[f]) + ((c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*
Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]
]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2*Sqrt[f])
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Sym
bol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_S
ymbol] := Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
```

1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1]) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]

Rule 734

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && (!RationalQ[m] || LtQ[m, 1]) && !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]

Rule 812

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m + 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !RationalQ[m])) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 814

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c*d*p) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 843

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

Rule 1021

Int[((g_.) + (h_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a

*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]

Rule 1033

Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]

Rule 1070

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((B*c*f*(2*p + 2*q + 3) + C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*(C*(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3)) + (2*p*(c*d - a*f)*(C*(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(-(b*c*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))))*x + (p*(-(b*f))*(C*(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))))*x^2, x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]

Rule 1078

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f, 0]

Rule 6725

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx+cx^2)^{3/2}}{x^3(d-fx^2)} dx &= \int \left(\frac{(a+bx+cx^2)^{3/2}}{dx^3} + \frac{f(a+bx+cx^2)^{3/2}}{d^2x} + \frac{f^2x(a+bx+cx^2)^{3/2}}{d^2(d-fx^2)} \right) dx \\
&= \frac{\int \frac{(a+bx+cx^2)^{3/2}}{x^3} dx}{d} + \frac{f \int \frac{(a+bx+cx^2)^{3/2}}{x} dx}{d^2} + \frac{f^2 \int \frac{x(a+bx+cx^2)^{3/2}}{d-fx^2} dx}{d^2} \\
&= -\frac{(a+bx+cx^2)^{3/2}}{2dx^2} + \frac{3 \int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x^2} dx}{4d} + \frac{f \int \frac{\sqrt{a+bx+cx^2} \left(\frac{3bd}{2} + 3(cd+af)x + \frac{3}{2}bfx^2 \right)}{d-fx^2} dx}{3d^2} \\
&= -\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx} + \frac{f(b^2+8ac+2bcx)\sqrt{a+bx+cx^2}}{8cd^2} - \frac{(8c^2d+b^2f+8cd^2)\sqrt{a+bx+cx^2}}{8cd^2} \\
&= -\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx} + \frac{f(b^2+8ac+2bcx)\sqrt{a+bx+cx^2}}{8cd^2} - \frac{(8c^2d+b^2f+8cd^2)\sqrt{a+bx+cx^2}}{8cd^2} \\
&= -\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx} + \frac{f(b^2+8ac+2bcx)\sqrt{a+bx+cx^2}}{8cd^2} - \frac{(8c^2d+b^2f+8cd^2)\sqrt{a+bx+cx^2}}{8cd^2} \\
&= -\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx} + \frac{f(b^2+8ac+2bcx)\sqrt{a+bx+cx^2}}{8cd^2} - \frac{(8c^2d+b^2f+8cd^2)\sqrt{a+bx+cx^2}}{8cd^2} \\
&= -\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx} + \frac{f(b^2+8ac+2bcx)\sqrt{a+bx+cx^2}}{8cd^2} - \frac{(8c^2d+b^2f+8cd^2)\sqrt{a+bx+cx^2}}{8cd^2}
\end{aligned}$$

Mathematica [A] time = 0.97, size = 303, normalized size = 0.49

$$\frac{(4a(2af+3cd)+3b^2d) \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right) - 4(af+b(-\sqrt{d})\sqrt{f+cd})^{3/2} \tanh^{-1}\left(\frac{2a\sqrt{f-b\sqrt{d}+b}\sqrt{f-x-2c}\sqrt{dx}}{2\sqrt{a+x(b+cx)}\sqrt{af+b(-\sqrt{d})\sqrt{f+cd}}}\right) + 4(af+b\sqrt{d}\sqrt{f+cd})^{3/2} \tanh^{-1}\left(\frac{-2(a\sqrt{f+c}\sqrt{dx})-b(\sqrt{d}+\sqrt{fx})}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f+cd}}}\right) + \frac{2d(2a+5bx)\sqrt{a+x(b+cx)}}{x^2}}{8d^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + c*x^2)^(3/2)/(x^3*(d - f*x^2)), x]

[Out]
$$\begin{aligned}
& -1/8*((2*d*(2*a + 5*b*x)*\text{Sqrt}[a + x*(b + c*x)]/x^2 + ((3*b^2*d + 4*a*(3*c*d + 2*a*f))*\text{ArcTanh}[(2*a + b*x)/(2*\text{Sqrt}[a]*\text{Sqrt}[a + x*(b + c*x)])))/\text{Sqrt}[a] \\
& - (4*(c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f)^(3/2)*\text{ArcTanh}[(-b*\text{Sqrt}[d]) + 2*a*\text{Sqrt}[f] - 2*c*\text{Sqrt}[d]*x + b*\text{Sqrt}[f]*x)/(2*\text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{Sqrt}[a + x*(b + c*x)])))/\text{Sqrt}[f] + (4*(c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f)^(3/2)*\text{ArcTanh}[(-2*(a*\text{Sqrt}[f] + c*\text{Sqrt}[d]*x) - b*(\text{Sqrt}[d] + \text{Sqrt}[f]*x))/(2*\text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f)*\text{Sqrt}[a + x*(b + c*x)])))/\text{Sqrt}[f])/d^2
\end{aligned}$$

IntegrateAlgebraic [C] time = 1.17, size = 597, normalized size = 0.97

$$\frac{(-2*a - 5*b*x)*\text{Sqrt}[a + b*x + c*x^2]/(4*d*x^2) + ((-3*b^2*d - 12*a*c*d - 8*a^2*f)*\text{ArcTanh}[(-\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2)]/\text{Sqrt}[a]]/(4*\text{Sqrt}[a]*d^2) - \text{RootSum}[b^2*d - a^2*f - 4*b*\text{Sqrt}[c]*d*\#1 + 4*c*d*\#1^2 + 2*a*f*\#1^2]}{8d^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x + c*x^2)^(3/2)/(x^3*(d - f*x^2)), x]

[Out]
$$\begin{aligned}
& ((-2*a - 5*b*x)*\text{Sqrt}[a + b*x + c*x^2]/(4*d*x^2) + ((-3*b^2*d - 12*a*c*d - 8*a^2*f)*\text{ArcTanh}[(-\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2)]/\text{Sqrt}[a]]/(4*\text{Sqrt}[a]*d^2) - \text{RootSum}[b^2*d - a^2*f - 4*b*\text{Sqrt}[c]*d*\#1 + 4*c*d*\#1^2 + 2*a*f*\#1^2]
\end{aligned}$$

- f*#1^4 & , (2*b^2*c*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*c^2*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a*b^2*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*a^2*c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^3*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 4*b*c^(3/2)*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 4*a*b*Sqrt[c]*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + c^2*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + b^2*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + 2*a*c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + a^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) &]/(2*d^2)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(3/2)/x^3/(-f*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage2

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(3/2)/x^3/(-f*x^2+d),x, algorithm="giac")

[Out] sage2

maple [B] time = 0.02, size = 5056, normalized size = 8.23

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x+a)^(3/2)/x^3/(-f*x^2+d),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(cx^2 + bx + a)^{\frac{3}{2}}}{(fx^2 - d)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(3/2)/x^3/(-f*x^2+d),x, algorithm="maxima")

[Out] -integrate((c*x^2 + b*x + a)^(3/2)/((f*x^2 - d)*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^2 + bx + a)^{3/2}}{x^3 (d - fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x + c*x^2)^(3/2)/(x^3*(d - f*x^2)),x)

[Out] `int((a + b*x + c*x^2)^(3/2)/(x^3*(d - f*x^2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a\sqrt{a+bx+cx^2}}{-dx^3+fx^5} dx - \int \frac{bx\sqrt{a+bx+cx^2}}{-dx^3+fx^5} dx - \int \frac{cx^2\sqrt{a+bx+cx^2}}{-dx^3+fx^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+b*x+a)**(3/2)/x**3/(-f*x**2+d), x)`

[Out] `-Integral(a*sqrt(a + b*x + c*x**2)/(-d*x**3 + f*x**5), x) - Integral(b*x*sqrt(a + b*x + c*x**2)/(-d*x**3 + f*x**5), x) - Integral(c*x**2*sqrt(a + b*x + c*x**2)/(-d*x**3 + f*x**5), x)`

$$3.91 \quad \int \frac{(a+bx+cx^2)^{3/2}}{1-x^2} dx$$

Optimal. Leaf size=189

$$-\frac{(12ac + 3b^2 + 8c^2) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}} - \frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2} - \frac{1}{2}(a-b+c)^{3/2} \tanh^{-1}\left(\frac{2a+x(b-2c)}{2\sqrt{a-b+c}\sqrt{a+bx+cx^2}}\right)$$

Rubi [A] time = 0.28, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 24, number of rules / integrand size = 0.250, Rules used = {978, 1078, 621, 206, 1033, 724}

$$-\frac{(12ac + 3b^2 + 8c^2) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}} - \frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2} - \frac{1}{2}(a-b+c)^{3/2} \tanh^{-1}\left(\frac{2a+x(b-2c)-b}{2\sqrt{a-b+c}\sqrt{a+bx+cx^2}}\right) + \frac{1}{2}(a+b+c)^{3/2} \tanh^{-1}\left(\frac{2a+x(b+2c)+b}{2\sqrt{a+b+c}\sqrt{a+bx+cx^2}}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + c*x^2)^(3/2)/(1 - x^2), x]

[Out] -((5*b + 2*c*x)*Sqrt[a + b*x + c*x^2])/4 - ((a - b + c)^(3/2)*ArcTanh[(2*a - b + (b - 2*c)*x)/(2*Sqrt[a - b + c]*Sqrt[a + b*x + c*x^2])])/2 - ((3*b^2 + 12*a*c + 8*c^2)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]) + ((a + b + c)^(3/2)*ArcTanh[(2*a + b + (b + 2*c)*x)/(2*Sqrt[a + b + c]*Sqrt[a + b*x + c*x^2])])/2

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 978

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((b*(3*p + 2*q) + 2*c*(p + q)*x)*(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^(q + 1))/(2*f*(p + q)*(2*p + 2*q + 1)), x] - Dist[1/(2*f*(p + q)*(2*p + 2*q + 1)), Int[(a + b*x + c*x^2)^(p - 2)*(d + f*x^2)^q*Simp[b^2*d*(p - 1)*(2*p + q) - (p + q)*(b^2*d*(1 - p) - 2*a*(c*d - a*f*(2*p + 2*q + 1)) - (2*b*(c*d - a*f)*(1 - p)*(2*p + q) - 2*(p + q)*b*(2*c*d*(2*p + q) - (c*d + a*f)*(2*p + 2*q + 1)))*x + (b^2*f*p*(1 - p) + 2*c*(p + q)*(c*d*(2*p - 1) - a*f*(4*p + 2*q - 1)))*x^2, x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*q + 1, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/(q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(a + bx + cx^2)^{3/2}}{1 - x^2} dx &= -\frac{1}{4}(5b + 2cx)\sqrt{a + bx + cx^2} + \frac{1}{2} \int \frac{\frac{1}{4}(8a^2 + 5b^2 + 4ac) + 4b(a + c)x + \frac{1}{4}(3b^2 + 12ac)}{(1 - x^2)\sqrt{a + bx + cx^2}} dx \\ &= -\frac{1}{4}(5b + 2cx)\sqrt{a + bx + cx^2} - \frac{1}{2} \int \frac{\frac{1}{4}(-8a^2 - 5b^2 - 4ac) + \frac{1}{4}(-3b^2 - 12ac - 8c^2)}{(1 - x^2)\sqrt{a + bx + cx^2}} dx \\ &= -\frac{1}{4}(5b + 2cx)\sqrt{a + bx + cx^2} - \frac{1}{2}(a - b + c)^2 \int \frac{1}{(-1 - x)\sqrt{a + bx + cx^2}} dx + \frac{1}{2}(a + b + c) \int \frac{1}{(1 - x)\sqrt{a + bx + cx^2}} dx \\ &= -\frac{1}{4}(5b + 2cx)\sqrt{a + bx + cx^2} - \frac{(3b^2 + 12ac + 8c^2) \tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right)}{8\sqrt{c}} + (a - b + c) \operatorname{arctanh}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right) \\ &= -\frac{1}{4}(5b + 2cx)\sqrt{a + bx + cx^2} - \frac{1}{2}(a - b + c)^{3/2} \tanh^{-1}\left(\frac{2a - b + (b - 2c)x}{2\sqrt{a - b + c}\sqrt{a + bx + cx^2}}\right) \end{aligned}$$

Mathematica [A] time = 0.58, size = 181, normalized size = 0.96

$$\frac{1}{8} \left(\frac{(4c(3a + 2c) + 3b^2) \tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right)}{\sqrt{c}} - 2(5b + 2cx)\sqrt{a + x(b + cx)} - 4(a - b + c)^{3/2} \tanh^{-1}\left(\frac{2a + b(x - 1) - 2cx}{2\sqrt{a - b + c}\sqrt{a + x(b + cx)}}\right) + 4(a + b + c)^{3/2} \tanh^{-1}\left(\frac{2a + bx + b + 2cx}{2\sqrt{a + b + c}\sqrt{a + x(b + cx)}}\right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(1 - x^2), x]
```

```
[Out] (-2*(5*b + 2*c*x)*Sqrt[a + x*(b + c*x)] - 4*(a - b + c)^(3/2)*ArcTanh[(2*a + b*(-1 + x) - 2*c*x)/(2*Sqrt[a - b + c]*Sqrt[a + x*(b + c*x)])] - ((3*b^2 + 4*c*(3*a + 2*c))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + 4*(a + b + c)^(3/2)*ArcTanh[(2*a + b + b*x + 2*c*x)/(2*Sqrt[a + b + c]*Sqrt[a + x*(b + c*x)])])/8
```

IntegrateAlgebraic [A] time = 0.82, size = 314, normalized size = 1.66

$$\frac{(12ac + 3b^2 + 8c^2) \log\left(\frac{-2\sqrt{c}\sqrt{a + bx + cx^2} + b + 2cx}{\sqrt{c}}\right) + \frac{1}{4}(-5b - 2cx)\sqrt{a + bx + cx^2} + (a\sqrt{-a - b - c} + c\sqrt{-a - b - c} + b\sqrt{-a - b - c}) \tan^{-1}\left(\frac{\sqrt{a + bx + cx^2}}{\sqrt{-a - b - c}} - \frac{\sqrt{c}x}{\sqrt{-a - b - c}} + \frac{\sqrt{c}}{\sqrt{-a - b - c}}\right) + (a\sqrt{a + b - c} + c\sqrt{a + b - c} - b\sqrt{a + b - c}) \tan^{-1}\left(\frac{\sqrt{a + bx + cx^2}}{\sqrt{a + b - c}} + \frac{\sqrt{c}x}{\sqrt{a + b - c}} + \frac{\sqrt{c}}{\sqrt{a + b - c}}\right)}{8\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)^(3/2)/(1 - x^2), x]
```

```
[Out] ((-5*b - 2*c*x)*Sqrt[a + b*x + c*x^2])/4 + (a*Sqrt[-a - b - c] + b*Sqrt[-a - b - c] + Sqrt[-a - b - c]*c)*ArcTan[Sqrt[c]/Sqrt[-a - b - c]] - (Sqrt[c]*x
```

$$\frac{1}{\sqrt{-a-b-c}} + \frac{\sqrt{a+bx+cx^2}}{\sqrt{-a-b-c}} + (a\sqrt{-a-b-c} - b\sqrt{-a+b-c} + \sqrt{-a+b-c}c)\operatorname{ArcTan}\left[\frac{\sqrt{c}}{\sqrt{-a+b-c}}\right] + \frac{(\sqrt{c}x)/\sqrt{-a+b-c} - \sqrt{a+bx+cx^2}/\sqrt{-a+b-c}}{1} + \frac{((3b^2 + 12ac + 8c^2)\operatorname{Log}[b + 2cx - 2\sqrt{c}\sqrt{a+bx+cx^2}])}{(8\sqrt{c})}$$

fricas [A] time = 164.30, size = 2579, normalized size = 13.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(3/2)/(-x^2+1),x, algorithm="fricas")

[Out] $\frac{1}{16}((3b^2 + 12ac + 8c^2)\sqrt{c}\log(-8c^2x^2 - 8b^2cx - b^2 + 4\sqrt{c^2x^2 + bx + a}(2cx + b)\sqrt{c} - 4ac) + 4((a - b)c + c^2)\sqrt{c}\log(a - b + c)\log(-((b^2 + 4(a - 2b)c + 8c^2)x^2 - 4\sqrt{c^2x^2 + bx + a}((b - 2c)x + 2a - b)\sqrt{a - b + c} + 8a^2 - 8ab + b^2 + 4ac + 2(4ab - 3b^2 - 4(a - b)c)x)/(x^2 + 2x + 1)) + 4((a + b)c + c^2)\sqrt{c}\log(a + b + c)\log(-((b^2 + 4(a + 2b)c + 8c^2)x^2 + 4\sqrt{c^2x^2 + bx + a}((b + 2c)x + 2a + b)\sqrt{a + b + c} + 8a^2 + 8ab + b^2 + 4ac + 2(4ab + 3b^2 + 4(a + b)c)x)/(x^2 - 2x + 1)) - 4(2c^2x + 5bc)\sqrt{c^2x^2 + bx + a}/c, -1/16(8((a - b)c + c^2)\sqrt{-a + b - c}\operatorname{arctan}(-1/2\sqrt{c^2x^2 + bx + a}((b - 2c)x + 2a - b)\sqrt{-a + b - c}/((a - b)c + c^2)x^2 + a^2 - ab + ac + (ab - b^2 + bc)x)) - (3b^2 + 12ac + 8c^2)\sqrt{c}\log(-8c^2x^2 - 8b^2cx - b^2 + 4\sqrt{c^2x^2 + bx + a}(2cx + b)\sqrt{c} - 4ac) - 4((a + b)c + c^2)\sqrt{c}\log(a - b + c)\log(-((b^2 + 4(a + 2b)c + 8c^2)x^2 + 4\sqrt{c^2x^2 + bx + a}((b + 2c)x + 2a + b)\sqrt{a + b + c} + 8a^2 + 8ab + b^2 + 4ac + 2(4ab + 3b^2 + 4(a + b)c)x)/(x^2 - 2x + 1)) + 4(2c^2x + 5bc)\sqrt{c^2x^2 + bx + a}/c, -1/16(8((a + b)c + c^2)\sqrt{-a - b - c}\operatorname{arctan}(1/2\sqrt{c^2x^2 + bx + a}((b + 2c)x + 2a + b)\sqrt{-a - b - c}/(((a + b)c + c^2)x^2 + a^2 + ab + ac + (ab + b^2 + bc)x)) - (3b^2 + 12ac + 8c^2)\sqrt{c}\log(-8c^2x^2 - 8b^2cx - b^2 + 4\sqrt{c^2x^2 + bx + a}(2cx + b)\sqrt{c} - 4ac) - 4((a - b)c + c^2)\sqrt{c}\log(a - b + c)\log(-((b^2 + 4(a - 2b)c + 8c^2)x^2 - 4\sqrt{c^2x^2 + bx + a}((b - 2c)x + 2a - b)\sqrt{a - b + c} + 8a^2 - 8ab + b^2 + 4ac + 2(4ab - 3b^2 - 4(a - b)c)x)/(x^2 + 2x + 1)) + 4(2c^2x + 5bc)\sqrt{c^2x^2 + bx + a}/c, -1/16(8((a - b)c + c^2)\sqrt{-a + b - c}\operatorname{arctan}(-1/2\sqrt{c^2x^2 + bx + a}((b - 2c)x + 2a - b)\sqrt{-a + b - c}/(((a - b)c + c^2)x^2 + a^2 - ab + ac + (ab - b^2 + bc)x)) + 8((a + b)c + c^2)\sqrt{-a - b - c}\operatorname{arctan}(1/2\sqrt{c^2x^2 + bx + a}((b + 2c)x + 2a + b)\sqrt{-a - b - c}/(((a + b)c + c^2)x^2 + a^2 + ab + ac + (ab + b^2 + bc)x)) - (3b^2 + 12ac + 8c^2)\sqrt{c}\log(-8c^2x^2 - 8b^2cx - b^2 + 4\sqrt{c^2x^2 + bx + a}(2cx + b)\sqrt{c} - 4ac) + 4(2c^2x + 5bc)\sqrt{c^2x^2 + bx + a}/c, 1/8((3b^2 + 12ac + 8c^2)\sqrt{-c}\operatorname{arctan}(1/2\sqrt{c^2x^2 + bx + a}(2cx + b)\sqrt{-c}/(c^2x^2 + bcx + ac)) + 2((a - b)c + c^2)\sqrt{a - b + c}\log(-((b^2 + 4(a - 2b)c + 8c^2)x^2 - 4\sqrt{c^2x^2 + bx + a}((b - 2c)x + 2a - b)\sqrt{a - b + c} + 8a^2 - 8ab + b^2 + 4ac + 2(4ab - 3b^2 - 4(a - b)c)x)/(x^2 + 2x + 1)) + 2((a + b)c + c^2)\sqrt{a + b + c}\log(-((b^2 + 4(a + 2b)c + 8c^2)x^2 + 4\sqrt{c^2x^2 + bx + a}((b + 2c)x + 2a + b)\sqrt{a + b + c} + 8a^2 + 8ab + b^2 + 4ac + 2(4ab + 3b^2 + 4(a + b)c)x)/(x^2 - 2x + 1)) - 2(2c^2x + 5bc)\sqrt{c^2x^2 + bx + a}/c, -1/8(4((a - b)c + c^2)\sqrt{-a + b - c}\operatorname{arctan}(-1/2\sqrt{c^2x^2 + bx + a}((b - 2c)x + 2a - b)\sqrt{-a + b - c}/(((a - b)c + c^2)x^2 + a^2 - ab + ac + (ab - b^2 + bc)x)) - (3b^2 + 12ac + 8c^2)\sqrt{-c}\operatorname{arctan}(1/2\sqrt{c^2x^2 + bx + a}(2cx + b)\sqrt{-c}/(c^2x^2 + bcx + ac)) - 2((a + b)c + c^2)\sqrt{a + b + c}\log(-((b^2 + 4(a + 2b)c + 8c^2)x^2 + 4\sqrt{c^2x^2 + bx + a}((b + 2c)x + 2a + b)\sqrt{a + b + c} + 8a^2 + 8ab + b^2 + 4ac + 2(4ab + 3b^2 + 4(a + b)c)x)/(x^2 - 2x + 1)) + 2(2c^2x + 5bc)\sqrt{c^2x^2 + bx + a}/c, -1/8(4$

```

*((a + b)*c + c^2)*sqrt(-a - b - c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*((b +
2*c)*x + 2*a + b)*sqrt(-a - b - c)/(((a + b)*c + c^2)*x^2 + a^2 + a*b + a*c
+ (a*b + b^2 + b*c)*x)) - (3*b^2 + 12*a*c + 8*c^2)*sqrt(-c)*arctan(1/2*sqrt
(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*((a -
b)*c + c^2)*sqrt(a - b + c)*log(-((b^2 + 4*(a - 2*b)*c + 8*c^2)*x^2 - 4*sqrt
(c*x^2 + b*x + a)*((b - 2*c)*x + 2*a - b)*sqrt(a - b + c) + 8*a^2 - 8*a*b
+ b^2 + 4*a*c + 2*(4*a*b - 3*b^2 - 4*(a - b)*c)*x)/(x^2 + 2*x + 1)) + 2*(2*
c^2*x + 5*b*c)*sqrt(c*x^2 + b*x + a))/c, -1/8*(4*((a - b)*c + c^2)*sqrt(-a
+ b - c)*arctan(-1/2*sqrt(c*x^2 + b*x + a)*((b - 2*c)*x + 2*a - b)*sqrt(-a
+ b - c)/(((a - b)*c + c^2)*x^2 + a^2 - a*b + a*c + (a*b - b^2 + b*c)*x)) +
4*((a + b)*c + c^2)*sqrt(-a - b - c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*((b
+ 2*c)*x + 2*a + b)*sqrt(-a - b - c)/(((a + b)*c + c^2)*x^2 + a^2 + a*b + a
*c + (a*b + b^2 + b*c)*x)) - (3*b^2 + 12*a*c + 8*c^2)*sqrt(-c)*arctan(1/2*sqrt
(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(2*c
^2*x + 5*b*c)*sqrt(c*x^2 + b*x + a))/c]

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(3/2)/(-x^2+1),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type

maple [B] time = 0.02, size = 1346, normalized size = 7.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x+a)^(3/2)/(-x^2+1),x)

[Out]
$$\begin{aligned}
 & -1/2*c^{(3/2)}*\ln((1/2*b-c+c*(x+1))/c^{(1/2)}+((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^{(1/2)}) \\
 & -1/2*c^{(3/2)}*\ln((1/2*b+c+c*(x-1))/c^{(1/2)}+((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^{(1/2)}) \\
 & +1/2*a*((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^{(1/2)}+1/2*c*((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^{(1/2)} \\
 & -5/8*((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^{(1/2)}*b-1/2*a*((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^{(1/2)} \\
 & -1/2*c*((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^{(1/2)}-5/8*((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^{(1/2)}*b \\
 & +3/8*b/c^{(1/2)}*\ln((1/2*b-c+c*(x+1))/c^{(1/2)}+((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^{(1/2)}) \\
 & *a-3/8*b/c^{(1/2)}*\ln((1/2*b+c+c*(x-1))/c^{(1/2)}+((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^{(1/2)}) \\
 & *a-1/2*a*(a-b+c)^{(1/2)}*\ln((2*a-2*b+2*c+(b-2*c)*(x+1)+2*(a-b+c)^{(1/2)}*((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^{(1/2)}) \\
 & /((x+1))+3/4*b*\ln((1/2*b-c+c*(x+1))/c^{(1/2)}+((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^{(1/2)}) \\
 & *c^{(1/2)}+1/2*b*(a-b+c)^{(1/2)}*\ln((2*a-2*b+2*c+(b-2*c)*(x+1)+2*(a-b+c)^{(1/2)}*((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^{(1/2)}) \\
 & /((x+1))-1/2*c*(a-b+c)^{(1/2)}*\ln((2*a-2*b+2*c+(b-2*c)*(x+1)+2*(a-b+c)^{(1/2)}*((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^{(1/2)}) \\
 & /((x+1))+1/8*b*((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^{(1/2)}*x+1/16/c*((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^{(1/2)}*b^2-1/32/c^{(3/2)}*\ln((1/2*b-c+c*(x+1))/c^{(1/2)}+((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^{(1/2)}) \\
 & *b^3-1/4*c*((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^{(1/2)}*x+1/2*a*(a+b+c)^{(1/2)}*\ln((2*a+2*b+2*c+(b+2*c)*(x-1)+2*(a+b+c)^{(1/2)}*((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^{(1/2)}) \\
 & /((x-1))-3/4*b*\ln((1/2*b+c+c*(x-1))/c^{(1/2)}+((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^{(1/2)})*c^{(1/2)}+1/2*b*(a+b+c)^{(1/2)}*\ln((2*a+2*b+2*c+(b+2*c)*(x-1)+2*(a+b+c)^{(1/2)}*((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^{(1/2)}) \\
 & /((x-1))+1/2*c*(a+b+c)^{(1/2)}*\ln((2*a+2*b+2*c+(b+2*c)*(x-1)+2*(a+b+c)^{(1/2)}*((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^{(1/2)}) \\
 & /((x-1))-1/8*b*((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^{(1/2)}*x-1/16/c*((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^{(1/2)}*b^2+1/32/c^{(3/2)}*\ln((1/2*b+c+c*(x-1))/c^{(1/2)}+((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^{(1/2)}) \\
 & *b^3-1/4*c*((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^{(1/2)}*x-3/16/c^{(1/2)}*\ln((1/2*b-c+c*(x+1))/c^{(1/2)}+((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^{(1/2)}) \\
 & *b^2-3/4*c^{(1/2)}*\ln((1/2*b-c+c*(x+1))/c^{(1/2)}+((x+1)^2*c+(b-2*c)
 \end{aligned}$$

```
*(x+1)+a-b+c)^(1/2))*a-3/4*c^(1/2)*ln((1/2*b+c+c*(x-1))/c^(1/2))+((x-1)^2*c+
(b+2*c)*(x-1)+a+b+c)^(1/2))*a-3/16/c^(1/2)*ln((1/2*b+c+c*(x-1))/c^(1/2))+((x
-1)^2*c+(b+2*c)*(x-1)+a+b+c)^(1/2))*b^2+1/6*((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)
^(3/2)-1/6*((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^(3/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(3/2)/(-x^2+1),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{(cx^2 + bx + a)^{3/2}}{x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(a + b*x + c*x^2)^(3/2)/(x^2 - 1),x)

[Out] -int((a + b*x + c*x^2)^(3/2)/(x^2 - 1), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a\sqrt{a + bx + cx^2}}{x^2 - 1} dx - \int \frac{bx\sqrt{a + bx + cx^2}}{x^2 - 1} dx - \int \frac{cx^2\sqrt{a + bx + cx^2}}{x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)**(3/2)/(-x**2+1),x)

[Out] -Integral(a*sqrt(a + b*x + c*x**2)/(x**2 - 1), x) - Integral(b*x*sqrt(a + b*x + c*x**2)/(x**2 - 1), x) - Integral(c*x**2*sqrt(a + b*x + c*x**2)/(x**2 - 1), x)

$$3.92 \quad \int \frac{\sqrt{-1-x+x^2}}{1-x^2} dx$$

Optimal. Leaf size=75

$$-\frac{1}{2} \tan^{-1}\left(\frac{3-x}{2\sqrt{x^2-x-1}}\right) + \tanh^{-1}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right) + \frac{1}{2} \tanh^{-1}\left(\frac{3x+1}{2\sqrt{x^2-x-1}}\right)$$

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {990, 621, 206, 1033, 724, 204}

$$-\frac{1}{2} \tan^{-1}\left(\frac{3-x}{2\sqrt{x^2-x-1}}\right) + \tanh^{-1}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right) + \frac{1}{2} \tanh^{-1}\left(\frac{3x+1}{2\sqrt{x^2-x-1}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-1 - x + x^2]/(1 - x^2), x]

[Out] -ArcTan[(3 - x)/(2*Sqrt[-1 - x + x^2])]/2 + ArcTanh[(1 - 2*x)/(2*Sqrt[-1 - x + x^2])] + ArcTanh[(1 + 3*x)/(2*Sqrt[-1 - x + x^2])]/2

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 990

Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (f_.)*(x_)^2), x_Symbol] :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*f - b*f*x)/(Sqrt[a + b*x + c*x^2]*(d + f*x^2)), x], x] /; FreeQ[{a, b, c, d, f}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1033

Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/(q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f}

, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{-1-x+x^2}}{1-x^2} dx &= -\int \frac{1}{\sqrt{-1-x+x^2}} dx - \int \frac{x}{(1-x^2)\sqrt{-1-x+x^2}} dx \\ &= -\left(\frac{1}{2} \int \frac{1}{(-1-x)\sqrt{-1-x+x^2}} dx\right) - \frac{1}{2} \int \frac{1}{(1-x)\sqrt{-1-x+x^2}} dx - 2 \operatorname{Subst}\left(\int \frac{1}{4-x^2} dx, x, \frac{3-x}{\sqrt{-1-x+x^2}}\right) \\ &= \tanh^{-1}\left(\frac{1-2x}{2\sqrt{-1-x+x^2}}\right) + \operatorname{Subst}\left(\int \frac{1}{-4-x^2} dx, x, \frac{3-x}{\sqrt{-1-x+x^2}}\right) + \operatorname{Subst}\left(\int \frac{1}{4-x^2} dx, x, \frac{3-x}{\sqrt{-1-x+x^2}}\right) \\ &= -\frac{1}{2} \tan^{-1}\left(\frac{3-x}{2\sqrt{-1-x+x^2}}\right) + \tanh^{-1}\left(\frac{1-2x}{2\sqrt{-1-x+x^2}}\right) + \frac{1}{2} \tanh^{-1}\left(\frac{1+3x}{2\sqrt{-1-x+x^2}}\right) \end{aligned}$$

Mathematica [A] time = 0.04, size = 75, normalized size = 1.00

$$-\frac{1}{2} \tan^{-1}\left(\frac{3-x}{2\sqrt{x^2-x-1}}\right) + \tanh^{-1}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right) + \frac{1}{2} \tanh^{-1}\left(\frac{3x+1}{2\sqrt{x^2-x-1}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-1 - x + x^2]/(1 - x^2), x]

[Out] -1/2*ArcTan[(3 - x)/(2*Sqrt[-1 - x + x^2])] + ArcTanh[(1 - 2*x)/(2*Sqrt[-1 - x + x^2])] + ArcTanh[(1 + 3*x)/(2*Sqrt[-1 - x + x^2])]/2

IntegrateAlgebraic [A] time = 0.14, size = 57, normalized size = 0.76

$$\log\left(2\sqrt{x^2-x-1}-2x+1\right) + \tan^{-1}\left(\sqrt{x^2-x-1}-x+1\right) + \tanh^{-1}\left(-\sqrt{x^2-x-1}+x+1\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[-1 - x + x^2]/(1 - x^2), x]

[Out] ArcTan[1 - x + Sqrt[-1 - x + x^2]] + ArcTanh[1 + x - Sqrt[-1 - x + x^2]] + Log[1 - 2*x + 2*Sqrt[-1 - x + x^2]]

fricas [A] time = 0.42, size = 70, normalized size = 0.93

$$\arctan(-x + \sqrt{x^2-x-1} + 1) - \frac{1}{2} \log(-x + \sqrt{x^2-x-1}) + \frac{1}{2} \log(-x + \sqrt{x^2-x-1} - 2) + \log(-2x + 2\sqrt{x^2-x-1} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-x-1)^(1/2)/(-x^2+1), x, algorithm="fricas")

[Out] arctan(-x + sqrt(x^2 - x - 1) + 1) - 1/2*log(-x + sqrt(x^2 - x - 1)) + 1/2*log(-x + sqrt(x^2 - x - 1) - 2) + log(-2*x + 2*sqrt(x^2 - x - 1) + 1)

giac [A] time = 0.20, size = 73, normalized size = 0.97

$$\arctan(-x + \sqrt{x^2-x-1} + 1) - \frac{1}{2} \log\left(\left| -x + \sqrt{x^2-x-1} \right|\right) + \frac{1}{2} \log\left(\left| -x + \sqrt{x^2-x-1} - 2 \right|\right) + \log\left(\left| -2x + 2\sqrt{x^2-x-1} + 1 \right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-x-1)^(1/2)/(-x^2+1), x, algorithm="giac")

[Out] $\arctan(-x + \sqrt{x^2 - x - 1}) + 1 - 1/2 \cdot \log(\text{abs}(-x + \sqrt{x^2 - x - 1})) + 1/2 \cdot \log(\text{abs}(-x + \sqrt{x^2 - x - 1} - 2)) + \log(\text{abs}(-2x + 2\sqrt{x^2 - x - 1}) + 1)$

maple [A] time = 0.02, size = 102, normalized size = 1.36

$$\frac{\operatorname{arctanh}\left(\frac{-3x-1}{2\sqrt{-3x+(x+1)^2-2}}\right)}{2} + \frac{\operatorname{arctan}\left(\frac{x-3}{2\sqrt{x+(x-1)^2-2}}\right)}{2} - \frac{3\ln\left(x-\frac{1}{2}+\sqrt{-3x+(x+1)^2-2}\right)}{4} - \frac{\ln\left(x-\frac{1}{2}+\sqrt{x+(x-1)^2-2}\right)}{4} - \frac{\sqrt{x+(x-1)^2-2}}{2} + \frac{\sqrt{-3x+(x+1)^2-2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x^2-x-1)^{(1/2)} / (-x^2+1), x)$

[Out] $-1/2 \cdot ((x-1)^2+x-2)^{(1/2)} - 1/4 \cdot \ln(-1/2+x+((x-1)^2+x-2)^{(1/2)}) + 1/2 \cdot \arctan(1/2 \cdot (-3+x) / ((x-1)^2+x-2)^{(1/2)}) + 1/2 \cdot ((x+1)^2-3x-2)^{(1/2)} - 3/4 \cdot \ln(-1/2+x+((x+1)^2-3x-2)^{(1/2)}) - 1/2 \cdot \operatorname{arctanh}(1/2 \cdot (-1-3x) / ((x+1)^2-3x-2)^{(1/2)})$

maxima [A] time = 0.97, size = 83, normalized size = 1.11

$$\frac{1}{2} \arcsin\left(\frac{2\sqrt{5}x}{5|2x-2|} - \frac{6\sqrt{5}}{5|2x-2|}\right) - \log\left(x + \sqrt{x^2 - x - 1} - \frac{1}{2}\right) - \frac{1}{2} \log\left(\frac{2\sqrt{x^2 - x - 1}}{|2x+2|} + \frac{2}{|2x+2|} - \frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((x^2-x-1)^{(1/2)} / (-x^2+1), x, \text{algorithm}="maxima")$

[Out] $1/2 \cdot \arcsin(2/5 \cdot \sqrt{5} \cdot x / \text{abs}(2x - 2) - 6/5 \cdot \sqrt{5} / \text{abs}(2x - 2)) - \log(x + \sqrt{x^2 - x - 1} - 1/2) - 1/2 \cdot \log(2 \cdot \sqrt{x^2 - x - 1} / \text{abs}(2x + 2) + 2 / \text{abs}(2x + 2) - 3/2)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$- \int \frac{\sqrt{x^2 - x - 1}}{x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(-(x^2 - x - 1)^{(1/2)} / (x^2 - 1), x)$

[Out] $-\text{int}((x^2 - x - 1)^{(1/2)} / (x^2 - 1), x)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{\sqrt{x^2 - x - 1}}{x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((x**2-x-1)**(1/2) / (-x**2+1), x)$

[Out] $-\text{Integral}(\sqrt{x**2 - x - 1} / (x**2 - 1), x)$

$$3.93 \quad \int \frac{(x+x^2)^{3/2}}{1+x^2} dx$$

Optimal. Leaf size=130

$$\frac{1}{4}\sqrt{x^2+x}(2x+5)+\sqrt{1+\sqrt{2}} \tan^{-1}\left(\frac{-x+\sqrt{2}+1}{\sqrt{2}(1+\sqrt{2})\sqrt{x^2+x}}\right)-\sqrt{\sqrt{2}-1} \tanh^{-1}\left(\frac{-x-\sqrt{2}+1}{\sqrt{2}(\sqrt{2}-1)\sqrt{x^2+x}}\right)-\frac{5}{4} \tanh^{-1}\left(\frac{x}{\sqrt{x^2+x}}\right)$$

Rubi [A] time = 0.16, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {978, 1078, 620, 206, 12, 1036, 1030, 207, 203}

$$\frac{1}{4}\sqrt{x^2+x}(2x+5)+\sqrt{1+\sqrt{2}} \tan^{-1}\left(\frac{-x+\sqrt{2}+1}{\sqrt{2}(1+\sqrt{2})\sqrt{x^2+x}}\right)-\sqrt{\sqrt{2}-1} \tanh^{-1}\left(\frac{-x-\sqrt{2}+1}{\sqrt{2}(\sqrt{2}-1)\sqrt{x^2+x}}\right)-\frac{5}{4} \tanh^{-1}\left(\frac{x}{\sqrt{x^2+x}}\right)$$

Antiderivative was successfully verified.

[In] Int[(x + x^2)^(3/2)/(1 + x^2), x]

[Out] ((5 + 2*x)*Sqrt[x + x^2])/4 + Sqrt[1 + Sqrt[2]]*ArcTan[(1 + Sqrt[2] - x)/(Sqrt[2*(1 + Sqrt[2]])*Sqrt[x + x^2])] - Sqrt[-1 + Sqrt[2]]*ArcTanh[(1 - Sqrt[2] - x)/(Sqrt[2*(-1 + Sqrt[2]])*Sqrt[x + x^2])] - (5*ArcTanh[x/Sqrt[x + x^2]])/4

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[Rt[b, 2]*x]/Rt[-a, 2]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 620

Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]

Rule 978

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((b*(3*p + 2*q) + 2*c*(p + q)*x)*(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^(q + 1))/(2*f*(p + q)*(2*p + 2*q + 1)), x] - Dist[1/(2*f*(p + q)*(2*p + 2*q + 1)), Int[(a + b*x + c*x^2)^(p - 2)*(d + f*x^2)^q*Simp[b^2*d*(p - 1)*(2*p + q) - (p + q)*(b^2*d*(1 - p) - 2*a*(c*d - a*f*(2*p + 2*q + 1))], x], x]

))) - (2*b*(c*d - a*f)*(1 - p)*(2*p + q) - 2*(p + q)*b*(2*c*d*(2*p + q) - (c*d + a*f)*(2*p + 2*q + 1))) * x + (b^2*f*p*(1 - p) + 2*c*(p + q)*(c*d*(2*p - 1) - a*f*(4*p + 2*q - 1))) * x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*q + 1, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]

Rule 1030

Int[((g_) + (h_)*(x_))/(((a_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]

Rule 1036

Int[((g_) + (h_)*(x_))/(((a_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist[1/(2*q), Int[Simp[-(a*h*e) - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[-(a*h*e) - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && NegQ[-(a*c)]

Rule 1078

Int[((A_) + (B_)*(x_) + (C_)*(x_)^2)/(((a_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{(x+x^2)^{3/2}}{1+x^2} dx &= \frac{1}{4}(5+2x)\sqrt{x+x^2} - \frac{1}{2} \int \frac{\frac{5}{4} + 4x + \frac{5x^2}{4}}{(1+x^2)\sqrt{x+x^2}} dx \\
 &= \frac{1}{4}(5+2x)\sqrt{x+x^2} - \frac{1}{2} \int \frac{4x}{(1+x^2)\sqrt{x+x^2}} dx - \frac{5}{8} \int \frac{1}{\sqrt{x+x^2}} dx \\
 &= \frac{1}{4}(5+2x)\sqrt{x+x^2} - \frac{5}{4} \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \frac{x}{\sqrt{x+x^2}}\right) - 2 \int \frac{x}{(1+x^2)\sqrt{x+x^2}} dx \\
 &= \frac{1}{4}(5+2x)\sqrt{x+x^2} - \frac{5}{4} \tanh^{-1}\left(\frac{x}{\sqrt{x+x^2}}\right) + \frac{\int \frac{-1+(-1-\sqrt{2})x}{(1+x^2)\sqrt{x+x^2}} dx}{\sqrt{2}} - \frac{\int \frac{-1+(-1+\sqrt{2})x}{(1+x^2)\sqrt{x+x^2}} dx}{\sqrt{2}} \\
 &= \frac{1}{4}(5+2x)\sqrt{x+x^2} - \frac{5}{4} \tanh^{-1}\left(\frac{x}{\sqrt{x+x^2}}\right) + (-2+\sqrt{2}) \text{Subst}\left(\int \frac{1}{2(1-\sqrt{2})+x^2} dx, x, \right. \\
 &= \frac{1}{4}(5+2x)\sqrt{x+x^2} + \sqrt{1+\sqrt{2}} \tan^{-1}\left(\frac{1+\sqrt{2}-x}{\sqrt{2(1+\sqrt{2})}\sqrt{x+x^2}}\right) - \sqrt{-1+\sqrt{2}} \tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)
 \end{aligned}$$

Mathematica [C] time = 0.16, size = 120, normalized size = 0.92

$$\frac{\sqrt{x}\sqrt{x+1}\left(2\sqrt{x+1}x^{3/2}+5\sqrt{x+1}\sqrt{x}+4(-1+i)^{3/2}\tan^{-1}\left(\sqrt{-1+i}\sqrt{\frac{x}{x+1}}\right)-5\sinh^{-1}(\sqrt{x})+4(1+i)^{3/2}\tanh^{-1}\left(\sqrt{1+i}\sqrt{\frac{x}{x+1}}\right)\right)}{4\sqrt{x(x+1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(x + x^2)^(3/2)/(1 + x^2),x]

[Out] (Sqrt[x]*Sqrt[1 + x]*(5*Sqrt[x]*Sqrt[1 + x] + 2*x^(3/2)*Sqrt[1 + x] - 5*ArcSinh[Sqrt[x]] + 4*(-1 + I)^(3/2)*ArcTan[Sqrt[-1 + I]*Sqrt[x/(1 + x)]] + 4*(1 + I)^(3/2)*ArcTanh[Sqrt[1 + I]*Sqrt[x/(1 + x)]]))/(4*Sqrt[x*(1 + x)])

IntegrateAlgebraic [C] time = 0.43, size = 103, normalized size = 0.79

$$\frac{1}{4}\sqrt{x^2+x}(2x+5) + \sqrt{2+2i} \tan^{-1}\left(\frac{\sqrt{-\frac{1}{2}-\frac{i}{2}}\sqrt{x^2+x}}{x}\right) + \sqrt{2-2i} \tan^{-1}\left(\frac{\sqrt{-\frac{1}{2}+\frac{i}{2}}\sqrt{x^2+x}}{x}\right) - \frac{5}{4} \tanh^{-1}\left(\frac{\sqrt{x^2+x}}{x}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x + x^2)^(3/2)/(1 + x^2),x]

[Out] ((5 + 2*x)*Sqrt[x + x^2])/4 + Sqrt[2 + 2*I]*ArcTan[(Sqrt[-1/2 - I/2]*Sqrt[x + x^2])/x] + Sqrt[2 - 2*I]*ArcTan[(Sqrt[-1/2 + I/2]*Sqrt[x + x^2])/x] - (5 *ArcTanh[Sqrt[x + x^2]/x])/4

fricas [B] time = 0.46, size = 777, normalized size = 5.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x)^(3/2)/(x^2+1),x, algorithm="fricas")

[Out] -1/8*8^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)*log(8*x^2 - 8*sqrt(x^2 + x)*x + 2*(8^(1/4)*sqrt(x^2 + x)*(sqrt(2) - 1) - 8^(1/4)*(sqrt(2)*x - x - 1))*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4) + 1/8*8^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)*log(8*x^2 - 8*sqrt(x^2 + x)*x - 2*(8^(1/4)*sqrt(x^2 + x)*(sqrt(2) - 1) - 8^(1/4)*(sqrt(2)*x - x - 1))*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4) + 1/2*8^(1/4)*sqrt(2)*sqrt(2*sqrt(2) + 4)*arctan(1/7*sqrt(2)*(sqrt(2)*(5*x + 1) + 6*x + 4) + 1/112*sqrt(8*x^2 - 8*sqrt(x^2 + x)*x - 2*(8^(1/4)*sqrt(x^2 + x)*(sqrt(2) - 1) - 8^(1/4)*(sqrt(2)*x - x - 1))*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4)*(8*sqrt(2)*(5*sqrt(2) + 6) + (8^(3/4)*(5*sqrt(2) + 6) + 8*8^(1/4)*(2*sqrt(2) + 1))*sqrt(2*sqrt(2) + 4) + 64*sqrt(2) + 32) - 1/7*sqrt(x^2 + x)*(sqrt(2)*(5*sqrt(2) + 6) + 8*sqrt(2) + 4) + 1/7*sqrt(2)*(8*x + 3) + 1/56*(8^(3/4)*(sqrt(2)*(5*x + 1) + 6*x + 4) - sqrt(x^2 + x)*(8^(3/4)*(5*sqrt(2) + 6) + 8*8^(1/4)*(2*sqrt(2) + 1)) + 8*8^(1/4)*(sqrt(2)*(2*x - 1) + x + 3))*sqrt(2*sqrt(2) + 4) + 4/7*x + 5/7) + 1/2*8^(1/4)*sqrt(2)*sqrt(2*sqrt(2) + 4)*arctan(-1/7*sqrt(2)*(sqrt(2)*(5*x + 1) + 6*x + 4) - 1/112*sqrt(8*x^2 - 8*sqrt(x^2 + x)*x + 2*(8^(1/4)*sqrt(x^2 + x)*(sqrt(2) - 1) - 8^(1/4)*(sqrt(2)*x - x - 1))*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4)*(8*sqrt(2)*(5*sqrt(2) + 6) - (8^(3/4)*(5*sqrt(2) + 6) + 8*8^(1/4)*(2*sqrt(2) + 1))*sqrt(2*sqrt(2) + 4) + 64*sqrt(2) + 32) + 1/7*sqrt(x^2 + x)*(sqrt(2)*(5*sqrt(2) + 6) + 8*sqrt(2) + 4) - 1/7*sqrt(2)*(8*x + 3) + 1/56*(8^(3/4)*(sqrt(2)*(5*x + 1) + 6*x + 4) - sqrt(x^2 + x)*(8^(3/4)*(5*sqrt(2) + 6) + 8*8^(1/4)*(2*sqrt(2) + 1)) + 8*8^(1/4)*(sqrt(2)*(2*x - 1) + x + 3))*sqrt(2*sqrt(2) + 4) - 4/7*x - 5/7) + 1/4*sqrt(x^2 + x)*(2*x + 5) + 5/8*log(-2*x + 2*sqrt(x^2 + x) - 1)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x)^(3/2)/(x^2+1),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);;OUTPUT:Unable to divide, perhaps due to rounding er

ror%{}{poly1 [29378258633931653019799718485334848549113596143372369654301
4566614744867108514065050877682658325195312500, -181983623107889624631149356
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3281250, -457037569568074726029024220476354238878560215202443954711967761282
5585897607153309996504109115565063476562500, 5657088028436513895060368238114
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33975389261008891579718157298328760901400260223388671875, -18501639301691189
171876668611027555220095054439393285921280605772669698963742122921653330793
61994541992187500000, 211543560165990204169085849967411852854582551443377700
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99112, -65203241]{}}, [5]{}{}+{}{}{poly1 [2497527834892881694485372868168218
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, 0, -31106, -99112, -65203241]{}}, [2]{}{}+{}{}{poly1 [-2829165046514258991518
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646859750070859837003681504660562347747802734375, 92622410573620422001390283
280266603413021660935063007466600699398333831523463749530142976259339105900
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203241]{}}, [1]{}{}+{}{}{poly1 [1457442570281580782727937412636382766061047
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9463951678445919411830447649751834787235187730954876190185546875, -118464124
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265619064625] : [1, 0, -31106, -99112, -65203241]{}}, [0]{}{} Error: Bad Argument
Value

maple [B] time = 0.14, size = 789, normalized size = 6.07

$$\frac{\sqrt{x+1} \arcsin\left(\frac{\sqrt{x+1} \sqrt{x-1}}{\sqrt{2}}\right) - \frac{5}{8} \sqrt{x+1} + \frac{5}{4} \sqrt{x-1}}{\sqrt{x+1} \sqrt{x-1} \sqrt{x^2+1}} + \frac{\frac{1}{2} \sqrt{x+1} \sqrt{x-1} \sqrt{x^2+1}}{\sqrt{x+1} \sqrt{x-1} \sqrt{x^2+1}} + \frac{\frac{1}{2} \sqrt{x+1} \sqrt{x-1} \sqrt{x^2+1}}{\sqrt{x+1} \sqrt{x-1} \sqrt{x^2+1}} + \frac{\frac{1}{2} \sqrt{x+1} \sqrt{x-1} \sqrt{x^2+1}}{\sqrt{x+1} \sqrt{x-1} \sqrt{x^2+1}} + \frac{\frac{1}{2} \sqrt{x+1} \sqrt{x-1} \sqrt{x^2+1}}{\sqrt{x+1} \sqrt{x-1} \sqrt{x^2+1}} + \frac{\frac{1}{2} \sqrt{x+1} \sqrt{x-1} \sqrt{x^2+1}}{\sqrt{x+1} \sqrt{x-1} \sqrt{x^2+1}} + \frac{\frac{1}{2} \sqrt{x+1} \sqrt{x-1} \sqrt{x^2+1}}{\sqrt{x+1} \sqrt{x-1} \sqrt{x^2+1}} + \frac{\frac{1}{2} \sqrt{x+1} \sqrt{x-1} \sqrt{x^2+1}}{\sqrt{x+1} \sqrt{x-1} \sqrt{x^2+1}} + \frac{\frac{1}{2} \sqrt{x+1} \sqrt{x-1} \sqrt{x^2+1}}{\sqrt{x+1} \sqrt{x-1} \sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+x)^(3/2)/(x^2+1), x)

```
[Out] 1/2*x*(x^2+x)^(1/2)+5/4*(x^2+x)^(1/2)-5/8*ln(1/2+x+(x^2+x)^(1/2))+1/2*(4*(-
2^(1/2)-1+x)^2/(1-x-2^(1/2))^2-3*2^(1/2)*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2+4
+3*2^(1/2))^(1/2)*2^(1/2)*((-2+2*2^(1/2))^(1/2)*arctan(1/2*(-2+2*2^(1/2))^(
1/2)*((3*2^(1/2)-4)*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2+12*2^(1/2)+17))^(1/2
)*(24*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2+17*2^(1/2)*(-2^(1/2)-1+x)^2/(1-x-2^(
1/2))^2-2^(1/2))*(-2^(1/2)-1+x)/(1-x-2^(1/2))*(3*2^(1/2)-4)/((-2^(1/2)-1+x)
^4/(1-x-2^(1/2))^4-34*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2+1)*((1+2^(1/2))^(1/2
)*2^(1/2)-2*(-2+2*2^(1/2))^(1/2)*arctan(1/2*(-2+2*2^(1/2))^(1/2)*((3*2^(1/2
)-4)*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2+12*2^(1/2)+17))^(1/2)*(24*(-2^(1/2)
-1+x)^2/(1-x-2^(1/2))^2+17*2^(1/2)*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2-2^(1/2)
)*(-2^(1/2)-1+x)/(1-x-2^(1/2))*(3*2^(1/2)-4)/((-2^(1/2)-1+x)^4/(1-x-2^(1/2)
)^4-34*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2+1))*((1+2^(1/2))^(1/2)-4*arctanh(1/2
*(4*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2-3*2^(1/2)*(-2^(1/2)-1+x)^2/(1-x-2^(1/2)
))^2+4+3*2^(1/2))^(1/2)/(1+2^(1/2))^(1/2))*2^(1/2)+6*arctanh(1/2*(4*(-2^(1/2)
-1+x)^2/(1-x-2^(1/2))^2-3*2^(1/2)*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2+4+3*2^(
1/2))^(1/2)/(1+2^(1/2))^(1/2)))/(-3*2^(1/2)*(-2^(1/2)-1+x)^2/(1-x-2^(1/2)
)^2-4*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2-3*2^(1/2)-4)/((-2^(1/2)-1+x)/(1-x-2^(
1/2))+1)^2)^(1/2)/((-2^(1/2)-1+x)/(1-x-2^(1/2))+1)/(3*2^(1/2)-4)/(1+2^(1/2)
))^^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2 + x)^{\frac{3}{2}}}{x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)^(3/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + x)^(3/2)/(x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x^2 + x)^{3/2}}{x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2)^(3/2)/(x^2 + 1), x)
```

```
[Out] int((x + x^2)^(3/2)/(x^2 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x(x + 1))^{\frac{3}{2}}}{x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x)**(3/2)/(x**2+1), x)
```

```
[Out] Integral((x*(x + 1))**(3/2)/(x**2 + 1), x)
```

$$3.94 \quad \int \frac{x^4}{\sqrt{a+bx+cx^2}(d-fx^2)} dx$$

Optimal. Leaf size=369

$$\frac{(3b^2 - 4ac) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}f} + \frac{3b\sqrt{a+bx+cx^2}}{4c^2f} + \frac{d^{3/2} \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2f^2\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} + \frac{d^{3/2} \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^2\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{d \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2} - \frac{x\sqrt{a+bx+cx^2}}{2cf}$$

Rubi [A] time = 0.81, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6725, 621, 206, 742, 640, 984, 724}

$$\frac{(3b^2 - 4ac) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}f} + \frac{3b\sqrt{a+bx+cx^2}}{4c^2f} + \frac{d^{3/2} \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2f^2\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} + \frac{d^{3/2} \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^2\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{d \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2} - \frac{x\sqrt{a+bx+cx^2}}{2cf}$$

Antiderivative was successfully verified.

[In] Int[x^4/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)), x]

[Out] (3*b*Sqrt[a + b*x + c*x^2])/(4*c^2*f) - (x*Sqrt[a + b*x + c*x^2])/(2*c*f) - (d*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*f^2) - ((3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2)*f) + (d^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2]))/(2*f^2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + (d^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2]))/(2*f^2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 984

```
Int[1/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[1/2, Int[1/((a - Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[1/2, Int[1/((a + Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{a+bx+cx^2}(d-fx^2)} dx = \int \left(-\frac{d}{f^2\sqrt{a+bx+cx^2}} - \frac{x^2}{f\sqrt{a+bx+cx^2}} + \frac{d^2}{f^2\sqrt{a+bx+cx^2}(d-fx^2)} \right) dx$$

$$= -\frac{d \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{f^2} + \frac{d^2 \int \frac{1}{\sqrt{a+bx+cx^2}(d-fx^2)} dx}{f^2} - \frac{\int \frac{x^2}{\sqrt{a+bx+cx^2}} dx}{f}$$

$$= -\frac{x\sqrt{a+bx+cx^2}}{2cf} - \frac{(2d) \text{Subst}\left(\int \frac{1}{4c-x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f^2} + \frac{d^2 \int \frac{1}{(d-\sqrt{d}\sqrt{fx})\sqrt{a+bx+cx^2}} dx}{2f^2}$$

$$= \frac{3b\sqrt{a+bx+cx^2}}{4c^2f} - \frac{x\sqrt{a+bx+cx^2}}{2cf} - \frac{d \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2} - \frac{d^2 \text{Subst}\left(\int \frac{1}{d-\sqrt{d}\sqrt{fx}} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{2f^2}$$

$$= \frac{3b\sqrt{a+bx+cx^2}}{4c^2f} - \frac{x\sqrt{a+bx+cx^2}}{2cf} - \frac{d \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2} + \frac{d^{3/2} \tanh^{-1}\left(\frac{d-\sqrt{d}\sqrt{fx}}{d}\right)}{2f^2}$$

$$= \frac{3b\sqrt{a+bx+cx^2}}{4c^2f} - \frac{x\sqrt{a+bx+cx^2}}{2cf} - \frac{d \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2} - \frac{(3b^2 - 4ac) \tanh^{-1}\left(\frac{d-\sqrt{d}\sqrt{fx}}{d}\right)}{2f^2}$$

Mathematica [A] time = 1.71, size = 300, normalized size = 0.81

$$\frac{(-4acf+3b^2f+8c^2d) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right) - \frac{2f(2cx-3b)\sqrt{a+x(b+cx)}}{c^2} + \frac{4d^{3/2} \tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f+cd}}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f+cd}}} + \frac{4d^{3/2} \tanh^{-1}\left(\frac{-2a\sqrt{f}+b(\sqrt{d}-\sqrt{f}x)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b(-\sqrt{d})\sqrt{f+cd}}}\right)}{\sqrt{af+b(-\sqrt{d})\sqrt{f+cd}}}}{8f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```


[Out]
$$\frac{((-2*f*(-3*b + 2*c*x))*\text{Sqrt}[a + x*(b + c*x)]/c^2 - ((8*c^2*d + 3*b^2*f - 4*a*c*f)*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + x*(b + c*x)])])/c^{(5/2)} + (4*d^{(3/2)}*\text{ArcTanh}[(b*\text{Sqrt}[d] + 2*a*\text{Sqrt}[f] + 2*c*\text{Sqrt}[d]*x + b*\text{Sqrt}[f]*x)/(2*\text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f)*\text{Sqrt}[a + x*(b + c*x)])))/\text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f] + (4*d^{(3/2)}*\text{ArcTanh}[(-2*a*\text{Sqrt}[f] + 2*c*\text{Sqrt}[d]*x + b*(\text{Sqrt}[d] - \text{Sqrt}[f]*x))/(2*\text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f)*\text{Sqrt}[a + x*(b + c*x)])))/\text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]}/(8*f^2)$$

IntegrateAlgebraic [C] time = 0.79, size = 263, normalized size = 0.71

$$\frac{d^2 \text{RootSum}\left[\#1^4(-f) + 2\#1^2 af + 4\#1^2 cd - 4\#1 b \sqrt{c} d - a^2 f + b^2 d \&, \frac{b \log(\#1 + \sqrt{a+bx+cx^2} - \sqrt{c}) - 2\#1 \sqrt{c} \log(-\#1 + \sqrt{a+bx+cx^2} - \sqrt{c})}{\#1^2 f - \#1 af - 2\#1 cd + b \sqrt{c} d}\right] \& + \frac{(-4acf + 3b^2 f + 8c^2 d) \log(-2c^{5/2} f^2 \sqrt{a+bx+cx^2} + bc^2 f^2 + 2c^2 f^2 x)}{8c^{5/2} f^2} + \frac{(3b - 2cx) \sqrt{a+bx+cx^2}}{4c^2 f}}{2f^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^4/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]

[Out]
$$\frac{((3*b - 2*c*x)*\text{Sqrt}[a + b*x + c*x^2])/(4*c^2*f) + ((8*c^2*d + 3*b^2*f - 4*a*c*f)*\text{Log}[b*c^2*f^2 + 2*c^3*f^2*x - 2*c^{(5/2)}*f^2*\text{Sqrt}[a + b*x + c*x^2]])/(8*c^{(5/2)}*f^2) - (d^2*\text{RootSum}[b^2*d - a^2*f - 4*b*\text{Sqrt}[c]*d*\#1 + 4*c*d*\#1^2 + 2*a*f*\#1^2 - f*\#1^4 \&, (b*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] - 2*\text{Sqrt}[c]*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1)/(b*\text{Sqrt}[c]*d - 2*c*d*\#1 - a*f*\#1 + f*\#1^3) \&])/(2*f^2)$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type

maple [A] time = 0.03, size = 516, normalized size = 1.40

$$\frac{d^2 \ln\left(\frac{2c^2 d^2 \sqrt{a+bx+cx^2} + (b^2 d^2 - a^2 f) \sqrt{a+bx+cx^2} + (2c^2 d^2 - a^2 f) \sqrt{a+bx+cx^2}}{2\sqrt{f} \sqrt{a+bx+cx^2} f^2}\right) + d^2 \ln\left(\frac{2c^2 d^2 \sqrt{a+bx+cx^2} + (b^2 d^2 - a^2 f) \sqrt{a+bx+cx^2} + (2c^2 d^2 - a^2 f) \sqrt{a+bx+cx^2}}{2\sqrt{f} \sqrt{a+bx+cx^2} f^2}\right)}{2\sqrt{f} \sqrt{a+bx+cx^2} f^2} + \frac{d \ln\left(\frac{a^2 + \sqrt{c^2 + 4bx + 4a}}{\sqrt{c^2 + 4bx + 4a}}\right) - 3b^2 \ln\left(\frac{a^2 + \sqrt{c^2 + 4bx + 4a}}{\sqrt{c^2 + 4bx + 4a}}\right) - d \ln\left(\frac{a^2 + \sqrt{c^2 + 4bx + 4a}}{\sqrt{c^2 + 4bx + 4a}}\right)}{2\sqrt{f} f^2} + \frac{\sqrt{c^2 + 4bx + 4a} + 3\sqrt{c^2 + 4bx + 4a}}{2cf} + \frac{3\sqrt{c^2 + 4bx + 4a}}{4c^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)

[Out]
$$-1/2*x*(c*x^2+b*x+a)^{(1/2)}/c/f+3/4*b*(c*x^2+b*x+a)^{(1/2)}/c^2/f-3/8/f*b^2/c^{(5/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})+1/2/f*a/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-1/f^2*d*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})/c^{(1/2)}-1/2/f^2*d^2/(d*f)^{(1/2)}/((a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)}*\ln((2*(a*f+c*d-(d*f)^{(1/2)}*b)/f+(b*f-2*(d*f)^{(1/2)}*c)*(x+(d*f)^{(1/2)}/f)/f+2*((a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)}*((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)}*c)*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)})/(x+(d*f)^{(1/2)}/f))+1/2/f^2*d^2/(d*f)^{(1/2)}/((a*f+c*d+(d*f)^{(1/2)}*b)/f)^{(1/2)}*\ln((2*(a*f+c*d+(d*f)^{(1/2)}*b)/f+(b*f+2*(d*f)^{(1/2)}*c)*(x-(d*f)^{(1/2)}/f)/f+2*((a*f+c*d+$$

$(d*f)^{(1/2)*b}/f)^{(1/2)*((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)*c)*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)*b)/f)^{(1/2)})/(x-(d*f)^{(1/2)}/f)}$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see 'assume?' for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2 -c*((b*sqrt(4*d*f)) / (2*f) + (c*d)/f+a) /f^2 zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{(d - f x^2) \sqrt{c x^2 + b x + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)

[Out] int(x^4/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4}{-d\sqrt{a + bx + cx^2} + fx^2\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)

[Out] -Integral(x**4/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)), x)

$$3.95 \quad \int \frac{x^3}{\sqrt{a+bx+cx^2}(d-fx^2)} dx$$

Optimal. Leaf size=287

$$\frac{b \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f} - \frac{d \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2f^{3/2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} + \frac{d \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{3/2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{\sqrt{a}}{cf}$$

Rubi [A] time = 0.63, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {6725, 640, 621, 206, 1033, 724}

$$\frac{b \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f} - \frac{d \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2f^{3/2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} + \frac{d \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{3/2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{\sqrt{a+bx+cx^2}}{cf}$$

Antiderivative was successfully verified.

[In] Int[x^3/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]

[Out] -(Sqrt[a + b*x + c*x^2]/(c*f)) + (b*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*f) - (d*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + b*x + c*x^2])])/(2*f^(3/2)*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f)) + (d*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + b*x + c*x^2])])/(2*f^(3/2)*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1033

Int[((g_.) + (h_.)*(x_))/((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q

), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]

Rule 6725

Int[(u_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^3}{\sqrt{a+bx+cx^2}(d-fx^2)} dx &= \int \left(-\frac{x}{f\sqrt{a+bx+cx^2}} + \frac{dx}{f\sqrt{a+bx+cx^2}(d-fx^2)} \right) dx \\ &= -\frac{\int \frac{x}{\sqrt{a+bx+cx^2}} dx}{f} + \frac{d \int \frac{x}{\sqrt{a+bx+cx^2}(d-fx^2)} dx}{f} \\ &= -\frac{\sqrt{a+bx+cx^2}}{cf} + \frac{b \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{2cf} + \frac{d \int \frac{1}{(-\sqrt{d}\sqrt{f-fx})\sqrt{a+bx+cx^2}} dx}{2f} + \frac{d \int \frac{1}{(\sqrt{d}\sqrt{f-fx})\sqrt{a+bx+cx^2}} dx}{2f} \\ &= -\frac{\sqrt{a+bx+cx^2}}{cf} + \frac{b \operatorname{Subst}\left(\int \frac{1}{4c-x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{cf} - \frac{d \operatorname{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^3-x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{2f} \\ &= -\frac{\sqrt{a+bx+cx^2}}{cf} + \frac{b \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f} - \frac{d \tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+(2c\sqrt{d}-b\sqrt{f})\sqrt{a+bx+cx^2}}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^{3/2}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} \end{aligned}$$

Mathematica [A] time = 1.16, size = 325, normalized size = 1.13

$$\frac{b\sqrt{f} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}} + \frac{d \tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{d \tanh^{-1}\left(\frac{-2a\sqrt{f}+b(\sqrt{d}-\sqrt{f}x)+2c\sqrt{d}x}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} - \frac{2\sqrt{f}x^2}{\sqrt{a+bx+cx^2}} - \frac{2b\sqrt{f}x}{c\sqrt{a+bx+cx^2}} - \frac{2a\sqrt{f}}{c\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)), x]

[Out] ((-2*a*Sqrt[f])/(c*Sqrt[a + x*(b + c*x)]) - (2*b*Sqrt[f]*x)/(c*Sqrt[a + x*(b + c*x)])) - (2*Sqrt[f]*x^2)/Sqrt[a + x*(b + c*x)] + (b*Sqrt[f]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/c^(3/2) + (d*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + x*(b + c*x)])]/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f] - (d*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + x*(b + c*x)])]/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f])/(2*f^(3/2))

IntegrateAlgebraic [C] time = 0.50, size = 224, normalized size = 0.78

$$\frac{d\operatorname{RootSum}\left[\#1^4(-f)+2\#1^2af+4\#1^2cd-4\#1b\sqrt{c}d-a^2f+b^2d\&, \frac{a\log(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x)-\#1^2\log(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x)}{\#1^3(-f)+\#1af+2\#1cd-b\sqrt{c}d}\&]}{2f} - \frac{b\log(-2c^{3/2}f\sqrt{a+bx+cx^2}+bcf+2c^2fx)}{2c^{3/2}f} - \frac{\sqrt{a+bx+cx^2}}{cf}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)), x]

```
[Out] -(Sqrt[a + b*x + c*x^2]/(c*f)) - (b*Log[b*c*f + 2*c^2*f*x - 2*c^(3/2)*f*Sqr
t[a + b*x + c*x^2]])/(2*c^(3/2)*f) - (d*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]
*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (a*Log[-(Sqrt[c]*x) + Sqrt[a +
b*x + c*x^2] - #1] - Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/
(-b*Sqrt[c]*d) + 2*c*d*#1 + a*f*#1 - f*#1^3) & ])/(2*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.02, size = 410, normalized size = 1.43

$$d \ln \left(\frac{\frac{2af+2ad-2\sqrt{d}b}{f} \frac{(bf-2\sqrt{d})\sqrt{cx+\frac{d}{f}}}{f} + 2\sqrt{\frac{af+cd-\sqrt{d}b}{f}} \sqrt{\left(\frac{cx+\frac{d}{f}}{f}\right)^2 + \frac{(bf-2\sqrt{d})\sqrt{cx+\frac{d}{f}}}{f} + \frac{af+cd-\sqrt{d}b}{f}}}{x+\frac{\sqrt{d}}{f}}}{2\sqrt{\frac{af+cd-\sqrt{d}b}{f}}f^2} \right) + d \ln \left(\frac{\frac{2af+2ad+2\sqrt{d}b}{f} \frac{(bf+2\sqrt{d})\sqrt{cx+\frac{d}{f}}}{f} + 2\sqrt{\frac{af+cd+\sqrt{d}b}{f}} \sqrt{\left(\frac{cx+\frac{d}{f}}{f}\right)^2 + \frac{(bf+2\sqrt{d})\sqrt{cx+\frac{d}{f}}}{f} + \frac{af+cd+\sqrt{d}b}{f}}}{x-\frac{\sqrt{d}}{f}}}{2\sqrt{\frac{af+cd+\sqrt{d}b}{f}}f^2} \right) + \frac{b \ln \left(\frac{cx+\frac{b}{2}}{\sqrt{c}} + \sqrt{cx^2+bx+a} \right)}{2c^2f} - \frac{\sqrt{cx^2+bx+a}}{cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -(c*x^2+b*x+a)^(1/2)/c/f+1/2/f*b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/2/f^2*d/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))+1/2/f^2*d/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2
-(c*((b*sqrt(4*d*f)) / (2*f) + (c*d)/f+a))
/f^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{(d - f x^2) \sqrt{c x^2 + b x + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)

[Out] int(x^3/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{x^3}{-d\sqrt{a + bx + cx^2} + fx^2\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d), x)

[Out] -Integral(x**3/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)), x)

$$3.96 \quad \int \frac{x^2}{\sqrt{a+bx+cx^2}(d-fx^2)} dx$$

Optimal. Leaf size=266

$$\frac{\sqrt{d} \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2f\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} + \frac{\sqrt{d} \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}$$

Rubi [A] time = 0.22, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1079, 621, 206, 984, 724}

$$\frac{\sqrt{d} \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2f\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} + \frac{\sqrt{d} \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}$$

Antiderivative was successfully verified.

[In] Int[x^2/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]

[Out] -(ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/(Sqrt[c]*f)) + (Sqrt[d]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]/(2*f*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f)) + (Sqrt[d]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]/(2*f*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 984

Int[1/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[1/2, Int[1/((a - Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[1/2, Int[1/((a + Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]

Rule 1079

Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x]

+ Dist[(A*c - a*C)/c, Int[1/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /;
FreeQ[{a, c, d, e, f, A, C}, x] && NeQ[e^2 - 4*d*f, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{\sqrt{a+bx+cx^2}(d-fx^2)} dx &= -\frac{\int \frac{1}{\sqrt{a+bx+cx^2}} dx}{f} + \frac{d \int \frac{1}{\sqrt{a+bx+cx^2}(d-fx^2)} dx}{f} \\ &= -\frac{2 \operatorname{Subst}\left(\int \frac{1}{4c-x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f} + \frac{d \int \frac{1}{(d-\sqrt{d}\sqrt{f}x)\sqrt{a+bx+cx^2}} dx}{2f} + \frac{d \int \frac{1}{(d+\sqrt{d}\sqrt{f}x)\sqrt{a+bx+cx^2}} dx}{2f} \\ &= -\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f} - \frac{d \operatorname{Subst}\left(\int \frac{1}{4cd^2-4bd^{3/2}\sqrt{f}+4adf-x^2} dx, x, \frac{-bd+2a\sqrt{d}\sqrt{f}-\sqrt{a+bx+cx^2}}{\sqrt{a+bx+cx^2}}\right)}{f} \\ &= -\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f} + \frac{\sqrt{d} \tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+(2c\sqrt{d}-b\sqrt{f})x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} + \frac{\sqrt{d} \tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+(2c\sqrt{d}+b\sqrt{f})x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f\sqrt{cd+b\sqrt{d}\sqrt{f}+af}} \end{aligned}$$

Mathematica [A] time = 0.43, size = 250, normalized size = 0.94

$$\frac{\sqrt{d} \left(\frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\sqrt{d}-b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} + \frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} \right) - \frac{2 \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}}}{2f}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)), x]

[Out] ((-2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + Sqrt[d]*(ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + 2*c*Sqrt[d]*x - b*Sqrt[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f] + ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]))/(2*f)

IntegrateAlgebraic [C] time = 0.39, size = 197, normalized size = 0.74

$$\frac{\log\left(\frac{-2\sqrt{c}f\sqrt{a+bx+cx^2}+bf+2cfx}{\sqrt{c}f}\right) - \frac{d\operatorname{RootSum}\left[\#1^4(-f)+2\#1^2af+4\#1^2cd-4\#1b\sqrt{c}d-a^2f+b^2d\&, \frac{b\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1\sqrt{c}\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)}{\#1^3f-\#1af-2\#1cd+b\sqrt{c}d}\right]}{2f}}{2f}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)), x]

[Out] Log[b*f + 2*c*f*x - 2*Sqrt[c]*f*Sqrt[a + b*x + c*x^2]]/(Sqrt[c]*f) - (d*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 &, (b*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*Sqrt[c]*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) &])/(2*f)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type

maple [A] time = 0.02, size = 399, normalized size = 1.50

$$\frac{d \ln \left(\frac{2af+2cd-2\sqrt{df}b}{f} \frac{(bf-2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right)}{f} + 2\sqrt{\frac{af+cd-\sqrt{df}b}{f}} \sqrt{\frac{(x+\frac{\sqrt{df}}{f})^2}{c} + \frac{(bf-2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right)}{f}}}{x+\frac{\sqrt{df}}{f}} \right)}{2\sqrt{df} \sqrt{\frac{af+cd-\sqrt{df}b}{f}}} + \frac{d \ln \left(\frac{2af+2cd+2\sqrt{df}b}{f} \frac{(bf+2\sqrt{df}c)\left(x-\frac{\sqrt{df}}{f}\right)}{f} + 2\sqrt{\frac{af+cd+\sqrt{df}b}{f}} \sqrt{\frac{(x-\frac{\sqrt{df}}{f})^2}{c} + \frac{(bf+2\sqrt{df}c)\left(x-\frac{\sqrt{df}}{f}\right)}{f}}}{x-\frac{\sqrt{df}}{f}} \right)}{2\sqrt{df} \sqrt{\frac{af+cd+\sqrt{df}b}{f}}} - \frac{\ln \left(\frac{cx+\frac{b}{\sqrt{c}} + \sqrt{cx^2+bx+a}}{\sqrt{c}f} \right)}{\sqrt{c}f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)

[Out]
$$\frac{-1/f \cdot \ln\left(\frac{c*x+1/2*b}{c}\right) + (c*x^2+b*x+a)^{1/2}/c - 1/2*d/(d*f)^{1/2}}{f} \cdot \frac{1}{(a*f+c*d-(d*f)^{1/2}*b)/f} \cdot \ln\left(\frac{2*(a*f+c*d-(d*f)^{1/2}*b)/f + (b*f-2*(d*f)^{1/2}*c)*(x+(d*f)^{1/2}/f)}{(a*f+c*d-(d*f)^{1/2}*b)/f}\right) + 2 \cdot \frac{1}{(a*f+c*d-(d*f)^{1/2}*b)/f} \cdot \frac{1}{(x+(d*f)^{1/2}/f)^2*c + (b*f-2*(d*f)^{1/2}*c)*(x+(d*f)^{1/2}/f)/f + (a*f+c*d-(d*f)^{1/2}*b)/f} \cdot \ln\left(\frac{2*(a*f+c*d+(d*f)^{1/2}*b)/f + (b*f+2*(d*f)^{1/2}*c)*(x-(d*f)^{1/2}/f)}{(a*f+c*d+(d*f)^{1/2}*b)/f}\right) + \frac{1/2*d/(d*f)^{1/2}}{f} \cdot \frac{1}{(a*f+c*d+(d*f)^{1/2}*b)/f} \cdot \ln\left(\frac{2*(a*f+c*d+(d*f)^{1/2}*b)/f + (b*f+2*(d*f)^{1/2}*c)*(x-(d*f)^{1/2}/f)}{(a*f+c*d+(d*f)^{1/2}*b)/f}\right) + \frac{(b*f+2*(d*f)^{1/2}*c)*(x-(d*f)^{1/2}/f)/f + (a*f+c*d+(d*f)^{1/2}*b)/f}{(x-(d*f)^{1/2}/f)}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2 -c*((b*sqrt(4*d*f))/(2*f) + (c*d)/f+a) /f^2 zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{(d - f x^2) \sqrt{c x^2 + b x + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)

[Out] int(x^2/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{-d\sqrt{a+bx+cx^2} + fx^2\sqrt{a+bx+cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)

[Out] -Integral(x**2/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)),
x)

$$3.97 \quad \int \frac{x}{\sqrt{a+bx+cx^2} (d-fx^2)} dx$$

Optimal. Leaf size=220

$$\frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}}\sqrt{f}+cd}\right)}{2\sqrt{f}\sqrt{af+b\sqrt{d}}\sqrt{f}+cd} - \frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f}+cd}\right)}{2\sqrt{f}\sqrt{af+b(-\sqrt{d})}\sqrt{f}+cd}$$

Rubi [A] time = 0.13, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1033, 724, 206}

$$\frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}}\sqrt{f}+cd}\right)}{2\sqrt{f}\sqrt{af+b\sqrt{d}}\sqrt{f}+cd} - \frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f}+cd}\right)}{2\sqrt{f}\sqrt{af+b(-\sqrt{d})}\sqrt{f}+cd}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]

[Out] -ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]/(2*Sqrt[f]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f)) + ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]/(2*Sqrt[f]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1033

Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/(q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]

Rubi steps

$$\begin{aligned} \int \frac{x}{\sqrt{a+bx+cx^2}(d-fx^2)} dx &= \frac{1}{2} \int \frac{1}{(-\sqrt{d}\sqrt{f}-fx)\sqrt{a+bx+cx^2}} dx + \frac{1}{2} \int \frac{1}{(\sqrt{d}\sqrt{f}-fx)\sqrt{a+bx+cx^2}} dx \\ &= -\text{Subst} \left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2} dx, x, \frac{b\sqrt{d}\sqrt{f}-2af-(-2c\sqrt{d}\sqrt{f})x}{\sqrt{a+bx+cx^2}} \right) \\ &= -\frac{\tanh^{-1}\left(\frac{b\sqrt{d}\sqrt{f}-2af-(-2c\sqrt{d}\sqrt{f})x}{2\sqrt{f}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{f}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} - \frac{\tanh^{-1}\left(\frac{-b\sqrt{d}\sqrt{f}-2af-(2c\sqrt{d}\sqrt{f}+bf)x}{2\sqrt{f}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{f}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}} \end{aligned}$$

Mathematica [A] time = 0.17, size = 211, normalized size = 0.96

$$\frac{\frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+b(\sqrt{d}-\sqrt{f}x)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} + \frac{\tanh^{-1}\left(\frac{-2(a\sqrt{f}+c\sqrt{d}x)-b(\sqrt{d}+\sqrt{f}x)}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}}{2\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)), x]

[Out] $-\frac{1}{2} \frac{(\text{ArcTanh}[-2a\sqrt{f} + 2c\sqrt{d}x + b(\sqrt{d} - \sqrt{f}x)] / (2\sqrt{cd - b\sqrt{d}\sqrt{f} + af}) \sqrt{a + x(b + cx)}) + \text{ArcTanh}[-2(a\sqrt{f} + c\sqrt{d}x) - b(\sqrt{d} + \sqrt{f}x)] / (2\sqrt{cd + b\sqrt{d}\sqrt{f} + af}) \sqrt{a + x(b + cx)}}{\sqrt{cd - b\sqrt{d}\sqrt{f} + af} \sqrt{cd + b\sqrt{d}\sqrt{f} + af} \sqrt{f}}$

IntegrateAlgebraic [C] time = 0.35, size = 149, normalized size = 0.68

$$-\frac{1}{2} \text{RootSum} \left[\#1^4(-f) + 2\#1^2af + 4\#1^2cd - 4\#1b\sqrt{cd} - a^2f + b^2d \&, \frac{a \log(-\#1 + \sqrt{a+bx+cx^2} - \sqrt{cx}) - \#1^2 \log(-\#1 + \sqrt{a+bx+cx^2} - \sqrt{cx})}{\#1^3(-f) + \#1af + 2\#1cd - b\sqrt{cd}} \& \right]$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)), x]

[Out] $-\frac{1}{2} \frac{\text{RootSum}[b^2d - a^2f - 4b\sqrt{cd} + 4cd + 2af + f^2 - f^2 \&, (a \log(-\sqrt{c}x) + \sqrt{a+bx+cx^2} - \#1) - \log(-\sqrt{c}x) + \sqrt{a+bx+cx^2} - \#1)] / (-b\sqrt{cd} + 2cd + af - f^2 \&)]}{\sqrt{cd - b\sqrt{d}\sqrt{f} + af} \sqrt{cd + b\sqrt{d}\sqrt{f} + af} \sqrt{f}}$

fricas [B] time = 1.05, size = 2753, normalized size = 12.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d), x, algorithm="fricas")

[Out] $\frac{1}{4} \sqrt{(cd + af + (c^2d^2f + a^2f^3 - (b^2 - 2ac)d^2f^2) \sqrt{b^2d/(c^4d^4f + a^4f^5 - 2(b^2c^2 - 2ac^3)d^3f^2 + (b^4 - 4ab^2c + 6a^2c^2)d^2f^3 - 2(a^2b^2 - 2a^3c)d^2f^4)}) / (c^2d^2f + a^2f^3 - (b^2 - 2ac)d^2f^2)} \log((2b^2cdx + b^2d + 2(b^2df - (c^3d^3f + a^3f^4 - (b^2c - 3ac^2)d^2f^2 - (ab^2 - 3a^2c)d^2f^3) \sqrt{b^2d/(c^4d^4f + a^4f^5 - 2(b^2c^2 - 2ac^3)d^3f^2 + (b^4 - 4ab^2c + 6a^2c^2)d^2f^3 - 2(a^2b^2 - 2a^3c)d^2f^4)}) \sqrt{cx^2 + bx + a}) \sqrt{(cd + af + (c^2d^2f + a^2f^3 - (b^2 - 2ac)d^2f^2) \sqrt{b^2d/(c^4d^4f + a^4f^5 - 2(b^2c^2 - 2ac^3)d^3f^2 + (b^4 - 4ab^2c + 6a^2c^2)d^2f^3 - 2(a^2b^2 - 2a^3c)d^2f^4)}) \sqrt{cd - b\sqrt{d}\sqrt{f} + af} \sqrt{cd + b\sqrt{d}\sqrt{f} + af} \sqrt{f}}$

$$\begin{aligned} & \left. \left(\frac{2*(a^2*b^2 - 2*a^3*c)*d*f^4}{(c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)} - (2*a*c^2*d^2*f + 2*a^3*f^3 - 2*(a*b^2 - 2*a^2*c)*d*f^2 + (b*c^2*d^2*f + a^2*b*f^3 - (b^3 - 2*a*b*c)*d*f^2)*x) \right) \right) \sqrt{b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)} \\ & - \frac{1}{4} \sqrt{(c*d + a*f + (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*\sqrt{b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)}} \\ & \left. \left(\frac{2*b*c*d*x + b^2*d - 2*(b^2*d*f - (c^3*d^3*f + a^3*f^4 - (b^2*c - 3*a*c^2)*d^2*f^2 - (a*b^2 - 3*a^2*c)*d*f^3)*\sqrt{b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)}} \right) \right) \sqrt{c*x^2 + b*x + a} \sqrt{(c*d + a*f + (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*\sqrt{b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)}} \\ & \left. \left(\frac{2*a*c^2*d^2*f + 2*a^3*f^3 - 2*(a*b^2 - 2*a^2*c)*d*f^2 + (b*c^2*d^2*f + a^2*b*f^3 - (b^3 - 2*a*b*c)*d*f^2)*x) \right) \right) \sqrt{b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)} \\ & + \frac{1}{4} \sqrt{(c*d + a*f - (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*\sqrt{b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)}} \\ & \left. \left(\frac{2*b*c*d*x + b^2*d + 2*(b^2*d*f + (c^3*d^3*f + a^3*f^4 - (b^2*c - 3*a*c^2)*d^2*f^2 - (a*b^2 - 3*a^2*c)*d*f^3)*\sqrt{b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)}} \right) \right) \sqrt{c*x^2 + b*x + a} \sqrt{(c*d + a*f - (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*\sqrt{b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)}} \\ & \left. \left(\frac{2*a*c^2*d^2*f + 2*a^3*f^3 - 2*(a*b^2 - 2*a^2*c)*d*f^2 + (b*c^2*d^2*f + a^2*b*f^3 - (b^3 - 2*a*b*c)*d*f^2)*x) \right) \right) \sqrt{b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)} \\ & - \frac{1}{4} \sqrt{(c*d + a*f - (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*\sqrt{b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)}} \\ & \left. \left(\frac{2*b*c*d*x + b^2*d - 2*(b^2*d*f + (c^3*d^3*f + a^3*f^4 - (b^2*c - 3*a*c^2)*d^2*f^2 - (a*b^2 - 3*a^2*c)*d*f^3)*\sqrt{b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)}} \right) \right) \sqrt{c*x^2 + b*x + a} \sqrt{(c*d + a*f - (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*\sqrt{b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)}} \\ & \left. \left(\frac{2*a*c^2*d^2*f + 2*a^3*f^3 - 2*(a*b^2 - 2*a^2*c)*d*f^2 + (b*c^2*d^2*f + a^2*b*f^3 - (b^3 - 2*a*b*c)*d*f^2)*x) \right) \right) \sqrt{b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)} \\ & \left. \left(\frac{2*a*c^2*d^2*f + 2*a^3*f^3 - 2*(a*b^2 - 2*a^2*c)*d*f^2 + (b*c^2*d^2*f + a^2*b*f^3 - (b^3 - 2*a*b*c)*d*f^2)*x) \right) \right) \sqrt{b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)} \end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 0.52sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [B] time = 0.02, size = 354, normalized size = 1.61

$$\ln \left(\frac{\frac{2af+2cd-2\sqrt{df}b}{f} + \frac{(bf-2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right)}{f} + 2\sqrt{\frac{af+cd-\sqrt{df}b}{f}} \sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2 c + \frac{(bf-2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right)}{f} + \frac{af+cd-\sqrt{df}b}{f}}}{x+\frac{\sqrt{df}}{f}} \right) + \ln \left(\frac{\frac{2af+2cd+2\sqrt{df}b}{f} + \frac{(bf+2\sqrt{df}c)\left(x-\frac{\sqrt{df}}{f}\right)}{f} + 2\sqrt{\frac{af+cd+\sqrt{df}b}{f}} \sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2 c + \frac{(bf+2\sqrt{df}c)\left(x-\frac{\sqrt{df}}{f}\right)}{f} + \frac{af+cd+\sqrt{df}b}{f}}}{x-\frac{\sqrt{df}}{f}} \right)$$

$$2\sqrt{\frac{af+cd-\sqrt{df}b}{f}} f \quad 2\sqrt{\frac{af+cd+\sqrt{df}b}{f}} f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d), x)

[Out] 1/2/f/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))+1/2/f/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d), x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2 - (c*((b*sqrt(4*d*f)) / (2*f) + (c*d)/f+a)) /f^2 zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{(d - f x^2) \sqrt{c x^2 + b x + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)

[Out] int(x/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-d\sqrt{a + bx + cx^2} + fx^2\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d), x)

[Out] -Integral(x/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)), x)

$$3.98 \quad \int \frac{1}{\sqrt{a+bx+cx^2}(d-fx^2)} dx$$

Optimal. Leaf size=220

$$\frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2\sqrt{d}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} + \frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}$$

Rubi [A] time = 0.12, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {984, 724, 206}

$$\frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2\sqrt{d}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} + \frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]

[Out] ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]/(2*Sqrt[d]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f)) + ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]/(2*Sqrt[d]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 984

Int[1/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[1/2, Int[1/((a - Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[1/2, Int[1/((a + Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{a+bx+cx^2}(d-fx^2)} dx &= \frac{1}{2} \int \frac{1}{(d-\sqrt{d}\sqrt{fx})\sqrt{a+bx+cx^2}} dx + \frac{1}{2} \int \frac{1}{(d+\sqrt{d}\sqrt{fx})\sqrt{a+bx+cx^2}} dx \\ &= -\text{Subst} \left(\int \frac{1}{4cd^2 - 4bd^{3/2}\sqrt{f} + 4adf - x^2} dx, x, \frac{-bd + 2a\sqrt{d}\sqrt{f} - (2cd - b\sqrt{d}\sqrt{f})x}{\sqrt{a+bx+cx^2}} \right) \\ &= \frac{\tanh^{-1} \left(\frac{-bd + 2a\sqrt{d}\sqrt{f} - (2cd - b\sqrt{d}\sqrt{f})x}{2\sqrt{d}\sqrt{cd - b\sqrt{d}\sqrt{f} + af}\sqrt{a+bx+cx^2}} \right)}{2\sqrt{d}\sqrt{cd - b\sqrt{d}\sqrt{f} + af}} - \frac{\tanh^{-1} \left(\frac{-bd - 2a\sqrt{d}\sqrt{f} - (2cd + b\sqrt{d}\sqrt{f})x}{2\sqrt{d}\sqrt{cd + b\sqrt{d}\sqrt{f} + af}\sqrt{a+bx+cx^2}} \right)}{2\sqrt{d}\sqrt{cd + b\sqrt{d}\sqrt{f} + af}} \end{aligned}$$

Mathematica [A] time = 0.10, size = 209, normalized size = 0.95

$$\frac{\frac{\tanh^{-1} \left(\frac{2a\sqrt{f} + b\sqrt{d} + b\sqrt{fx} + 2c\sqrt{dx}}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} \right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} + \frac{\tanh^{-1} \left(\frac{-2a\sqrt{f} + b(\sqrt{d}-\sqrt{fx}) + 2c\sqrt{dx}}{2\sqrt{a+x(b+cx)}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} \right)}{\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}}{2\sqrt{d}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)), x]

[Out] (ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f] + ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f])/(2*Sqrt[d])

IntegrateAlgebraic [C] time = 0.35, size = 151, normalized size = 0.69

$$-\frac{1}{2} \text{RootSum} \left[\#1^4(-f) + 2\#1^2af + 4\#1^2cd - 4\#1b\sqrt{cd} - a^2f + b^2d \&, \frac{b \log(-\#1 + \sqrt{a+bx+cx^2} - \sqrt{cx}) - 2\#1\sqrt{c} \log(-\#1 + \sqrt{a+bx+cx^2} - \sqrt{cx})}{\#1^3f - \#1af - 2\#1cd + b\sqrt{cd}} \& \right]$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)), x]

[Out] -1/2*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 &, (b*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*Sqrt[c]*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) &]

fricas [B] time = 1.24, size = 2641, normalized size = 12.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d), x, algorithm="fricas")

[Out] 1/4*sqrt((c*d + a*f + (c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)*log((2*b*c*x + b^2 + 2*(b*c*d + a*b*f - (b*c^2*d^3 + a^2*b*d*f^2 - (b^3 - 2*a*b*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d + a*f + (c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3))

$$\begin{aligned} & (2*a^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)) - (2*a*c^2*d^2 + 2*a^3*f^2 - 2*(a*b^2 - 2*a^2*c)*d*f + (b*c^2*d^2 + a^2*b*f^2 - (b^3 - 2*a*b*c)*d*f)*x)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/x - 1/4*sqrt((c*d + a*f + (c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f))*log((2*b*c*x + b^2 - 2*(b*c*d + a*b*f - (b*c^2*d^3 + a^2*b*d*f^2 - (b^3 - 2*a*b*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d + a*f + (c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)) - (2*a*c^2*d^2 + 2*a^3*f^2 - 2*(a*b^2 - 2*a^2*c)*d*f + (b*c^2*d^2 + a^2*b*f^2 - (b^3 - 2*a*b*c)*d*f)*x)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/x + 1/4*sqrt((c*d + a*f - (c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f))*log((2*b*c*x + b^2 + 2*(b*c*d + a*b*f + (b*c^2*d^3 + a^2*b*d*f^2 - (b^3 - 2*a*b*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d + a*f - (c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)) + (2*a*c^2*d^2 + 2*a^3*f^2 - 2*(a*b^2 - 2*a^2*c)*d*f + (b*c^2*d^2 + a^2*b*f^2 - (b^3 - 2*a*b*c)*d*f)*x)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/x - 1/4*sqrt((c*d + a*f - (c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f))*log((2*b*c*x + b^2 - 2*(b*c*d + a*b*f + (b*c^2*d^3 + a^2*b*d*f^2 - (b^3 - 2*a*b*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))*sqrt(c*x^2 + b*x + a)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)) + (2*a*c^2*d^2 + 2*a^3*f^2 - 2*(a*b^2 - 2*a^2*c)*d*f + (b*c^2*d^2 + a^2*b*f^2 - (b^3 - 2*a*b*c)*d*f)*x)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/x$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 0.58sym2pol
 y/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argum
 ent Value

maple [B] time = 0.02, size = 358, normalized size = 1.63

$$\frac{\ln\left(\frac{\frac{2af+2cd-2\sqrt{df}b}{f} + \frac{(bf-2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right)}{f} + 2\sqrt{\frac{af+cd-\sqrt{df}b}{f}} \sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2 c + \frac{(bf-2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right) + af+cd-\sqrt{df}b}{f}}}{x+\frac{\sqrt{df}}{f}}\right)}{2\sqrt{df} \sqrt{\frac{af+cd-\sqrt{df}b}{f}}}$$

$$+ \frac{\ln\left(\frac{\frac{2af+2cd+2\sqrt{df}b}{f} + \frac{(bf+2\sqrt{df}c)\left(x-\frac{\sqrt{df}}{f}\right)}{f} + 2\sqrt{\frac{af+cd+\sqrt{df}b}{f}} \sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2 c + \frac{(bf+2\sqrt{df}c)\left(x-\frac{\sqrt{df}}{f}\right) + af+cd+\sqrt{df}b}{f}}}{x-\frac{\sqrt{df}}{f}}\right)}{2\sqrt{df} \sqrt{\frac{af+cd+\sqrt{df}b}{f}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d), x)
[Out] -1/2/(d*f)^(1/2)/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))+1/2/(d*f)^(1/2)/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d), x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2 -(c*((b*sqrt(4*d*f)) / (2*f) + (c*d)/f+a)) /f^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(d - f x^2) \sqrt{c x^2 + b x + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
[Out] int(1/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-d\sqrt{a + bx + cx^2} + fx^2\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d), x)
[Out] -Integral(1/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)), x)
```

$$3.99 \quad \int \frac{1}{x\sqrt{a+bx+cx^2}(d-fx^2)} dx$$

Optimal. Leaf size=267

$$\frac{\sqrt{f} \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2d\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} + \frac{\sqrt{f} \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d}$$

Rubi [A] time = 0.66, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6725, 724, 206, 1033}

$$\frac{\sqrt{f} \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2d\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} + \frac{\sqrt{f} \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]

[Out] -(ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/(Sqrt[a]*d)) - (Sqrt[f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + b*x + c*x^2]])/(2*d*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f)) + (Sqrt[f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + b*x + c*x^2]])/(2*d*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1033

Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]

Rule 6725

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a+bx+cx^2}(d-fx^2)} dx &= \int \left(\frac{1}{dx\sqrt{a+bx+cx^2}} - \frac{fx}{d\sqrt{a+bx+cx^2}(-d+fx^2)} \right) dx \\
&= \frac{\int \frac{1}{x\sqrt{a+bx+cx^2}} dx}{d} - \frac{f \int \frac{x}{\sqrt{a+bx+cx^2}(-d+fx^2)} dx}{d} \\
&= \frac{2 \operatorname{Subst} \left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}} \right)}{d} - \frac{f \int \frac{1}{(-\sqrt{d}\sqrt{f}+fx)\sqrt{a+bx+cx^2}} dx}{2d} - \frac{f \int \frac{1}{(\sqrt{d}\sqrt{f}+fx)\sqrt{a+bx+cx^2}} dx}{2d} \\
&= -\frac{\operatorname{tanh}^{-1} \left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}} \right)}{\sqrt{a}d} + \frac{f \operatorname{Subst} \left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2} dx, x, \frac{-b\sqrt{d}\sqrt{f}+2ax}{\sqrt{a+bx+cx^2}} \right)}{d} \\
&= -\frac{\operatorname{tanh}^{-1} \left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}} \right)}{\sqrt{a}d} - \frac{\sqrt{f} \operatorname{tanh}^{-1} \left(\frac{b\sqrt{d}-2a\sqrt{f}+(2c\sqrt{d}-b\sqrt{f})x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}} \right)}{2d\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} + \frac{\sqrt{f} \operatorname{tanh}^{-1} \left(\frac{b\sqrt{d}+2a\sqrt{f}+(2c\sqrt{d}+b\sqrt{f})x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}} \right)}{2d\sqrt{cd+b\sqrt{d}\sqrt{f}+af}}
\end{aligned}$$

Mathematica [A] time = 0.44, size = 252, normalized size = 0.94

$$\frac{\sqrt{f} \left(\frac{\operatorname{tanh}^{-1} \left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} \right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{\operatorname{tanh}^{-1} \left(\frac{-2a\sqrt{f}+b\sqrt{d}-b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} \right)}{\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} \right)}{2d} - \frac{2 \operatorname{tanh}^{-1} \left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}} \right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[a + b*x + c*x^2]*(d - f*x^2)), x]

[Out] ((-2*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/Sqrt[a] + Sqrt[f]*(-(ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + 2*c*Sqrt[d]*x - b*Sqrt[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f] + ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]))/(2*d)

IntegrateAlgebraic [C] time = 0.39, size = 199, normalized size = 0.75

$$\frac{2 \operatorname{tanh}^{-1} \left(\frac{\sqrt{c}x}{\sqrt{a}} - \frac{\sqrt{a+bx+cx^2}}{\sqrt{a}} \right)}{\sqrt{a}d} - \frac{f \operatorname{RootSum} \left[\#1^4(-f) + 2\#1^2af + 4\#1^2cd - 4\#1b\sqrt{c}d - a^2f + b^2d\&, \frac{a \log(-\#1 + \sqrt{a+bx+cx^2} - \sqrt{c}x) - \#1^2 \log(-\#1 + \sqrt{a+bx+cx^2} - \sqrt{c}x)}{\#1^3(-f) + \#1af + 2\#1cd - b\sqrt{c}d} \& \right]}{2d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*Sqrt[a + b*x + c*x^2]*(d - f*x^2)), x]

[Out] (2*ArcTanh[(Sqrt[c]*x)/Sqrt[a] - Sqrt[a + b*x + c*x^2]/Sqrt[a]])/(Sqrt[a]*d) - (f*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 &, (a*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(-(b*Sqrt[c]*d) + 2*c*d*#1 + a*f*#1 - f*#1^3) &])/(2*d)

fricas [B] time = 70.19, size = 5995, normalized size = 22.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d), x, algorithm="fricas")

```
[Out] [1/4*(a*d*sqrt((c*d*f + a*f^2 + (c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*  
f)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4  
- 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d  
^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f))*log((2*b*c*f^2*x + b^2*f^2 + 2*(b^  
2*d*f^2 - (c^3*d^5 + a^3*d^2*f^3 - (b^2*c - 3*a*c^2)*d^4*f - (a*b^2 - 3*a^2  
*c)*d^3*f^2)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^  
6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3  
)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d*f + a*f^2 + (c^2*d^4 + a^2*d^2*f^2 - (b  
^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c  
^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d  
^4*f^3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)) - (2*a*c^2*d^3*f +  
2*a^3*d*f^3 - 2*(a*b^2 - 2*a^2*c)*d^2*f^2 + (b*c^2*d^3*f + a^2*b*d*f^3 - (b^  
3 - 2*a*b*c)*d^2*f^2)*x)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2  
- 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*  
a^3*c)*d^4*f^3))/x) - a*d*sqrt((c*d*f + a*f^2 + (c^2*d^4 + a^2*d^2*f^2 - (b  
^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*  
c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*  
d^4*f^3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f))*log((2*b*c*f^2*x  
+ b^2*f^2 - 2*(b^2*d*f^2 - (c^3*d^5 + a^3*d^2*f^3 - (b^2*c - 3*a*c^2)*d^4*f  
- (a*b^2 - 3*a^2*c)*d^3*f^2)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*  
c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 -  
2*a^3*c)*d^4*f^3)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d*f + a*f^2 + (c^2*d^4 +  
a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2  
*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2  
*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)) -  
(2*a*c^2*d^3*f + 2*a^3*d*f^3 - 2*(a*b^2 - 2*a^2*c)*d^2*f^2 + (b*c^2*d^3*f  
+ a^2*b*d*f^3 - (b^3 - 2*a*b*c)*d^2*f^2)*x)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3  
*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2  
- 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3))/x) + a*d*sqrt((c*d*f + a*f^2 - (c^2*d^4  
+ a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 -  
2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^  
2*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f))*  
log((2*b*c*f^2*x + b^2*f^2 + 2*(b^2*d*f^2 + (c^3*d^5 + a^3*d^2*f^3 - (b^2*c  
- 3*a*c^2)*d^4*f - (a*b^2 - 3*a^2*c)*d^3*f^2)*sqrt(b^2*f^3/(c^4*d^7 + a^4*  
d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f  
^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d*f + a  
*f^2 - (c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7  
+ a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)  
*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2 -  
2*a*c)*d^3*f)) + (2*a*c^2*d^3*f + 2*a^3*d*f^3 - 2*(a*b^2 - 2*a^2*c)*d^2*f^  
2 + (b*c^2*d^3*f + a^2*b*d*f^3 - (b^3 - 2*a*b*c)*d^2*f^2)*x)*sqrt(b^2*f^3/(  
c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*  
a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3))/x) - a*d*sqrt((c*d*f +  
a*f^2 - (c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7  
+ a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2  
)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2  
- 2*a*c)*d^3*f))*log((2*b*c*f^2*x + b^2*f^2 - 2*(b^2*d*f^2 + (c^3*d^5 + a^3  
*d^2*f^3 - (b^2*c - 3*a*c^2)*d^4*f - (a*b^2 - 3*a^2*c)*d^3*f^2)*sqrt(b^2*f^  
3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c +  
6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))*sqrt(c*x^2 + b*x + a  
)*sqrt((c*d*f + a*f^2 - (c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)*sqrt(  
b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b  
^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a^2  
*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)) + (2*a*c^2*d^3*f + 2*a^3*d*f^3 - 2*(a*b^2  
- 2*a^2*c)*d^2*f^2 + (b*c^2*d^3*f + a^2*b*d*f^3 - (b^3 - 2*a*b*c)*d^2*f^2)*  
x)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4  
- 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3))/x) + 2  
*sqrt(a)*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x +  
2*a)*sqrt(a) + 8*a^2)/x^2))/(a*d), 1/4*(a*d*sqrt((c*d*f + a*f^2 + (c^2*d^4
```


giac [F] time = 0.00, size = 0, normalized size = 0.00

sage2

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")

[Out] sage2

maple [A] time = 0.02, size = 391, normalized size = 1.46

$$\ln \left(\frac{\frac{2af+2cd-2\sqrt{df}b}{f} + \frac{(bf-2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right)}{f} + 2\sqrt{\frac{af+cd-\sqrt{df}b}{f}} \sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2 + \frac{(bf-2\sqrt{df}c)\left(x+\frac{\sqrt{df}}{f}\right) + af+cd-\sqrt{df}b}{f}}}{x+\frac{\sqrt{df}}{f}} \right) + \ln \left(\frac{\frac{2af+2cd+2\sqrt{df}b}{f} + \frac{(bf+2\sqrt{df}c)\left(x-\frac{\sqrt{df}}{f}\right)}{f} + 2\sqrt{\frac{af+cd+\sqrt{df}b}{f}} \sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2 + \frac{(bf+2\sqrt{df}c)\left(x-\frac{\sqrt{df}}{f}\right) + af+cd+\sqrt{df}b}{f}}}{x-\frac{\sqrt{df}}{f}} \right) - \ln \left(\frac{bx+2a+2\sqrt{cx^2+bx+a}\sqrt{d}}{x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)

[Out] 1/2/d/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))+1/2/d/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))-1/d/a^(1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{cx^2+bx+a}(fx^2-d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] -integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x(d-fx^2)\sqrt{cx^2+bx+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(d-f*x^2)*(a+b*x+c*x^2)^(1/2)),x)

[Out] int(1/(x*(d-f*x^2)*(a+b*x+c*x^2)^(1/2)),x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-dx\sqrt{a+bx+cx^2}+fx^3\sqrt{a+bx+cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)

[Out] -Integral(1/(-d*x*sqrt(a + b*x + c*x**2) + f*x**3*sqrt(a + b*x + c*x**2)), x)

$$3.100 \quad \int \frac{1}{x^2 \sqrt{a+bx+cx^2} (d-fx^2)} dx$$

Optimal. Leaf size=291

$$\frac{b \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d} + \frac{f \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2d^{3/2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} + \frac{f \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{3/2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{\sqrt{a+bx+cx^2}}{adx}$$

Rubi [A] time = 0.66, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 28, number of rules / integrand size = 0.179, Rules used = {6725, 730, 724, 206, 984}

$$\frac{b \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d} + \frac{f \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2d^{3/2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} + \frac{f \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{3/2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{\sqrt{a+bx+cx^2}}{adx}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]

[Out] -(Sqrt[a + b*x + c*x^2]/(a*d*x)) + (b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*a^(3/2)*d) + (f*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + b*x + c*x^2]])/(2*d^(3/2)*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f) + (f*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + b*x + c*x^2]])/(2*d^(3/2)*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 730

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]

Rule 984

Int[1/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[1/2, Int[1/((a - Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[1/2, Int[1/((a + Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]

Rule 6725

Int[(u_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{x^2 \sqrt{a+bx+cx^2} (d-fx^2)} dx &= \int \left(\frac{1}{dx^2 \sqrt{a+bx+cx^2}} + \frac{f}{d \sqrt{a+bx+cx^2} (d-fx^2)} \right) dx \\ &= \frac{\int \frac{1}{x^2 \sqrt{a+bx+cx^2}} dx}{d} + \frac{f \int \frac{1}{\sqrt{a+bx+cx^2} (d-fx^2)} dx}{d} \\ &= -\frac{\sqrt{a+bx+cx^2}}{adx} - \frac{b \int \frac{1}{x \sqrt{a+bx+cx^2}} dx}{2ad} + \frac{f \int \frac{1}{(d-\sqrt{d}\sqrt{fx}) \sqrt{a+bx+cx^2}} dx}{2d} + \frac{f \int \frac{1}{(d+\sqrt{d}\sqrt{fx}) \sqrt{a+bx+cx^2}} dx}{2d} \\ &= -\frac{\sqrt{a+bx+cx^2}}{adx} + \frac{b \operatorname{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{ad} - \frac{f \operatorname{Subst}\left(\int \frac{1}{4cd^2-4} dx, x, \frac{b\sqrt{d}-2a\sqrt{fx}}{\sqrt{a+bx+cx^2}}\right)}{2d} + \frac{f \operatorname{Subst}\left(\int \frac{1}{4cd^2-4} dx, x, \frac{b\sqrt{d}+2a\sqrt{fx}}{\sqrt{a+bx+cx^2}}\right)}{2d} \\ &= -\frac{\sqrt{a+bx+cx^2}}{adx} + \frac{b \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d} + \frac{f \tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{fx}+(2c\sqrt{d}-b)\sqrt{a+bx+cx^2}}{2\sqrt{cd-b\sqrt{d}}\sqrt{fx}+af}\right)}{2d^{3/2}\sqrt{cd-b\sqrt{d}}\sqrt{fx}} + \frac{f \tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{fx}+(2c\sqrt{d}+b)\sqrt{a+bx+cx^2}}{2\sqrt{cd+b\sqrt{d}}\sqrt{fx}+af}\right)}{2d^{3/2}\sqrt{cd+b\sqrt{d}}\sqrt{fx}} \end{aligned}$$

Mathematica [A] time = 1.03, size = 325, normalized size = 1.12

$$\frac{b\sqrt{d} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{a^{3/2}} + \frac{f \tanh^{-1}\left(\frac{2a\sqrt{fx}+b\sqrt{d}+b\sqrt{fx}+2c\sqrt{d}x}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}}\sqrt{fx}+cd}\right)}{\sqrt{af+b\sqrt{d}}\sqrt{fx}+cd} + \frac{f \tanh^{-1}\left(\frac{-2a\sqrt{fx}+b(\sqrt{d}-\sqrt{fx})+2c\sqrt{d}x}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{fx}+cd}\right)}{\sqrt{af+b(-\sqrt{d})}\sqrt{fx}+cd} - \frac{2b\sqrt{d}}{a\sqrt{a+bx+cx^2}} - \frac{2c\sqrt{d}x}{a\sqrt{a+bx+cx^2}} - \frac{2\sqrt{d}}{x\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]

[Out] ((-2*b*Sqrt[d])/(a*Sqrt[a + x*(b + c*x)]) - (2*Sqrt[d])/(x*Sqrt[a + x*(b + c*x)]) - (2*c*Sqrt[d]*x)/(a*Sqrt[a + x*(b + c*x)]) + (b*Sqrt[d]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/a^(3/2) + (f*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + x*(b + c*x)])]/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f] + (f*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + x*(b + c*x)])]/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f))/(2*d^(3/2))

IntegrateAlgebraic [C] time = 0.52, size = 227, normalized size = 0.78

$$\frac{f \operatorname{RootSum}\left[\#1^4(-f) + 2\#1^2af + 4\#1^2cd - 4\#1b\sqrt{cd} - a^2f + b^2d \&, \frac{b \log\left(\frac{-\#1 + \sqrt{a+bx+cx^2} - \sqrt{cx}}{2\sqrt{a+bx+cx^2} - \sqrt{cx}}\right) - 2\#1\sqrt{c} \log\left(\frac{-\#1 + \sqrt{a+bx+cx^2} - \sqrt{cx}}{2\sqrt{a+bx+cx^2} - \sqrt{cx}}\right)}{\#1^3f - \#1af - 2\#1cd + b\sqrt{cd}} \& \right]}{2d} - \frac{b \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a}} - \frac{\sqrt{a+bx+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\sqrt{a+bx+cx^2}}{adx}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^2*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]

[Out] -(Sqrt[a + b*x + c*x^2]/(a*d*x)) - (b*ArcTanh[(Sqrt[c]*x)/Sqrt[a] - Sqrt[a + b*x + c*x^2]/Sqrt[a]])/(a^(3/2)*d) - (f*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 &, (b*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*Sqrt[c]*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) &])/(2*d)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 1.11sym2pol
y/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argum
ent Value

maple [A] time = 0.02, size = 427, normalized size = 1.47

$$\frac{f \ln \left(\frac{2af+2ad-2\sqrt{d}b}{f} + \frac{(bf-2\sqrt{d})\sqrt{a+\frac{\sqrt{d}}{f}}}{f} + 2\sqrt{\frac{bf+cd+\sqrt{d}b}{f}} \sqrt{\frac{(x+\frac{\sqrt{d}}{f})^2 + (bf-2\sqrt{d})\sqrt{a+\frac{\sqrt{d}}{f}}}{f} + \frac{bf+cd+\sqrt{d}b}{f}}}{x+\frac{\sqrt{d}}{f}} \right) + f \ln \left(\frac{2af+2ad+2\sqrt{d}b}{f} + \frac{(bf+2\sqrt{d})\sqrt{a-\frac{\sqrt{d}}{f}}}{f} + 2\sqrt{\frac{bf+cd+\sqrt{d}b}{f}} \sqrt{\frac{(x-\frac{\sqrt{d}}{f})^2 + (bf+2\sqrt{d})\sqrt{a-\frac{\sqrt{d}}{f}}}{f} + \frac{bf+cd+\sqrt{d}b}{f}}}{x-\frac{\sqrt{d}}{f}} \right)}{2\sqrt{df} \sqrt{\frac{bf+cd+\sqrt{d}b}{f}d}} + \frac{b \ln \left(\frac{bx+2a+2\sqrt{cx^2+bx+a}\sqrt{d}}{x} \right) - \frac{\sqrt{cx^2+bx+a}}{adx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)

[Out] $-1/2*f/d/(d*f)^{(1/2)}/((a*f+c*d-(d*f)^{(1/2)*b}/f)^{(1/2)}*\ln((2*(a*f+c*d-(d*f)^{(1/2)*b})/f+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+2*((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b}/f)^{(1/2)})/(x+(d*f)^{(1/2)/f})-(c*x^2+b*x+a)^{(1/2)}/a/d/x+1/2/d*b/a^{(3/2)}*\ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)*a^{(1/2)})/x)+1/2*f/d/(d*f)^{(1/2)}/((a*f+c*d+(d*f)^{(1/2)*b}/f)^{(1/2)}*\ln((2*(a*f+c*d+(d*f)^{(1/2)*b})/f+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f})/f+2*((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c})*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b}/f)^{(1/2)})/(x-(d*f)^{(1/2)/f}))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{cx^2+bx+a}(fx^2-d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] -integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2 (d - f x^2) \sqrt{c x^2 + b x + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)

[Out] `int(1/(x^2*(d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-dx^2\sqrt{a+bx+cx^2} + fx^4\sqrt{a+bx+cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d), x)`

[Out] `-Integral(1/(-d*x**2*sqrt(a + b*x + c*x**2) + f*x**4*sqrt(a + b*x + c*x**2)), x)`

$$3.101 \quad \int \frac{1}{x^3 \sqrt{a+bx+cx^2} (d-fx^2)} dx$$

Optimal. Leaf size=376

$$\frac{(3b^2 - 4ac) \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} - \frac{f^{3/2} \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2d^2\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} + \frac{f^{3/2} \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^2\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{f \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^2} - \frac{\sqrt{a+bx+cx^2}}{2adx^2}$$

Rubi [A] time = 0.73, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 28, number of rules / integrand size = 0.214, Rules used = {6725, 744, 806, 724, 206, 1033}

$$\frac{(3b^2 - 4ac) \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} - \frac{f^{3/2} \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2d^2\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}} + \frac{f^{3/2} \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^2\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{f \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^2} - \frac{\sqrt{a+bx+cx^2}}{2adx^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]

[Out] -Sqrt[a + b*x + c*x^2]/(2*a*d*x^2) + (3*b*Sqrt[a + b*x + c*x^2])/(4*a^2*d*x) - ((3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(5/2)*d) - (f*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(Sqrt[a]*d^2) - (f^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f)) + (f^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 744

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x, x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])

Rule 806

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f

+ d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]

Rule 1033

Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]

Rule 6725

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{x^3 \sqrt{a+bx+cx^2} (d-fx^2)} dx &= \int \left(\frac{1}{dx^3 \sqrt{a+bx+cx^2}} + \frac{f}{d^2 x \sqrt{a+bx+cx^2}} + \frac{f^2 x}{d^2 \sqrt{a+bx+cx^2} (d-fx^2)} \right) dx \\ &= \frac{\int \frac{1}{x^3 \sqrt{a+bx+cx^2}} dx}{d} + \frac{f \int \frac{1}{x \sqrt{a+bx+cx^2}} dx}{d^2} + \frac{f^2 \int \frac{x}{\sqrt{a+bx+cx^2} (d-fx^2)} dx}{d^2} \\ &= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} - \frac{\int \frac{\frac{3b}{2}+cx}{x^2 \sqrt{a+bx+cx^2}} dx}{2ad} - \frac{(2f) \text{Subst} \left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}} \right)}{d^2} \\ &= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2 dx} - \frac{f \tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}} \right)}{\sqrt{a} d^2} + \frac{(3b^2 - 4ac) \tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}} \right)}{8a^{5/2} d} \\ &= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2 dx} - \frac{f \tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}} \right)}{\sqrt{a} d^2} - \frac{f^{3/2} \tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}} \right)}{8a^{5/2} d} \end{aligned}$$

Mathematica [A] time = 2.01, size = 314, normalized size = 0.84

$$\frac{2\sqrt{a} \left(\frac{2a^2 f^{3/2} \tanh^{-1} \left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}}\sqrt{f+cd}} \right) - 2a^2 f^{3/2} \tanh^{-1} \left(\frac{-2a\sqrt{f}+b(\sqrt{d}-\sqrt{f}x)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b(-\sqrt{d})}\sqrt{f+cd}} \right) - \frac{d(2a-3bx)\sqrt{a+x(b+cx)}}{x^2} \right) + (4a(cd-2af)-3b^2d) \tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}} \right)}{8a^{5/2}d^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*Sqrt[a + b*x + c*x^2]*(d - f*x^2)), x]

[Out] ((-3*b^2*d + 4*a*(c*d - 2*a*f))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[a]*(-(d*(2*a - 3*b*x)*Sqrt[a + x*(b + c*x)]/x^2) + (2*a^2*f^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f])*

$x)/(2*\text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{Sqrt}[a + x*(b + c*x)])]/\text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f] - (2*a^2*f^(3/2)*\text{ArcTanh}[(-2*a*\text{Sqrt}[f] + 2*c*\text{Sqrt}[d]*x + b*(\text{Sqrt}[d] - \text{Sqrt}[f]*x))/(2*\text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f]*\text{Sqrt}[a + x*(b + c*x)])]/\text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f])/(8*a^(5/2)*d^2)$

IntegrateAlgebraic [C] time = 0.84, size = 251, normalized size = 0.67

$$\frac{f^2 \text{RootSum}\left[\#1^4(-f) + 2\#1^2 a f + 4\#1^2 c d - 4\#1 b \sqrt{c} d - a^2 f + b^2 d \&, \frac{a \log(\#1 + \sqrt{a + b x + c x^2} - \sqrt{c} x) - \#1^2 \log(\#1 + \sqrt{a + b x + c x^2} - \sqrt{c} x)}{\#1^3(-f) + \#1 a f + 2\#1 c d - b \sqrt{c} d}\right]}{2d^2} + \frac{(3bx - 2a)\sqrt{a + bx + cx^2}}{4a^2 dx^2} + \frac{(-8a^2 f + 4acd - 3b^2 d) \tanh^{-1}\left(\frac{\sqrt{a + b x + c x^2} - \sqrt{c} x}{\sqrt{a}}\right)}{4a^{5/2} d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
[Out] ((-2*a + 3*b*x)*Sqrt[a + b*x + c*x^2])/(4*a^2*d*x^2) + ((-3*b^2*d + 4*a*c*d - 8*a^2*f)*ArcTanh[(-Sqrt[c]*x) + Sqrt[a + b*x + c*x^2])/Sqrt[a])/(4*a^(5/2)*d^2) - (f^2*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 &, (a*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(-(b*Sqrt[c]*d) + 2*c*d*#1 + a*f*#1 - f*#1^3) & ])/(2*d^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x);OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 1.43sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.02, size = 519, normalized size = 1.38

$$\frac{f \ln\left(\frac{\sqrt{\frac{2f+2a+2\sqrt{d}}{f}} \sqrt{\frac{b^2+2a\sqrt{d}}{f}} \sqrt{\frac{c^2+2a\sqrt{d}}{f}} \sqrt{\frac{d^2+2a\sqrt{d}}{f}}}{\sqrt{\frac{2f+2a+2\sqrt{d}}{f}} \sqrt{\frac{b^2+2a\sqrt{d}}{f}} \sqrt{\frac{c^2+2a\sqrt{d}}{f}} \sqrt{\frac{d^2+2a\sqrt{d}}{f}}}\right)}{2\sqrt{\frac{2f+2a+2\sqrt{d}}{f}} d^2} + \frac{f \ln\left(\frac{\sqrt{\frac{2f+2a+2\sqrt{d}}{f}} \sqrt{\frac{b^2+2a\sqrt{d}}{f}} \sqrt{\frac{c^2+2a\sqrt{d}}{f}} \sqrt{\frac{d^2+2a\sqrt{d}}{f}}}{\sqrt{\frac{2f+2a+2\sqrt{d}}{f}} \sqrt{\frac{b^2+2a\sqrt{d}}{f}} \sqrt{\frac{c^2+2a\sqrt{d}}{f}} \sqrt{\frac{d^2+2a\sqrt{d}}{f}}}\right)}{2\sqrt{\frac{2f+2a+2\sqrt{d}}{f}} d^2} - \frac{f \ln\left(\frac{b+2a\sqrt{d}}{\sqrt{d}}\right) + c \ln\left(\frac{c+2a\sqrt{d}}{\sqrt{d}}\right) - 2b \ln\left(\frac{b+2a\sqrt{d}}{\sqrt{d}}\right) - 2c \ln\left(\frac{c+2a\sqrt{d}}{\sqrt{d}}\right) + \frac{3\sqrt{c^2+bx+a}}{4a^2 dx} - \frac{\sqrt{c^2+bx+a}}{2ad x^2}}{\sqrt{d} d^2} + \frac{c \ln\left(\frac{b+2a\sqrt{d}}{\sqrt{d}}\right) - 2b \ln\left(\frac{b+2a\sqrt{d}}{\sqrt{d}}\right) - 2c \ln\left(\frac{c+2a\sqrt{d}}{\sqrt{d}}\right)}{2a^2 d} - \frac{3\sqrt{c^2+bx+a}}{8a^2 d} + \frac{3\sqrt{c^2+bx+a}}{4a^2 dx} - \frac{\sqrt{c^2+bx+a}}{2ad x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
[Out] 1/2*f/d^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f + (b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)/(x+(d*f)^(1/2)/f))+1/2*f/d^2/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f + (b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)/(x-(d*f)^(1/2)/f))-1/2*(c*x^2+b*x+a)^(1/2)/a/d/x^2+3/4*b*(c*x^2+b*x+a)^(1/2)
```

$\left. \right) / a^{2/d} x^{-3/8} d^{b^2/a^{5/2}} \ln\left(\frac{(bx+2a+2\sqrt{cx^2+bx+a})^{1/2} a^{1/2}}{x}\right) + 1/2 d^c / a^{3/2} \ln\left(\frac{(bx+2a+2\sqrt{cx^2+bx+a})^{1/2} a^{1/2}}{x}\right) - f/d^{2/a^{1/2}} \ln\left(\frac{(bx+2a+2\sqrt{cx^2+bx+a})^{1/2} a^{1/2}}{x}\right)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{cx^2 + bx + a} (fx^2 - d)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] -integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3 (d - fx^2) \sqrt{cx^2 + bx + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)

[Out] int(1/(x^3*(d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-dx^3 \sqrt{a + bx + cx^2} + fx^5 \sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)

[Out] -Integral(1/(-d*x**3*sqrt(a + b*x + c*x**2) + f*x**5*sqrt(a + b*x + c*x**2)), x)

$$3.102 \quad \int \frac{x^4}{(a+bx+cx^2)^{3/2}(d-fx^2)} dx$$

Optimal. Leaf size=466

$$\frac{2d^2(b(b^2f - c(3af + cd)) - cx(2acf + b^2(-f) + 2c^2d))}{f^2(b^2 - 4ac)\sqrt{a+bx+cx^2}(b^2df - (af + cd)^2)} + \frac{2d(b+2cx)}{f^2(b^2 - 4ac)\sqrt{a+bx+cx^2}} + \frac{2b\sqrt{a+bx+cx^2}}{cf(b^2 - 4ac)} - \frac{f}{f^2(b^2 - 4ac)}$$

Rubi [A] time = 1.35, antiderivative size = 466, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 28, number of rules / integrand size = 0.321, Rules used = {6725, 613, 738, 640, 621, 206, 975, 1033, 724}

$$\frac{2d^2(b(b^2f - c(3af + cd)) - cx(2acf + b^2(-f) + 2c^2d))}{f^2(b^2 - 4ac)\sqrt{a+bx+cx^2}(b^2df - (af + cd)^2)} + \frac{2d(b+2cx)}{f^2(b^2 - 4ac)\sqrt{a+bx+cx^2}} + \frac{2b\sqrt{a+bx+cx^2}}{cf(b^2 - 4ac)} - \frac{f}{f^2(b^2 - 4ac)} - \frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{a+bx+cx^2}}\right)}{c^{3/2}f} + \frac{d^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f} + (2a\sqrt{d} + \sqrt{f}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2f(af+b(-\sqrt{d})\sqrt{f}+cd)^{3/2}} + \frac{d^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f} + (b\sqrt{f} + 2\sqrt{d}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f(af+b\sqrt{d}\sqrt{f}+cd)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^4/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]

[Out] (-2*x*(2*a + b*x))/((b^2 - 4*a*c)*f*Sqrt[a + b*x + c*x^2]) + (2*d*(b + 2*c*x))/((b^2 - 4*a*c)*f^2*Sqrt[a + b*x + c*x^2]) - (2*d^2*(b*(b^2*f - c*(c*d + 3*a*f)) - c*(2*c^2*d - b^2*f + 2*a*c*f)*x))/((b^2 - 4*a*c)*f^2*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) + (2*b*Sqrt[a + b*x + c*x^2])/(c*(b^2 - 4*a*c)*f) - ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/(c^(3/2)*f) + (d^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2]))/(2*f*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (d^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2]))/(2*f*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2))

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 613

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-3/2), x_Symbol] := Simp[(-2*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 621

Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 640

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p+1))/(2*c*(p+1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 724

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2

$*a*e - b*d - (2*c*d - b*e)*x)/\text{Sqrt}[a + b*x + c*x^2], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[2*c*d - b*e, 0]$

Rule 738

$\text{Int}[\{(d_.) + (e_.)*(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^{(p_.)}, x_Symbol] :> \text{Simp}[\{(d + e*x)^{(m - 1)}*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^{(p + 1)}\}/\{(p + 1)*(b^2 - 4*a*c)\}, x] + \text{Dist}[1/\{(p + 1)*(b^2 - 4*a*c)\}, \text{Int}[(d + e*x)^{(m - 2)}*\text{Simp}[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^{(p + 1)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - 2 - b*d*e + a*e^2, 0] \&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{LtQ}[p, -1] \&\& \text{GtQ}[m, 1] \&\& \text{IntQuadraticQ}[a, b, c, d, e, m, p, x]$

Rule 975

$\text{Int}[\{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^{(p_.)}*((d_.) + (f_.)*(x_.)^2)^{(q_.)}, x_Symbol] :> \text{Simp}[\{(b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*f)))*x*(a + b*x + c*x^2)^{(p + 1)}*(d + f*x^2)^{(q + 1)}\}/\{(b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)\}, x] - \text{Dist}[1/\{(b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)\}, \text{Int}[(a + b*x + c*x^2)^{(p + 1)}*(d + f*x^2)^q*\text{Simp}[2*c*(b^2*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f)))*(p + q + 2) - (2*c^2*d + b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2*p + 2*q + 5)*x^2, x], x], x] /; \text{FreeQ}[\{a, b, c, d, f, q\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{LtQ}[p, -1] \&\& \text{NeQ}[b^2*d*f + (c*d - a*f)^2, 0] \&\& !(IntegerQ[p] \&\& ILtQ[q, -1]) \&\& !IGtQ[q, 0]$

Rule 1033

$\text{Int}[\{(g_.) + (h_.)*(x_.)\}/\{(a_.) + (c_.)*(x_.)^2\}*\text{Sqrt}[\{(d_.) + (e_.)*(x_.) + (f_.)*(x_.)^2\}], x_Symbol] :> \text{With}[\{q = \text{Rt}[-(a*c), 2]\}, \text{Dist}[h/2 + (c*g)/(2*q), \text{Int}[1/\{(-q + c*x)*\text{Sqrt}[d + e*x + f*x^2]\}, x], x] + \text{Dist}[h/2 - (c*g)/(2*q), \text{Int}[1/\{(q + c*x)*\text{Sqrt}[d + e*x + f*x^2]\}, x], x]] /; \text{FreeQ}[\{a, c, d, e, f, g, h\}, x] \&\& \text{NeQ}[e^2 - 4*d*f, 0] \&\& \text{PosQ}[-(a*c)]$

Rule 6725

$\text{Int}[(u_)/\{(a_.) + (b_.)*(x_.)^{(n_.)}\}, x_Symbol] :> \text{With}[\{v = \text{RationalFunctionExpand}[u/(a + b*x^n), x]\}, \text{Int}[v, x] /; \text{SumQ}[v]] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{IGtQ}[n, 0]$

Rubi steps

$$\int \frac{x^4}{(a + bx + cx^2)^{3/2} (d - fx^2)} dx = \int \left(-\frac{d}{f^2 (a + bx + cx^2)^{3/2}} - \frac{x^2}{f (a + bx + cx^2)^{3/2}} + \frac{d^2}{f^2 (a + bx + cx^2)^{3/2} (d - fx^2)} \right) dx$$

$$= -\frac{d \int \frac{1}{(a+bx+cx^2)^{3/2}} dx}{f^2} + \frac{d^2 \int \frac{1}{(a+bx+cx^2)^{3/2}(d-fx^2)} dx}{f^2} - \frac{\int \frac{x^2}{(a+bx+cx^2)^{3/2}} dx}{f}$$

$$= -\frac{2x(2a + bx)}{(b^2 - 4ac) f \sqrt{a + bx + cx^2}} + \frac{2d(b + 2cx)}{(b^2 - 4ac) f^2 \sqrt{a + bx + cx^2}} - \frac{2d^2 (b (b^2 f - d))}{(b^2 - 4ac) f^3 \sqrt{a + bx + cx^2}}$$

$$= -\frac{2x(2a + bx)}{(b^2 - 4ac) f \sqrt{a + bx + cx^2}} + \frac{2d(b + 2cx)}{(b^2 - 4ac) f^2 \sqrt{a + bx + cx^2}} - \frac{2d^2 (b (b^2 f - d))}{(b^2 - 4ac) f^3 \sqrt{a + bx + cx^2}}$$

$$= -\frac{2x(2a + bx)}{(b^2 - 4ac) f \sqrt{a + bx + cx^2}} + \frac{2d(b + 2cx)}{(b^2 - 4ac) f^2 \sqrt{a + bx + cx^2}} - \frac{2d^2 (b (b^2 f - d))}{(b^2 - 4ac) f^3 \sqrt{a + bx + cx^2}}$$

$$= -\frac{2x(2a + bx)}{(b^2 - 4ac) f \sqrt{a + bx + cx^2}} + \frac{2d(b + 2cx)}{(b^2 - 4ac) f^2 \sqrt{a + bx + cx^2}} - \frac{2d^2 (b (b^2 f - d))}{(b^2 - 4ac) f^3 \sqrt{a + bx + cx^2}}$$

Mathematica [A] time = 1.41, size = 562, normalized size = 1.21

$$\frac{f \left(a \sqrt{b^2 - 4ac} \operatorname{tanh}^{-1} \left(\frac{bx + 2a}{\sqrt{a + bx + cx^2}} \right) - 2 \sqrt{c} (a(b - 2cx) + bcx^2) \sqrt{a + bx + cx^2} \right)}{a^{3/2} (4ac - b^2)} + \frac{\left(\frac{(b^2 - 4ac)(ef + \sqrt{d} \sqrt{f + cd}) \operatorname{tanh}^{-1} \left(\frac{-2a \sqrt{f} + (\sqrt{b^2 - 4ac} + 2a \sqrt{c})}{2 \sqrt{(ef + cd)(a + bx + cx^2)}} \right) + (4ac - b^2)(ef + \sqrt{d} \sqrt{f + cd}) \operatorname{tanh}^{-1} \left(\frac{-2(a \sqrt{f} + \sqrt{c} b) + (\sqrt{b^2 - 4ac} + \sqrt{c} f)}{2 \sqrt{(ef + cd)(a + bx + cx^2)}} \right)}{\sqrt{ef + \sqrt{d} \sqrt{f + cd}}} \right)}{2(b^2 - 4ac)(ef + cd - b^2 df)} + \frac{2d(b + 2cx)}{(b^2 - 4ac) \sqrt{a + bx + cx^2}} - \frac{2f^2(-2ac + b^2 + bcx)}{d(b^2 - 4ac) \sqrt{a + bx + cx^2}} - \frac{2d^2(-bc(3af + cd) - 2a^2c(af + cd) + b^2 f + b^2 cf x)}{(b^2 - 4ac) \sqrt{a + bx + cx^2} (b^2 df - a^2 cf^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)), x]
[Out] ((2*d*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]) - (2*f*x^3*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]) - (2*d^2*(b^3*f - b*c*(c*d + 3*a*f) + b^2*c*f*x - 2*c^2*(c*d + a*f)*x))/((b^2 - 4*a*c)*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + x*(b + c*x)]) + (f*(-2*Sqrt[c]*(b*c*x^2 + a*(b - 2*c*x))*Sqrt[a + x*(b + c*x)] + a*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(a*c^(3/2)*(-b^2 + 4*a*c)) + (d^(3/2)*f*((b^2 - 4*a*c)*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f] + ((-b^2 + 4*a*c)*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]))/(2*(b^2 - 4*a*c)*(-b^2*d*f) + (c*d + a*f)^2))/f^2
```

IntegrateAlgebraic [C] time = 1.54, size = 469, normalized size = 1.01

$$\frac{d^2 \operatorname{RootSum} \left[\frac{1}{3}(-f) + 2d^2bf + 4d^2cd - 4d^2b\sqrt{c}d - d^2f + b^2db, x^3 - 3d^2bx + 2ad^2 - d^2f \right]}{2f(b^2f^2 + 2ad^2f + b^2(-d)f + c^2d^2)} + \frac{2(b^2bf - 2b^2cfx + a^2b^2fx + 3d^2bd - 2a^2cdx - ab^2d + 4ab^2cdx + b^4(-d))}{c(b^2 - b^2)\sqrt{a + bx + cx^2} (b^2f^2 + 2ad^2f + b^2(-d)f + c^2d^2)} + \frac{\log(-2c^2\sqrt{a + bx + cx^2} + bcf + 2c^2fx)}{c^2f}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)), x]
[Out] (-2*(-(a*b^3*d) + 3*a^2*b*c*d + a^3*b*f - b^4*d*x + 4*a*b^2*c*d*x - 2*a^2*c^2*d*x + a^2*b^2*f*x - 2*a^3*c*f*x))/(c*(-b^2 + 4*a*c)*(c^2*d^2 - b^2*d*f) +
```

$$2*a*c*d*f + a^2*f^2)*\text{Sqrt}[a + b*x + c*x^2]) + \text{Log}[b*c*f + 2*c^2*f*x - 2*c^{(3/2)}*f*\text{Sqrt}[a + b*x + c*x^2]]/(c^{(3/2)}*f) - (d^2*\text{RootSum}[b^2*d - a^2*f - 4*b*\text{Sqrt}[c]*d*\#1 + 4*c*d*\#1^2 + 2*a*f*\#1^2 - f*\#1^4 \& , (b*c*d*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] + 2*a*b*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] - 2*c^{(3/2)}*d*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] *\#1 - 2*a*\text{Sqrt}[c]*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] *\#1 - b*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] *\#1^2)/(b*\text{Sqrt}[c]*d - 2*c*d*\#1 - a*f*\#1 + f*\#1^3) \&])/(2*f*(c^2*d^2 - b^2*d*f + 2*a*c*d*f + a^2*f^2))$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type

maple [B] time = 0.02, size = 1648, normalized size = 3.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)

[Out]
$$\frac{1}{f*x/c/(c*x^2+b*x+a)^{(1/2)}-1/2/f*b/c^2/(c*x^2+b*x+a)^{(1/2)}-1/f*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x-1/2/f*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}-1/f/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-4/f^2*d/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*c*x-2/f^2*d/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*b+1/2/f*d^2/(d*f)^{(1/2)}/(a*f+c*d-(d*f)^{(1/2)}*b)/((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)}*c)*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)}+2/f^2*d^2/(a*f+c*d-(d*f)^{(1/2)}*b)/(4*a*c-b^2)/((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)}*c)*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)}*x*c^2-1/f*d^2/(d*f)^{(1/2)}/(a*f+c*d-(d*f)^{(1/2)}*b)/(4*a*c-b^2)/((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)}*c)*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)}*x*b*c+1/f^2*d^2/(a*f+c*d-(d*f)^{(1/2)}*b)/(4*a*c-b^2)/((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)}*c)*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)}*b^2-1/2/f*d^2/(d*f)^{(1/2)}/(a*f+c*d-(d*f)^{(1/2)}*b)/((a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)}*\ln((2*(a*f+c*d-(d*f)^{(1/2)}*b)/f+(b*f-2*(d*f)^{(1/2)}*c)*(x+(d*f)^{(1/2)}/f)/f+2*((a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)}*((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)}*c)*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)})/(x+(d*f)^{(1/2)}/f))-1/2/f*d^2/(d*f)^{(1/2)}/(a*f+c*d+(d*f)^{(1/2)}*b)/((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)}*c)*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)}*b)/f)^{(1/2)}+2/f^2*d^2/(a*f+c*d+(d*f)^{(1/2)}*b)/(4*a*c-b^2)/((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)}*c)*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)}*b)/f)^{(1/2)}*x*c^2+1/f*d^2/(d*f)^{(1/2)}/(a*f+c*d+(d*f)^{(1/2)}*b)/(4*a*c-b^2)/((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)}*c)*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)}*b)/f)^{(1/2)}*x*b*c+1/f^2*d^2/(a*f+c*d+(d*f)^{(1/2)}*b)/(4*a*c-b^2)/((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)}*c)*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)}*b)/f)^{(1/2)}$$

2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*b*c+1/2/f*d^2/(d*f)^(1/2)/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*b^2+1/2/f*d^2/(d*f)^(1/2)/(a*f+c*d+(d*f)^(1/2)*b)/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see 'assume?' for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2 - (c*((b*sqrt(4*d*f)) / (2*f) + (c*d)/f+a)) /f^2 zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{(d - f x^2) (c x^2 + b x + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)

[Out] int(x^4/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4}{-ad\sqrt{a+bx+cx^2}+afx^2\sqrt{a+bx+cx^2}-bdx\sqrt{a+bx+cx^2}+bfx^3\sqrt{a+bx+cx^2}-cdx^2\sqrt{a+bx+cx^2}+cfx^4\sqrt{a+bx+cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)

[Out] -Integral(x**4/(-a*d*sqrt(a + b*x + c*x**2) + a*f*x**2*sqrt(a + b*x + c*x**2) - b*d*x*sqrt(a + b*x + c*x**2) + b*f*x**3*sqrt(a + b*x + c*x**2) - c*d*x**2*sqrt(a + b*x + c*x**2) + c*f*x**4*sqrt(a + b*x + c*x**2)), x)

$$3.103 \quad \int \frac{x^3}{(a+bx+cx^2)^{3/2}(d-fx^2)} dx$$

Optimal. Leaf size=341

$$\frac{2d(a(2acf + b^2(-f) + 2c^2d) + bcx(cd - af))}{f(b^2 - 4ac)\sqrt{a + bx + cx^2}(b^2df - (af + cd)^2)} - \frac{2(2a + bx)}{f(b^2 - 4ac)\sqrt{a + bx + cx^2}} - \frac{d \tanh^{-1}\left(\frac{-2a\sqrt{f} + x(2c\sqrt{d} - b\sqrt{f})}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f} + \dots}\right)}{2\sqrt{f}(af + b(-\sqrt{d})\sqrt{f} + \dots)}$$

Rubi [A] time = 1.04, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {6725, 636, 1018, 1033, 724, 206}

$$\frac{2d(a(2acf + b^2(-f) + 2c^2d) + bcx(cd - af))}{f(b^2 - 4ac)\sqrt{a + bx + cx^2}(b^2df - (af + cd)^2)} - \frac{2(2a + bx)}{f(b^2 - 4ac)\sqrt{a + bx + cx^2}} - \frac{d \tanh^{-1}\left(\frac{-2a\sqrt{f} + x(2c\sqrt{d} - b\sqrt{f}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f} + cd}\right)}{2\sqrt{f}(af + b(-\sqrt{d})\sqrt{f} + cd)^{3/2}} + \frac{d \tanh^{-1}\left(\frac{2a\sqrt{f} + x(b\sqrt{f} + 2c\sqrt{d}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}}\sqrt{f} + cd}\right)}{2\sqrt{f}(af + b\sqrt{d}\sqrt{f} + cd)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^3/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)), x]

[Out] $(-2*(2*a + b*x))/((b^2 - 4*a*c)*f*\text{Sqrt}[a + b*x + c*x^2]) - (2*d*(a*(2*c^2*d - b^2*f + 2*a*c*f) + b*c*(c*d - a*f)*x))/((b^2 - 4*a*c)*f*(b^2*d*f - (c*d + a*f)^2)*\text{Sqrt}[a + b*x + c*x^2]) - (d*\text{ArcTanh}[(b*\text{Sqrt}[d] - 2*a*\text{Sqrt}[f] + (2*c*\text{Sqrt}[d] - b*\text{Sqrt}[f])*x)/(2*\text{Sqrt}[c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f)*\text{Sqrt}[a + b*x + c*x^2]])/(2*\text{Sqrt}[f]*(c*d - b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f)^(3/2)) + (d*\text{ArcTanh}[(b*\text{Sqrt}[d] + 2*a*\text{Sqrt}[f] + (2*c*\text{Sqrt}[d] + b*\text{Sqrt}[f])*x)/(2*\text{Sqrt}[c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f)*\text{Sqrt}[a + b*x + c*x^2]])/(2*\text{Sqrt}[f]*(c*d + b*\text{Sqrt}[d]*\text{Sqrt}[f] + a*f)^(3/2))$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 636

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1018

Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x))/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q)*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*

```
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
+ 5)*x^2, x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] && !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x^3}{(a+bx+cx^2)^{3/2}(d-fx^2)} dx &= \int \left(-\frac{x}{f(a+bx+cx^2)^{3/2}} + \frac{dx}{f(a+bx+cx^2)^{3/2}(d-fx^2)} \right) dx \\ &= -\frac{\int \frac{x}{(a+bx+cx^2)^{3/2}} dx}{f} + \frac{d \int \frac{x}{(a+bx+cx^2)^{3/2}(d-fx^2)} dx}{f} \\ &= -\frac{2(2a+bx)}{(b^2-4ac)f\sqrt{a+bx+cx^2}} - \frac{2d(a(2c^2d-b^2f+2acf)+bc(cd-af)x)}{(b^2-4ac)f(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} \\ &= -\frac{2(2a+bx)}{(b^2-4ac)f\sqrt{a+bx+cx^2}} - \frac{2d(a(2c^2d-b^2f+2acf)+bc(cd-af)x)}{(b^2-4ac)f(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} \\ &= -\frac{2(2a+bx)}{(b^2-4ac)f\sqrt{a+bx+cx^2}} - \frac{2d(a(2c^2d-b^2f+2acf)+bc(cd-af)x)}{(b^2-4ac)f(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} \\ &= -\frac{2(2a+bx)}{(b^2-4ac)f\sqrt{a+bx+cx^2}} - \frac{2d(a(2c^2d-b^2f+2acf)+bc(cd-af)x)}{(b^2-4ac)f(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} \end{aligned}$$

Mathematica [A] time = 1.20, size = 414, normalized size = 1.21

$$\frac{1}{2} \left(\frac{8a^2f + 4a^2(bf+2cd) - 4ab(d-b-3c) - 4b^2d}{(b^2-4ac)\sqrt{a+bx+cx^2}} \frac{d \log(\sqrt{d}\sqrt{f-fx})}{\sqrt{f}(af+b\sqrt{d}\sqrt{f+cd})} - \frac{d \log(\sqrt{d}\sqrt{f+fx})}{\sqrt{f}(af+b(-\sqrt{d})\sqrt{f+cd})} + \frac{d \log(\sqrt{d}(2\sqrt{a+3(b^2+cx)}\sqrt{af+b(-\sqrt{d})\sqrt{f+cd}} + 2a\sqrt{f-b\sqrt{d}} + b\sqrt{f+2c\sqrt{d}x})}{\sqrt{f}(af+b(-\sqrt{d})\sqrt{f+cd})} + \frac{d \log(\sqrt{d}(2(\sqrt{a+3(b^2+cx)}\sqrt{af+b\sqrt{d}\sqrt{f+cd}} + a\sqrt{f+c\sqrt{d}x}) + b(\sqrt{d} + \sqrt{fx})))}{\sqrt{f}(af+b\sqrt{d}\sqrt{f+cd})} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] ((8*a^3*f - 4*b^3*d*x - 4*a*b*d*(b - 3*c*x) + 4*a^2*(2*c*d + b*f*x))/((b^2
- 4*a*c)*(-(c^2*d^2) - 2*a*c*d*f + f*(b^2*d - a^2*f))*Sqrt[a + x*(b + c*x)]
```

$$\begin{aligned} & - (d \cdot \text{Log}[\text{Sqrt}[d] \cdot \text{Sqrt}[f] - f \cdot x]) / (\text{Sqrt}[f] \cdot (c \cdot d + b \cdot \text{Sqrt}[d] \cdot \text{Sqrt}[f] + a \cdot f) \\ & \wedge (3/2)) - (d \cdot \text{Log}[\text{Sqrt}[d] \cdot \text{Sqrt}[f] + f \cdot x]) / (\text{Sqrt}[f] \cdot (c \cdot d - b \cdot \text{Sqrt}[d] \cdot \text{Sqrt}[f] \\ & + a \cdot f) \wedge (3/2)) + (d \cdot \text{Log}[\text{Sqrt}[d] \cdot (-b \cdot \text{Sqrt}[d]) + 2 \cdot a \cdot \text{Sqrt}[f] - 2 \cdot c \cdot \text{Sqrt}[d] \cdot x \\ & + b \cdot \text{Sqrt}[f] \cdot x + 2 \cdot \text{Sqrt}[c \cdot d - b \cdot \text{Sqrt}[d] \cdot \text{Sqrt}[f] + a \cdot f] \cdot \text{Sqrt}[a + x \cdot (b + c \cdot x)] \\ &)) / (\text{Sqrt}[f] \cdot (c \cdot d - b \cdot \text{Sqrt}[d] \cdot \text{Sqrt}[f] + a \cdot f) \wedge (3/2)) + (d \cdot \text{Log}[\text{Sqrt}[d] \cdot (b \cdot (\text{Sqrt}[d] \\ & + \text{Sqrt}[f] \cdot x) + 2 \cdot (a \cdot \text{Sqrt}[f] + c \cdot \text{Sqrt}[d] \cdot x + \text{Sqrt}[c \cdot d + b \cdot \text{Sqrt}[d] \cdot \text{Sqrt}[f] \\ & + a \cdot f] \cdot \text{Sqrt}[a + x \cdot (b + c \cdot x)]))]) / (\text{Sqrt}[f] \cdot (c \cdot d + b \cdot \text{Sqrt}[d] \cdot \text{Sqrt}[f] + a \cdot f) \\ & \wedge (3/2))) / 2 \end{aligned}$$

IntegrateAlgebraic [C] time = 0.96, size = 424, normalized size = 1.24

$$\frac{d \cdot \text{RootSum}\left[\#1^3(-f) + 2\#1^2af + 4\#1cd - 4\#1b\sqrt{cd - a^2f + b^2de}, \frac{-a^2d \log\left(\frac{\#1 + \sqrt{cd - a^2f + b^2de}}{\sqrt{cd - a^2f + b^2de}}\right) - a^2f \log\left(\frac{\#1 + \sqrt{cd - a^2f + b^2de}}{\sqrt{cd - a^2f + b^2de}}\right) + a^2d \log\left(\frac{\#1 + \sqrt{cd - a^2f + b^2de}}{\sqrt{cd - a^2f + b^2de}}\right) - a^2f \log\left(\frac{\#1 + \sqrt{cd - a^2f + b^2de}}{\sqrt{cd - a^2f + b^2de}}\right)}{\#1^3 - a^2f - 2a^2cd + b^2de}\right] + \frac{2(2a^2f + a^2bf + 2a^2cd - ab^2d + 3abcdx - b^3dx)}{(b^2 - 4ac)\sqrt{a + bx + cx^2}(-a^2f^2 - 2acdf + b^2df - c^2d^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]

[Out] (2*(-(a*b^2*d) + 2*a^2*c*d + 2*a^3*f - b^3*d*x + 3*a*b*c*d*x + a^2*b*f*x))/((b^2 - 4*a*c)*(-(c^2*d^2) + b^2*d*f - 2*a*c*d*f - a^2*f^2)*Sqrt[a + b*x + c*x^2]) + (d*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (b^2*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*b*Sqrt[c]*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - a*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) &])/(2*(c^2*d^2 - b^2*d*f + 2*a*c*d*f + a^2*f^2))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Valueint() Bad Argument Type

maple [B] time = 0.02, size = 1480, normalized size = 4.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)

[Out] 1/f/c/(c*x^2+b*x+a)^(1/2)+2/f*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+1/f*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-1/2/f*d/(a*f+c*d-(d*f)^(1/2)*b)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)-2/f^2*d/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*(d*f)^(1/2)*x*c^2+1/f*d/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)

```
(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*b*c-1/f^2*d/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*(d*f)^(1/2)*b*c+1/2/f*d/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b^2+1/2/f*d/(a*f+c*d-(d*f)^(1/2)*b)/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))-1/2/f*d/(a*f+c*d+(d*f)^(1/2)*b)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)+2/f^2*d/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*x*b*c+1/f^2*d/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*(d*f)^(1/2)*b*c+1/2/f*d/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2
-(c*((b*sqrt(4*d*f)))/(2*f) + (c*d)/f+a))
/f^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{(d - f x^2) (c x^2 + b x + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int(x^3/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3}{-ad\sqrt{a+bx+cx^2} + afx^2\sqrt{a+bx+cx^2} - bdx\sqrt{a+bx+cx^2} + bfx^3\sqrt{a+bx+cx^2} - cdx^2\sqrt{a+bx+cx^2} + cfx^4\sqrt{a+bx+cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x**3/(-a*d*sqrt(a + b*x + c*x**2) + a*f*x**2*sqrt(a + b*x + c*x**
2) - b*d*x*sqrt(a + b*x + c*x**2) + b*f*x**3*sqrt(a + b*x + c*x**2) - c*d*x
**2*sqrt(a + b*x + c*x**2) + c*f*x**4*sqrt(a + b*x + c*x**2)), x)
```


$$3.104 \quad \int \frac{x^2}{(a+bx+cx^2)^{3/2}(d-fx^2)} dx$$

Optimal. Leaf size=297

$$\frac{2(cx(b^2d - 2a(af + cd)) + ab(cd - af))}{(b^2 - 4ac)\sqrt{a + bx + cx^2}(b^2df - (af + cd)^2)} + \frac{\sqrt{d} \tanh^{-1}\left(\frac{-2a\sqrt{f} + x(2c\sqrt{d} - b\sqrt{f}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2(af + b(-\sqrt{d})\sqrt{f} + cd)^{3/2}} + \frac{\sqrt{d} \tanh^{-1}\left(\frac{2a\sqrt{f}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2(af + b\sqrt{d}\sqrt{f} + cd)^{3/2}}$$

Rubi [A] time = 0.45, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1065, 1033, 724, 206}

$$\frac{2(cx(b^2d - 2a(af + cd)) + ab(cd - af))}{(b^2 - 4ac)\sqrt{a + bx + cx^2}(b^2df - (af + cd)^2)} + \frac{\sqrt{d} \tanh^{-1}\left(\frac{-2a\sqrt{f} + x(2c\sqrt{d} - b\sqrt{f}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2(af + b(-\sqrt{d})\sqrt{f} + cd)^{3/2}} + \frac{\sqrt{d} \tanh^{-1}\left(\frac{2a\sqrt{f} + x(b\sqrt{f} + 2c\sqrt{d}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2(af + b\sqrt{d}\sqrt{f} + cd)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^2/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]

[Out] (2*(a*b*(c*d - a*f) + c*(b^2*d - 2*a*(c*d + a*f))*x)/((b^2 - 4*a*c)*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) + (Sqrt[d]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (Sqrt[d]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1033

Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]

Rule 1065

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1)*((A*c - a*C)*(-(b*(c*d + a*f))) + (A*b)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(2*a*f)) + C*(b^2*d - 2*a*(c*d - a*f))))*x)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[(-2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d)*(-(b*f)))]*(p + 1) +

```
(b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(a*f*(p + 1) -
c*d*(p + 2)) - (2*f*((A*c - a*C)*(-(b*(c*d + a*f))) + (A*b)*(2*c^2*d + b^2*
f - c*(2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f) - a*(c*
C*d - a*C*f)))*(b*f*(p + 1)))*x - c*f*(b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f)
- a*(c*C*d - a*C*f))*(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d
, f, A, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*
d - a*f)^2, 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]
```

Rubi steps

$$\int \frac{x^2}{(a + bx + cx^2)^{3/2} (d - fx^2)} dx = \frac{2(abcd - af) + c(b^2d - 2a(cd + af))x}{(b^2 - 4ac)(b^2df - (cd + af)^2)\sqrt{a + bx + cx^2}} + \frac{2 \int \frac{-\frac{1}{2}(b^2 - 4ac)d(cd + af) + \frac{1}{2}b(b^2 - 4ac)}{\sqrt{a + bx + cx^2}(d - fx^2)} dx}{(b^2 - 4ac)(b^2df - (cd + af)^2)}$$

$$= \frac{2(abcd - af) + c(b^2d - 2a(cd + af))x}{(b^2 - 4ac)(b^2df - (cd + af)^2)\sqrt{a + bx + cx^2}} - \frac{(\sqrt{d}\sqrt{f}) \int \frac{1}{(-\sqrt{d}\sqrt{f} - fx)\sqrt{a + bx + cx^2}} dx}{2(cd - b\sqrt{d}\sqrt{f} + af)}$$

$$= \frac{2(abcd - af) + c(b^2d - 2a(cd + af))x}{(b^2 - 4ac)(b^2df - (cd + af)^2)\sqrt{a + bx + cx^2}} + \frac{(\sqrt{d}\sqrt{f}) \text{Subst}\left(\int \frac{1}{4cdf - 4b\sqrt{d}\sqrt{f}x + a^2} dx\right)}{2(cd - b\sqrt{d}\sqrt{f} + af)}$$

$$= \frac{2(abcd - af) + c(b^2d - 2a(cd + af))x}{(b^2 - 4ac)(b^2df - (cd + af)^2)\sqrt{a + bx + cx^2}} + \frac{\sqrt{d} \tanh^{-1}\left(\frac{b\sqrt{d} - 2a\sqrt{f} + (2c\sqrt{d}\sqrt{f} - b)\sqrt{a + bx + cx^2}}{2\sqrt{cd - b\sqrt{d}\sqrt{f} + af}}\right)}{2(cd - b\sqrt{d}\sqrt{f} + af)}$$

Mathematica [A] time = 0.40, size = 352, normalized size = 1.19

$$\frac{2 \left(\frac{a^2 f (b + 2cx) + acd(2cx - b) - b^2 c dx}{\sqrt{a + x(b + cx)}} + \frac{\sqrt{d}(b^2 - 4ac)(af + b\sqrt{d}\sqrt{f} + cd) \tanh^{-1}\left(\frac{-2a\sqrt{f} + b(\sqrt{d} - \sqrt{f}x) + 2c\sqrt{d}x}{2\sqrt{a + x(b + cx)}\sqrt{af + b(-\sqrt{d})\sqrt{f} + cd}}\right)}{4\sqrt{af + b(-\sqrt{d})\sqrt{f} + cd}} + \frac{\sqrt{d}(4ac - b^2)(af + b(-\sqrt{d})\sqrt{f} + cd) \tanh^{-1}\left(\frac{-2(a\sqrt{f} + c\sqrt{d}x) - b(\sqrt{d} + \sqrt{f}x)}{2\sqrt{a + x(b + cx)}\sqrt{af + b\sqrt{d}\sqrt{f} + cd}}\right)}{4\sqrt{af + b\sqrt{d}\sqrt{f} + cd}} \right)}{(b^2 - 4ac)((af + cd)^2 - b^2df)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)), x]
[Out] (2*((-(b^2*c*d*x) + a*c*d*(-b + 2*c*x) + a^2*f*(b + 2*c*x))/Sqrt[a + x*(b +
c*x)] + ((b^2 - 4*a*c)*Sqrt[d]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2
*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[
d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(4*Sqrt[c*d - b*Sqrt[d]*Sqrt[f]
+ a*f]) + ((-b^2 + 4*a*c)*Sqrt[d]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-
2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqr
t[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(4*Sqrt[c*d + b*Sqrt[d]*Sqrt[f]
+ a*f])))/((b^2 - 4*a*c)*(-(b^2*d*f) + (c*d + a*f)^2))
```

IntegrateAlgebraic [C] time = 0.87, size = 388, normalized size = 1.31

$$\frac{d\text{RootSum}\left[\#1^4(-f) + 2\#1^2af + 4\#1^2cd - 4\#1b\sqrt{cd} - a^2f + b^2d\#1, \frac{\#1^2(-b)\log\left(\frac{-\#1 + \sqrt{a + b\sqrt{d}\sqrt{f} + cd} - \sqrt{c}}{\#1 + \sqrt{a + b\sqrt{d}\sqrt{f} + cd} - \sqrt{c}}\right) - 2\#1c^2\log\left(\frac{\#1 + \sqrt{a + b\sqrt{d}\sqrt{f} + cd} - \sqrt{c}}{\#1 + \sqrt{a + b\sqrt{d}\sqrt{f} + cd} - \sqrt{c}}\right) + 2\#1a\sqrt{f}\log\left(\frac{\#1 + \sqrt{a + b\sqrt{d}\sqrt{f} + cd} - \sqrt{c}}{\#1 + \sqrt{a + b\sqrt{d}\sqrt{f} + cd} - \sqrt{c}}\right) + 2ab\log\left(\frac{\#1 + \sqrt{a + b\sqrt{d}\sqrt{f} + cd} - \sqrt{c}}{\#1 + \sqrt{a + b\sqrt{d}\sqrt{f} + cd} - \sqrt{c}}\right) - \frac{2(a^2bf + 2a^2cfx - abcd + 2a^2dx - b^2cdx)}{(b^2 - 4ac)\sqrt{a + bx + cx^2}(-a^2f^2 - 2acdf + b^2df - c^2d^2)}\right]}{2(a^2f^2 + 2acdf + b^2(-d)f + c^2d^2)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)), x]
[Out] (-2*((-(a*b*c*d) + a^2*b*f - b^2*c*d*x + 2*a*c^2*d*x + 2*a^2*c*f*x))/((b^2 -
4*a*c)*(-(c^2*d^2) + b^2*d*f - 2*a*c*d*f - a^2*f^2)*Sqrt[a + b*x + c*x^2])
- (d*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 -
```

$$f^4 \& , (b*c*d*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] + 2*a*b*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] - 2*c^{(3/2)}*d*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1 - 2*a*\text{Sqrt}[c]*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1 - b*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1^2)/(b*\text{Sqrt}[c]*d - 2*c*d*\#1 - a*f*\#1 + f*\#1^3) \&])/(2*(c^2*d^2 - b^2*d*f + 2*a*c*d*f + a^2*f^2))$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Valueint() Bad Argument Type

maple [B] time = 0.02, size = 1427, normalized size = 4.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)

[Out]
$$-2/f*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}+1/2*d/(d*f)^{(1/2)}/(a*f+c*d-(d*f)^{(1/2)}*b)/((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)}*c)*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)}+2*d/f/(a*f+c*d-(d*f)^{(1/2)}*b)/(4*a*c-b^2)/((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)}*c)*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)}*x*c^2-d/(d*f)^{(1/2)}/(a*f+c*d-(d*f)^{(1/2)}*b)/(4*a*c-b^2)/((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)}*c)*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)}*x*b*c+d/f/(a*f+c*d-(d*f)^{(1/2)}*b)/(4*a*c-b^2)/((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)}*c)*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)}*b*c-1/2*d/(d*f)^{(1/2)}/(a*f+c*d-(d*f)^{(1/2)}*b)/(4*a*c-b^2)/((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)}*c)*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)}*b^2-1/2*d/(d*f)^{(1/2)}/(a*f+c*d-(d*f)^{(1/2)}*b)/((a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)}*ln((2*(a*f+c*d-(d*f)^{(1/2)}*b)/f+(b*f-2*(d*f)^{(1/2)}*c)*(x+(d*f)^{(1/2)}/f)/f+2*((a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)}*((x+(d*f)^{(1/2)}/f)^2*c+(b*f-2*(d*f)^{(1/2)}*c)*(x+(d*f)^{(1/2)}/f)/f+(a*f+c*d-(d*f)^{(1/2)}*b)/f)^{(1/2)})/(x+(d*f)^{(1/2)}/f))-1/2*d/(d*f)^{(1/2)}/(a*f+c*d+(d*f)^{(1/2)}*b)/((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)}*c)*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)}*b)/f)^{(1/2)}*x*c^2+d/(d*f)^{(1/2)}/(a*f+c*d+(d*f)^{(1/2)}*b)/(4*a*c-b^2)/((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)}*c)*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)}*b)/f)^{(1/2)}*x*b*c+d/f/(a*f+c*d+(d*f)^{(1/2)}*b)/(4*a*c-b^2)/((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)}*c)*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)}*b)/f)^{(1/2)}*b*c+1/2*d/(d*f)^{(1/2)}/(a*f+c*d+(d*f)^{(1/2)}*b)/(4*a*c-b^2)/((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)}*c)*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)}*b)/f)^{(1/2)}*b^2+1/2*d/(d*f)^{(1/2)}/(a*f+c*d+(d*f)^{(1/2)}*b)/((a*f+c*d+(d*f)^{(1/2)}*b)/f)^{(1/2)}*ln((2*(a*f+c*d+(d*f)^{(1/2)}*b)/f+(b*f+2*(d*f)^{(1/2)}*c)*(x-(d*f)^{(1/2)}/f)/f+2*((a*f+c*d+(d*f)^{(1/2)}*b)/f)^{(1/2)}*((x-(d*f)^{(1/2)}/f)^2*c+(b*f+2*(d*f)^{(1/2)}*c)*(x-(d*f)^{(1/2)}/f)/f+(a*f+c*d+(d*f)^{(1/2)}*b)/f)^{(1/2)})/(x-(d*f)^{(1/2)}/f))$$

$$\frac{1}{2}c) * (x - (d*f)^{1/2}/f) / f + 2 * ((a*f + c*d + (d*f)^{1/2} * b) / f)^{1/2} * ((x - (d*f)^{1/2} / f)^{1/2} * c + (b*f + 2 * (d*f)^{1/2} * c) * (x - (d*f)^{1/2} / f) / f + (a*f + c*d + (d*f)^{1/2} * b) / f)^{1/2} / (x - (d*f)^{1/2} / f)$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see 'help assume?' for more details)Is ((c*sqrt(4*d*f))/(2*f^2) + b/(2*f)) ^2 - (c*((b*sqrt(4*d*f)) / (2*f) + (c*d)/f+a)) / f^2 zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{(d - f x^2) (c x^2 + b x + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)

[Out] int(x^2/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{-ad\sqrt{a+bx+cx^2} + afx^2\sqrt{a+bx+cx^2} - bdx\sqrt{a+bx+cx^2} + bfx^3\sqrt{a+bx+cx^2} - cdx^2\sqrt{a+bx+cx^2} + cfx^4\sqrt{a+bx+cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)

[Out] -Integral(x**2/(-a*d*sqrt(a + b*x + c*x**2) + a*f*x**2*sqrt(a + b*x + c*x**2) - b*d*x*sqrt(a + b*x + c*x**2) + b*f*x**3*sqrt(a + b*x + c*x**2) - c*d*x**2*sqrt(a + b*x + c*x**2) + c*f*x**4*sqrt(a + b*x + c*x**2)), x)

$$3.105 \quad \int \frac{x}{(a+bx+cx^2)^{3/2}(d-fx^2)} dx$$

Optimal. Leaf size=299

$$\frac{2(a(2acf + b^2(-f) + 2c^2d) + bcx(cd - af))}{(b^2 - 4ac)\sqrt{a + bx + cx^2}(b^2df - (af + cd)^2)} - \frac{\sqrt{f} \tanh^{-1}\left(\frac{-2a\sqrt{f} + x(2c\sqrt{d} - b\sqrt{f}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f+cd}}\right)}{2(af + b(-\sqrt{d})\sqrt{f} + cd)^{3/2}} + \frac{\sqrt{f} \tanh^{-1}\left(\frac{2a\sqrt{f} + x(b\sqrt{f} + 2c\sqrt{d}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}}\sqrt{f+cd}}\right)}{2(af + b\sqrt{d}\sqrt{f} + cd)^{3/2}}$$

Rubi [A] time = 0.40, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, number of rules / integrand size = 0.154, Rules used = {1018, 1033, 724, 206}

$$\frac{2(a(2acf + b^2(-f) + 2c^2d) + bcx(cd - af))}{(b^2 - 4ac)\sqrt{a + bx + cx^2}(b^2df - (af + cd)^2)} - \frac{\sqrt{f} \tanh^{-1}\left(\frac{-2a\sqrt{f} + x(2c\sqrt{d} - b\sqrt{f}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})}\sqrt{f+cd}}\right)}{2(af + b(-\sqrt{d})\sqrt{f} + cd)^{3/2}} + \frac{\sqrt{f} \tanh^{-1}\left(\frac{2a\sqrt{f} + x(b\sqrt{f} + 2c\sqrt{d}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}}\sqrt{f+cd}}\right)}{2(af + b\sqrt{d}\sqrt{f} + cd)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]

[Out] (-2*(a*(2*c^2*d - b^2*f + 2*a*c*f) + b*c*(c*d - a*f)*x))/((b^2 - 4*a*c)*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) - (Sqrt[f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (Sqrt[f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1018

Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x))/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p + q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] && !(IntegerQ[p] && ILtQ[q, -1])

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{x}{(a + bx + cx^2)^{3/2} (d - fx^2)} dx = -\frac{2(a(2c^2d - b^2f + 2acf) + bc(cd - af)x)}{(b^2 - 4ac)(b^2df - (cd + af)^2)\sqrt{a + bx + cx^2}} + \frac{2 \int \frac{\frac{1}{2}b(b^2 - 4ac)df - \frac{1}{2}(b^2 - 4ac)f(d - fx^2)}{\sqrt{a + bx + cx^2} (d - fx^2)} dx}{(b^2 - 4ac)(b^2df - (cd + af)^2)}$$

$$= -\frac{2(a(2c^2d - b^2f + 2acf) + bc(cd - af)x)}{(b^2 - 4ac)(b^2df - (cd + af)^2)\sqrt{a + bx + cx^2}} + \frac{f \int \frac{1}{(-\sqrt{d}\sqrt{f} - fx)\sqrt{a + bx + cx^2}} dx}{2(cd - b\sqrt{d}\sqrt{f} + af)}$$

$$= -\frac{2(a(2c^2d - b^2f + 2acf) + bc(cd - af)x)}{(b^2 - 4ac)(b^2df - (cd + af)^2)\sqrt{a + bx + cx^2}} - \frac{f \operatorname{Subst}\left(\int \frac{1}{4cdf - 4b\sqrt{d}f^{3/2} + 4cdx} dx, x, \frac{b\sqrt{d} - 2a\sqrt{f} + 2\sqrt{cd - b\sqrt{d}\sqrt{f} + af}}{2\sqrt{cd - b\sqrt{d}\sqrt{f} + af}}\right)}{2(cd - b\sqrt{d}\sqrt{f} + af)}$$

Mathematica [A] time = 0.35, size = 356, normalized size = 1.19

$$\frac{2 \left(\frac{2a^2cf + a(b^2(-f) - bcfx + 2c^2d) + bc^2dx}{\sqrt{a + x(b + cx)}} - \frac{\sqrt{f}(b^2 - 4ac)(af + b\sqrt{d}\sqrt{f} + cd) \tanh^{-1}\left(\frac{-2a\sqrt{f} + b(\sqrt{d} - \sqrt{f}x) + 2c\sqrt{d}x}{2\sqrt{a + x(b + cx)}\sqrt{af + b(-\sqrt{d})\sqrt{f} + cd}}\right)}{4\sqrt{af + b(-\sqrt{d})\sqrt{f} + cd}} + \frac{\sqrt{f}(4ac - b^2)(af + b(-\sqrt{d})\sqrt{f} + cd) \tanh^{-1}\left(\frac{-2(a\sqrt{f} + c\sqrt{d}x) - b(\sqrt{d} + \sqrt{f}x)}{2\sqrt{a + x(b + cx)}\sqrt{af + b\sqrt{d}\sqrt{f} + cd}}\right)}{4\sqrt{af + b\sqrt{d}\sqrt{f} + cd}} \right)}{(b^2 - 4ac)((af + cd)^2 - b^2df)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)), x]
```

```
[Out] (2*((2*a^2*c*f + b*c^2*d*x + a*(2*c^2*d - b^2*f - b*c*f*x))/Sqrt[a + x*(b + c*x)] - ((b^2 - 4*a*c)*Sqrt[f]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])))/(4*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f)) + ((-b^2 + 4*a*c)*Sqrt[f]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])))/(4*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f)))/((b^2 - 4*a*c)*(-(b^2*d*f) + (c*d + a*f)^2))
```

IntegrateAlgebraic [C] time = 0.91, size = 419, normalized size = 1.40

$$\frac{\sqrt{\operatorname{RootSum}\left[\#1^4(-f) + 2\#1^2af + 4\#1^2cd - 4\#1b\sqrt{cd} - a^2f + b^2df, \frac{-a^2d \operatorname{Log}\left[\#1 + \sqrt{a + b\sqrt{cd} - \sqrt{c}}\right] + a^2f \operatorname{Log}\left[\#1 + \sqrt{a + b\sqrt{cd} - \sqrt{c}}\right] + a^2d \operatorname{Log}\left[\#1 + \sqrt{a + b\sqrt{cd} - \sqrt{c}}\right] + b^2d \operatorname{Log}\left[\#1 + \sqrt{a + b\sqrt{cd} - \sqrt{c}}\right] + a^2d \operatorname{Log}\left[\#1 + \sqrt{a + b\sqrt{cd} - \sqrt{c}}\right] + 2\#1b\sqrt{cd} \operatorname{Log}\left[\#1 + \sqrt{a + b\sqrt{cd} - \sqrt{c}}\right]}{\#1^2f - 4\#1af - 2\#1cd + b^2d}\right]}{2(a^2f^2 + 2acdf + b^2(-df) + c^2d^2)} - \frac{2(2a^2cf - ab^2f - abcfx + 2ac^2d + bc^2d)}{(4ac - b^2)\sqrt{a + bx + cx^2}(a^2f^2 + 2acdf + b^2(-df) + c^2d^2)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)), x]
```

```
[Out] (-2*(2*a*c^2*d - a*b^2*f + 2*a^2*c*f + b*c^2*d*x - a*b*c*f*x))/((-b^2 + 4*a*c)*(c^2*d^2 - b^2*d*f + 2*a*c*d*f + a^2*f^2)*Sqrt[a + b*x + c*x^2]) + (f*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4])
```

```
& , (b^2*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*b*Sqrt[c]*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - a*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ])/(2*(c^2*d^2 - b^2*d*f + 2*a*c*d*f + a^2*f^2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Valueint() Bad Argument Type

maple [B] time = 0.02, size = 1360, normalized size = 4.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] -1/2/(a*f+c*d-(d*f)^(1/2)*b)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)-2/f/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*(d*f)^(1/2)*x*c^2+1/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*b*c-1/f/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b*c+1/2/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b^2+1/2/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/((x+(d*f)^(1/2)/f)-1/2/(a*f+c*d+(d*f)^(1/2)*b)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)+2/f/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*(d*f)^(1/2)*x*c^2+1/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*x*b*c+1/f/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*b^2+1/2/(a*f+c*d+(d*f)^(1/2)*b)/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/((x-(d*f)^(1/2)/f))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see 'assume?' for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2 -c*((b*sqrt(4*d*f)) / (2*f) + (c*d)/f+a) /f^2 zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{(d - f x^2) (c x^2 + b x + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)

[Out] int(x/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-ad\sqrt{a+bx+cx^2} + afx^2\sqrt{a+bx+cx^2} - bdx\sqrt{a+bx+cx^2} + bfx^3\sqrt{a+bx+cx^2} - cdx^2\sqrt{a+bx+cx^2} + cf x^4\sqrt{a+bx+cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)

[Out] -Integral(x/(-a*d*sqrt(a + b*x + c*x**2) + a*f*x**2*sqrt(a + b*x + c*x**2) - b*d*x*sqrt(a + b*x + c*x**2) + b*f*x**3*sqrt(a + b*x + c*x**2) - c*d*x**2*sqrt(a + b*x + c*x**2) + c*f*x**4*sqrt(a + b*x + c*x**2)), x)

$$3.106 \quad \int \frac{1}{(a+bx+cx^2)^{3/2}(d-fx^2)} dx$$

Optimal. Leaf size=310

$$\frac{2(b(b^2f - c(3af + cd)) - cx(2acf + b^2(-f) + 2c^2d))}{(b^2 - 4ac)\sqrt{a + bx + cx^2}(b^2df - (af + cd)^2)} + \frac{f \tanh^{-1}\left(\frac{-2a\sqrt{f} + x(2c\sqrt{d} - b\sqrt{f}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2\sqrt{d}(af + b(-\sqrt{d})\sqrt{f} + cd)^{3/2}} + \frac{f \tanh^{-1}\left(\frac{2a\sqrt{f} + x(b\sqrt{f} + 2c\sqrt{d}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}(af + b\sqrt{d}\sqrt{f} + cd)^{3/2}}$$

Rubi [A] time = 0.41, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {975, 1033, 724, 206}

$$\frac{2(b(b^2f - c(3af + cd)) - cx(2acf + b^2(-f) + 2c^2d))}{(b^2 - 4ac)\sqrt{a + bx + cx^2}(b^2df - (af + cd)^2)} + \frac{f \tanh^{-1}\left(\frac{-2a\sqrt{f} + x(2c\sqrt{d} - b\sqrt{f}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2\sqrt{d}(af + b(-\sqrt{d})\sqrt{f} + cd)^{3/2}} + \frac{f \tanh^{-1}\left(\frac{2a\sqrt{f} + x(b\sqrt{f} + 2c\sqrt{d}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}(af + b\sqrt{d}\sqrt{f} + cd)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]

[Out] (-2*(b*(b^2*f - c*(c*d + 3*a*f)) - c*(2*c^2*d - b^2*f + 2*a*c*f)*x))/((b^2 - 4*a*c)*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) + (f*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + b*x + c*x^2]])/(2*Sqrt[d]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (f*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + b*x + c*x^2]])/(2*Sqrt[d]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 975

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*f)))*x*(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1))/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^2*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] && (!IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]

Rule 1033

Int[((g_.) + (h_.)*(x_))/((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q

), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]

Rubi steps

$$\int \frac{1}{(a + bx + cx^2)^{3/2} (d - fx^2)} dx = -\frac{2(b(b^2f - c(cd + 3af)) - c(2c^2d - b^2f + 2acf)x)}{(b^2 - 4ac)(b^2df - (cd + af)^2)\sqrt{a + bx + cx^2}} - \frac{2 \int \frac{\frac{1}{2}(b^2 - 4ac)f(cd + af) - \dots}{\sqrt{a + bx + cx^2}} dx}{(b^2 - 4ac)(b^2df - (cd + af)^2)}$$

$$= -\frac{2(b(b^2f - c(cd + 3af)) - c(2c^2d - b^2f + 2acf)x)}{(b^2 - 4ac)(b^2df - (cd + af)^2)\sqrt{a + bx + cx^2}} - \frac{f^{3/2} \int \frac{1}{(-\sqrt{d}\sqrt{f} - fx)\sqrt{a - bx - cx^2}} dx}{2\sqrt{d}(cd - b\sqrt{d}\sqrt{f})}$$

$$= -\frac{2(b(b^2f - c(cd + 3af)) - c(2c^2d - b^2f + 2acf)x)}{(b^2 - 4ac)(b^2df - (cd + af)^2)\sqrt{a + bx + cx^2}} + \frac{f^{3/2} \text{Subst}\left(\int \frac{1}{4cdf - 4b\sqrt{d}\sqrt{f} - fx^2} dx\right)}{2\sqrt{d}(cd - b\sqrt{d}\sqrt{f})}$$

$$= -\frac{2(b(b^2f - c(cd + 3af)) - c(2c^2d - b^2f + 2acf)x)}{(b^2 - 4ac)(b^2df - (cd + af)^2)\sqrt{a + bx + cx^2}} + \frac{f \tanh^{-1}\left(\frac{b\sqrt{d} - 2a\sqrt{f}}{2\sqrt{cd - b\sqrt{d}\sqrt{f}}}\right)}{2\sqrt{d}(cd - b\sqrt{d}\sqrt{f})}$$

Mathematica [A] time = 0.40, size = 360, normalized size = 1.16

$$2 \left(\frac{f(b^2 - 4ac)(af + b\sqrt{d}\sqrt{f} + cd) \tanh^{-1}\left(\frac{-2a\sqrt{f} + b(\sqrt{d} - \sqrt{f}x) + 2c\sqrt{d}x}{2\sqrt{a + x(b + cx)}\sqrt{af + b(-\sqrt{d})\sqrt{f} + cd}}\right)}{4\sqrt{d}\sqrt{af + b(-\sqrt{d})\sqrt{f} + cd}} + \frac{f(4ac - b^2)(af + b(-\sqrt{d})\sqrt{f} + cd) \tanh^{-1}\left(\frac{-2(a\sqrt{f} + c\sqrt{d}x) - b(\sqrt{d} + \sqrt{f}x)}{2\sqrt{a + x(b + cx)}\sqrt{af + b\sqrt{d}\sqrt{f} + cd}}\right)}{4\sqrt{d}\sqrt{af + b\sqrt{d}\sqrt{f} + cd}} + \frac{-bc(3af + cd) - 2c^2x(af + cd) + b^3f + b^2cfx}{\sqrt{a + x(b + cx)}} \right) / ((b^2 - 4ac)((af + cd)^2 - b^2df))$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)), x]

[Out] (2*((b^3*f - b*c*(c*d + 3*a*f) + b^2*c*f*x - 2*c^2*(c*d + a*f)*x)/Sqrt[a + x*(b + c*x)] + ((b^2 - 4*a*c)*f*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + x*(b + c*x)])])/(4*Sqrt[d]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f) + ((-b^2 + 4*a*c)*f*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + x*(b + c*x)])])/(4*Sqrt[d]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f)))/((b^2 - 4*a*c)*(-(b^2*d*f) + (c*d + a*f)^2))

IntegrateAlgebraic [C] time = 0.88, size = 392, normalized size = 1.26

$$\frac{f \text{RootSum}\left[\#1^4(-f) + 2\#1^2af + 4\#1^2cd - 4\#1b\sqrt{c}d - a^2f + b^2d\&\amp; \frac{\#1^2(-b)\log(-\#1 + \sqrt{a + b\sqrt{c}x^2} - \sqrt{c}) - 2\#1^2d\log(-\#1 + \sqrt{a + b\sqrt{c}x^2} - \sqrt{c}) + b\sqrt{c}\log(-\#1 + \sqrt{a + b\sqrt{c}x^2} - \sqrt{c}) - 2\#1a\sqrt{f}\log(-\#1 + \sqrt{a + b\sqrt{c}x^2} - \sqrt{c}) + 2abf\log(-\#1 + \sqrt{a + b\sqrt{c}x^2} - \sqrt{c})}{\#1^3f - \#1af - 2\#1cd + b\sqrt{c}d}\&\amp; \right]}{2(a^2f^2 + 2acdf + b^2(-d)f + c^2d^2)} - \frac{2(-3abcf - 2ac^2fx + b^2f + b^2cfx - b^2d - 2a^2dx)}{(b^2 - 4ac)\sqrt{a + bx + cx^2}(-a^2f^2 - 2acdf + b^2df - c^2d^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)), x]

[Out] (-2*(-(b*c^2*d) + b^3*f - 3*a*b*c*f - 2*c^3*d*x + b^2*c*f*x - 2*a*c^2*f*x))/((b^2 - 4*a*c)*(-(c^2*d^2) + b^2*d*f - 2*a*c*d*f - a^2*f^2)*Sqrt[a + b*x + c*x^2]) - (f*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (b*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*a*b*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*c^(3/2)*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*a*Sqrt[c]*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - b*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x +

$c*x^2] - \#1*\#1^2)/(b*\text{Sqrt}[c]*d - 2*c*d*\#1 - a*f*\#1 + f*\#1^3) \&])/(2*(c^2*d^2 - b^2*d*f + 2*a*c*d*f + a^2*f^2))$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
i,const vecteur & l) Error: Bad Argument Valueint() Bad Argument Type

maple [B] time = 0.02, size = 1376, normalized size = 4.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)

[Out] $\frac{1}{2} \frac{1}{(d*f)^{1/2}} \frac{1}{(a*f+c*d-(d*f)^{1/2}*b)*f} \frac{1}{((x+(d*f)^{1/2}/f)^2*c+(b*f-2*(d*f)^{1/2}*c)*(x+(d*f)^{1/2}/f)/f+(a*f+c*d-(d*f)^{1/2}*b)/f)^{1/2}+2/(a*f+c*d-(d*f)^{1/2}*b)/(4*a*c-b^2)} \frac{1}{((x+(d*f)^{1/2}/f)^2*c+(b*f-2*(d*f)^{1/2}*c)*(x+(d*f)^{1/2}/f)/f+(a*f+c*d-(d*f)^{1/2}*b)/f)^{1/2}} * x*c^2-1/(d*f)^{1/2} \frac{1}{(a*f+c*d-(d*f)^{1/2}*b)/(4*a*c-b^2)} \frac{1}{((x+(d*f)^{1/2}/f)^2*c+(b*f-2*(d*f)^{1/2}*c)*(x+(d*f)^{1/2}/f)/f+(a*f+c*d-(d*f)^{1/2}*b)/f)^{1/2}} * x*b*c*f+1/(a*f+c*d-(d*f)^{1/2}*b)/(4*a*c-b^2) \frac{1}{((x+(d*f)^{1/2}/f)^2*c+(b*f-2*(d*f)^{1/2}*c)*(x+(d*f)^{1/2}/f)/f+(a*f+c*d-(d*f)^{1/2}*b)/f)^{1/2}} * b*c-1/2 \frac{1}{(d*f)^{1/2}} \frac{1}{(a*f+c*d-(d*f)^{1/2}*b)/(4*a*c-b^2)} \frac{1}{((x+(d*f)^{1/2}/f)^2*c+(b*f-2*(d*f)^{1/2}*c)*(x+(d*f)^{1/2}/f)/f+(a*f+c*d-(d*f)^{1/2}*b)/f)^{1/2}} * b^2*f-1/2 \frac{1}{(d*f)^{1/2}} \frac{1}{(a*f+c*d-(d*f)^{1/2}*b)*f} \frac{1}{((a*f+c*d-(d*f)^{1/2}*b)/f)^{1/2}} * \ln((2*(a*f+c*d-(d*f)^{1/2}*b)/f+(b*f-2*(d*f)^{1/2}*c)*(x+(d*f)^{1/2}/f)/f+2*((a*f+c*d-(d*f)^{1/2}*b)/f)^{1/2}) \frac{1}{((x+(d*f)^{1/2}/f)^2*c+(b*f-2*(d*f)^{1/2}*c)*(x+(d*f)^{1/2}/f)/f+(a*f+c*d-(d*f)^{1/2}*b)/f)^{1/2}} \frac{1}{(x+(d*f)^{1/2}/f)} -1/2 \frac{1}{(d*f)^{1/2}} \frac{1}{(a*f+c*d+(d*f)^{1/2}*b)*f} \frac{1}{((x-(d*f)^{1/2}/f)^2*c+(b*f+2*(d*f)^{1/2}*c)*(x-(d*f)^{1/2}/f)/f+(a*f+c*d+(d*f)^{1/2}*b)/f)^{1/2}+2/(a*f+c*d+(d*f)^{1/2}*b)/(4*a*c-b^2)} \frac{1}{((x-(d*f)^{1/2}/f)^2*c+(b*f+2*(d*f)^{1/2}*c)*(x-(d*f)^{1/2}/f)/f+(a*f+c*d+(d*f)^{1/2}*b)/f)^{1/2}} * x*c^2+1/(d*f)^{1/2} \frac{1}{(a*f+c*d+(d*f)^{1/2}*b)/(4*a*c-b^2)} \frac{1}{((x-(d*f)^{1/2}/f)^2*c+(b*f+2*(d*f)^{1/2}*c)*(x-(d*f)^{1/2}/f)/f+(a*f+c*d+(d*f)^{1/2}*b)/f)^{1/2}} * x*b*c*f+1/(a*f+c*d+(d*f)^{1/2}*b)/(4*a*c-b^2) \frac{1}{((x-(d*f)^{1/2}/f)^2*c+(b*f+2*(d*f)^{1/2}*c)*(x-(d*f)^{1/2}/f)/f+(a*f+c*d+(d*f)^{1/2}*b)/f)^{1/2}} * b*c+1/2 \frac{1}{(d*f)^{1/2}} \frac{1}{(a*f+c*d+(d*f)^{1/2}*b)/(4*a*c-b^2)} \frac{1}{((x-(d*f)^{1/2}/f)^2*c+(b*f+2*(d*f)^{1/2}*c)*(x-(d*f)^{1/2}/f)/f+(a*f+c*d+(d*f)^{1/2}*b)/f)^{1/2}} * b^2*f+1/2 \frac{1}{(d*f)^{1/2}} \frac{1}{(a*f+c*d+(d*f)^{1/2}*b)*f} \frac{1}{((a*f+c*d+(d*f)^{1/2}*b)/f)^{1/2}} * \ln((2*(a*f+c*d+(d*f)^{1/2}*b)/f+(b*f+2*(d*f)^{1/2}*c)*(x-(d*f)^{1/2}/f)/f+2*((a*f+c*d+(d*f)^{1/2}*b)/f)^{1/2}) \frac{1}{((x-(d*f)^{1/2}/f)^2*c+(b*f+2*(d*f)^{1/2}*c)*(x-(d*f)^{1/2}/f)/f+(a*f+c*d+(d*f)^{1/2}*b)/f)^{1/2}} \frac{1}{(x-(d*f)^{1/2}/f)}$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see 'assume?' for more details)Is ((c*sqrt(4*d*f))/(2*f^2) +b/(2*f)) ^2 -c*((b*sqrt(4*d*f)) / (2*f) + (c*d)/f+a) /f^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(d - f x^2) (c x^2 + b x + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)

[Out] int(1/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-ad\sqrt{a+bx+cx^2} + afx^2\sqrt{a+bx+cx^2} - bdx\sqrt{a+bx+cx^2} + bfx^3\sqrt{a+bx+cx^2} - cdx^2\sqrt{a+bx+cx^2} + cfx^4\sqrt{a+bx+cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)

[Out] -Integral(1/(-a*d*sqrt(a + b*x + c*x**2) + a*f*x**2*sqrt(a + b*x + c*x**2) - b*d*x*sqrt(a + b*x + c*x**2) + b*f*x**3*sqrt(a + b*x + c*x**2) - c*d*x**2*sqrt(a + b*x + c*x**2) + c*f*x**4*sqrt(a + b*x + c*x**2)), x)

$$3.107 \quad \int \frac{1}{x(a+bx+cx^2)^{3/2}(d-fx^2)} dx$$

Optimal. Leaf size=394

$$\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{a^{3/2}d} - \frac{2f(a(2acf+b^2(-f)+2c^2d)+bcx(cd-af))}{d(b^2-4ac)\sqrt{a+bx+cx^2}(b^2df-(af+cd)^2)} + \frac{2(-2ac+b^2+bcx)}{ad(b^2-4ac)\sqrt{a+bx+cx^2}} - \frac{f^{3/2}}{d(b^2-4ac)\sqrt{a+bx+cx^2}}$$

Rubi [A] time = 1.18, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 28, number of rules / integrand size = 0.250, Rules used = {6725, 740, 12, 724, 206, 1018, 1033}

$$\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{a^{3/2}d} - \frac{2f(a(2acf+b^2(-f)+2c^2d)+bcx(cd-af))}{d(b^2-4ac)\sqrt{a+bx+cx^2}(b^2df-(af+cd)^2)} + \frac{2(-2ac+b^2+bcx)}{ad(b^2-4ac)\sqrt{a+bx+cx^2}} - \frac{f^{3/2} \tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2d(af+b(-\sqrt{d})\sqrt{f}+cd)^{3/2}} + \frac{f^{3/2} \tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d(af+b\sqrt{d}\sqrt{f}+cd)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]

[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*d*Sqrt[a + b*x + c*x^2]) - (2*f*(a*(2*c^2*d - b^2*f + 2*a*c*f) + b*c*(c*d - a*f)*x))/((b^2 - 4*a*c)*d*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) - ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/(a^(3/2)*d) - (f^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + b*x + c*x^2])]/(2*d*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (f^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + b*x + c*x^2])]/(2*d*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 740

Int[((d_.) + (e_.)*(x_)^(m_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,

-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]

Rule 1018

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x))/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p + q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] && !(IntegerQ[p] && ILtQ[q, -1])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x(a+bx+cx^2)^{3/2}(d-fx^2)} dx &= \int \left(\frac{1}{dx(a+bx+cx^2)^{3/2}} - \frac{fx}{d(a+bx+cx^2)^{3/2}(-d+fx^2)} \right) dx \\ &= \frac{\int \frac{1}{x(a+bx+cx^2)^{3/2}} dx}{d} - \frac{f \int \frac{x}{(a+bx+cx^2)^{3/2}(-d+fx^2)} dx}{d} \\ &= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)d\sqrt{a+bx+cx^2}} - \frac{2f(a(2c^2d - b^2f + 2acf) + bc(cd - af)x)}{(b^2 - 4ac)d(b^2df - (cd + af)^2)\sqrt{a+bx+cx^2}} \\ &= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)d\sqrt{a+bx+cx^2}} - \frac{2f(a(2c^2d - b^2f + 2acf) + bc(cd - af)x)}{(b^2 - 4ac)d(b^2df - (cd + af)^2)\sqrt{a+bx+cx^2}} \\ &= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)d\sqrt{a+bx+cx^2}} - \frac{2f(a(2c^2d - b^2f + 2acf) + bc(cd - af)x)}{(b^2 - 4ac)d(b^2df - (cd + af)^2)\sqrt{a+bx+cx^2}} \\ &= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)d\sqrt{a+bx+cx^2}} - \frac{2f(a(2c^2d - b^2f + 2acf) + bc(cd - af)x)}{(b^2 - 4ac)d(b^2df - (cd + af)^2)\sqrt{a+bx+cx^2}} \end{aligned}$$

Mathematica [A] time = 0.87, size = 436, normalized size = 1.11

$$\frac{\frac{\tanh^{-1}\left(\frac{2a+bc}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{a^{3/2}} - \frac{2f(a(2acf+b^2(-f)+2c^2d)+bcx(cd-af))}{(b^2-4ac)\sqrt{a+x(b+cx)}(b^2df-(af+cd)^2)} + \frac{f^{3/2}\left((af+b(-\sqrt{d})\sqrt{f+cd})^{3/2} \tanh^{-1}\left(\frac{2a\sqrt{f+b(-\sqrt{d})\sqrt{f+cd}}+2c\sqrt{dx}}{2\sqrt{a+x(b+cx)}\sqrt{af+b(-\sqrt{d})\sqrt{f+cd}}}\right) + (af+b\sqrt{d}\sqrt{f+cd})^{3/2} \tanh^{-1}\left(\frac{2a\sqrt{f-b\sqrt{d}+b\sqrt{f}x-2c\sqrt{dx}}}{2\sqrt{a+x(b+cx)}\sqrt{af+b(-\sqrt{d})\sqrt{f+cd}}}\right)\right)}{2\sqrt{af+b(-\sqrt{d})\sqrt{f+cd}}\sqrt{af+b\sqrt{d}\sqrt{f+cd}}(af+cd)^2-b^2df}} + \frac{2(-2ac+b^2+bcx)}{a(b^2-4ac)\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]

[Out]
$$\left(\frac{(2(b^2 - 2ac + bcx))/(a(b^2 - 4ac)\sqrt{a + x(b + cx)}) - (2f(a(2c^2d - b^2f + 2ac^2f) + bc^2(cd - af) + b^2c(cd - af)x))/(b^2 - 4ac)(b^2d^2f - (cd + af)^2)\sqrt{a + x(b + cx)}}{a^{3/2}} + \frac{(f^{3/2}((cd + b\sqrt{d})\sqrt{f} + a)^{3/2}\text{ArcTanh}[-(b\sqrt{d}) + 2a\sqrt{f} - 2c\sqrt{d}x + b\sqrt{f}x]/(2\sqrt{cd - b\sqrt{d}\sqrt{f} + af})\sqrt{a + x(b + cx)}) + (cd - b\sqrt{d}\sqrt{f} + a)^{3/2}\text{ArcTanh}[(b\sqrt{d} + 2a\sqrt{f} + 2c\sqrt{d}x + b\sqrt{f}x)/(2\sqrt{cd + b\sqrt{d}\sqrt{f} + af})\sqrt{a + x(b + cx)})]}{2\sqrt{cd - b\sqrt{d}\sqrt{f} + af}\sqrt{cd + b\sqrt{d}\sqrt{f} + af}}\right)/d$$

IntegrateAlgebraic [C] time = 1.59, size = 498, normalized size = 1.26

$$\frac{f^2\text{RootSum}\left[\frac{1}{2d(x^2 + 2cdf + b^2(-d) + c^2d^2)}\right] + \frac{2f\sqrt{d}\sqrt{a + b\sqrt{d}\sqrt{f} + a}}{a^{3/2}} + \frac{2(2b^2d^2 - 4af^2c - 3ab^2f + 2ac^2d + b^2f + b^2cd - bc^2dx)}{a(b^2 - 4ac)\sqrt{a + bx + cx^2}(-b^2df - c^2d^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]

[Out]
$$\frac{(2(-(b^2c^2d) + 2a^2c^3d + b^4f - 4ab^2c^2f + 2a^2c^2f - bc^3d)x + b^3c^2fx - 3ab^2c^2fx)/(a(b^2 - 4ac)(-c^2d^2 + b^2d^2f - 2ac^2d^2f - a^2f^2)\sqrt{a + bx + cx^2}) + (2\text{ArcTanh}[(\sqrt{c}x)/\sqrt{a} - \sqrt{a + bx + cx^2}/\sqrt{a}])/(a^{3/2}d) + (f^2\text{RootSum}[b^2d - a^2f - 4b\sqrt{c}d\sqrt{c}d\sqrt{c} + 4c^2d\sqrt{c}d\sqrt{c} + 2a^2f\sqrt{c}d\sqrt{c} - f\sqrt{c}d\sqrt{c}d\sqrt{c} \& , (b^2d\text{Log}[-(\sqrt{c}x) + \sqrt{a + bx + cx^2} - \sqrt{c}d] + a^2f\text{Log}[-(\sqrt{c}x) + \sqrt{a + bx + cx^2} - \sqrt{c}d] - 2b\sqrt{c}d\text{Log}[-(\sqrt{c}x) + \sqrt{a + bx + cx^2} - \sqrt{c}d])\sqrt{c}d - c^2d\text{Log}[-(\sqrt{c}x) + \sqrt{a + bx + cx^2} - \sqrt{c}d])\sqrt{c}d - 2c^2d\sqrt{c}d\sqrt{c} - a^2f\sqrt{c}d\sqrt{c}d\sqrt{c} \&])/(2d(c^2d^2 - b^2d^2f + 2ac^2d^2f + a^2f^2))$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage2

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")

[Out] sage2

maple [B] time = 0.02, size = 1518, normalized size = 3.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)`

[Out]
$$-1/2/d/(a*f+c*d-(d*f)^{(1/2)*b})*f/((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*c*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}-2/d/(a*f+c*d-(d*f)^{(1/2)*b})/(4*a*c-b^2)/((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*c*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*(d*f)^{(1/2)*x*c^2+1/d/(a*f+c*d-(d*f)^{(1/2)*b})/(4*a*c-b^2)/((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*c*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*x*b*c*f-1/d/(a*f+c*d-(d*f)^{(1/2)*b})/(4*a*c-b^2)/((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*c*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*(d*f)^{(1/2)*b*c+1/2/d/(a*f+c*d-(d*f)^{(1/2)*b})/(4*a*c-b^2)/((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*c*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*b^2*f+1/2/d/(a*f+c*d-(d*f)^{(1/2)*b})*f/((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*\ln((2*(a*f+c*d-(d*f)^{(1/2)*b})/f+(b*f-2*(d*f)^{(1/2)*c})*c*(x+(d*f)^{(1/2)/f})/f+2*((a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)}*((x+(d*f)^{(1/2)/f})^2*c+(b*f-2*(d*f)^{(1/2)*c})*c*(x+(d*f)^{(1/2)/f})/f+(a*f+c*d-(d*f)^{(1/2)*b})/f)^{(1/2)})/(x+(d*f)^{(1/2)/f})-1/2/d/(a*f+c*d+(d*f)^{(1/2)*b})*f/((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c})*c*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}+2/d/(a*f+c*d+(d*f)^{(1/2)*b})/(4*a*c-b^2)/((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c})*c*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*(d*f)^{(1/2)*x*c^2+1/d/(a*f+c*d+(d*f)^{(1/2)*b})/(4*a*c-b^2)/((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c})*c*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*x*b*c*f+1/d/(a*f+c*d+(d*f)^{(1/2)*b})/(4*a*c-b^2)/((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c})*c*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*(d*f)^{(1/2)*b*c+1/2/d/(a*f+c*d+(d*f)^{(1/2)*b})/(4*a*c-b^2)/((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c})*c*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*\ln((2*(a*f+c*d+(d*f)^{(1/2)*b})/f+(b*f+2*(d*f)^{(1/2)*c})*c*(x-(d*f)^{(1/2)/f})/f+2*((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)}*((x-(d*f)^{(1/2)/f})^2*c+(b*f+2*(d*f)^{(1/2)*c})*c*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)})/(x-(d*f)^{(1/2)/f})+1/d/a/(c*x^2+b*x+a)^(1/2)-2/d*b/a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*c*x-1/d*b^2/a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-1/d/a^(3/2)*\ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{(cx^2 + bx + a)^{\frac{3}{2}}(fx^2 - d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")`

[Out] `-integrate(1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)*x), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x(d - fx^2)(cx^2 + bx + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)`

[Out] `int(1/(x*(d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-adx\sqrt{a+bx+cx^2} + afx^3\sqrt{a+bx+cx^2} - bdx^2\sqrt{a+bx+cx^2} + bfx^4\sqrt{a+bx+cx^2} - cdx^3\sqrt{a+bx+cx^2} + cfx^5\sqrt{a+bx+cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(1/(-a*d*x*sqrt(a + b*x + c*x**2) + a*f*x**3*sqrt(a + b*x + c*x**2) - b*d*x**2*sqrt(a + b*x + c*x**2) + b*f*x**4*sqrt(a + b*x + c*x**2) - c*d*x**3*sqrt(a + b*x + c*x**2) + c*f*x**5*sqrt(a + b*x + c*x**2)), x)
```

$$3.108 \quad \int \frac{1}{x^2(a+bx+cx^2)^{3/2}(d-fx^2)} dx$$

Optimal. Leaf size=454

$$\frac{3b \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{5/2}d} - \frac{(3b^2 - 8ac)\sqrt{a+bx+cx^2}}{a^2 dx(b^2 - 4ac)} - \frac{2f(b(b^2f - c(3af + cd)) - cx(2acf + b^2(-f) + 2c^2d))}{d(b^2 - 4ac)\sqrt{a+bx+cx^2}(b^2df - (af + cd)^2)}$$

Rubi [A] time = 1.19, antiderivative size = 454, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 28, number of rules / integrand size = 0.250, Rules used = {6725, 740, 806, 724, 206, 975, 1033}

$$\frac{(3b^2 - 8ac)\sqrt{a+bx+cx^2}}{a^2 dx(b^2 - 4ac)} + \frac{3b \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{5/2}d} - \frac{2f(b(b^2f - c(3af + cd)) - cx(2acf + b^2(-f) + 2c^2d))}{d(b^2 - 4ac)\sqrt{a+bx+cx^2}(b^2df - (af + cd)^2)} + \frac{2(-2ac + b^2 + bcx)}{adx(b^2 - 4ac)\sqrt{a+bx+cx^2}} + \frac{f^2 \tanh^{-1}\left(\frac{-2x\sqrt{f} + (2c\sqrt{a} - \sqrt{f}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2d^{3/2}(af + b(-\sqrt{d})\sqrt{f} + cd)^{3/2}} + \frac{f^2 \tanh^{-1}\left(\frac{2x\sqrt{f} + (b\sqrt{f} + 2c\sqrt{a}) + b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{3/2}(af + b\sqrt{d}\sqrt{f} + cd)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]

[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*d*x*Sqrt[a + b*x + c*x^2]) - (2*f*(b*(b^2*f - c*(c*d + 3*a*f)) - c*(2*c^2*d - b^2*f + 2*a*c*f)*x))/((b^2 - 4*a*c)*d*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) - ((3*b^2 - 8*a*c)*Sqrt[a + b*x + c*x^2])/(a^2*(b^2 - 4*a*c)*d*x) + (3*b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*a^(5/2)*d) + (f^2*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + b*x + c*x^2]])/(2*d^(3/2)*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (f^2*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f)*Sqrt[a + b*x + c*x^2]])/(2*d^(3/2)*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 740

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]

Rule 806

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b

```
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
+ d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 975

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] := Simp[((b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*
f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1))/((b^2 - 4*a*c)*(b^2*d
*f + (c*d - a*f)^2)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^2
*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1)
- c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d +
b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] && !(IntegerQ[
p] && ILtQ[q, -1]) && !IGtQ[q, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]], x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2 (a + bx + cx^2)^{3/2} (d - fx^2)} dx = \int \left(\frac{1}{dx^2 (a + bx + cx^2)^{3/2}} + \frac{f}{d (a + bx + cx^2)^{3/2} (d - fx^2)} \right) dx$$

$$= \frac{\int \frac{1}{x^2 (a + bx + cx^2)^{3/2}} dx}{d} + \frac{f \int \frac{1}{(a + bx + cx^2)^{3/2} (d - fx^2)} dx}{d}$$

$$= \frac{2 (b^2 - 2ac + bcx)}{a (b^2 - 4ac) dx \sqrt{a + bx + cx^2}} - \frac{2f (b (b^2 f - c(cd + 3af)) - c (2c^2 d - b^2 f))}{(b^2 - 4ac) d (b^2 d f - (cd + af)^2) \sqrt{a + bx + cx^2}}$$

$$= \frac{2 (b^2 - 2ac + bcx)}{a (b^2 - 4ac) dx \sqrt{a + bx + cx^2}} - \frac{2f (b (b^2 f - c(cd + 3af)) - c (2c^2 d - b^2 f))}{(b^2 - 4ac) d (b^2 d f - (cd + af)^2) \sqrt{a + bx + cx^2}}$$

$$= \frac{2 (b^2 - 2ac + bcx)}{a (b^2 - 4ac) dx \sqrt{a + bx + cx^2}} - \frac{2f (b (b^2 f - c(cd + 3af)) - c (2c^2 d - b^2 f))}{(b^2 - 4ac) d (b^2 d f - (cd + af)^2) \sqrt{a + bx + cx^2}}$$

$$= \frac{2 (b^2 - 2ac + bcx)}{a (b^2 - 4ac) dx \sqrt{a + bx + cx^2}} - \frac{2f (b (b^2 f - c(cd + 3af)) - c (2c^2 d - b^2 f))}{(b^2 - 4ac) d (b^2 d f - (cd + af)^2) \sqrt{a + bx + cx^2}}$$

Mathematica [A] time = 1.19, size = 488, normalized size = 1.07

$$\frac{3b(b^2 - 4ac) \operatorname{tanh}^{-1}\left(\frac{2a + bx}{2\sqrt{a}\sqrt{a + bx + cx^2}}\right) + \frac{2(8ac - 3b^2)\sqrt{a + bx + cx^2}}{a^2 x} - \frac{f^2 \left(\frac{(b^2 - 4ac)(ef + b\sqrt{d}\sqrt{d - fx^2}) \operatorname{tanh}^{-1}\left(\frac{2a\sqrt{d} - b\sqrt{d} + \sqrt{d}x - 2\sqrt{d}x}{2\sqrt{a + bx + cx^2}\sqrt{ef + b(-\sqrt{d})\sqrt{d - fx^2}}}\right) + \frac{(4ac - b^2)(ef + b(-\sqrt{d})\sqrt{d - fx^2}) \operatorname{tanh}^{-1}\left(\frac{2a\sqrt{d} + b\sqrt{d} + \sqrt{d}x + 2\sqrt{d}x}{2\sqrt{a + bx + cx^2}\sqrt{ef + b\sqrt{d}\sqrt{d - fx^2}}}\right)}{\sqrt{ef + b(-\sqrt{d})\sqrt{d - fx^2}}} \right)}{\sqrt{d}(af + cd)^2 - b^2 df}} + \frac{4(-2ac + b^2 + bcx)}{a^2 \sqrt{a + bx + cx^2}} - \frac{4f(-bc(3af + cd) - 2d^2 x(ef + cd) + b^3 f + d^2 c f x)}{\sqrt{a + bx + cx^2}(b^2 d f - (cd + af)^2)}}{2d(b^2 - 4ac)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
[Out] ((4*(b^2 - 2*a*c + b*c*x))/(a*x*Sqrt[a + x*(b + c*x)]) - (4*f*(b^3*f - b*c*(c*d + 3*a*f) + b^2*c*f*x - 2*c^2*(c*d + a*f)*x))/((b^2*d*f - (c*d + a*f)^2)*Sqrt[a + x*(b + c*x)]) + (2*(-3*b^2 + 8*a*c)*Sqrt[a + x*(b + c*x)]/(a^2*x) + (3*b*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/a^(5/2) - (f^2*((b^2 - 4*a*c)*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x]/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])))/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f] + ((-b^2 + 4*a*c)*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])))/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f))/(Sqrt[d]*(-(b^2*d*f) + (c*d + a*f)^2))/(2*(b^2 - 4*a*c)*d)
```

IntegrateAlgebraic [C] time = 2.92, size = 649, normalized size = 1.43

$$\frac{f^2 \operatorname{tanh}^{-1}\left(\frac{2a + bx}{2\sqrt{a}\sqrt{a + bx + cx^2}}\right) + \frac{2(8ac - 3b^2)\sqrt{a + bx + cx^2}}{a^2 x} - \frac{f^2 \left(\frac{(b^2 - 4ac)(ef + b\sqrt{d}\sqrt{d - fx^2}) \operatorname{tanh}^{-1}\left(\frac{2a\sqrt{d} - b\sqrt{d} + \sqrt{d}x - 2\sqrt{d}x}{2\sqrt{a + bx + cx^2}\sqrt{ef + b(-\sqrt{d})\sqrt{d - fx^2}}}\right) + \frac{(4ac - b^2)(ef + b(-\sqrt{d})\sqrt{d - fx^2}) \operatorname{tanh}^{-1}\left(\frac{2a\sqrt{d} + b\sqrt{d} + \sqrt{d}x + 2\sqrt{d}x}{2\sqrt{a + bx + cx^2}\sqrt{ef + b\sqrt{d}\sqrt{d - fx^2}}}\right)}{\sqrt{ef + b(-\sqrt{d})\sqrt{d - fx^2}}} \right)}{\sqrt{d}(af + cd)^2 - b^2 df}} + \frac{4(-2ac + b^2 + bcx)}{a^2 \sqrt{a + bx + cx^2}} - \frac{4f(-bc(3af + cd) - 2d^2 x(ef + cd) + b^3 f + d^2 c f x)}{\sqrt{a + bx + cx^2}(b^2 d f - (cd + af)^2)}}{2d(b^2 - 4ac)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
[Out] (a*b^2*c^2*d^2 - 4*a^2*c^3*d^2 - a*b^4*d*f + 6*a^2*b^2*c*d*f - 8*a^3*c^2*d*f + a^3*b^2*f^2 - 4*a^4*c*f^2 + 3*b^3*c^2*d^2*x - 10*a*b*c^3*d^2*x - 3*b^5*d*f*x + 16*a*b^3*c*d*f*x - 18*a^2*b*c^2*d*f*x + a^2*b^3*f^2*x - 4*a^3*b*c*f^2*x + 3*b^2*c^3*d^2*x^2 - 8*a*c^4*d^2*x^2 - 3*b^4*c*d*f*x^2 + 14*a*b^2*c^2
```

```
*d*f*x^2 - 12*a^2*c^3*d*f*x^2 + a^2*b^2*c*f^2*x^2 - 4*a^3*c^2*f^2*x^2)/(a^2
*(-b^2 + 4*a*c)*d*(c^2*d^2 - b^2*d*f + 2*a*c*d*f + a^2*f^2)*x*Sqrt[a + b*x
+ c*x^2]) - (3*b*ArcTanh[(Sqrt[c]*x)/Sqrt[a] - Sqrt[a + b*x + c*x^2]/Sqrt[a
]])/(a^(5/2)*d) - (f^2*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^
2 + 2*a*f*#1^2 - f*#1^4 & , (b*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2]
- #1] + 2*a*b*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*c^(3/2)
*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*a*Sqrt[c]*f*Log[-(
Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - b*f*Log[-(Sqrt[c]*x) + Sqrt[a
+ b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ])
/(2*d*(c^2*d^2 - b^2*d*f + 2*a*c*d*f + a^2*f^2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 1.84int()
Bad Argument Type
```

maple [B] time = 0.02, size = 1656, normalized size = 3.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] 1/2*f^2/d/(d*f)^(1/2)/(a*f+c*d-(d*f)^(1/2)*b)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2
*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)+2*f/d/
(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/
2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*c^2-f^2/d/(d*f
)^(1/2)/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(
d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*b*c+f/
d/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(
1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b*c-1/2*f^2/d/
(d*f)^(1/2)/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f
-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b^2-
1/2*f^2/d/(d*f)^(1/2)/(a*f+c*d-(d*f)^(1/2)*b)/((a*f+c*d-(d*f)^(1/2)*b)/f)^(
1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f
)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)
^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1
/2)/f))-1/d/a/x/(c*x^2+b*x+a)^(1/2)-3/2/d*b/a^2/(c*x^2+b*x+a)^(1/2)+3/d*b^2
/a^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*c*x+3/2/d*b^3/a^2/(4*a*c-b^2)/(c*x^2+b
*x+a)^(1/2)+3/2/d*b/a^(5/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-8
/d*c^2/a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-4/d*c/a/(4*a*c-b^2)/(c*x^2+b*x+a
)^(1/2)*b-1/2*f^2/d/(d*f)^(1/2)/(a*f+c*d+(d*f)^(1/2)*b)/((x-(d*f)^(1/2)/f)^
2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1
/2)+2*f/d/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2
```

$$\begin{aligned} &*(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)*x*c^2+ \\ &f^2/d/(d*f)^{(1/2)/(a*f+c*d+(d*f)^{(1/2)*b})/(4*a*c-b^2)/((x-(d*f)^{(1/2)/f})^2* \\ &c+(b*f+2*(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2) \\ &)*x*b*c+f/d/(a*f+c*d+(d*f)^{(1/2)*b})/(4*a*c-b^2)/((x-(d*f)^{(1/2)/f})^2*c+(b*f \\ &+2*(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)*b*c+ \\ &1/2*f^2/d/(d*f)^{(1/2)/(a*f+c*d+(d*f)^{(1/2)*b})/(4*a*c-b^2)/((x-(d*f)^{(1/2)/f} \\ &)^2*c+(b*f+2*(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(\\ &(1/2)*b^2+1/2*f^2/d/(d*f)^{(1/2)/(a*f+c*d+(d*f)^{(1/2)*b})/((a*f+c*d+(d*f)^{(1/ \\ &2)*b)/f)^{(1/2)*ln((2*(a*f+c*d+(d*f)^{(1/2)*b})/f+(b*f+2*(d*f)^{(1/2)*c}*(x-(d* \\ &f)^{(1/2)/f})/f+2*((a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)*((x-(d*f)^{(1/2)/f})^2*c+(b \\ &*f+2*(d*f)^{(1/2)*c}*(x-(d*f)^{(1/2)/f})/f+(a*f+c*d+(d*f)^{(1/2)*b})/f)^{(1/2)))/(\\ &x-(d*f)^{(1/2)/f)} \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{(cx^2 + bx + a)^{\frac{3}{2}}(fx^2 - d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d), x, algorithm="maxima")

[Out] -integrate(1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2 (d - f x^2) (c x^2 + b x + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)

[Out] int(1/(x^2*(d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-adx^2\sqrt{a+bx+cx^2}+afx^4\sqrt{a+bx+cx^2}-bdx^3\sqrt{a+bx+cx^2}+bfx^5\sqrt{a+bx+cx^2}-cdx^4\sqrt{a+bx+cx^2}+cfx^6\sqrt{a+bx+cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d), x)

[Out] -Integral(1/(-a*d*x**2*sqrt(a + b*x + c*x**2) + a*f*x**4*sqrt(a + b*x + c*x**2) - b*d*x**3*sqrt(a + b*x + c*x**2) + b*f*x**5*sqrt(a + b*x + c*x**2) - c*d*x**4*sqrt(a + b*x + c*x**2) + c*f*x**6*sqrt(a + b*x + c*x**2)), x)


```
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1067

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (C_)*(x_)^2)*((d_) +
(e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((C*(b*f*p - c*e*(2*p +
q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + e*x + f*x^2)^(q +
1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)
*(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[
p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) - c*(C*e)*(2*p + 2*q + 3)) + (p + q +
1)*(b^2*C*d*f*p + a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(-2*A*f)*(2*p + 2*
q + 3))) + (2*p*(c*d - a*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e)*(2*p + 2*q + 3
)) + (p + q + 1)*(C*e*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2
) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))]*x + (p*(c*e - b*f)*(C*(c*e - b*f)*
(q + 1) - c*(C*e)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c
^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))]*x^
2, x], x] /; FreeQ[{a, b, c, d, e, f, A, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_) + (B_)*(x_) + (C_)*(x_)^2)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)
*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{x^2 \sqrt{a + bx + cx^2}}{d + ex + fx^2} dx = -\frac{(4ce - bf - 2cfx)\sqrt{a + bx + cx^2}}{4cf^2} - \frac{\int \frac{-\frac{1}{4}d(4bce - b^2f - 4acf) - \frac{1}{4}(8c^2de - b^2ef - 4acef + 4bc(e^2 - 2df))x + \dots}{\sqrt{a + bx + cx^2}(d + ex + fx^2)} dx}{2cf^2}$$

$$= -\frac{(4ce - bf - 2cfx)\sqrt{a + bx + cx^2}}{4cf^2} - \frac{\int \frac{-\frac{1}{4}df(4bce - b^2f - 4acf) - \frac{1}{4}d(b^2f^2 + 4cf(be - af) - 8c^2(e^2 - df)) + \dots}{\sqrt{a + bx + cx^2}} dx}{\sqrt{a + bx + cx^2}}$$

$$= -\frac{(4ce - bf - 2cfx)\sqrt{a + bx + cx^2}}{4cf^2} - \frac{(b^2f^2 + 4cf(be - af) - 8c^2(e^2 - df)) \text{Subst}\left(\int \frac{1}{4c} \dots}{4cf^3}\right)}{4cf^3}$$

$$= -\frac{(4ce - bf - 2cfx)\sqrt{a + bx + cx^2}}{4cf^2} - \frac{(b^2f^2 + 4cf(be - af) - 8c^2(e^2 - df)) \tanh^{-1}\left(\frac{\dots}{2\sqrt{\dots}}\right)}{8c^{3/2}f^3}$$

$$= -\frac{(4ce - bf - 2cfx)\sqrt{a + bx + cx^2}}{4cf^2} - \frac{(b^2f^2 + 4cf(be - af) - 8c^2(e^2 - df)) \tanh^{-1}\left(\frac{\dots}{2\sqrt{\dots}}\right)}{8c^{3/2}f^3}$$

Mathematica [A] time = 2.31, size = 552, normalized size = 0.73

Mathematica output showing various nested functions and radicals.

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
[Out] (((-b^2*f^2) + 4*c*f*(-(b*e) + a*f) + 8*c^2*(e^2 - d*f))*ArcTanh[(b + 2*c*x)
]/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])))/(8*c^(3/2)*f^3) + (f*Sqrt[e^2 - 4*d*f
]*(-4*c*e + b*f + 2*c*f*x)*Sqrt[a + x*(b + c*x)] + Sqrt[2]*c*(e^2 - 2*d*f +
e*Sqrt[e^2 - 4*d*f])*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f
- b*(e + Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])
*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e
Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b +
c*x)]]) + Sqrt[2]*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])*Sqrt[c*(e^2 - 2*d
*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]*ArcTanh
[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*
x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-
e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])))/(4*c*f^3*Sqrt[e^2 - 4*d*f
])
```

IntegrateAlgebraic [C] time = 1.45, size = 869, normalized size = 1.14

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
[Out] ((-4*c*e + b*f + 2*c*f*x)*Sqrt[a + b*x + c*x^2])/(4*c*f^2) + ((-8*c^2*e^2 +
8*c^2*d*f + 4*b*c*e*f + b^2*f^2 - 4*a*c*f^2)*Log[b*c + 2*c^2*x - 2*c^(3/2)
*Sqrt[a + b*x + c*x^2]])/(8*c^(3/2)*f^3) - RootSum[b^2*d - a*b*e + a^2*f -
4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 -
2*Sqrt[c]*e*#1^3 + f*#1^4 & , (-b*c*d*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x
+ c*x^2] - #1]) + a*c*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] +
b*c*d^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + b^2*d*e*f*Log[-(
Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*a*c*d*e*f*Log[-(Sqrt[c]*x) + S
qrt[a + b*x + c*x^2] - #1] - a*b*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*
x^2] - #1] + a^2*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*c
^(3/2)*d*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*c^(3/2)*
d^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*b*Sqrt[c]*d*e*f
*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*a*Sqrt[c]*d*f^2*Log[
-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - c*e^3*Log[-(Sqrt[c]*x) + Sq
rt[a + b*x + c*x^2] - #1]*#1^2 + 2*c*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x
+ c*x^2] - #1]*#1^2 + b*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1
]*#1^2 - b*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - a*e*
f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(2*b*Sqrt[c]*d - a
*Sqrt[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2*f*#1^3) &
]/f^3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT>Error: Bad Argument Type

maple [B] time = 0.02, size = 14815, normalized size = 19.47

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)

[Out] result too large to display

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \sqrt{cx^2 + bx + a}}{fx^2 + ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2),x)

[Out] int((x^2*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sqrt{a + bx + cx^2}}{d + ex + fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)

[Out] Integral(x**2*sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)

$$3.110 \quad \int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2} dx$$

Optimal. Leaf size=549

$$\frac{\left((e - \sqrt{e^2 - 4df}) (f(be - af) - c(e^2 - df)) + 2df(ce - bf) \right) \tanh^{-1} \left(\frac{4af + 2x(bf - c(e - \sqrt{e^2 - 4df})) - b(e - \sqrt{e^2 - 4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}} \right)}{\sqrt{2}f^2\sqrt{e^2 - 4df}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}}$$

Rubi [A] time = 7.03, antiderivative size = 549, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 28, number of rules / integrand size = 0.214, Rules used = {1019, 1076, 621, 206, 1032, 724}

$$\frac{\left((e - \sqrt{e^2 - 4df}) (f(be - af) - c(e^2 - df)) + 2df(ce - bf) \right) \tanh^{-1} \left(\frac{4af + 2x(bf - c(e - \sqrt{e^2 - 4df})) - b(e - \sqrt{e^2 - 4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}} \right)}{\sqrt{2}f^2\sqrt{e^2 - 4df}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}} + \frac{\left((\sqrt{e^2 - 4df} + e) (f(be - af) - c(e^2 - df)) + 2df(ce - bf) \right) \tanh^{-1} \left(\frac{4af + 2x(bf - c(e - \sqrt{e^2 - 4df})) - b(e - \sqrt{e^2 - 4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}} \right)}{\sqrt{2}f^2\sqrt{e^2 - 4df}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}} + \frac{(2ce - bf) \tanh^{-1} \left(\frac{bx}{2\sqrt{a+bx+cx^2}} \right)}{2\sqrt{e}f^2} + \frac{\sqrt{a+bx+cx^2}}{f}$$

Antiderivative was successfully verified.

[In] Int[(x*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2), x]

[Out] Sqrt[a + b*x + c*x^2]/f - ((2*c*e - b*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(2*Sqrt[c]*f^2) - ((2*d*f*(c*e - b*f) + (e - Sqrt[e^2 - 4*d*f])*(f*(b*e - a*f) - c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x]/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]))/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) + ((2*d*f*(c*e - b*f) + (e + Sqrt[e^2 - 4*d*f])*(f*(b*e - a*f) - c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x]/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]))/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1019

Int[((g_) + (h_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[(h*(a + b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[h*p*(b*d - a*e) + a*(h*e - 2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(h*e - 2*g*f)*(p + q + 1))*x + (h*p*(c*e - b*f) + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x] /; Fr

eeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]

Rule 1032

Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]

Rule 1076

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

Rubi steps

$$\int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2} dx = \frac{\sqrt{a+bx+cx^2}}{f} - \frac{\int \frac{\frac{bd}{2} + \frac{1}{2}(2cd+be-2af)x + \frac{1}{2}(2ce-bf)x^2}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx}{f}$$

$$= \frac{\sqrt{a+bx+cx^2}}{f} - \frac{\int \frac{\frac{bdf}{2} - \frac{1}{2}d(2ce-bf) + (\frac{1}{2}f(2cd+be-2af) - \frac{1}{2}e(2ce-bf))x}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx}{f^2} - \frac{(2ce-bf) \int \frac{1}{\sqrt{a+bx+cx^2}}}{2f^2}$$

$$= \frac{\sqrt{a+bx+cx^2}}{f} - \frac{(2ce-bf) \text{Subst}\left(\int \frac{1}{4c-x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f^2} - \frac{\left(2f\left(\frac{bdf}{2} - \frac{1}{2}d(2ce-bf)\right) - \left(\frac{1}{2}\right)\right)}{2f^2}$$

$$= \frac{\sqrt{a+bx+cx^2}}{f} - \frac{(2ce-bf) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2} + \frac{\left(2\left(2f\left(\frac{bdf}{2} - \frac{1}{2}d(2ce-bf)\right) - \left(\frac{1}{2}\right)\right)\right)}{\sqrt{2}f^2\sqrt{a+bx+cx^2}}$$

$$= \frac{\sqrt{a+bx+cx^2}}{f} - \frac{(2ce-bf) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2} - \frac{(2f(cde-bdf) + (e - \sqrt{e^2 - 4df}))}{\sqrt{2}f^2\sqrt{a+bx+cx^2}}$$

Mathematica [A] time = 1.78, size = 496, normalized size = 0.90

$$\frac{\sqrt{e^2-4df}\sqrt{a+bx+cx^2} - \sqrt{e^2-4df}\sqrt{(2af-b(\sqrt{e^2-4df}+e)) + c(\sqrt{e^2-4df}-2df+e)} \tanh^{-1}\left(\frac{bf + (\sqrt{e^2-4df} - e)x + (2af - b(\sqrt{e^2-4df} + e))}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) - \sqrt{e^2-4df}\sqrt{(2af + b(\sqrt{e^2-4df} + b(-e)) + c(-\sqrt{e^2-4df} - 2df + e))} \tanh^{-1}\left(\frac{bf + (\sqrt{e^2-4df} - e)x + (2af + b(\sqrt{e^2-4df} + b(-e)) + c(-\sqrt{e^2-4df} - 2df + e))}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{4f^2\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2), x]
[Out] -1/2*((2*c*e - b*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]) / (Sqrt[c]*f^2) + (4*f*Sqrt[e^2 - 4*d*f]*Sqrt[a + x*(b + c*x)] - Sqrt[2]*(e + Sqrt[e^2 - 4*d*f])*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])*Sqrt[a + x*(b +
```

$$c*x))]] - \text{Sqrt}[2]*(-e + \text{Sqrt}[e^2 - 4*d*f])*\text{Sqrt}[f*(-(b*e) + 2*a*f + b*\text{Sqrt}[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*\text{Sqrt}[e^2 - 4*d*f])]*\text{ArcTanh}[(4*a*f + 2*c*(-e + \text{Sqrt}[e^2 - 4*d*f])*x + b*(-e + \text{Sqrt}[e^2 - 4*d*f] + 2*f*x))/(2*\text{Sqrt}[2]*\text{Sqrt}[f*(-(b*e) + 2*a*f + b*\text{Sqrt}[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*\text{Sqrt}[e^2 - 4*d*f])])*\text{Sqrt}[a + x*(b + c*x)])]/(4*f^2*\text{Sqrt}[e^2 - 4*d*f])$$

IntegrateAlgebraic [C] time = 0.86, size = 646, normalized size = 1.18

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]

[Out] $\frac{\text{Sqrt}[a + b*x + c*x^2]/f + ((2*c*e - b*f)*\text{Log}[b + 2*c*x - 2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2]])/(2*\text{Sqrt}[c]*f^2) + \text{RootSum}[b^2*d - a*b*e + a^2*f - 4*b*\text{Sqrt}[c]*d*\#1 + 2*a*\text{Sqrt}[c]*e*\#1 + 4*c*d*\#1^2 + b*e*\#1^2 - 2*a*f*\#1^2 - 2*\text{Sqrt}[c]*e*\#1^3 + f*\#1^4 \& , (- (b*c*d*e*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]) + a*c*e^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] + b^2*d*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] - a*c*d*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] - a*b*e*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] + a^2*f^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] + 2*c^(3/2)*d*e*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1 - 2*b*\text{Sqrt}[c]*d*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1 - c*e^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1^2 + c*d*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1^2 + b*e*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1^2 - a*f^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1^2)/(2*b*\text{Sqrt}[c]*d - a*\text{Sqrt}[c]*e - 4*c*d*\#1 - b*e*\#1 + 2*a*f*\#1 + 3*\text{Sqrt}[c]*e*\#1^2 - 2*f*\#1^3) \&]/f^2$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [B] time = 0.02, size = 10138, normalized size = 18.47

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)

[Out] result too large to display

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x \sqrt{c x^2 + b x + a}}{f x^2 + e x + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2),x)

[Out] int((x*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sqrt{a + b x + c x^2}}{d + e x + f x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)

[Out] Integral(x*sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)

$$3.111 \quad \int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2} dx$$

Optimal. Leaf size=431

$$\frac{\sqrt{f(2af - b(e - \sqrt{e^2 - 4df})) + c(-e\sqrt{e^2 - 4df} - 2df + e^2)} \tanh^{-1}\left(\frac{4af + 2x(bf - c(e - \sqrt{e^2 - 4df})) - b(e - \sqrt{e^2 - 4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - c^2}}\right)}{\sqrt{2}f\sqrt{e^2 - 4df}}$$

Rubi [A] time = 0.65, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {989, 621, 206, 1032, 724}

$$\frac{\sqrt{f(2af - b(e - \sqrt{e^2 - 4df})) + c(-e\sqrt{e^2 - 4df} - 2df + e^2)} \tanh^{-1}\left(\frac{4af + 2x(bf - c(e - \sqrt{e^2 - 4df})) - b(e - \sqrt{e^2 - 4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - c^2}}\right)}{\sqrt{2}f\sqrt{e^2 - 4df}} + \frac{\sqrt{f(2af - b(\sqrt{e^2 - 4df} + e)) + c(e\sqrt{e^2 - 4df} - 2df + e^2)} \tanh^{-1}\left(\frac{4af + 2x(bf - c(\sqrt{e^2 - 4df} + e)) - b(\sqrt{e^2 - 4df} + e)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - c^2}}\right)}{\sqrt{2}f\sqrt{e^2 - 4df}} + \frac{\sqrt{c} \tanh^{-1}\left(\frac{bx+2c}{2\sqrt{a+bx+cx^2}}\right)}{f}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2), x]

[Out] (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f - (Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x]/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]))/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]) + (Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x]/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]))/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 989

Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^2), x_Symbol] :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*f + (c*e - b*f)*x)/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2} dx = -\frac{\int \frac{cd-af+(ce-bf)x}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx}{f} + \frac{c \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{f}$$

$$= \frac{(2c) \text{Subst}\left(\int \frac{1}{4c-x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right) - (2f(cd-af) - (ce-bf)(e - \sqrt{e^2-4df})) \int \frac{1}{(e-\sqrt{e^2-4df})\sqrt{a+bx+cx^2}} dx}{f\sqrt{e^2-4df}}$$

$$= \frac{\sqrt{c} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f} + \frac{(2(2f(cd-af) - (ce-bf)(e - \sqrt{e^2-4df}))) \text{Subst}\left(\int \frac{1}{16c-x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{\sqrt{e^2-4df}}$$

$$= \frac{\sqrt{c} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f} - \frac{\sqrt{c}(e^2 - 2df - e\sqrt{e^2-4df}) + f(2af - b(e - \sqrt{e^2-4df}))}{\sqrt{2}f\sqrt{e^2-4df}}$$

Mathematica [A] time = 0.74, size = 417, normalized size = 0.97

$$\frac{\sqrt{f(2af - b(\sqrt{e^2-4df} + e)) + c(e\sqrt{e^2-4df} - 2df + e^2)} \tanh^{-1}\left(\frac{4af + (\sqrt{e^2-4df} + 2f) - 2c(\sqrt{e^2-4df} + e)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{(2af - b(\sqrt{e^2-4df} + e)) + c(e\sqrt{e^2-4df} - 2df + e^2)}}\right) - \sqrt{f(2af + b\sqrt{e^2-4df} + b(-e) + c(-e\sqrt{e^2-4df} - 2df + e^2))} \tanh^{-1}\left(\frac{4af + (\sqrt{e^2-4df} + 2f) - 2c(\sqrt{e^2-4df} + e)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{(2af + b\sqrt{e^2-4df} + b(-e) + c(-e\sqrt{e^2-4df} - 2df + e^2))}}\right) + \frac{\sqrt{c} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}}{\sqrt{2}f\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2), x]
[Out] (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/f + (Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])] - Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + x*(b + c*x)])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f])
```

IntegrateAlgebraic [C] time = 0.00, size = 402, normalized size = 0.93

$$\frac{\text{RootSum}\left[\#1^4 f^3 - 2\#1^3 \sqrt{c} e - 2\#1^2 a f + \#1^2 b e + 4\#1^2 c d + 2\#1 a \sqrt{c} e - 4\#1 b \sqrt{c} d + a^2 f - a b e + b^2 d e, \frac{\#1^2 \log(-\#1 + \sqrt{a+bx+cx^2} - \sqrt{c}) + \#1^2 \log(-\#1 + \sqrt{a+bx+cx^2} + \sqrt{c}) - 2\#1^2 d \log(-\#1 + \sqrt{a+bx+cx^2} - \sqrt{c}) + 2\#1^2 d \log(-\#1 + \sqrt{a+bx+cx^2} + \sqrt{c}) - a c \log(-\#1 + \sqrt{a+bx+cx^2} - \sqrt{c}) - 2\#1 a \sqrt{c} \log(-\#1 + \sqrt{a+bx+cx^2} - \sqrt{c})}{-2\#1^3 \sqrt{c}^2 \sqrt{e^2-4df} - \#1 b e - 4\#1 c d + 2\#1^2 d}\right] e}{f} - \frac{\sqrt{c} \log(-2\sqrt{c} f \sqrt{a+bx+cx^2} + b f + 2c f)}{f}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2), x]
[Out] -((Sqrt[c]*Log[b*f + 2*c*f*x - 2*Sqrt[c]*f*Sqrt[a + b*x + c*x^2]])/f) + RootSum[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*c*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*c^(3/2)*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*a*Sqrt[c]*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 +
```


$c*e*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1^2 - b*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]*\#1^2)/(2*b*\text{Sqrt}[c]*d - a*\text{Sqrt}[c]*e - 4*c*d*\#1 - b*e*\#1 + 2*a*f*\#1 + 3*\text{Sqrt}[c]*e*\#1^2 - 2*f*\#1^3) \&]/f$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
i,const vecteur & l) Error: Bad Argument Value

maple [B] time = 0.02, size = 6019, normalized size = 13.97

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)

[Out] result too large to display

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{fx^2 + ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x + c*x^2)^(1/2)/(d + e*x + f*x^2),x)

[Out] int((a + b*x + c*x^2)^(1/2)/(d + e*x + f*x^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx + cx^2}}{d + ex + fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d), x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)
```

$$3.112 \quad \int \frac{\sqrt{a+bx+cx^2}}{x(d+ex+fx^2)} dx$$

Optimal. Leaf size=523

$$\frac{(cd(e - \sqrt{e^2 - 4df}) - f(2bd - a(\sqrt{e^2 - 4df} + e))) \tanh^{-1} \left(\frac{4af + 2x(bf - c(e - \sqrt{e^2 - 4df})) - b(e - \sqrt{e^2 - 4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}} \right)}{\sqrt{2}d\sqrt{e^2 - 4df}\sqrt{f(2af - b(e - \sqrt{e^2 - 4df})) + c(-e\sqrt{e^2 - 4df} - 2df + e^2)}}$$

Rubi [A] time = 3.70, antiderivative size = 521, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6728, 734, 843, 621, 206, 724, 1019, 1076, 1032}

$$\frac{(-af(\sqrt{e^2 - 4df} + e) + 2bdf - cd(e - \sqrt{e^2 - 4df})) \tanh^{-1} \left(\frac{4af + 2x(bf - c(e - \sqrt{e^2 - 4df})) - b(e - \sqrt{e^2 - 4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}} \right) + (-af(e - \sqrt{e^2 - 4df}) + 2bdf - cd(\sqrt{e^2 - 4df} + e)) \tanh^{-1} \left(\frac{4af + 2x(bf - c(e - \sqrt{e^2 - 4df})) - b(e - \sqrt{e^2 - 4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}} \right) + \sqrt{a} \tanh^{-1} \left(\frac{2+bx}{2\sqrt{a+bx+cx^2}} \right)}{\sqrt{2}d\sqrt{e^2 - 4df}\sqrt{f(2af - b(e - \sqrt{e^2 - 4df})) + c(-e\sqrt{e^2 - 4df} - 2df + e^2)} + \sqrt{2}d\sqrt{e^2 - 4df}\sqrt{f(2af - b(\sqrt{e^2 - 4df} + e)) + c(e\sqrt{e^2 - 4df} - 2df + e^2)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x + c*x^2]/(x*(d + e*x + f*x^2)), x]

[Out] -((Sqrt[a]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d) - ((2*b*d*f - c*d*(e - Sqrt[e^2 - 4*d*f]) - a*f*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x]/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]))/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))]) + ((2*b*d*f - a*f*(e - Sqrt[e^2 - 4*d*f]) - c*d*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x]/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]))/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 734

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]

, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && (!RationalQ[m] || LtQ[m, 1]) &&
& !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]

Rule 843

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

Rule 1019

Int[((g_.) + (h_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[(h*(a + b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[h*p*(b*d - a*e) + a*(h*e - 2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(h*e - 2*g*f)*(p + q + 1))*x + (h*p*(c*e - b*f) + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]

Rule 1032

Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]

Rule 1076

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

Rule 6728

Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx+cx^2}}{x(d+ex+fx^2)} dx &= \int \left(\frac{\sqrt{a+bx+cx^2}}{x} + \frac{(-e-fx)\sqrt{a+bx+cx^2}}{d(d+ex+fx^2)} \right) dx \\
&= \frac{\int \frac{\sqrt{a+bx+cx^2}}{x} dx}{d} + \frac{\int \frac{(-e-fx)\sqrt{a+bx+cx^2}}{d+ex+fx^2} dx}{d} \\
&= -\frac{\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}} dx}{2d} - \frac{\int \frac{-\frac{1}{2}(bd-2ae)f - \frac{1}{2}f(2cd-be-2af)x + \frac{1}{2}bf^2x^2}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx}{df} \\
&= \frac{a \int \frac{1}{x\sqrt{a+bx+cx^2}} dx}{d} - \frac{\int \frac{-\frac{1}{2}bdf^2 - \frac{1}{2}(bd-2ae)f^2 + \left(-\frac{1}{2}bef^2 - \frac{1}{2}f^2(2cd-be-2af)\right)x}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx}{df^2} \\
&= -\frac{(2a) \text{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d} - \frac{(2bdf - af(e - \sqrt{e^2 - 4df}) - cd(e + \sqrt{e^2 - 4df}))}{d\sqrt{e^2 - 4df}} \\
&= -\frac{\sqrt{a} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d} + \frac{(2(2bdf - af(e - \sqrt{e^2 - 4df})) - cd(e + \sqrt{e^2 - 4df}))}{d\sqrt{e^2 - 4df}} \\
&= -\frac{\sqrt{a} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d} - \frac{(2bdf - cd(e - \sqrt{e^2 - 4df}) - af(e + \sqrt{e^2 - 4df}))}{\sqrt{2}d\sqrt{e^2 - 4df}\sqrt{ce^2 - 2cdf - b}}
\end{aligned}$$

Mathematica [A] time = 1.31, size = 454, normalized size = 0.87

$$\frac{(\sqrt{e^2 - 4df} - e)\sqrt{f(2af - b(\sqrt{e^2 - 4df} + e))} + c(e\sqrt{e^2 - 4df} - 2df + e) \tanh^{-1}\left(\frac{bf + e(\sqrt{e^2 - 4df} + e) - 2a(\sqrt{e^2 - 4df} + e)}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right) + (\sqrt{e^2 - 4df} + e)\sqrt{f(2af + b(\sqrt{e^2 - 4df} + b(-e))} + c(-e\sqrt{e^2 - 4df} - 2df + e) \tanh^{-1}\left(\frac{bf + e(\sqrt{e^2 - 4df} + e) - 2a(\sqrt{e^2 - 4df} + e)}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{2}df\sqrt{e^2 - 4df}} + \frac{\sqrt{a} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*x + c*x^2]/(x*(d + e*x + f*x^2)),x]

[Out] -((Sqrt[a]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/d) + ((-e + Sqrt[e^2 - 4*d*f])*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])] + (e + Sqrt[e^2 - 4*d*f])*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + x*(b + c*x)])])/((2*Sqrt[2]*d*f*Sqrt[e^2 - 4*d*f])

IntegrateAlgebraic [C] time = 0.64, size = 474, normalized size = 0.91

$$\frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{a+bx+cx^2}}\right)}{d} - \frac{\text{RootSum}\left[11^3f^3 - 201^3\sqrt{c}e - 201^3df + 81^3bc + 481^3cd + 2011a\sqrt{c}e - 4811b\sqrt{c}d + e^2f - abc + f^2dk, \frac{e^2 + b\sqrt{a+bx+cx^2} - c}{e^2 + b\sqrt{a+bx+cx^2} - c}\right]}{d} - \frac{\sqrt{a} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a + b*x + c*x^2]/(x*(d + e*x + f*x^2)),x]

[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[c]*x)/Sqrt[a] - Sqrt[a + b*x + c*x^2]/Sqrt[a])/d - RootSum[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b^2*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*c*d*Log[-(Sqrt[c]*x) + Sqrt[a

```
a + b*x + c*x^2] - #1] - a*b*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*b*Sqrt[c]*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*a*Sqrt[c]*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - a*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(2*b*Sqrt[c]*d - a*Sqrt[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2*f*#1^3) & ]/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Evaluation time: 7.77sym2poly/r2sym(const ge
n & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.02, size = 6460, normalized size = 12.35

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/x/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{(fx^2 + ex + d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 + e*x + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{x(fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/(x*(d + e*x + f*x^2)),x)
```

```
[Out] int((a + b*x + c*x^2)^(1/2)/(x*(d + e*x + f*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx + cx^2}}{x(d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)**(1/2)/x/(f*x**2+e*x+d),x)

[Out] Integral(sqrt(a + b*x + c*x**2)/(x*(d + e*x + f*x**2)), x)

3.113 $\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex+fx^2)} dx$

Optimal. Leaf size=736

$$\frac{f(a(e\sqrt{e^2-4df}-2df+e^2)-bd(\sqrt{e^2-4df}+e)+2cd^2)\tanh^{-1}\left(\frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+e^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{f(2af-b(e-\sqrt{e^2-4df}))+c(-e\sqrt{e^2-4df}-2df+e^2)}}$$

Rubi [A] time = 3.48, antiderivative size = 736, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 30, number of rules / integrand size = 0.333, Rules used = {6728, 732, 843, 621, 206, 724, 734, 1019, 1076, 1032}

$$\frac{f(e(\sqrt{e^2-4df}-2df+e^2)-bd(\sqrt{e^2-4df}+e)+2cd^2)\tanh^{-1}\left(\frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+e^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{f(2af-b(e-\sqrt{e^2-4df}))+c(-e\sqrt{e^2-4df}-2df+e^2)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(x^2*(d + e*x + f*x^2)), x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/(d*x)) - (b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[a]*d) + (Sqrt[a]*e*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d^2 + (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/d - (b*e*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*d^2) - ((2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*d^2) - (f*(2*c*d^2 - b*d*(e + Sqrt[e^2 - 4*d*f]) + a*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f])*Sqrt[a + b*x + c*x^2])]/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))]) + (f*(2*c*d^2 - b*d*(e - Sqrt[e^2 - 4*d*f]) + a*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f])*Sqrt[a + b*x + c*x^2])]/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
```



```
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1]) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && (!RationalQ[m] || LtQ[m, 1]) && !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1019

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[(h*(a + b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[h*p*(b*d - a*e) + a*(h*e - 2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(h*e - 2*g*f)*(p + q + 1))*x + (h*p*(c*e - b*f) + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex+fx^2)} dx = \int \left(\frac{\sqrt{a+bx+cx^2}}{dx^2} - \frac{e\sqrt{a+bx+cx^2}}{d^2x} + \frac{(e^2-df+efx)\sqrt{a+bx+cx^2}}{d^2(d+ex+fx^2)} \right) dx$$

$$= \frac{\int \frac{(e^2-df+efx)\sqrt{a+bx+cx^2}}{d+ex+fx^2} dx}{d^2} + \frac{\int \frac{\sqrt{a+bx+cx^2}}{x^2} dx}{d} - \frac{e \int \frac{\sqrt{a+bx+cx^2}}{x} dx}{d^2}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx} + \frac{\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}} dx}{2d} + \frac{e \int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}} dx}{2d^2} - \frac{\int \frac{\frac{1}{2}f(bde-2ae^2+2adf)+\frac{1}{2}f(2cde-b^2)}{\sqrt{a+bx+cx^2}} dx}{d^2}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx} + \frac{b \int \frac{1}{x\sqrt{a+bx+cx^2}} dx}{2d} + \frac{c \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{d} - \frac{(ae) \int \frac{1}{x\sqrt{a+bx+cx^2}} dx}{d^2} - \frac{(be)}{d^2}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx} - \frac{b \operatorname{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d} + \frac{(2c) \operatorname{Subst}\left(\int \frac{1}{4c-x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{d}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx} - \frac{b \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}d} + \frac{\sqrt{a}e \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2} + \frac{\sqrt{c} \operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{d}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx} - \frac{b \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}d} + \frac{\sqrt{a}e \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2} + \frac{\sqrt{c} \operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{d}$$

Mathematica [A] time = 1.68, size = 520, normalized size = 0.71

$$\frac{(2ae-b)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}d} - \frac{4ef\sqrt{e^2-4d*df}\sqrt{a+bx+cx^2} + \sqrt{2e}\sqrt{e^2-4d*df}\sqrt{2df-e}\sqrt{(2df-b)(\sqrt{e^2-4d*df}+e)} + c\sqrt{e^2-4d*df}\sqrt{2df+e}}{4ef\sqrt{e^2-4d*df}} + \frac{\sqrt{a}e \operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2} + \frac{\sqrt{c} \operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(x^2*(d + e*x + f*x^2)),x]
[Out] ((-(b*d) + 2*a*e)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/(2*Sqrt[a]*d^2) - (4*d*f*Sqrt[e^2 - 4*d*f]*Sqrt[a + x*(b + c*x)] + Sqrt[2]*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*x)*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])] + Sqrt[2]*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]*x)*ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/(4*d^2*f*Sqrt[e^2 - 4*d*f]*x)
```

IntegrateAlgebraic [C] time = 0.88, size = 589, normalized size = 0.80

$$\frac{\operatorname{RootSum}\left[11^7 - 20^7\sqrt{c} - 20^7ef + 81^7bc + 80^7d + 2014\sqrt{c}e - 4013\sqrt{c}d + e^7 - 40e + 1^700c\right]}{2\sqrt{a}d} - \frac{\sqrt{a}e \operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2} + \frac{\sqrt{c} \operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x + c*x^2]/(x^2*(d + e*x + f*x^2)),x]
[Out] -(Sqrt[a + b*x + c*x^2]/(d*x)) + ((-(b*d) + 2*a*e)*ArcTanh[(-Sqrt[c]*x) + Sqrt[a + b*x + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^2) + RootSum[b^2*d - a*b*e + a^
```

$$2*f - 4*b*\text{Sqrt}[c]*d**1 + 2*a*\text{Sqrt}[c]*e**1 + 4*c*d**1^2 + b*e**1^2 - 2*a*f**1^2 - 2*\text{Sqrt}[c]*e**1^3 + f**1^4 \& , (- (b*c*d^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]) + b^2*d*e*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] - a*b*e^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] + a^2*e*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1] + 2*c^(3/2)*d^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]**1 - 2*b*\text{Sqrt}[c]*d*e*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]**1 + 2*a*\text{Sqrt}[c]*e^2*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]**1 - 2*a*\text{Sqrt}[c]*d*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]**1 + b*d*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]**1^2 - a*e*f*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]**1^2)/(2*b*\text{Sqrt}[c]*d - a*\text{Sqrt}[c]*e - 4*c*d**1 - b*e**1 + 2*a*f**1 + 3*\text{Sqrt}[c]*e**1^2 - 2*f**1^3) \&]/d^2$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.02, size = 6765, normalized size = 9.19

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x+a)^(1/2)/x^2/(f*x^2+e*x+d),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{(fx^2 + ex + d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 + e*x + d)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{x^2 (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x + c*x^2)^(1/2)/(x^2*(d + e*x + f*x^2)),x)

```
[Out] int((a + b*x + c*x^2)^(1/2)/(x^2*(d + e*x + f*x^2)), x)
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/x**2/(f*x**2+e*x+d), x)
```

```
[Out] Timed out
```

$$3.114 \quad \int \frac{x^3}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx$$

Optimal. Leaf size=545

$$\frac{b \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (2def - (e^2 - df)(e - \sqrt{e^2 - 4df})) \tanh^{-1}\left(\frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{2c^{3/2}f} \sqrt{2} f^2 \sqrt{e^2 - 4df} \sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}$$

Rubi [A] time = 3.72, antiderivative size = 545, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 30, number of rules / integrand size = 0.200, Rules used = {6728, 621, 206, 640, 1032, 724}

$$\frac{b \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (2def - (e^2 - df)(e - \sqrt{e^2 - 4df})) \tanh^{-1}\left(\frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right) + (2def - (e^2 - df)(\sqrt{e^2 - 4df} + e)) \tanh^{-1}\left(\frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right) e \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) + \frac{\sqrt{a+bx+cx^2}}{cf}}{\sqrt{2} f^2 \sqrt{e^2 - 4df} \sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]

[Out] Sqrt[a + b*x + c*x^2]/(c*f) - (e*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/(Sqrt[c]*f^2) - (b*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/(2*c^(3/2)*f) - ((2*d*e*f - (e^2 - d*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x]/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]))/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) + ((2*d*e*f - (e^2 - d*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x]/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]))/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx = \int \left(-\frac{e}{f^2\sqrt{a+bx+cx^2}} + \frac{x}{f\sqrt{a+bx+cx^2}} + \frac{de+(e^2-df)x}{f^2\sqrt{a+bx+cx^2}(d+ex+fx^2)} \right) dx$$

$$= \frac{\int \frac{de+(e^2-df)x}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx}{f^2} - \frac{e \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{f^2} + \frac{\int \frac{x}{\sqrt{a+bx+cx^2}} dx}{f}$$

$$= \frac{\sqrt{a+bx+cx^2}}{cf} - \frac{(2e) \text{Subst}\left(\int \frac{1}{4c-x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f^2} - \frac{b \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{2cf}$$

$$= \frac{\sqrt{a+bx+cx^2}}{cf} - \frac{e \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2} - \frac{b \text{Subst}\left(\int \frac{1}{4c-x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{cf}$$

$$= \frac{\sqrt{a+bx+cx^2}}{cf} - \frac{e \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2} - \frac{b \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f}$$

Mathematica [A] time = 2.20, size = 550, normalized size = 1.01

$$\frac{bf \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}} + \frac{\sqrt{2}\left(\frac{e(e^2-3df)}{\sqrt{2-4df}} - df + e^2\right) \tanh^{-1}\left(\frac{4ef + (\sqrt{2-4df} - c + 2f)\sqrt{2-4df}}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{(2af + b(\sqrt{2-4df} - c) + c\sqrt{2-4df} - 2df + e^2)}} + \frac{\sqrt{2}\left(e^2\sqrt{2-4df} - df\sqrt{2-4df} - 3def + e^3\right) \tanh^{-1}\left(\frac{4ef - (\sqrt{2-4df} + 2f)\sqrt{2-4df}}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2-4df}\sqrt{(2af - b(\sqrt{2-4df} + c) + c\sqrt{2-4df} - 2df + e^2)}} + \frac{2e \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}} - \frac{2f\sqrt{a+bx+cx^2}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
[Out] -1/2*((-2*f*Sqrt[a + x*(b + c*x)])/c + (2*e*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + (b*f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/c^(3/2) + (Sqrt[2]*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f])])]*Sqrt[a + x*(b + c*x)])))/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f])]) + (Sqrt[2]*(e^2 - d*f - (e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f])])]*Sqrt[a + x*(b + c*x)]))
```

)]))/Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]/f^2

IntegrateAlgebraic [C] time = 0.74, size = 430, normalized size = 0.79

$$\frac{\text{RootSum}\left[\frac{4f^3 - 2d^2\sqrt{c} - 2d^2af + d^2be + 4d^2cd + 2d^2\sqrt{c}e - 4d^2\sqrt{c}d + d^2f - abe + d^2de}{f^2}, \frac{-d^2\sqrt{c}\log\left(\frac{4f + \sqrt{4d^2c^2 - c^2}}{4f - \sqrt{4d^2c^2 - c^2}}\right) + d^2\sqrt{c}\log\left(\frac{4f + \sqrt{4d^2c^2 - c^2}}{4f - \sqrt{4d^2c^2 - c^2}}\right) + 2d^2\sqrt{c}\log\left(\frac{4f + \sqrt{4d^2c^2 - c^2}}{4f - \sqrt{4d^2c^2 - c^2}}\right) + \text{off}\log\left(\frac{4f + \sqrt{4d^2c^2 - c^2}}{4f - \sqrt{4d^2c^2 - c^2}}\right) + d^2\log\left(\frac{4f + \sqrt{4d^2c^2 - c^2}}{4f - \sqrt{4d^2c^2 - c^2}}\right)}{2d^2\sqrt{c} + 2d^2af - 4d^2cd - d^2\sqrt{c}d}\right], \frac{(bf + 2c)\log\left(\frac{-2c^2\sqrt{a + bx + cx^2} + bx + 2c^2}{2c^2f}\right) + \sqrt{a + bx + cx^2}}{cf}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]

[Out] Sqrt[a + b*x + c*x^2]/(c*f) + ((2*c*e + b*f)*Log[b*c + 2*c^2*x - 2*c^(3/2)*Sqrt[a + b*x + c*x^2]]/(2*c^(3/2)*f^2) - RootSum[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*Sqrt[c]*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(2*b*Sqrt[c]*d - a*Sqrt[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2*f*#1^3) &]/f^2

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type

maple [B] time = 0.03, size = 3131, normalized size = 5.74

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)

[Out] (c*x^2+b*x+a)^(1/2)/c/f-1/2*b/c^(3/2)/f*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/f^2*e*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+1/2/f^2*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*d-1/2/f^3*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)

[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{cx^2 + bx + a} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)

[Out] int(x^3/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a + bx + cx^2} (d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)

[Out] Integral(x**3/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)

$$3.115 \quad \int \frac{x^2}{\sqrt{a+bx+cx^2} (d+ex+fx^2)} dx$$

Optimal. Leaf size=463

$$\frac{(-e\sqrt{e^2-4df}-2df+e^2) \tanh^{-1}\left(\frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right) (2df-e(\sqrt{e^2-4df}+e))}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} - \frac{(2df-e(\sqrt{e^2-4df}+e))}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2}}$$

Rubi [A] time = 3.44, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 30, number of rules / integrand size = 0.167, Rules used = {1077, 621, 206, 1032, 724}

$$\frac{(-e\sqrt{e^2-4df}-2df+e^2) \tanh^{-1}\left(\frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} - \frac{(2df-e(\sqrt{e^2-4df}+e)) \tanh^{-1}\left(\frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} + \frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}$$

Antiderivative was successfully verified.

[In] Int[x^2/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)), x]

[Out] ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/(Sqrt[c]*f) - ((e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x]/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]))/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - ((2*d*f - e*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x]/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]))/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1032

Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]

Rule 1077

Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C - b*C*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{\sqrt{a+bx+cx^2} (d+ex+fx^2)} dx &= \frac{\int \frac{1}{\sqrt{a+bx+cx^2}} dx}{f} + \frac{\int \frac{-d-ex}{\sqrt{a+bx+cx^2} (d+ex+fx^2)} dx}{f} \\ &= \frac{2 \operatorname{Subst}\left(\int \frac{1}{4c-x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f} + \frac{(e^2 - 2df - e\sqrt{e^2 - 4df}) \int \frac{1}{(e-\sqrt{e^2 - 4df})}}{f\sqrt{e^2 - 4df}} \\ &= \frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f} - \frac{(2(e^2 - 2df - e\sqrt{e^2 - 4df})) \operatorname{Subst}\left(\int \frac{1}{16af^2 - 2c^2x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f} \\ &= \frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f} - \frac{(e^2 - 2df - e\sqrt{e^2 - 4df}) \tanh^{-1}\left(\frac{4af-b}{2\sqrt{2}\sqrt{ce^2 - 2cdf - bef}}\right)}{\sqrt{2}f\sqrt{e^2 - 4df}\sqrt{ce^2 - 2cdf - bef}} \end{aligned}$$

Mathematica [A] time = 0.95, size = 468, normalized size = 1.01

$$\frac{\sqrt{2}(e\sqrt{e^2-4df}-2df+e^2)\tanh^{-1}\left(\frac{4af-b(\sqrt{e^2-4df}+e-2fx)-2cx(\sqrt{e^2-4df}+e)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{f(2af-b(\sqrt{e^2-4df}+e))+c(e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{e^2-4df}\sqrt{f(2af-b(\sqrt{e^2-4df}+e))+c(e\sqrt{e^2-4df}-2df+e^2)}} + \frac{\sqrt{2}\left(\frac{2df-e^2}{\sqrt{e^2-4df}}+e\right)\tanh^{-1}\left(\frac{4af+b(\sqrt{e^2-4df}-e+2fx)+2cx(\sqrt{e^2-4df}-e)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{f(2af-b(\sqrt{e^2-4df}-e))+c(-e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{f(2af+b(\sqrt{e^2-4df}-e))+c(-e\sqrt{e^2-4df}-2df+e^2)}} + \frac{2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]

[Out] ((2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + (Sqrt[2]*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])*Sqrt[a + x*(b + c*x)]])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]) + (Sqrt[2]*(e + (-e^2 + 2*d*f)/Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))])*Sqrt[a + x*(b + c*x)]])/(Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))])/(2*f)

IntegrateAlgebraic [C] time = 0.53, size = 324, normalized size = 0.70

$$\frac{\operatorname{RootSum}\left[\#1^4 f - 2\#1^3 \sqrt{c} e - 2\#1^2 a f + \#1^2 b e + 4\#1^2 c d + 2\#1 a \sqrt{c} e - 4\#1 b \sqrt{c} d + a^2 f - a b e + b^2 d e, \frac{\#1^2 e \log\left(\#1 + \sqrt{a+bx+cx^2} - \sqrt{c}\right) - 2\#1 \sqrt{c} d \log\left(\#1 + \sqrt{a+bx+cx^2} - \sqrt{c}\right) + b d \log\left(\#1 + \sqrt{a+bx+cx^2} - \sqrt{c}\right) - e \log\left(\#1 + \sqrt{a+bx+cx^2} - \sqrt{c}\right)}{-2\#1^3 f + 3\#1^2 \sqrt{c} e + 2\#1 a f - \#1 b e - 4\#1 c d - a \sqrt{c} e + 2\#1 \sqrt{c} d}\right]}{f} - \frac{\log\left(-2\sqrt{c} f \sqrt{a+bx+cx^2} + b f + 2c f x\right)}{\sqrt{c} f}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]

[Out] -(Log[b*f + 2*c*f*x - 2*Sqrt[c]*f*Sqrt[a + b*x + c*x^2]]/(Sqrt[c]*f)) + RootSum[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1

$$\begin{aligned} & \sqrt{2 + b e^{\#1} - 2 a f^{\#1} - 2 \sqrt{c} e^{\#1} + f^{\#1}} \&, (b d \log[-(\sqrt{c} x) + \sqrt{a + b x + c x^2} - \#1] - a e \log[-(\sqrt{c} x) + \sqrt{a + b x + c x^2} - \#1] - 2 \sqrt{c} d \log[-(\sqrt{c} x) + \sqrt{a + b x + c x^2} - \#1] \#1 + e \log[-(\sqrt{c} x) + \sqrt{a + b x + c x^2} - \#1] \#1^2) / (2 b \sqrt{c} d - a \sqrt{c} e - 4 c d \#1 - b e^{\#1} + 2 a f^{\#1} + 3 \sqrt{c} e^{\#1} - 2 f^{\#1} \#1^3) \&] / f \end{aligned}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
i,const vecteur & l) Error: Bad Argument Value

maple [B] time = 0.02, size = 2321, normalized size = 5.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)

[Out]
$$\begin{aligned} & \frac{1}{c^{1/2}} \frac{1}{f} \ln\left(\frac{c x + 1/2 b}{c^{1/2} + (c x^2 + b x + a)^{1/2}} + \frac{1}{2} \frac{f^2}{f^2} \frac{1}{(((-4 d f + e^2)^{1/2} b f + (-4 d f + e^2)^{1/2} c e + 2 a f^2 - b e f - 2 c d f + c e^2) / f^2)^{1/2}} \right) \ln\left(\frac{(-4 d f + e^2)^{1/2} b f + (-4 d f + e^2)^{1/2} c e + 2 a f^2 - b e f - 2 c d f + c e^2}{f^2} + \frac{1}{f} \frac{(-c(-4 d f + e^2)^{1/2} + b f - c e) (x + 1/2 (e + (-4 d f + e^2)^{1/2}))}{f} + \frac{1}{2} \frac{2^{1/2} ((-4 d f + e^2)^{1/2} b f + (-4 d f + e^2)^{1/2} c e + 2 a f^2 - b e f - 2 c d f + c e^2) / f^2}{(4 (x + 1/2 (e + (-4 d f + e^2)^{1/2})) / f)^2} \frac{c + 4 / f (-c(-4 d f + e^2)^{1/2} + b f - c e) (x + 1/2 (e + (-4 d f + e^2)^{1/2}))}{f} + 2 \frac{(-(-4 d f + e^2)^{1/2} b f + (-4 d f + e^2)^{1/2} c e + 2 a f^2 - b e f - 2 c d f + c e^2) / f^2}{(x + 1/2 (e + (-4 d f + e^2)^{1/2})) / f} \right) e^{-1 / (-4 d f + e^2)^{1/2}} \frac{2^{1/2}}{(((-4 d f + e^2)^{1/2} b f + (-4 d f + e^2)^{1/2} c e + 2 a f^2 - b e f - 2 c d f + c e^2) / f^2)^{1/2}} \ln\left(\frac{(-4 d f + e^2)^{1/2} b f + (-4 d f + e^2)^{1/2} c e + 2 a f^2 - b e f - 2 c d f + c e^2}{f^2} + \frac{1}{f} \frac{(-c(-4 d f + e^2)^{1/2} + b f - c e) (x + 1/2 (e + (-4 d f + e^2)^{1/2}))}{f} + \frac{1}{2} \frac{2^{1/2} ((-4 d f + e^2)^{1/2} b f + (-4 d f + e^2)^{1/2} c e + 2 a f^2 - b e f - 2 c d f + c e^2) / f^2}{(4 (x + 1/2 (e + (-4 d f + e^2)^{1/2})) / f)^2} \frac{c + 4 / f (-c(-4 d f + e^2)^{1/2} + b f - c e) (x + 1/2 (e + (-4 d f + e^2)^{1/2}))}{f} + 2 \frac{(-(-4 d f + e^2)^{1/2} b f + (-4 d f + e^2)^{1/2} c e + 2 a f^2 - b e f - 2 c d f + c e^2) / f^2}{(x + 1/2 (e + (-4 d f + e^2)^{1/2})) / f} \right) e^{-2 + 1/2 / f^2} \frac{2^{1/2}}{(((-4 d f + e^2)^{1/2} b f + (-4 d f + e^2)^{1/2} c e + 2 a f^2 - b e f - 2 c d f + c e^2) / f^2)^{1/2}} \ln\left(\frac{(-4 d f + e^2)^{1/2} b f + (-4 d f + e^2)^{1/2} c e + 2 a f^2 - b e f - 2 c d f + c e^2}{f^2} + \frac{1}{f} \frac{(-c(-4 d f + e^2)^{1/2} + b f - c e) (x + 1/2 (e + (-4 d f + e^2)^{1/2}))}{f} + \frac{1}{2} \frac{2^{1/2} ((-4 d f + e^2)^{1/2} b f + (-4 d f + e^2)^{1/2} c e + 2 a f^2 - b e f - 2 c d f + c e^2) / f^2}{(4 (x + 1/2 (e + (-4 d f + e^2)^{1/2})) / f)^2} \frac{c + 4 / f (-c(-4 d f + e^2)^{1/2} + b f - c e) (x + 1/2 (e + (-4 d f + e^2)^{1/2}))}{f} + 2 \frac{(-(-4 d f + e^2)^{1/2} b f + (-4 d f + e^2)^{1/2} c e + 2 a f^2 - b e f - 2 c d f + c e^2) / f^2}{(x + 1/2 (e + (-4 d f + e^2)^{1/2})) / f} \right) \end{aligned}$$

$$2-b*ef-2*c*d*f+c*e^2)/f^2)^{(1/2)}*\ln(\left(\left(\left(-4*d*f+e^2\right)^{(1/2)}*b*f-\left(-4*d*f+e^2\right)^{(1/2)}*c*e+2*a*f^2-b*ef-2*c*d*f+c*e^2\right)/f^2+\left(c*\left(-4*d*f+e^2\right)^{(1/2)}+b*f-c*e\right)/f*\left(x-1/2*\left(-e+\left(-4*d*f+e^2\right)^{(1/2)}\right)/f\right)+1/2*2^{(1/2)}*\left(\left(-4*d*f+e^2\right)^{(1/2)}*b*f-\left(-4*d*f+e^2\right)^{(1/2)}*c*e+2*a*f^2-b*ef-2*c*d*f+c*e^2\right)/f^2\right)^{(1/2)}*(4*\left(x-1/2*\left(-e+\left(-4*d*f+e^2\right)^{(1/2)}\right)/f\right)^2*c+4*\left(c*\left(-4*d*f+e^2\right)^{(1/2)}+b*f-c*e\right)/f*\left(x-1/2*\left(-e+\left(-4*d*f+e^2\right)^{(1/2)}\right)/f\right)+2*\left(\left(-4*d*f+e^2\right)^{(1/2)}*b*f-\left(-4*d*f+e^2\right)^{(1/2)}*c*e+2*a*f^2-b*ef-2*c*d*f+c*e^2\right)/f^2\right)^{(1/2)}\right)/\left(x-1/2*\left(-e+\left(-4*d*f+e^2\right)^{(1/2)}\right)/f\right))*e+1/\left(-4*d*f+e^2\right)^{(1/2)}/f*2^{(1/2)}/\left(\left(-4*d*f+e^2\right)^{(1/2)}*b*f-\left(-4*d*f+e^2\right)^{(1/2)}*c*e+2*a*f^2-b*ef-2*c*d*f+c*e^2\right)/f^2)^{(1/2)}*\ln(\left(\left(\left(-4*d*f+e^2\right)^{(1/2)}*b*f-\left(-4*d*f+e^2\right)^{(1/2)}*c*e+2*a*f^2-b*ef-2*c*d*f+c*e^2\right)/f^2+\left(c*\left(-4*d*f+e^2\right)^{(1/2)}+b*f-c*e\right)/f*\left(x-1/2*\left(-e+\left(-4*d*f+e^2\right)^{(1/2)}\right)/f\right)+1/2*2^{(1/2)}*\left(\left(-4*d*f+e^2\right)^{(1/2)}*b*f-\left(-4*d*f+e^2\right)^{(1/2)}*c*e+2*a*f^2-b*ef-2*c*d*f+c*e^2\right)/f^2\right)^{(1/2)}*(4*\left(x-1/2*\left(-e+\left(-4*d*f+e^2\right)^{(1/2)}\right)/f\right)^2*c+4*\left(c*\left(-4*d*f+e^2\right)^{(1/2)}+b*f-c*e\right)/f*\left(x-1/2*\left(-e+\left(-4*d*f+e^2\right)^{(1/2)}\right)/f\right)+2*\left(\left(-4*d*f+e^2\right)^{(1/2)}*b*f-\left(-4*d*f+e^2\right)^{(1/2)}*c*e+2*a*f^2-b*ef-2*c*d*f+c*e^2\right)/f^2\right)^{(1/2)}\right)/\left(x-1/2*\left(-e+\left(-4*d*f+e^2\right)^{(1/2)}\right)/f\right))*d-1/2/\left(-4*d*f+e^2\right)^{(1/2)}/f*2^{(1/2)}/\left(\left(-4*d*f+e^2\right)^{(1/2)}*b*f-\left(-4*d*f+e^2\right)^{(1/2)}*c*e+2*a*f^2-b*ef-2*c*d*f+c*e^2\right)/f^2)^{(1/2)}*\ln(\left(\left(\left(-4*d*f+e^2\right)^{(1/2)}*b*f-\left(-4*d*f+e^2\right)^{(1/2)}*c*e+2*a*f^2-b*ef-2*c*d*f+c*e^2\right)/f^2+\left(c*\left(-4*d*f+e^2\right)^{(1/2)}+b*f-c*e\right)/f*\left(x-1/2*\left(-e+\left(-4*d*f+e^2\right)^{(1/2)}\right)/f\right)+1/2*2^{(1/2)}*\left(\left(-4*d*f+e^2\right)^{(1/2)}*b*f-\left(-4*d*f+e^2\right)^{(1/2)}*c*e+2*a*f^2-b*ef-2*c*d*f+c*e^2\right)/f^2\right)^{(1/2)}*(4*\left(x-1/2*\left(-e+\left(-4*d*f+e^2\right)^{(1/2)}\right)/f\right)^2*c+4*\left(c*\left(-4*d*f+e^2\right)^{(1/2)}+b*f-c*e\right)/f*\left(x-1/2*\left(-e+\left(-4*d*f+e^2\right)^{(1/2)}\right)/f\right)+2*\left(\left(-4*d*f+e^2\right)^{(1/2)}*b*f-\left(-4*d*f+e^2\right)^{(1/2)}*c*e+2*a*f^2-b*ef-2*c*d*f+c*e^2\right)/f^2\right)^{(1/2)}\right)/\left(x-1/2*\left(-e+\left(-4*d*f+e^2\right)^{(1/2)}\right)/f\right))*e^2$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{cx^2 + bx + a} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)

[Out] int(x^2/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a + bx + cx^2} (d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)

[Out] Integral(x**2/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)

$$3.116 \quad \int \frac{x}{\sqrt{a+bx+cx^2} (d+ex+fx^2)} dx$$

Optimal. Leaf size=402

$$\frac{(e - \sqrt{e^2 - 4df}) \tanh^{-1} \left(\frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} \right) (\sqrt{e^2 - 4df} + e) \tanh^{-1} \left(\frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))+b(\sqrt{e^2-4df}+e)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} \right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2} \quad \sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

Rubi [A] time = 0.96, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1032, 724, 206}

$$\frac{(e - \sqrt{e^2 - 4df}) \tanh^{-1} \left(\frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} \right) (\sqrt{e^2 - 4df} + e) \tanh^{-1} \left(\frac{4af+2x(bf-c(\sqrt{e^2-4df}+e))-b(\sqrt{e^2-4df}+e)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} \right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2} \quad \sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)), x]

[Out] ((e - Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x]/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]))/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - ((e + Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x]/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]))/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1032

Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]

Rubi steps

$$\int \frac{x}{\sqrt{a+bx+cx^2} (d+ex+fx^2)} dx = - \left(\left(-1 - \frac{e}{\sqrt{e^2-4df}} \right) \int \frac{1}{(e + \sqrt{e^2-4df} + 2fx) \sqrt{a+bx+cx^2}} dx \right) +$$

$$= - \left(\left(2 \left(1 - \frac{e}{\sqrt{e^2-4df}} \right) \right) \text{Subst} \left(\int \frac{1}{16af^2 - 8bf(e - \sqrt{e^2-4df}) + 4c} \right. \right.$$

$$\left. \left. \left(1 - \frac{e}{\sqrt{e^2-4df}} \right) \tanh^{-1} \left(\frac{4af-b(e-\sqrt{e^2-4df})+2(bf-c(e-\sqrt{e^2-4df}))x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}} \sqrt{a+bx+cx^2}} \right) \right) \right)$$

$$= - \frac{\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}}{\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}}$$

Mathematica [A] time = 0.97, size = 407, normalized size = 1.01

$$\frac{(\sqrt{e^2-4df+e}) \tanh^{-1} \left(\frac{4af-b(\sqrt{e^2-4df+e-2fx})-2cx(\sqrt{e^2-4df+e})}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f(2af-b(\sqrt{e^2-4df+e})+c(e\sqrt{e^2-4df}-2df+e^2))} \right)}{\sqrt{e^2-4df}\sqrt{f(2af-b(\sqrt{e^2-4df+e})+c(e\sqrt{e^2-4df}-2df+e^2))}} - \frac{\left(1 - \frac{e}{\sqrt{e^2-4df}} \right) \tanh^{-1} \left(\frac{4af+b(\sqrt{e^2-4df-e+2fx})+2cx(\sqrt{e^2-4df-e})}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f(2af+b(\sqrt{e^2-4df-e})+c(-e\sqrt{e^2-4df}-2df+e^2))} \right)}{\sqrt{f(2af+b(\sqrt{e^2-4df-e})+c(-e\sqrt{e^2-4df}-2df+e^2))}}}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]

[Out] $-\left(\left(\left(e + \sqrt{e^2 - 4df}\right) \text{ArcTanh}\left[\frac{4af - 2c(e + \sqrt{e^2 - 4df})x - b(e + \sqrt{e^2 - 4df} - 2fx)}{2\sqrt{2}\sqrt{c(e^2 - 2df + e\sqrt{e^2 - 4df}) + f(2af - b(e + \sqrt{e^2 - 4df}))}}\right] \sqrt{a + x(b + cx)}\right) / \left(\sqrt{e^2 - 4df} \sqrt{c(e^2 - 2df + e\sqrt{e^2 - 4df}) + f(2af - b(e + \sqrt{e^2 - 4df}))}\right) - \left(\left(1 - \frac{e}{\sqrt{e^2 - 4df}}\right) \text{ArcTanh}\left[\frac{4af + 2c(-e + \sqrt{e^2 - 4df})x + b(-e + \sqrt{e^2 - 4df} + 2fx)}{2\sqrt{2}\sqrt{c(e^2 - 2df - e\sqrt{e^2 - 4df}) + f(2af + b(-e + \sqrt{e^2 - 4df}))}}\right] \sqrt{a + x(b + cx)}\right) / \left(\sqrt{e^2 - 4df} \sqrt{c(e^2 - 2df - e\sqrt{e^2 - 4df}) + f(2af + b(-e + \sqrt{e^2 - 4df}))}\right) / \sqrt{2}\right)$

IntegrateAlgebraic [C] time = 0.43, size = 204, normalized size = 0.51

$$\text{RootSum} \left[\#1^4 f - 2\#1^3 \sqrt{ce} - 2\#1^2 af + \#1^2 be + 4\#1^2 cd + 2\#1 a \sqrt{ce} - 4\#1 b \sqrt{cd} + a^2 f - abe + b^2 d \&, \frac{\#1^2 \log(-\#1 + \sqrt{a+bx+cx^2} - \sqrt{cx}) - a \log(-\#1 + \sqrt{a+bx+cx^2} - \sqrt{cx})}{2\#1^3 f - 3\#1^2 \sqrt{ce} - 2\#1 af + \#1 be + 4\#1 cd + a \sqrt{ce} - 2b \sqrt{cd}} \& \right]$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]

[Out] $\text{RootSum}[b^2d - a*be + a^2f - 4*b*\text{Sqrt}[c]*d*\#1 + 2*a*\text{Sqrt}[c]*e*\#1 + 4*c*d*\#1^2 + b*e*\#1^2 - 2*a*f*\#1^2 - 2*\text{Sqrt}[c]*e*\#1^3 + f*\#1^4 \&, (- (a*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1]) + \text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + b*x + c*x^2] - \#1])*\#1^2) / (-2*b*\text{Sqrt}[c]*d + a*\text{Sqrt}[c]*e + 4*c*d*\#1 + b*e*\#1 - 2*a*f*\#1 - 3*\text{Sqrt}[c]*e*\#1^2 + 2*f*\#1^3) \&]$

fricas [B] time = 14.50, size = 11311, normalized size = 28.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] $\frac{1}{4}\sqrt{2}\sqrt{(2cd^2 - bde + ae^2 - 2adf + (c^2d^2e^2 - bcd* e^3 + ac*e^4 - 4a^2d*f^3 + (4abde + a^2e^2 - 4(b^2 - 2ac)*d^2)*f^2 - (4c^2d^3 - 4b*c*d^2*e + a*b*e^3 - (b^2 - 6ac)*d*e^2)*f)*\sqrt{(b^2*d^2 - 2a*b*d*e + a^2e^2)/(c^4d^4e^2 - 2b*c^3d^3e^3 - 2a*b*c^2d*e^2 - 2a*b*d*e + a^2e^2)}}$

$$\begin{aligned}
& \cdot 2e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)) / (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)) * \log(- (2*b^2*d^3 - 4*a*b*d^2*e + 2*a^2*d*e^2 + \sqrt{2}*(b^2*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4 - 4*(b^2*d^3 - 2*a*b*d^2*e + a^2*d*e^2)*f + (2*c^3*d^4*e^2 - 3*b*c^2*d^3*e^3 - 2*a*b*c*d*e^5 + a^2*c*e^6 + 8*a^3*d^2*f^4 + (b^2*c + 3*a*c^2)*d^2*e^4 - 2*(2*a^2*b*d^2*e + 3*a^3*d*e^2 - 4*(a*b^2 - 3*a^2*c)*d^3)*f^3 + (5*a^2*b*d*e^3 + a^3*e^4 - 8*(b^2*c - 3*a*c^2)*d^4 + 4*(b^3 - 2*a*b*c)*d^3*e - 2*(5*a*b^2 - 11*a^2*c)*d^2*e^2)*f^2 - (8*c^3*d^5 - 12*b*c^2*d^4*e + a^2*b*e^5 + 2*(b^2*c + 9*a*c^2)*d^3*e^2 + (b^3 - 10*a*b*c)*d^2*e^3 - 2*(a*b^2 - 4*a^2*c)*d*e^4)*f)*\sqrt{(b^2*d^2 - 2*a*b*d*e + a^2*e^2)} / (c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)) * \sqrt{c*x^2 + b*x + a} * \sqrt{(2*c*d^2 - b*d*e + a*e^2 - 2*a*d*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*\sqrt{(b^2*d^2 - 2*a*b*d*e + a^2*e^2)} / (c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)) / (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)) + (4*b*c*d^3 + a*b*d*e^2 - (b^2 + 4*a*c)*d^2*e)*x + (2*a*c^2*d^3*e^2 - 2*a*b*c*d^2*e^3 + 2*a^2*c*d*e^4 - 8*a^3*d^2*f^3 + 2*(4*a^2*b*d^2*e + a^3*d*e^2 - 4*(a*b^2 - 2*a^2*c)*d^3)*f^2 - 2*(4*a*c^2*d^4 - 4*a*b*c*d^3*e + a^2*b*d*e^3 - (a*b^2 - 6*a^2*c)*d^2*e^2)*f + (b*c^2*d^3*e^2 - b^2*c*d^2*e^3 + a*b*c*d*e^4 - 4*a^2*b*d^2*f^3 + (4*a*b^2*d^2*e + a^2*b*d*e^2 - 4*(b^3 - 2*a*b*c)*d^3)*f^2 - (4*b*c^2*d^4 - 4*b^2*c*d^3*e + a*b^2*d*e^3 - (b^3 - 6*a*b*c)*d^2*e^2)*f)*x)*\sqrt{(b^2*d^2 - 2*a*b*d*e + a^2*e^2)} / (c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)) / x) - 1/4*\sqrt{2}*\sqrt{(2*c*d^2 - b*d*e + a*e^2 - 2*a*d*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*\sqrt{(b^2*d^2 - 2*a*b*d*e + a^2*e^2)} / (c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)) / x)
\end{aligned}$$

$$\begin{aligned}
& 2) * d^2 * e^3 - 2 * (a * b^2 * c - 2 * a^2 * c^2) * d * e^4) * f)) / (c^2 * d^2 * e^2 - b * c * d * e^3 + \\
& a * c * e^4 - 4 * a^2 * d * f^3 + (4 * a * b * d * e + a^2 * e^2 - 4 * (b^2 - 2 * a * c) * d^2) * f^2 - \\
& (4 * c^2 * d^3 - 4 * b * c * d^2 * e + a * b * e^3 - (b^2 - 6 * a * c) * d * e^2) * f)) * \log(- (2 * b^2 * d \\
& ^3 - 4 * a * b * d^2 * e + 2 * a^2 * d * e^2 - \sqrt{2} * (b^2 * d^2 * e^2 - 2 * a * b * d * e^3 + a^2 * e \\
& ^4 - 4 * (b^2 * d^3 - 2 * a * b * d^2 * e + a^2 * d * e^2) * f + (2 * c^3 * d^4 * e^2 - 3 * b * c^2 * d^3 \\
& * e^3 - 2 * a * b * c * d * e^5 + a^2 * c * e^6 + 8 * a^3 * d^2 * f^4 + (b^2 * c + 3 * a * c^2) * d^2 * e^4 \\
& - 2 * (2 * a^2 * b * d^2 * e + 3 * a^3 * d * e^2 - 4 * (a * b^2 - 3 * a^2 * c) * d^3) * f^3 + (5 * a^2 * \\
& b * d * e^3 + a^3 * e^4 - 8 * (b^2 * c - 3 * a * c^2) * d^4 + 4 * (b^3 - 2 * a * b * c) * d^3 * e - 2 * (\\
& 5 * a * b^2 - 11 * a^2 * c) * d^2 * e^2) * f^2 - (8 * c^3 * d^5 - 12 * b * c^2 * d^4 * e + a^2 * b * e^5 \\
& + 2 * (b^2 * c + 9 * a * c^2) * d^3 * e^2 + (b^3 - 10 * a * b * c) * d^2 * e^3 - 2 * (a * b^2 - 4 * a^2 \\
& * c) * d * e^4) * f) * \sqrt{(b^2 * d^2 - 2 * a * b * d * e + a^2 * e^2) / (c^4 * d^4 * e^2 - 2 * b * c^3 * d \\
& ^3 * e^3 - 2 * a * b * c^2 * d * e^5 + a^2 * c^2 * e^6 - 4 * a^4 * d * f^5 + (b^2 * c^2 + 2 * a * c^3) * \\
& d^2 * e^4 + (8 * a^3 * b * d * e + a^4 * e^2 - 8 * (a^2 * b^2 - 2 * a^3 * c) * d^2) * f^4 - 2 * (a^3 * \\
& b * e^3 + 2 * (b^4 - 4 * a * b^2 * c + 6 * a^2 * c^2) * d^3 - 4 * (a * b^3 - a^2 * b * c) * d^2 * e + (\\
& a^2 * b^2 + 6 * a^3 * c) * d * e^2) * f^3 - (8 * (b^2 * c^2 - 2 * a * c^3) * d^4 - 8 * (b^3 * c - a * b \\
& * c^2) * d^3 * e - (b^4 - 20 * a * b^2 * c + 22 * a^2 * c^2) * d^2 * e^2 + 2 * (a * b^3 - 5 * a^2 * b * \\
& c) * d * e^3 - (a^2 * b^2 + 2 * a^3 * c) * e^4) * f^2 - 2 * (2 * c^4 * d^5 - 4 * b * c^3 * d^4 * e + a^2 \\
& * b * c * e^5 + (b^2 * c^2 + 6 * a * c^3) * d^3 * e^2 + (b^3 * c - 5 * a * b * c^2) * d^2 * e^3 - 2 * (\\
& a * b^2 * c - 2 * a^2 * c^2) * d * e^4) * f)) * \sqrt{c * x^2 + b * x + a} * \sqrt{(2 * c * d^2 - b * d * \\
& e + a * e^2 - 2 * a * d * f - (c^2 * d^2 * e^2 - b * c * d * e^3 + a * c * e^4 - 4 * a^2 * d * f^3 + (4 \\
& * a * b * d * e + a^2 * e^2 - 4 * (b^2 - 2 * a * c) * d^2) * f^2 - (4 * c^2 * d^3 - 4 * b * c * d^2 * e + \\
& a * b * e^3 - (b^2 - 6 * a * c) * d * e^2) * f) * \sqrt{(b^2 * d^2 - 2 * a * b * d * e + a^2 * e^2) / (c^4 \\
& * d^4 * e^2 - 2 * b * c^3 * d^3 * e^3 - 2 * a * b * c^2 * d * e^5 + a^2 * c^2 * e^6 - 4 * a^4 * d * f^5 + \\
& (b^2 * c^2 + 2 * a * c^3) * d^2 * e^4 + (8 * a^3 * b * d * e + a^4 * e^2 - 8 * (a^2 * b^2 - 2 * a^3 * c) \\
&) * d^2) * f^4 - 2 * (a^3 * b * e^3 + 2 * (b^4 - 4 * a * b^2 * c + 6 * a^2 * c^2) * d^3 - 4 * (a * b^3 \\
& - a^2 * b * c) * d^2 * e + (a^2 * b^2 + 6 * a^3 * c) * d * e^2) * f^3 - (8 * (b^2 * c^2 - 2 * a * c^3) * \\
& d^4 - 8 * (b^3 * c - a * b * c^2) * d^3 * e - (b^4 - 20 * a * b^2 * c + 22 * a^2 * c^2) * d^2 * e^2 + \\
& 2 * (a * b^3 - 5 * a^2 * b * c) * d * e^3 - (a^2 * b^2 + 2 * a^3 * c) * e^4) * f^2 - 2 * (2 * c^4 * d^5 \\
& - 4 * b * c^3 * d^4 * e + a^2 * b * c * e^5 + (b^2 * c^2 + 6 * a * c^3) * d^3 * e^2 + (b^3 * c - 5 * a * \\
& b * c^2) * d^2 * e^3 - 2 * (a * b^2 * c - 2 * a^2 * c^2) * d * e^4) * f)) / (c^2 * d^2 * e^2 - b * c * d * e \\
& ^3 + a * c * e^4 - 4 * a^2 * d * f^3 + (4 * a * b * d * e + a^2 * e^2 - 4 * (b^2 - 2 * a * c) * d^2) * f^2 - \\
& (4 * c^2 * d^3 - 4 * b * c * d^2 * e + a * b * e^3 - (b^2 - 6 * a * c) * d * e^2) * f)) + (4 * b * c * \\
& d^3 + a * b * d * e^2 - (b^2 + 4 * a * c) * d^2 * e) * x + (2 * a * c^2 * d^3 * e^2 - 2 * a * b * c * d^2 * e \\
& ^3 + 2 * a^2 * c * d * e^4 - 8 * a^3 * d^2 * f^3 + 2 * (4 * a^2 * b * d^2 * e + a^3 * d * e^2 - 4 * (a * b^2 \\
& - 2 * a^2 * c) * d^3) * f^2 - 2 * (4 * a * c^2 * d^4 - 4 * a * b * c * d^3 * e + a^2 * b * d * e^3 - (a * b \\
& ^2 - 6 * a^2 * c) * d^2 * e^2) * f + (b * c^2 * d^3 * e^2 - b^2 * c * d^2 * e^3 + a * b * c * d * e^4 - 4 \\
& * a^2 * b * d^2 * f^3 + (4 * a * b^2 * d^2 * e + a^2 * b * d * e^2 - 4 * (b^3 - 2 * a * b * c) * d^3) * f^2 \\
& - (4 * b * c^2 * d^4 - 4 * b^2 * c * d^3 * e + a * b^2 * d * e^3 - (b^3 - 6 * a * b * c) * d^2 * e^2) * f) * \\
& x) * \sqrt{(b^2 * d^2 - 2 * a * b * d * e + a^2 * e^2) / (c^4 * d^4 * e^2 - 2 * b * c^3 * d^3 * e^3 - 2 * \\
& a * b * c^2 * d * e^5 + a^2 * c^2 * e^6 - 4 * a^4 * d * f^5 + (b^2 * c^2 + 2 * a * c^3) * d^2 * e^4 + (\\
& 8 * a^3 * b * d * e + a^4 * e^2 - 8 * (a^2 * b^2 - 2 * a^3 * c) * d^2) * f^4 - 2 * (a^3 * b * e^3 + 2 * (\\
& b^4 - 4 * a * b^2 * c + 6 * a^2 * c^2) * d^3 - 4 * (a * b^3 - a^2 * b * c) * d^2 * e + (a^2 * b^2 + 6 \\
& * a^3 * c) * d * e^2) * f^3 - (8 * (b^2 * c^2 - 2 * a * c^3) * d^4 - 8 * (b^3 * c - a * b * c^2) * d^3 * e \\
& - (b^4 - 20 * a * b^2 * c + 22 * a^2 * c^2) * d^2 * e^2 + 2 * (a * b^3 - 5 * a^2 * b * c) * d * e^3 - \\
& (a^2 * b^2 + 2 * a^3 * c) * e^4) * f^2 - 2 * (2 * c^4 * d^5 - 4 * b * c^3 * d^4 * e + a^2 * b * c * e^5 + \\
& (b^2 * c^2 + 6 * a * c^3) * d^3 * e^2 + (b^3 * c - 5 * a * b * c^2) * d^2 * e^3 - 2 * (a * b^2 * c - 2 \\
& * a^2 * c^2) * d * e^4) * f)) / x)
\end{aligned}$$

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.01, size = 1516, normalized size = 3.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)`

[Out]
$$-1/2/(-4*d*f+e^2)^{(1/2)}/f*2^{(1/2)}/((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*\ln(((b*f-c*e-(-4*d*f+e^2)^{(1/2)}*c)*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c+4*(b*f-c*e-(-4*d*f+e^2)^{(1/2)}*c)*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)})/(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)))*e^{-1/2}/f*2^{(1/2)}/((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*\ln(((b*f-c*e-(-4*d*f+e^2)^{(1/2)}*c)*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c+4*(b*f-c*e-(-4*d*f+e^2)^{(1/2)}*c)*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)})/(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f))-1/2/f*2^{(1/2)}/((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*\ln(((b*f-c*e+(-4*d*f+e^2)^{(1/2)}*c)*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c+4*(b*f-c*e+(-4*d*f+e^2)^{(1/2)}*c)*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)})/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f))+1/2/(-4*d*f+e^2)^{(1/2)}/f*2^{(1/2)}/((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*\ln(((b*f-c*e+(-4*d*f+e^2)^{(1/2)}*c)*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c+4*(b*f-c*e+(-4*d*f+e^2)^{(1/2)}*c)*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)})/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f))*e$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see 'assume?' for more details) Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{cx^2 + bx + a} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)`

[Out] `int(x/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a + bx + cx^2} (d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d), x)
```

```
[Out] Integral(x/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

$$3.117 \quad \int \frac{1}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx$$

Optimal. Leaf size=374

$$\frac{\sqrt{2} f \tanh^{-1} \left(\frac{4af+2x(bf-c(\sqrt{e^2-4df}+e))-b(\sqrt{e^2-4df}+e)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} \right)}{\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} - \frac{\sqrt{2} f \tanh^{-1} \left(\frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} \right)}{\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

Rubi [A] time = 0.31, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, number of rules / integrand size = 0.111, Rules used = {983, 724, 206}

$$\frac{\sqrt{2} f \tanh^{-1} \left(\frac{4af+2x(bf-c(\sqrt{e^2-4df}+e))-b(\sqrt{e^2-4df}+e)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} \right)}{\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} - \frac{\sqrt{2} f \tanh^{-1} \left(\frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} \right)}{\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]

[Out] -((Sqrt[2]*f*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x]/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]))/(Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) + (Sqrt[2]*f*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x]/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]))/(Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 983

Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c)/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e - b*f, 0] && PosQ[b^2 - 4*a*c]

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx = \frac{(2f) \int \frac{1}{(e-\sqrt{e^2-4df}+2fx)\sqrt{a+bx+cx^2}} dx}{\sqrt{e^2-4df}} - \frac{(2f) \int \frac{1}{(e+\sqrt{e^2-4df}+2fx)\sqrt{a+bx+cx^2}} dx}{\sqrt{e^2-4df}}$$

$$= \frac{(4f) \operatorname{Subst} \left(\int \frac{1}{16af^2-8bf(e-\sqrt{e^2-4df})+4c(e-\sqrt{e^2-4df})^2-x^2} dx, x, \frac{4af-b(e-\sqrt{e^2-4df})}{\sqrt{e^2-4df}} \right)}{\sqrt{e^2-4df}}$$

$$= \frac{\sqrt{2} f \operatorname{tanh}^{-1} \left(\frac{4af-b(e-\sqrt{e^2-4df})+2(bf-c(e-\sqrt{e^2-4df}))x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}} \right)}{\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} + \frac{\sqrt{2} f}{\sqrt{e^2-4df}}$$

Mathematica [A] time = 0.79, size = 376, normalized size = 1.01

$$\sqrt{2} f \frac{\left(\operatorname{tanh}^{-1} \left(\frac{4af-b(\sqrt{e^2-4df}+e-2fx)-2cx(\sqrt{e^2-4df}+e)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f(2af-b(\sqrt{e^2-4df}+e))+c(e\sqrt{e^2-4df}-2df+e^2)}} \right) - \operatorname{tanh}^{-1} \left(\frac{4af+b(\sqrt{e^2-4df}-c+2fx)+2cx(\sqrt{e^2-4df}-e)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f(2af+b(\sqrt{e^2-4df}+b(-e))+(-e\sqrt{e^2-4df}-2df+e^2))}} \right) \right)}{\sqrt{f(2af-b(\sqrt{e^2-4df}+e))+c(e\sqrt{e^2-4df}-2df+e^2)}\sqrt{f(2af+b(\sqrt{e^2-4df}+b(-e))+(-e\sqrt{e^2-4df}-2df+e^2))}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]

[Out] (Sqrt[2]*f*(ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f]))*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])/Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))] - ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f]))*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + x*(b + c*x)])/Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]))/Sqrt[e^2 - 4*d*f]

IntegrateAlgebraic [C] time = 0.00, size = 211, normalized size = 0.56

$$-\operatorname{RootSum} \left[\#1^4 f - 2\#1^3 \sqrt{c} e - 2\#1^2 a f + \#1^2 b e + 4\#1^2 c d + 2\#1 a \sqrt{c} e - 4\#1 b \sqrt{c} d + a^2 f - a b e + b^2 d \&, \frac{b \log(-\#1 + \sqrt{a + b x + c x^2} - \sqrt{c} x) - 2\#1 \sqrt{c} \log(-\#1 + \sqrt{a + b x + c x^2} - \sqrt{c} x)}{-2\#1^3 f + 3\#1^2 \sqrt{c} e + 2\#1 a f - \#1 b e - 4\#1 c d - a \sqrt{c} e + 2 b \sqrt{c} d} \& \right]$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]

[Out] -RootSum[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 &, (b*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*Sqrt[c]*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1)/(2*b*Sqrt[c]*d - a*Sqrt[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2*f*#1^3) &]

fricas [B] time = 17.21, size = 11287, normalized size = 30.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")

```
[Out] 1/4*sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f + (c^2*d^2*e^2 - b*c*d*
e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f
^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2
*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^
5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e
+ a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^
2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^
2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20
*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2
*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 +
6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*
e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a
^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b
^2 - 6*a*c)*d*e^2)*f))*log((2*(b^2*d - a*b*e)*f^2 + sqrt(2)*(c^2*d*e^3 - 4*
a*b*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*b*e^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^
2 + a*c*e^3)*f - (c^3*d^3*e^3 - b*c^2*d^2*e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^
2 - a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3*e^3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*
a*b^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^4 - a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*
d^3*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3*a*b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^
3*d^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e^4 - a^2*c*e^5 - (3*b^2*c - 5*a*c^2)
*d^2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^
3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d
^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b
*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a
^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*
c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c
)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2
*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a
*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2 + b*x + a)*sqrt((c*e^2 + 2*a*f^2
- (2*c*d + b*e)*f + (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*
a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a
*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*
d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (
b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)
*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 -
a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d
^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 +
2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 -
4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b
*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^
3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2
- (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d
*e - a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^2 - (4*c^2*d*e - b*c*e^2)*f)*x - (8*
a^3*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - 4*(a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*
a*c^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a
*c^2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f + (4*a^2*b*d*f^4 - (4*a*b^2*d*e +
a^2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3 + (4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*
b^2*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 - (b*c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c
*e^4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3
*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^
2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*
e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^
2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c
^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)
*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*
b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*
b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x) - 1/4*sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*
c*d + b*e)*f + (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*
e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3
```


$$\begin{aligned}
& - (b^2 - 6ac)d^2e^2) * f) * \sqrt{(c^2e^2 - 2b^2c^2ef + b^2f^2) / (c^4d^4e^2 - 2b^2c^3d^3e^3 - 2a^2b^2c^2d^2e^4 + a^2c^2e^6 - 4a^4d^4f^5 + (b^2c^2 + 2a^2c^3)d^2e^4 + (8a^3b^2d^2e + a^4e^2 - 8(a^2b^2 - 2a^3c)d^2) * f^4 - 2(a^3b^2e^3 + 2(b^4 - 4a^2b^2c + 6a^2c^2)d^3 - 4(a^2b^3 - a^2b^2c)d^2e + (a^2b^2 + 6a^3c)d^2e^2) * f^3 - (8(b^2c^2 - 2a^2c^3)d^4 - 8(b^3c - a^2b^2c)d^3e - (b^4 - 20a^2b^2c + 22a^2c^2)d^2e^2 + 2(a^2b^3 - 5a^2b^2c)d^2e^3 - (a^2b^2 + 2a^3c)e^4) * f^2 - 2(2c^4d^5 - 4b^2c^3d^4e + a^2b^2c^2d^3e^2 + (b^2c^2 + 6a^2c^3)d^3e^2 + (b^3c - 5a^2b^2c)d^2e^3 - 2(a^2b^2c - 2a^2c^2)d^2e^4) * f)) / (c^2d^2e^2 - b^2c^2d^2e^3 + ac^2e^4 - 4a^2d^2f^3 + (4a^2b^2d^2e + a^2e^2 - 4(b^2 - 2ac)d^2) * f^2 - (4c^2d^3 - 4b^2c^2d^2e + a^2b^2e^3 - (b^2 - 6ac)d^2e^2) * f) * \log((2(b^2d - ab^2e) * f^2 - \sqrt{2}(c^2d^2e^3 - 4a^2b^2d^2f^3 + (4b^2c^2d^2 + 4a^2c^2d^2e + ab^2e^2) * f^2 - (4c^2d^2e + b^2c^2d^2e^2 + ac^2e^3) * f - (c^3d^3e^3 - b^2c^2d^2e^4 + ac^2d^2e^5 + 4(2a^2b^2d^2 - a^3d^2e) * f^4 + (2a^2b^2d^2e^2 + a^3e^3 + 8(b^3 - 2a^2b^2c)d^3 - 4(3a^2b^2 - a^2c)d^2e) * f^3 + (8b^2c^2d^4 - a^2b^2e^4 - 4(3b^2c - ac^2)d^3e - 2(b^3 - 10a^2b^2c)d^2e^2 + (3a^2b^2 - 5a^2c)d^2e^3) * f^2 - (4c^3d^4e - 2b^2c^2d^3e^2 + 4a^2b^2c^2d^2e^4 - a^2c^2e^5 - (3b^2c - 5a^2c^2)d^2e^3) * f) * \sqrt{(c^2e^2 - 2b^2c^2ef + b^2f^2) / (c^4d^4e^2 - 2b^2c^3d^3e^3 - 2a^2b^2c^2d^2e^4 + a^2c^2e^6 - 4a^4d^4f^5 + (b^2c^2 + 2a^2c^3)d^2e^4 + (8a^3b^2d^2e + a^4e^2 - 8(a^2b^2 - 2a^3c)d^2) * f^4 - 2(a^3b^2e^3 + 2(b^4 - 4a^2b^2c + 6a^2c^2)d^3 - 4(a^2b^3 - a^2b^2c)d^2e + (a^2b^2 + 6a^3c)d^2e^2) * f^3 - (8(b^2c^2 - 2a^2c^3)d^4 - 8(b^3c - a^2b^2c)d^3e - (b^4 - 20a^2b^2c + 22a^2c^2)d^2e^2 + 2(a^2b^3 - 5a^2b^2c)d^2e^3 - (a^2b^2 + 2a^3c)e^4) * f^2 - 2(2c^4d^5 - 4b^2c^3d^4e + a^2b^2c^2d^3e^2 + (b^2c^2 + 6a^2c^3)d^3e^2 + (b^3c - 5a^2b^2c)d^2e^3 - 2(a^2b^2c - 2a^2c^2)d^2e^4) * f)) * \sqrt{cx^2 + bx + a} * \sqrt{(c^2e^2 + 2a^2f^2 - (2c^2d + b^2e) * f + (c^2d^2e^2 - b^2c^2d^2e^3 + ac^2e^4 - 4a^2d^2f^3 + (4a^2b^2d^2e + a^2e^2 - 4(b^2 - 2ac)d^2) * f^2 - (4c^2d^3 - 4b^2c^2d^2e + ab^2e^3 - (b^2 - 6ac)d^2e^2) * f) * \sqrt{(c^2e^2 - 2b^2c^2ef + b^2f^2) / (c^4d^4e^2 - 2b^2c^3d^3e^3 - 2a^2b^2c^2d^2e^4 + a^2c^2e^6 - 4a^4d^4f^5 + (b^2c^2 + 2a^2c^3)d^2e^4 + (8a^3b^2d^2e + a^4e^2 - 8(a^2b^2 - 2a^3c)d^2) * f^4 - 2(a^3b^2e^3 + 2(b^4 - 4a^2b^2c + 6a^2c^2)d^3 - 4(a^2b^3 - a^2b^2c)d^2e + (a^2b^2 + 6a^3c)d^2e^2) * f^3 - (8(b^2c^2 - 2a^2c^3)d^4 - 8(b^3c - a^2b^2c)d^3e - (b^4 - 20a^2b^2c + 22a^2c^2)d^2e^2 + 2(a^2b^3 - 5a^2b^2c)d^2e^3 - (a^2b^2 + 2a^3c)e^4) * f^2 - 2(2c^4d^5 - 4b^2c^3d^4e + a^2b^2c^2d^3e^2 + (b^2c^2 + 6a^2c^3)d^3e^2 + (b^3c - 5a^2b^2c)d^2e^3 - 2(a^2b^2c - 2a^2c^2)d^2e^4) * f)) / (c^2d^2e^2 - b^2c^2d^2e^3 + ac^2e^4 - 4a^2d^2f^3 + (4a^2b^2d^2e + a^2e^2 - 4(b^2 - 2ac)d^2) * f^2 - (4c^2d^3 - 4b^2c^2d^2e + ab^2e^3 - (b^2 - 6ac)d^2e^2) * f) - 2(b^2c^2d^2e - ac^2e^2) * f + ((4b^2c^2d - b^2e) * f^2 - (4c^2d^2e - b^2c^2d^2e^3 + ab^2e^4) * f) * x - (8a^3d^2f^4 - 2(4a^2b^2d^2e + a^3e^2 - 4(a^2b^2 - 2a^2c)d^2) * f^3 + 2(4a^2c^2d^3 - 4a^2b^2c^2d^2e + a^2b^2e^3 - (a^2b^2 - 6a^2c)d^2e^2) * f^2 - 2(ac^2d^2e^2 - ab^2c^2d^2e^3 + a^2c^2e^4) * f + (4a^2b^2d^2f^4 - (4a^2b^2d^2e + a^2b^2e^2 - 4(b^3 - 2a^2b^2c)d^2) * f^3 + (4b^2c^2d^3 - 4b^2c^2d^2e + ab^2e^3 - (b^3 - 6a^2b^2c)d^2e^2) * f^2 - (b^2c^2d^2e^2 - b^2c^2d^2e^3 + ab^2c^2e^4) * f) * x) * \sqrt{(c^2e^2 - 2b^2c^2ef + b^2f^2) / (c^4d^4e^2 - 2b^2c^3d^3e^3 - 2a^2b^2c^2d^2e^4 + a^2c^2e^6 - 4a^4d^4f^5 + (b^2c^2 + 2a^2c^3)d^2e^4 + (8a^3b^2d^2e + a^4e^2 - 8(a^2b^2 - 2a^3c)d^2) * f^4 - 2(a^3b^2e^3 + 2(b^4 - 4a^2b^2c + 6a^2c^2)d^3 - 4(a^2b^3 - a^2b^2c)d^2e + (a^2b^2 + 6a^3c)d^2e^2) * f^3 - (8(b^2c^2 - 2a^2c^3)d^4 - 8(b^3c - a^2b^2c)d^3e - (b^4 - 20a^2b^2c + 22a^2c^2)d^2e^2 + 2(a^2b^3 - 5a^2b^2c)d^2e^3 - (a^2b^2 + 2a^3c)e^4) * f^2 - 2(2c^4d^5 - 4b^2c^3d^4e + a^2b^2c^2d^3e^2 + (b^2c^2 + 6a^2c^3)d^3e^2 + (b^3c - 5a^2b^2c)d^2e^3 - 2(a^2b^2c - 2a^2c^2)d^2e^4) * f)) / x) + 1/4 * \sqrt{2} * \sqrt{(c^2e^2 + 2a^2f^2 - (2c^2d + b^2e) * f - (c^2d^2e^2 - b^2c^2d^2e^3 + ac^2e^4 - 4a^2d^2f^3 + (4a^2b^2d^2e + a^2e^2 - 4(b^2 - 2ac)d^2) * f^2 - (4c^2d^3 - 4b^2c^2d^2e + ab^2e^3 - (b^2 - 6ac)d^2e^2) * f) * \sqrt{(c^2e^2 - 2b^2c^2ef + b^2f^2) / (c^4d^4e^2 - 2b^2c^3d^3e^3 - 2a^2b^2c^2d^2e^4 + a^2c^2e^6 - 4a^4d^4f^5 + (b^2c^2 + 2a^2c^3)d^2e^4 + (8a^3b^2d^2e + a^4e^2 - 8(a^2b^2 - 2a^3c)d^2) * f^4 - 2(a^3b^2e^3 + 2(b^4 - 4a^2b^2c + 6a^2c^2)d^3 - 4(a^2b^3 - a^2b^2c)d^2e + (a^2b^2 + 6a^3c)d^2e^2) * f^3 - (8(b^2c^2 - 2a^2c^3)d^4 - 8(b^3c - a^2b^2c)d^3e - (b^4 - 20a^2b^2c + 22a^2c^2)d^2e^2 + 2(a^2b^3 - 5a^2b^2c)d^2e^3 - (a^2b^2 + 2a^3c)e^4) * f^2 - 2(2c^4d^5 - 4b^2c^3d^4e + a^2b^2c^2d^3e^2 + (b^2c^2 + 6a^2c^3)d^3e^2 + (b^3c - 5a^2b^2c)d^2e^3 - 2(a^2b^2c - 2a^2c^2)d^2e^4) * f)) / x)
\end{aligned}$$

$$\begin{aligned}
& ^4e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c \\
& + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)* \\
& f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a* \\
& b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^ \\
& 3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a \\
& *c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4 \\
&)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2* \\
& e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 \\
& - 6*a*c)*d*e^2)*f))*log((2*(b^2*d - a*b*e)*f^2 + sqrt(2)*(c^2*d*e^3 - 4*a*b \\
& *d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*b*e^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^2 + \\
& a*c*e^3)*f + (c^3*d^3*e^3 - b*c^2*d^2*e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^2 - \\
& a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3*e^3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*a*b \\
& ^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^4 - a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*d^3 \\
& *e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3*a*b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^3*d \\
& ^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e^4 - a^2*c*e^5 - (3*b^2*c - 5*a*c^2)*d^ \\
& 2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e \\
& ^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2* \\
& e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^ \\
& 3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2* \\
& b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2 \\
&)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d \\
& *e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b* \\
& c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^ \\
& 2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2 + b*x + a)*sqrt((c*e^2 + 2*a*f^2 - \\
& (2*c*d + b*e)*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b \\
& *d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b* \\
& e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4 \\
& *e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2 \\
& *c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^ \\
& 2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^ \\
& 2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 \\
& - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(\\
& a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4* \\
& b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^ \\
& 2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + \\
& a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - \\
& (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d*e \\
& - a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^2 - (4*c^2*d*e - b*c*e^2)*f)*x + (8*a^3 \\
& *d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - 4*(a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*a*c \\
& ^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a*c^ \\
& 2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f + (4*a^2*b*d*f^4 - (4*a*b^2*d*e + a^ \\
& 2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3 + (4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*b^2 \\
& *e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 - (b*c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c*e^ \\
& 4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^ \\
& 3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e \\
& ^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 \\
& + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b \\
& ^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2) \\
& *d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d* \\
& e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c \\
& *e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2 \\
& *c - 2*a^2*c^2)*d*e^4)*f)))/x) - 1/4*sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d \\
& + b*e)*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + \\
& a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - \\
& (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - \\
& 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + \\
& 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 \\
& - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c) \\
& *d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b
\end{aligned}$$

$$\begin{aligned} &^3c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 \\ &- 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3* \\ &d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2 \\ &*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e \\ &^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2 \\ &*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f))*log((2*(b^2*d - a*b \\ &*e)*f^2 - sqrt(2)*(c^2*d*e^3 - 4*a*b*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*b*e \\ &^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^2 + a*c*e^3)*f + (c^3*d^3*e^3 - b*c^2*d^2* \\ &e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^2 - a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3*e^ \\ &3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*a*b^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^4 - \\ &a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*d^3*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3*a* \\ &b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^3*d^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e^4 \\ &- a^2*c*e^5 - (3*b^2*c - 5*a*c^2)*d^2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b \\ &^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4* \\ &a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^ \\ &2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 \\ &- 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 \\ &- 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^ \\ &2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2 \\ &*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (\\ &b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2 \\ &+ b*x + a)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f - (c^2*d^2*e^2 - b*c*d* \\ &e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f \\ &^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2 \\ &*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^ \\ &5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e \\ &+ a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^ \\ &2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^ \\ &2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20 \\ &*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2 \\ &*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + \\ &6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d* \\ &e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a \\ &^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b \\ &^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d*e - a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^2 - \\ &(4*c^2*d*e - b*c*e^2)*f)*x + (8*a^3*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - 4*(\\ &a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*a*c^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a \\ &*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a*c^2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f \\ &+ (4*a^2*b*d*f^4 - (4*a*b^2*d*e + a^2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3 + \\ &(4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*b^2*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 - (b* \\ &c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c*e^4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f + b^ \\ &2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a \\ &^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^ \\ &2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 \\ &- 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 \\ &- 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^ \\ &2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2* \\ &(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b \\ &^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x \end{aligned}$$

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.01, size = 761, normalized size = 2.03

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d), x)`

[Out]
$$\frac{1}{(-4df+e^2)^{1/2}2^{1/2}} \frac{1}{((2af^2-bef-2cdf+ce^2-(-4df+e^2)^{1/2})bf+(-4df+e^2)^{1/2}ce)/f^2)^{1/2}} \ln\left(\frac{(bf-ce-(-4df+e^2)^{1/2})c(x+1/2(e+(-4df+e^2)^{1/2}))/f)}{f+(2af^2-bef-2cdf+ce^2-(-4df+e^2)^{1/2})bf+(-4df+e^2)^{1/2}ce)/f^2+1/22^{1/2}((2af^2-bef-2cdf+ce^2-(-4df+e^2)^{1/2})bf+(-4df+e^2)^{1/2}ce)/f^2)^{1/2}}\right) \frac{4(x+1/2(e+(-4df+e^2)^{1/2}))/f}{f^2c+4(bf-ce-(-4df+e^2)^{1/2})c} \frac{(x+1/2(e+(-4df+e^2)^{1/2}))/f}{f+2(2af^2-bef-2cdf+ce^2-(-4df+e^2)^{1/2})bf+(-4df+e^2)^{1/2}ce)/f^2)^{1/2}}{(x+1/2(e+(-4df+e^2)^{1/2}))/f)} - \frac{1}{(-4df+e^2)^{1/2}2^{1/2}} \frac{1}{((2af^2-bef-2cdf+ce^2+(-4df+e^2)^{1/2})bf-(-4df+e^2)^{1/2}ce)/f^2)^{1/2}} \ln\left(\frac{(bf-ce+(-4df+e^2)^{1/2})c(x-1/2(-e+(-4df+e^2)^{1/2}))/f)}{f+(2af^2-bef-2cdf+ce^2+(-4df+e^2)^{1/2})bf-(-4df+e^2)^{1/2}ce)/f^2+1/22^{1/2}((2af^2-bef-2cdf+ce^2+(-4df+e^2)^{1/2})bf-(-4df+e^2)^{1/2}ce)/f^2)^{1/2}}\right) \frac{4(x-1/2(-e+(-4df+e^2)^{1/2}))/f}{f^2c+4(bf-ce+(-4df+e^2)^{1/2})c} \frac{(x-1/2(-e+(-4df+e^2)^{1/2}))/f}{f+2(2af^2-bef-2cdf+ce^2+(-4df+e^2)^{1/2})bf-(-4df+e^2)^{1/2}ce)/f^2)^{1/2}}{(x-1/2(-e+(-4df+e^2)^{1/2}))/f)}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d), x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see 'assume?' for more details) Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{cx^2+bx+a} \sqrt{fx^2+ex+d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)`

[Out] `int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx+cx^2} \sqrt{d+ex+fx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d), x)`

[Out] `Integral(1/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)`

$$3.118 \quad \int \frac{1}{x\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx$$

Optimal. Leaf size=451

$$\frac{f(\sqrt{e^2-4df}+e) \tanh^{-1}\left(\frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right) + f(e-\sqrt{e^2-4df}) \tanh^{-1}\left(\frac{4af+2x(bf-c(e+\sqrt{e^2-4df}))-b(e+\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)+bef-2cdf+ce^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2} + \sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)+bef-2cdf+ce^2}}$$

Rubi [A] time = 2.63, antiderivative size = 451, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 30, number of rules / integrand size = 0.133, Rules used = {6728, 724, 206, 1032}

$$\frac{f(\sqrt{e^2-4df}+e) \tanh^{-1}\left(\frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right) + f(e-\sqrt{e^2-4df}) \tanh^{-1}\left(\frac{4af+2x(bf-c(e+\sqrt{e^2-4df}))-b(e+\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)+bef-2cdf+ce^2}}\right) + \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2} + \sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)+bef-2cdf+ce^2} + \sqrt{ad}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)), x]

[Out] -(ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/(Sqrt[a]*d)) + (f*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x]/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]))/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - (f*(e - Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x]/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]))/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1032

Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]

Rule 6728

Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx &= \int \left(\frac{1}{dx\sqrt{a+bx+cx^2}} + \frac{-e-fx}{d\sqrt{a+bx+cx^2}(d+ex+fx^2)} \right) dx \\
 &= \frac{\int \frac{1}{x\sqrt{a+bx+cx^2}} dx}{d} + \frac{\int \frac{-e-fx}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx}{d} \\
 &= \frac{2 \operatorname{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d} - \frac{\left(f\left(1 - \frac{e}{\sqrt{e^2-4df}}\right)\right) \int \frac{1}{(e+\sqrt{e^2-4df}+2f)}}{d} \\
 &= -\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d} + \frac{\left(2f\left(1 - \frac{e}{\sqrt{e^2-4df}}\right)\right) \operatorname{Subst}\left(\int \frac{1}{16af^2-8bf(e+\sqrt{e^2-4df}}}\right)}{\sqrt{a}d} \\
 &= -\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d} + \frac{f\left(1 + \frac{e}{\sqrt{e^2-4df}}\right) \tanh^{-1}\left(\frac{4af-b(e-\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2}}\right)}{\sqrt{2}d\sqrt{ce^2-2cdf-bef+2af^2}}
 \end{aligned}$$

Mathematica [A] time = 2.41, size = 450, normalized size = 1.00

$$\frac{\sqrt{2}f \left(\frac{\left(\sqrt{e^2-4df} - e \right) \tanh^{-1} \left(\frac{4af-b(\sqrt{e^2-4df}+e-2fx)-2cx(\sqrt{e^2-4df}+e)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f(2af-b(\sqrt{e^2-4df}+e))+c(e\sqrt{e^2-4df}-2df+e^2)}} \right)}{\sqrt{f(2af-b(\sqrt{e^2-4df}+e))+c(e\sqrt{e^2-4df}-2df+e^2)}} \right) + \frac{\left(\sqrt{e^2-4df} + e \right) \tanh^{-1} \left(\frac{4af+b(\sqrt{e^2-4df}-e+2fx)+2cx(\sqrt{e^2-4df}-e)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f(2af+b(\sqrt{e^2-4df}+b(-e))+c(-e\sqrt{e^2-4df}-2df+e^2)}} \right)}{\sqrt{f(2af+b(\sqrt{e^2-4df}+b(-e))+c(-e\sqrt{e^2-4df}-2df+e^2)}} \right)}{2d} - \frac{2 \tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}} \right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]

[Out] ((-2*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/Sqrt[a] + (Sqrt[2]*f*(((-e + Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)]))/Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))] + ((e + Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + x*(b + c*x)]))/Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])])))/Sqrt[e^2 - 4*d*f])/(2*d)

IntegrateAlgebraic [C] time = 0.54, size = 326, normalized size = 0.72

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{c}x - \sqrt{a+bx+cx^2}}{\sqrt{a}} \right)}{\sqrt{a}} - \frac{\operatorname{RootSum} \left[\#1^4 f - 2\#1^3 \sqrt{c}e - 2\#1^2 af + \#1^2 be + 4\#1^2 cd + 2\#1a\sqrt{c}e - 4\#1b\sqrt{c}d + a^2 f - abe + b^2 d \&, \frac{\#1^2 f \log \left(\#1 + \sqrt{a+bx+cx^2} - \sqrt{c}x \right) - 2\#1 \sqrt{c} \log \left(\#1 + \sqrt{a+bx+cx^2} - \sqrt{c}x \right) + b \log \left(\#1 + \sqrt{a+bx+cx^2} - \sqrt{c}x \right) - f \log \left(\#1 + \sqrt{a+bx+cx^2} - \sqrt{c}x \right)}{2\#1^3 f - 3\#1^2 \sqrt{c}e - 2\#1af + \#1be + 4\#1cd + a\sqrt{c}e - 2b\sqrt{c}d} \& \right]}{d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]

[Out] (2*ArcTanh[(Sqrt[c]*x)/Sqrt[a] - Sqrt[a + b*x + c*x^2]/Sqrt[a]])/(Sqrt[a]*d) - RootSum[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 &, (b*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*Sqrt[c]*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(-2*b*Sq

$\text{rt}[c]*d + a*\text{Sqrt}[c]*e + 4*c*d*#1 + b*e*#1 - 2*a*f*#1 - 3*\text{Sqrt}[c]*e*#1^2 + 2*f*#1^3) \&]/d$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.02, size = 859, normalized size = 1.90

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)

[Out]
$$\begin{aligned} & -2*f/(e+(-4*d*f+e^2)^{(1/2)})/(-4*d*f+e^2)^{(1/2)}*2^{(1/2)}/((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*\ln(((b*f-c*e-(-4*d*f+e^2)^{(1/2)}*c)*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c+4*(b*f-c*e-(-4*d*f+e^2)^{(1/2)}*c)*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)})/(x+1/2*(e+(-4*d*f+e^2)^{(1/2)}))/f)-2*f/(-e+(-4*d*f+e^2)^{(1/2)})/(-4*d*f+e^2)^{(1/2)}*2^{(1/2)}/((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*\ln(((b*f-c*e+(-4*d*f+e^2)^{(1/2)}*c)*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2+1/2*2^{(1/2)}*((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)^2*c+4*(b*f-c*e+(-4*d*f+e^2)^{(1/2)}*c)*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)})/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}))/f)+4*f/(-e+(-4*d*f+e^2)^{(1/2)})/(e+(-4*d*f+e^2)^{(1/2)})/a^{(1/2)}*\ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)})*a^{(1/2)})/x) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a}(fx^2 + ex + d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x \sqrt{cx^2 + bx + a} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)

[Out] int(1/(x*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sqrt{a + bx + cx^2} (d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)

[Out] Integral(1/(x*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)

$$3.119 \quad \int \frac{1}{x^2 \sqrt{a+bx+cx^2} (d+ex+fx^2)} dx$$

Optimal. Leaf size=543

$$\frac{b \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right) f\left(e\sqrt{e^2-4df}-2df+e^2\right) \tanh^{-1}\left(\frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{2a^{3/2}d} - \frac{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{f(2af-b(e-\sqrt{e^2-4df}))}+c(-e\sqrt{e^2-4df}-2df+e^2)}}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{f(2af-b(e-\sqrt{e^2-4df}))}+c(-e\sqrt{e^2-4df}-2df+e^2)}} + \frac{e \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right) - \frac{\sqrt{a+bx+cx^2}}{adx}}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{f(2af-b(e-\sqrt{e^2-4df}))}+c(-e\sqrt{e^2-4df}-2df+e^2)}} + \frac{e \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right) - \frac{\sqrt{a+bx+cx^2}}{adx}}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}}$$

Rubi [A] time = 4.59, antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 30, number of rules / integrand size = 0.167, Rules used = {6728, 730, 724, 206, 1032}

$$\frac{b \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right) - \frac{\sqrt{a+bx+cx^2}}{adx}}{2a^{3/2}d} - \frac{f(e\sqrt{e^2-4df}-2df+e^2) \tanh^{-1}\left(\frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{f(2af-b(e-\sqrt{e^2-4df}))}+c(-e\sqrt{e^2-4df}-2df+e^2)}} + \frac{f(-e\sqrt{e^2-4df}-2df+e^2) \tanh^{-1}\left(\frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}} + \frac{e \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right) - \frac{\sqrt{a+bx+cx^2}}{adx}}{\sqrt{a}d^2} - \frac{\sqrt{a+bx+cx^2}}{adx}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]

[Out] -(sqrt[a + b*x + c*x^2]/(a*d*x)) + (b*ArcTanh[(2*a + b*x)/(2*sqrt[a]*sqrt[a + b*x + c*x^2])])/(2*a^(3/2)*d) + (e*ArcTanh[(2*a + b*x)/(2*sqrt[a]*sqrt[a + b*x + c*x^2])])/(sqrt[a]*d^2) - (f*(e^2 - 2*d*f + e*sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e - sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - sqrt[e^2 - 4*d*f]))*x]/(2*sqrt[2]*sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*sqrt[e^2 - 4*d*f]*sqrt[a + b*x + c*x^2])])/(sqrt[2]*d^2*sqrt[e^2 - 4*d*f]*sqrt[c*(e^2 - 2*d*f - e*sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - sqrt[e^2 - 4*d*f]))]) + (f*(e^2 - 2*d*f - e*sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e + sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + sqrt[e^2 - 4*d*f]))*x]/(2*sqrt[2]*sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*sqrt[e^2 - 4*d*f]*sqrt[a + b*x + c*x^2])])/(sqrt[2]*d^2*sqrt[e^2 - 4*d*f]*sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*sqrt[e^2 - 4*d*f]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 730

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]

Rule 1032

Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*sqrt[d + e*x + f*x^2]), x],

```
x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2 \sqrt{a + bx + cx^2} (d + ex + fx^2)} dx = \int \left(\frac{1}{dx^2 \sqrt{a + bx + cx^2}} - \frac{e}{d^2 x \sqrt{a + bx + cx^2}} + \frac{e^2 - df + efx}{d^2 \sqrt{a + bx + cx^2} (d + ex + fx^2)} \right) dx$$

$$= \frac{\int \frac{e^2 - df + efx}{\sqrt{a + bx + cx^2} (d + ex + fx^2)} dx}{d^2} + \frac{\int \frac{1}{x^2 \sqrt{a + bx + cx^2}} dx}{d} - \frac{e \int \frac{1}{x \sqrt{a + bx + cx^2}} dx}{d^2}$$

$$= -\frac{\sqrt{a + bx + cx^2}}{adx} - \frac{b \int \frac{1}{x \sqrt{a + bx + cx^2}} dx}{2ad} + \frac{(2e) \text{Subst} \left(\int \frac{1}{4a - x^2} dx, x, \frac{2a + bx}{\sqrt{a + bx + cx^2}} \right)}{d^2}$$

$$= -\frac{\sqrt{a + bx + cx^2}}{adx} + \frac{e \tanh^{-1} \left(\frac{2a + bx}{2\sqrt{a} \sqrt{a + bx + cx^2}} \right)}{\sqrt{a} d^2} + \frac{b \text{Subst} \left(\int \frac{1}{4a - x^2} dx, x, \frac{2a + bx}{\sqrt{a + bx + cx^2}} \right)}{ad}$$

$$= -\frac{\sqrt{a + bx + cx^2}}{adx} + \frac{b \tanh^{-1} \left(\frac{2a + bx}{2\sqrt{a} \sqrt{a + bx + cx^2}} \right)}{2a^{3/2}d} + \frac{e \tanh^{-1} \left(\frac{2a + bx}{2\sqrt{a} \sqrt{a + bx + cx^2}} \right)}{\sqrt{a} d^2}$$

Mathematica [A] time = 1.41, size = 533, normalized size = 0.98

$$\frac{b d \tanh^{-1} \left(\frac{2a + bx}{2\sqrt{a} \sqrt{a + bx + cx^2}} \right)}{a^{3/2} d^2} + \frac{\sqrt{2} f (e \sqrt{e^2 - 4df} + 2df - e^2) \tanh^{-1} \left(\frac{4ef + b(\sqrt{e^2 - 4df} + e - 2f) - 2c(\sqrt{e^2 - 4df} - e)}{2\sqrt{2} \sqrt{a + bx + cx^2} \sqrt{(2ef - b(\sqrt{e^2 - 4df} + e)) + (e \sqrt{e^2 - 4df} - 2df + e^2)}} \right)}{\sqrt{e^2 - 4df} \sqrt{f(2ef - b(\sqrt{e^2 - 4df} + e)) + (e \sqrt{e^2 - 4df} - 2df + e^2)}} + \frac{\sqrt{2} f (-2 - 2df + e) \tanh^{-1} \left(\frac{4ef + b(\sqrt{e^2 - 4df} + e + 2f) + 2c(\sqrt{e^2 - 4df} - e)}{2\sqrt{2} \sqrt{a + bx + cx^2} \sqrt{(2ef + b(\sqrt{e^2 - 4df} - e)) + (-e \sqrt{e^2 - 4df} - 2df + e^2)}} \right)}{\sqrt{f(2ef + b(\sqrt{e^2 - 4df} - e)) + (-e \sqrt{e^2 - 4df} - 2df + e^2)}} + \frac{2d \sqrt{a + bx + cx^2}}{ax} - \frac{2e \tanh^{-1} \left(\frac{2a + bx}{2\sqrt{a} \sqrt{a + bx + cx^2}} \right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)), x]
[Out] -1/2*((2*d*Sqrt[a + x*(b + c*x)])/(a*x) - (b*d*ArcTanh[(2*a + b*x)/(2*Sqrt[a]
*Sqrt[a + x*(b + c*x)]))]/a^(3/2) - (2*e*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*S
qrt[a + x*(b + c*x)])))/Sqrt[a] + (Sqrt[2]*f*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4
*d*f])*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4
*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(
2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])))/(Sqrt[e^2 - 4*
d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^
2 - 4*d*f]))]) + (Sqrt[2]*f*(e + (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*ArcTanh[
(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x)
)/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e
+ Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])))/Sqrt[c*(e^2 - 2*d*f - e*Sqr
t[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))])/d^2
```

IntegrateAlgebraic [C] time = 0.77, size = 430, normalized size = 0.79

$$\frac{\text{RootSum} \left[81^2 f^3 - 281^2 \sqrt{c} e - 281^2 a f + 81^2 b c + 481^2 c d + 281 a \sqrt{c} e - 481 b \sqrt{c} d + a^2 f^2 - a b c + b^2 d c, \frac{81^2 f \log \left(\frac{41 + \sqrt{41 b^2 c^2 - c^2}}{41 - \sqrt{41 b^2 c^2 - c^2}} \right) - 81 f \log \left(\frac{41 + \sqrt{41 b^2 c^2 - c^2}}{41 - \sqrt{41 b^2 c^2 - c^2}} \right) + 2 a \sqrt{c} \log \left(\frac{41 + \sqrt{41 b^2 c^2 - c^2}}{41 - \sqrt{41 b^2 c^2 - c^2}} \right) + 2 a \sqrt{c} \log \left(\frac{41 + \sqrt{41 b^2 c^2 - c^2}}{41 - \sqrt{41 b^2 c^2 - c^2}} \right) \right]}{281^2 f^3 - 281^2 \sqrt{c} e - 281^2 a f + 81^2 b c + 481^2 c d + 281 a \sqrt{c} e - 481 b \sqrt{c} d + a^2 f^2 - a b c + b^2 d c} \right] + \frac{(2a + b) \tanh^{-1} \left(\frac{\sqrt{41 b^2 c^2 - c^2}}{41} \right)}{a^{3/2} d} - \frac{\sqrt{a + bx + cx^2}}{adx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
[Out] -(Sqrt[a + b*x + c*x^2]/(a*d*x)) + ((b*d + 2*a*e)*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2])/Sqrt[a]])/(a^(3/2)*d^2) + RootSum[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - b*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*Sqrt[c]*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*Sqrt[c]*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(-2*b*Sqrt[c]*d + a*Sqrt[c]*e + 4*c*d*#1 + b*e*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]/d^2
fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
[Out] Timed out
maple [B] time = 0.02, size = 983, normalized size = 1.81
```

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
[Out] 4*f^2/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(((b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2)))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2)))/f)^2*c+4*(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2)))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2)))/f)+4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a/x*(c*x^2+b*x+a)^(1/2)-2*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*b/a^(3/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-4*f^2/(-e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(((b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2)))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2)))/f)^2*c+4*(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2)))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2)))/f)
```

$f+e^2)^{1/2})/f))+16*f^2*e/(-e+(-4*d*f+e^2)^{1/2})^2/(e+(-4*d*f+e^2)^{1/2})^2/a^{1/2}*ln((b*x+2*a+2*(c*x^2+b*x+a)^{1/2}*a^{1/2})/x)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a} (fx^2 + ex + d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2 \sqrt{cx^2 + bx + a} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)

[Out] int(1/(x^2*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{a + bx + cx^2} (d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)

[Out] Integral(1/(x**2*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)

$$3.120 \quad \int \frac{1}{x^3 \sqrt{a+bx+cx^2} (d+ex+fx^2)} dx$$

Optimal. Leaf size=679

$$\frac{(3b^2 - 4ac) \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right) - be \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right) + \frac{3b\sqrt{a+bx+cx^2}}{4a^2 dx} - \frac{(e^2 - df) \tanh^{-1}\left(\frac{2a}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a} d^3}}{8a^{5/2}d} - \frac{be \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d^2} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2 dx} - \frac{(e^2 - df) \tanh^{-1}\left(\frac{2a}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a} d^3}$$

Rubi [A] time = 11.23, antiderivative size = 679, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 30, number of rules / integrand size = 0.233, Rules used = {6728, 744, 806, 724, 206, 730, 1032}

$$\frac{(3b^2 - 4ac) \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right) - be \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right) + \frac{3b\sqrt{a+bx+cx^2}}{4a^2 dx} - \frac{(e^2 - df) \tanh^{-1}\left(\frac{2a}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a} d^3}}{8a^{5/2}d} - \frac{be \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d^2} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2 dx} - \frac{(e^2 - df) \tanh^{-1}\left(\frac{2a}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a} d^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]

[Out] -Sqrt[a + b*x + c*x^2]/(2*a*d*x^2) + (3*b*Sqrt[a + b*x + c*x^2])/(4*a^2*d*x) + (e*Sqrt[a + b*x + c*x^2])/(a*d^2*x) - ((3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(5/2)*d) - (b*e*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*a^(3/2)*d^2) - ((e^2 - d*f)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(Sqrt[a]*d^3) + (f*(2*e^3 - 4*d*e*f - (e^2 - d*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - (f*(2*e^3 - 4*d*e*f - (e^2 - d*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 730

Int[((d_.) + (e_.)*(x_)^(m_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x, x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(2*n_.)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 \sqrt{a+bx+cx^2} (d+ex+fx^2)} dx &= \int \left(\frac{1}{dx^3 \sqrt{a+bx+cx^2}} - \frac{e}{d^2 x^2 \sqrt{a+bx+cx^2}} + \frac{e^2 - df}{d^3 x \sqrt{a+bx+cx^2}} + \right. \\
&= \frac{\int \frac{-e(e^2-2df)-f(e^2-df)x}{\sqrt{a+bx+cx^2} (d+ex+fx^2)} dx}{d^3} + \frac{\int \frac{1}{x^3 \sqrt{a+bx+cx^2}} dx}{d} - \frac{e \int \frac{1}{x^2 \sqrt{a+bx+cx^2}} dx}{d^2} + \\
&= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} + \frac{e\sqrt{a+bx+cx^2}}{ad^2x} - \frac{\int \frac{\frac{3b}{2}+cx}{x^2 \sqrt{a+bx+cx^2}} dx}{2ad} + \frac{(be) \int \frac{1}{x \sqrt{a+bx+cx^2}} dx}{2a} \\
&= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} + \frac{e\sqrt{a+bx+cx^2}}{ad^2x} - \frac{(e^2-df)}{ad^2} \\
&= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} + \frac{e\sqrt{a+bx+cx^2}}{ad^2x} - \frac{be \tanh^{-1}}{ad^2} \\
&= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} + \frac{e\sqrt{a+bx+cx^2}}{ad^2x} - \frac{(3b^2-4a)}{ad^2}
\end{aligned}$$

Mathematica [A] time = 2.11, size = 669, normalized size = 0.99

$$\frac{d^2 \left((4ac-3b^2) \operatorname{tanh}^{-1} \left(\frac{2bx}{\sqrt{a+bx+cx^2}} \right) + \sqrt{d+ex+fx^2} \sqrt{a+bx+cx^2} \right) - 4d^2 c \operatorname{tanh}^{-1} \left(\frac{2bx}{\sqrt{a+bx+cx^2}} \right) - \frac{4d^2 \sqrt{a+bx+cx^2}}{2ax} - \frac{8(d^2-df) \operatorname{tanh}^{-1} \left(\frac{2bx}{\sqrt{a+bx+cx^2}} \right)}{\sqrt{a+bx+cx^2}} + \frac{4\sqrt{d+ex+fx^2} \left(\frac{d^2-2df}{\sqrt{a+bx+cx^2}} - df + e^2 \right) \operatorname{tanh}^{-1} \left(\frac{4ef \left(\sqrt{a+bx+cx^2} + 2f \right) + 2 \left(\sqrt{a+bx+cx^2} - e \right)}{2d\sqrt{a+bx+cx^2} \sqrt{d+ex+fx^2} + \left(\sqrt{a+bx+cx^2} - e \right) \left(-\sqrt{a+bx+cx^2} - 2df + e^2 \right)} \right) - 4\sqrt{d+ex+fx^2} \left(-d^2 \sqrt{a+bx+cx^2} + df \sqrt{a+bx+cx^2} - 2df + e^2 \right) \operatorname{tanh}^{-1} \left(\frac{4ef \left(\sqrt{a+bx+cx^2} + 2f \right) + 2 \left(\sqrt{a+bx+cx^2} - e \right)}{2d\sqrt{a+bx+cx^2} \sqrt{d+ex+fx^2} + \left(\sqrt{a+bx+cx^2} - e \right) \left(-\sqrt{a+bx+cx^2} - 2df + e^2 \right)} \right)}{\sqrt{d+ex+fx^2} \sqrt{a+bx+cx^2} \left((2d+e) \sqrt{a+bx+cx^2} + e \left(\sqrt{a+bx+cx^2} - 2df + e^2 \right) \right)} + \frac{8d\sqrt{a+bx+cx^2}}{a^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]

[Out] $\left((-4d^2 \operatorname{Sqrt}[a + x(b + cx)]) / (ax^2) + (8de \operatorname{Sqrt}[a + x(b + cx)]) / (ax^3) - (4bdde \operatorname{ArcTanh}[(2a + bx) / (2 \operatorname{Sqrt}[a] \operatorname{Sqrt}[a + x(b + cx)])]) / a^{3/2} - (8(e^2 - df) \operatorname{ArcTanh}[(2a + bx) / (2 \operatorname{Sqrt}[a] \operatorname{Sqrt}[a + x(b + cx)])]) / \operatorname{Sqrt}[a] + (d^2(6 \operatorname{Sqrt}[a] b \operatorname{Sqrt}[a + x(b + cx)] + (-3b^2x + 4acx) \operatorname{ArcTanh}[(2a + bx) / (2 \operatorname{Sqrt}[a] \operatorname{Sqrt}[a + x(b + cx)])]) / a^{5/2} - (4 \operatorname{Sqrt}[2] f (e^3 - 3de f - e^2 \operatorname{Sqrt}[e^2 - 4df] + df \operatorname{Sqrt}[e^2 - 4df]) \operatorname{ArcTanh}[(4af - 2c(e + \operatorname{Sqrt}[e^2 - 4df])x - b(e + \operatorname{Sqrt}[e^2 - 4df]) - 2fx) / (2 \operatorname{Sqrt}[2] \operatorname{Sqrt}[c(e^2 - 2df + e \operatorname{Sqrt}[e^2 - 4df]) + f(2af - b(e + \operatorname{Sqrt}[e^2 - 4df]))] \operatorname{Sqrt}[a + x(b + cx)])]) / (\operatorname{Sqrt}[e^2 - 4df] \operatorname{Sqrt}[c(e^2 - 2df + e \operatorname{Sqrt}[e^2 - 4df]) + f(2af - b(e + \operatorname{Sqrt}[e^2 - 4df]))]) + (4 \operatorname{Sqrt}[2] f (e^2 - df + (e(e^2 - 3df)) / \operatorname{Sqrt}[e^2 - 4df]) \operatorname{ArcTanh}[(4af + 2c(-e + \operatorname{Sqrt}[e^2 - 4df])x + b(-e + \operatorname{Sqrt}[e^2 - 4df]) + 2fx) / (2 \operatorname{Sqrt}[2] \operatorname{Sqrt}[c(e^2 - 2df - e \operatorname{Sqrt}[e^2 - 4df]) + f(2af + b(-e + \operatorname{Sqrt}[e^2 - 4df]))] \operatorname{Sqrt}[a + x(b + cx)])]) / \operatorname{Sqrt}[c(e^2 - 2df - e \operatorname{Sqrt}[e^2 - 4df]) + f(2af + b(-e + \operatorname{Sqrt}[e^2 - 4df]))]) / (8d^3) \right)$

IntegrateAlgebraic [C] time = 1.48, size = 558, normalized size = 0.82

$$\frac{8d^2 \sqrt{a+bx+cx^2} \operatorname{tanh}^{-1} \left(\frac{2bx}{\sqrt{a+bx+cx^2}} \right) - 4d^2 c \operatorname{tanh}^{-1} \left(\frac{2bx}{\sqrt{a+bx+cx^2}} \right) - \frac{4d^2 \sqrt{a+bx+cx^2}}{2ax} - \frac{8(d^2-df) \operatorname{tanh}^{-1} \left(\frac{2bx}{\sqrt{a+bx+cx^2}} \right)}{\sqrt{a+bx+cx^2}} + \frac{4\sqrt{d+ex+fx^2} \left(\frac{d^2-2df}{\sqrt{a+bx+cx^2}} - df + e^2 \right) \operatorname{tanh}^{-1} \left(\frac{4ef \left(\sqrt{a+bx+cx^2} + 2f \right) + 2 \left(\sqrt{a+bx+cx^2} - e \right)}{2d\sqrt{a+bx+cx^2} \sqrt{d+ex+fx^2} + \left(\sqrt{a+bx+cx^2} - e \right) \left(-\sqrt{a+bx+cx^2} - 2df + e^2 \right)} \right) - 4\sqrt{d+ex+fx^2} \left(-d^2 \sqrt{a+bx+cx^2} + df \sqrt{a+bx+cx^2} - 2df + e^2 \right) \operatorname{tanh}^{-1} \left(\frac{4ef \left(\sqrt{a+bx+cx^2} + 2f \right) + 2 \left(\sqrt{a+bx+cx^2} - e \right)}{2d\sqrt{a+bx+cx^2} \sqrt{d+ex+fx^2} + \left(\sqrt{a+bx+cx^2} - e \right) \left(-\sqrt{a+bx+cx^2} - 2df + e^2 \right)} \right)}{\sqrt{d+ex+fx^2} \sqrt{a+bx+cx^2} \left((2d+e) \sqrt{a+bx+cx^2} + e \left(\sqrt{a+bx+cx^2} - 2df + e^2 \right) \right)} + \frac{8d\sqrt{a+bx+cx^2}}{a^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^3*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]

```
[Out] ((-2*a*d + 3*b*d*x + 4*a*e*x)*Sqrt[a + b*x + c*x^2])/(4*a^2*d^2*x^2) + ((-3
*b^2*d^2 + 4*a*c*d^2 - 4*a*b*d*e - 8*a^2*e^2 + 8*a^2*d*f)*ArcTanh[(-(Sqrt[c
]*x) + Sqrt[a + b*x + c*x^2])/Sqrt[a]])/(4*a^(5/2)*d^3) - RootSum[b^2*d - a
*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2
- 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b*e^3*Log[-(Sqrt[c]*x) + Sqrt
[a + b*x + c*x^2] - #1] - 2*b*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2
] - #1] - a*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a*d*f^2*
Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*Sqrt[c]*e^3*Log[-(Sqrt[c
]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 4*Sqrt[c]*d*e*f*Log[-(Sqrt[c]*x) +
Sqrt[a + b*x + c*x^2] - #1]*#1 + e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c
*x^2] - #1]*#1^2 - d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2
)/(-2*b*Sqrt[c]*d + a*Sqrt[c]*e + 4*c*d*#1 + b*e*#1 - 2*a*f*#1 - 3*Sqrt[c]*
e*#1^2 + 2*f*#1^3) & ]/d^3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

[Out] Timed out

maple [B] time = 0.02, size = 1296, normalized size = 1.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] -8*f^3/(e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-b*e*f-
2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(
((b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2
-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2
*2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(
1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e-(-
4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c
*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)+16*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+
e^2)^(1/2))^2/a/x*(c*x^2+b*x+a)^(1/2)-8*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+
(-4*d*f+e^2)^(1/2))^2*b/a^(3/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/
x)+2*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a/x^2*(c*x^2+b*x+a)^(
1/2)-3*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*b/a^2/x*(c*x^2+b*x+
a)^(1/2)+3/2*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*b^2/a^(5/2)*l
n((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-2*f/(-e+(-4*d*f+e^2)^(1/2))/(e
+(-4*d*f+e^2)^(1/2))*c/a^(3/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x
)-8*f^3/(-e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-b*e*
f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*l
n(((b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*
f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2+
1/2*2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^
```


$$2)^{(1/2)} * c * e / f^2)^{(1/2)} * (4 * (x - 1/2 * (-e + (-4 * d * f + e^2)^{(1/2)})) / f)^2 * c + 4 * (b * f - c * e + (-4 * d * f + e^2)^{(1/2)} * c) * (x - 1/2 * (-e + (-4 * d * f + e^2)^{(1/2)})) / f / f + 2 * (2 * a * f^2 - b * e * f - 2 * c * d * f + c * e^2 + (-4 * d * f + e^2)^{(1/2)} * b * f - (-4 * d * f + e^2)^{(1/2)} * c * e) / f^2)^{(1/2)} / (x - 1/2 * (-e + (-4 * d * f + e^2)^{(1/2)})) / f) - 64 * f^4 / (-e + (-4 * d * f + e^2)^{(1/2)})^3 / (e + (-4 * d * f + e^2)^{(1/2)})^3 / a^{(1/2)} * \ln((b * x + 2 * a + 2 * (c * x^2 + b * x + a)^{(1/2)} * a^{(1/2)}) / x) * d + 64 * f^3 / (-e + (-4 * d * f + e^2)^{(1/2)})^3 / (e + (-4 * d * f + e^2)^{(1/2)})^3 / a^{(1/2)} * \ln((b * x + 2 * a + 2 * (c * x^2 + b * x + a)^{(1/2)} * a^{(1/2)}) / x) * e^2$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a} (fx^2 + ex + d)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3 \sqrt{cx^2 + bx + a} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)

[Out] int(1/(x^3*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \sqrt{a + bx + cx^2} (d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)

[Out] Integral(1/(x**3*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)

3.121 $\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)} dx$

Optimal. Leaf size=779

$$\frac{2\left(cx\left((e^2-df)(abf-2ace+bcd)-de(-c(2af+be)+b^2f+2c^2d)\right)-(adf-ae^2+bde)\left(-c(2af+be)+b^2f+2c^2d\right)\right)}{f^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

Rubi [A] time = 14.17, antiderivative size = 779, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 30, number of rules / integrand size = 0.233, Rules used = {6728, 613, 636, 1016, 1032, 724, 206}

$$\frac{2\left(cx\left((e^2-df)(abf-2ace+bcd)-de(-c(2af+be)+b^2f+2c^2d)\right)-(adf-ae^2+bde)\left(-c(2af+be)+b^2f+2c^2d\right)\right)}{f^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

Antiderivative was successfully verified.

[In] Int[x^3/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

[Out] (2*(2*a + b*x))/((b^2 - 4*a*c)*f*Sqrt[a + b*x + c*x^2]) + (2*e*(b + 2*c*x))/((b^2 - 4*a*c)*f^2*Sqrt[a + b*x + c*x^2]) + (2*(c*d*e*(b*c*d - 2*a*c*e + a*b*f) - (b*d*e - a*e^2 + a*d*f)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*((b*c*d - 2*a*c*e + a*b*f)*(e^2 - d*f) - d*e*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))))*x)/((b^2 - 4*a*c)*f^2*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[a + b*x + c*x^2]) + ((2*d*(b*d - a*e)*f + (e - Sqrt[e^2 - 4*d*f])*(c*d^2 - b*d*e + a*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])*Sqrt[a + b*x + c*x^2]])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - ((2*d*(b*d - a*e)*f + (e + Sqrt[e^2 - 4*d*f])*(c*d^2 - b*d*e + a*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])*Sqrt[a + b*x + c*x^2]])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 613

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] := Simp[(-2*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 636

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] := Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0]

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol]
:= Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1016

```
Int[((g_.) + (h_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol]
:= Simp[((a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*c*d - 2*a*c*e + a*b*f))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol]
:= With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol]
:= With[{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)} dx &= \int \left(-\frac{e}{f^2(a+bx+cx^2)^{3/2}} + \frac{x}{f(a+bx+cx^2)^{3/2}} + \frac{de+(e^2-df)x}{f^2(a+bx+cx^2)^{3/2}} \right) dx \\
&= \frac{\int \frac{de+(e^2-df)x}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)} dx}{f^2} - \frac{e \int \frac{1}{(a+bx+cx^2)^{3/2}} dx}{f^2} + \frac{\int \frac{x}{(a+bx+cx^2)^{3/2}} dx}{f} \\
&= \frac{2(2a+bx)}{(b^2-4ac)f\sqrt{a+bx+cx^2}} + \frac{2e(b+2cx)}{(b^2-4ac)f^2\sqrt{a+bx+cx^2}} + \frac{2(cde(bc-d^2))}{(b^2-4ac)^2\sqrt{a+bx+cx^2}} \\
&= \frac{2(2a+bx)}{(b^2-4ac)f\sqrt{a+bx+cx^2}} + \frac{2e(b+2cx)}{(b^2-4ac)f^2\sqrt{a+bx+cx^2}} + \frac{2(cde(bc-d^2))}{(b^2-4ac)^2\sqrt{a+bx+cx^2}} \\
&= \frac{2(2a+bx)}{(b^2-4ac)f\sqrt{a+bx+cx^2}} + \frac{2e(b+2cx)}{(b^2-4ac)f^2\sqrt{a+bx+cx^2}} + \frac{2(cde(bc-d^2))}{(b^2-4ac)^2\sqrt{a+bx+cx^2}} \\
&= \frac{2(2a+bx)}{(b^2-4ac)f\sqrt{a+bx+cx^2}} + \frac{2e(b+2cx)}{(b^2-4ac)f^2\sqrt{a+bx+cx^2}} + \frac{2(cde(bc-d^2))}{(b^2-4ac)^2\sqrt{a+bx+cx^2}}
\end{aligned}$$

Mathematica [A] time = 2.62, size = 1066, normalized size = 1.37

Antiderivative was successfully verified.

[In] Integrate[x^3/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

[Out] ((4*(2*a^3*f + b^3*d*x + a^2*(-2*c*d - b*e + 2*c*e*x + b*f*x) + a*b*(-3*c*d*x + b*(d - e*x)))/((b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]) + (Sqrt[2]*(c*d^2*(-e + Sqrt[e^2 - 4*d*f]) + b*d*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + a*(-e^3 + 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f]))*Log[-e + Sqrt[e^2 - 4*d*f] - 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]) + (Sqrt[2]*(c*d^2*(e + Sqrt[e^2 - 4*d*f]) - b*d*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + a*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f]))*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]) - (Sqrt[2]*(c*d^2*(e + Sqrt[e^2 - 4*d*f]) - b*d*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + a*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f]))*Log[-4*a*f + 2*c*e*x + 2*c*Sqrt[e^2 - 4*d*f]*x + b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x) - 2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])*Sqrt[a + x*(b + c*x)])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]) - (Sqrt[2]*(c*d^2*(-e + Sqrt[e^2 - 4*d*f]) + b*d*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + a*(-e^3 + 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f]))*Log[b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x) + 2*(2*a*f - c*e*x + c*Sqrt[e^2 - 4*d*f]*x + Sqrt[2]*Sqrt[f*(-b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))*Sqrt[a + x*(b + c*x)])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]))/(2*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f)))

IntegrateAlgebraic [C] time = 1.69, size = 729, normalized size = 0.94

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
[Out] (2*(a*b^2*d - 2*a^2*c*d - a^2*b*e + 2*a^3*f + b^3*d*x - 3*a*b*c*d*x - a*b^2
*e*x + 2*a^2*c*e*x + a^2*b*f*x))/((b^2 - 4*a*c)*(c^2*d^2 - b*c*d*e + a*c*e^
2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2)*Sqrt[a + b*x + c*x^2]) + RootS
um[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2
+ b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b^2*d^2*Log[-(Sqr
t[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a*c*d^2*Log[-(Sqrt[c]*x) + Sqrt[a +
b*x + c*x^2] - #1] - 2*a*b*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -
#1] + a^2*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^2*d*f*Log[
-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*b*Sqrt[c]*d^2*Log[-(Sqrt[c]*
x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*a*Sqrt[c]*d*e*Log[-(Sqrt[c]*x) + Sq
rt[a + b*x + c*x^2] - #1]*#1 - c*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^
2] - #1]*#1^2 + b*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 -
a*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + a*d*f*Log[-(Sq
rt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(2*b*Sqrt[c]*d - a*Sqrt[c]*e -
4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2*f*#1^3) & ]/(c^2*d^2 -
b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.04, size = 14651, normalized size = 18.81

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{(cx^2 + bx + a)^{3/2} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)

[Out] int(x^3/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d), x)

[Out] Timed out

$$3.122 \quad \int \frac{x^2}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)} dx$$

Optimal. Leaf size=609

$$\frac{2(cx(-abe - 2a(cd - af) + b^2d) + a(abf - 2ace + bcd))}{(b^2 - 4ac)\sqrt{a + bx + cx^2}((cd - af)^2 - (bd - ae)(ce - bf))} \frac{f(2d(cd - af) - (e - \sqrt{e^2 - 4df})(bd - ae)) \operatorname{tanh}^{-1}\left(\frac{e - \sqrt{e^2 - 4df}}{\sqrt{2a^2 + \sqrt{e^2 - 4df}(ce - bf) - bcf - 2cdf + ce^2}}\right)}{\sqrt{2}\sqrt{e^2 - 4df}((cd - af)^2 - (bd - ae)(ce - bf))}$$

Rubi [A] time = 5.84, antiderivative size = 609, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, number of rules / integrand size = 0.133, Rules used = {1061, 1032, 724, 206}

$$\frac{2(cx(-abe - 2a(cd - af) + b^2d) + a(abf - 2ace + bcd))}{(b^2 - 4ac)\sqrt{a + bx + cx^2}((cd - af)^2 - (bd - ae)(ce - bf))} \frac{f(2d(cd - af) - (e - \sqrt{e^2 - 4df})(bd - ae)) \operatorname{tanh}^{-1}\left(\frac{e - \sqrt{e^2 - 4df}}{\sqrt{2a^2 + \sqrt{e^2 - 4df}(ce - bf) - bcf - 2cdf + ce^2}}\right)}{\sqrt{2}\sqrt{e^2 - 4df}((cd - af)^2 - (bd - ae)(ce - bf))} + \frac{f(2d(cd - af) - (\sqrt{e^2 - 4df} + e)(bd - ae)) \operatorname{tanh}^{-1}\left(\frac{4af + 2b(bf - e - \sqrt{e^2 - 4df}) - 3(e - \sqrt{e^2 - 4df})}{2\sqrt{2}\sqrt{a + bx + cx^2}\sqrt{2a^2 - \sqrt{e^2 - 4df}(ce - bf) - bcf - 2cdf + ce^2}}\right)}{\sqrt{2}\sqrt{e^2 - 4df}((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{2a^2 + \sqrt{e^2 - 4df}(ce - bf) - bcf - 2cdf + ce^2}}$$

Antiderivative was successfully verified.

[In] Int[x^2/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

[Out] (-2*(a*(b*c*d - 2*a*c*e + a*b*f) + c*(b^2*d - a*b*e - 2*a*(c*d - a*f))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[a + b*x + c*x^2] - (f*(2*d*(c*d - a*f) - (b*d - a*e)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f])*Sqrt[a + b*x + c*x^2]])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) + (f*(2*d*(c*d - a*f) - (b*d - a*e)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f])*Sqrt[a + b*x + c*x^2]])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1032

Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]

Rule 1061

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[(a + b*x + c*x^2)^(p +

```

1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b)
*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a
*f)) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))x)/((b^2 - 4*a*c)*((c*d - a*f)
^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d -
a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^q*Simp[(-2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(Plus[A])*c*e + a*C*e) + 2*(A*c*(c*
d - a*f) - a*(c*C*d - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)
*(2*a*c*e - b*(c*d + a*f)) + (A*b)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
+ q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b)*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(A*c*e + a*C*e)
+ 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4
)))*x - c*f*(b^2*(C*d + A*f) - b*(A*c*e + a*C*e) + 2*(A*c*(c*d - a*f) - a*(
c*C*d - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f
, A, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1]
&& NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && I
LtQ[q, -1]) && !IGtQ[q, 0]

```

Rubi steps

$$\int \frac{x^2}{(a + bx + cx^2)^{3/2} (d + ex + fx^2)} dx = -\frac{2(a(bcd - 2ace + abf) + c(b^2d - abe - 2a(cd - af))x)}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a + bx + cx^2}} - \frac{2 \int \frac{1}{(b^2 - 4ac)} dx}{(b^2 - 4ac)}$$

$$= -\frac{2(a(bcd - 2ace + abf) + c(b^2d - abe - 2a(cd - af))x)}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a + bx + cx^2}} + \frac{f(2d(c))}{(b^2 - 4ac)}$$

$$= -\frac{2(a(bcd - 2ace + abf) + c(b^2d - abe - 2a(cd - af))x)}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a + bx + cx^2}} - \frac{(2f(2d(c))}{(b^2 - 4ac)}$$

$$= -\frac{2(a(bcd - 2ace + abf) + c(b^2d - abe - 2a(cd - af))x)}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a + bx + cx^2}} - \frac{f(2d(cd))}{\sqrt{2}\sqrt{e^2}}$$

Mathematica [A] time = 6.59, size = 1097, normalized size = 1.80

```

Integrate[x^2/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

```

Antiderivative was successfully verified.

```

[In] Integrate[x^2/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
[Out] (-2*(e - (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*(2*b^2*f - 4*a*c*f - b*c*(e - Sqr
t[e^2 - 4*d*f]) + 2*c*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)*(a + b*x + c*x^2
)/((b^2 - 4*a*c)*f*(4*a*f^2 - 2*b*f*(e - Sqrt[e^2 - 4*d*f]) + c*(e - Sqrt[
e^2 - 4*d*f])^2)*(a + x*(b + c*x))^(3/2)) - (2*(e + (e^2 - 2*d*f)/Sqrt[e^2
- 4*d*f])*(2*b^2*f - 4*a*c*f - b*c*(e + Sqrt[e^2 - 4*d*f]) + 2*c*(b*f - c*(
e + Sqrt[e^2 - 4*d*f]))*x)*(a + b*x + c*x^2))/((b^2 - 4*a*c)*f*(4*a*f^2 - 2
*b*f*(e + Sqrt[e^2 - 4*d*f]) + c*(e + Sqrt[e^2 - 4*d*f])^2)*(a + x*(b + c*x
))^(3/2)) + (4*(b + 2*c*x)*(-(c*(a + b*x + c*x^2))/(b^2 - 4*a*c))^(3/2))/
(c*f*(a + x*(b + c*x))^(3/2)*Sqrt[1 - (b + 2*c*x)^2/(b^2 - 4*a*c)]) + (16*S
qrt[2]*f*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b
*f*Sqrt[e^2 - 4*d*f])*(e + (-e^2 + 2*d*f)/Sqrt[e^2 - 4*d*f])*(a + b*x + c*x
^2)^(3/2)*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) - (-2*b*f + 2*c*(e - S

```


$$\frac{\sqrt{e^2 - 4df} \sqrt{ax^2 + bx + c}}{(2\sqrt{2}\sqrt{ce^2 - 2cdf - b^2e + 2af^2 - c^2}\sqrt{e^2 - 4df} + b\sqrt{ce^2 - 4df})\sqrt{ax^2 + bx + c}} \left(\frac{16af^2 - 8b^2(e - \sqrt{e^2 - 4df}) + 4c(e - \sqrt{e^2 - 4df})^2}{(4af^2 - 2b^2(e - \sqrt{e^2 - 4df}) + c(e - \sqrt{e^2 - 4df})^2)(16af^2 - 8b^2(e - \sqrt{e^2 - 4df}) + 4c(e - \sqrt{e^2 - 4df})^2)(a + x(b + cx))^{3/2}} + \frac{16\sqrt{2}\sqrt{ce^2 - 2cdf - b^2e + 2af^2 - c^2}\sqrt{e^2 - 4df} - b\sqrt{ce^2 - 4df}}{(4af^2 - 2b^2(e - \sqrt{e^2 - 4df}) + c(e - \sqrt{e^2 - 4df})^2)(16af^2 - 8b^2(e - \sqrt{e^2 - 4df}) + 4c(e - \sqrt{e^2 - 4df})^2)(a + x(b + cx))^{3/2}} \right) \operatorname{ArcTanh}\left(\frac{4af - b(e + \sqrt{e^2 - 4df})}{(4af^2 - 2b^2(e - \sqrt{e^2 - 4df}) + c(e - \sqrt{e^2 - 4df})^2)(16af^2 - 8b^2(e - \sqrt{e^2 - 4df}) + 4c(e - \sqrt{e^2 - 4df})^2)(a + x(b + cx))^{3/2}}}\right)$$

IntegrateAlgebraic [C] time = 1.40, size = 570, normalized size = 0.94

$$\frac{\operatorname{RootSum}\left[\frac{a^2 f^2 - 2b^2 \sqrt{ce} - 2b^2 e f + 8c^2 e + 4b^2 c d + 281 a \sqrt{ce} - 481 b \sqrt{cd} + e^2 f - abe + f^2 ab}{-b^2 f + abf + 2adf - a^2 e + b^2(-df) + bcd - c^2 e}\right]}{\frac{2(\sqrt{2} b f - 2c^2 e + 2b^2 c f + abcd - abce - 2a^2 d + f^2 ab)}{(b^2 - 4ac)\sqrt{a + bx + cx^2}(\sqrt{e^2 - 4df} - b^2 e + 2af^2 - c^2)}} \sqrt{ax^2 + bx + c}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

[Out]
$$\frac{(-2*(a*b*c*d - 2*a^2*c*e + a^2*b*f + b^2*c*d*x - 2*a*c^2*d*x - a*b*c*e*x + 2*a^2*c*f*x))}{(b^2 - 4*a*c)*(c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2)*\sqrt{a + b*x + c*x^2}} + \operatorname{RootSum}[b^2*d - a*b*e + a^2*f - 4*b*\sqrt{c}*d*\#1 + 2*a*\sqrt{c}*e*\#1 + 4*c*d*\#1^2 + b*e*\#1^2 - 2*a*f*\#1^2 - 2*\sqrt{c}*e*\#1^3 + f*\#1^4 \& , (b*c*d^2*\operatorname{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1) - 2*a*b*d*f*\operatorname{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1 + a^2*e*f*\operatorname{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1 - 2*c^(3/2)*d^2*\operatorname{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1*\#1 + 2*a*\sqrt{c}*d*f*\operatorname{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1*\#1 + b*d*f*\operatorname{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1*\#1^2 - a*e*f*\operatorname{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1*\#1^2)/(2*b*\sqrt{c}*d - a*\sqrt{c}*e - 4*c*d*\#1 - b*e*\#1 + 2*a*f*\#1 + 3*\sqrt{c}*e*\#1^2 - 2*f*\#1^3) \&]/(-c^2*d^2) + b*c*d*e - a*c*e^2 - b^2*d*f + 2*a*c*d*f + a*b*e*f - a^2*f^2)$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage2

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] sage2

maple [B] time = 0.02, size = 11341, normalized size = 18.62

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)

[Out] result too large to display

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for more details)Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{(cx^2 + bx + a)^{3/2} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)

[Out] int(x^2/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)

[Out] Timed out

$$3.123 \quad \int \frac{x}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)} dx$$

Optimal. Leaf size=609

$$\frac{2(a(-2acf + b^2f - bce + 2c^2d) + cx(abf - 2ace + bcd))}{(b^2 - 4ac)\sqrt{a + bx + cx^2}((cd - af)^2 - (bd - ae)(ce - bf))} + \frac{f(2d(ce - bf) - (e - \sqrt{e^2 - 4df})(cd - af)) \tanh^{-1}\left(\frac{2d(ce - bf) - (e - \sqrt{e^2 - 4df})(cd - af)}{\sqrt{2}\sqrt{e^2 - 4df}((cd - af)^2 - (bd - ae)(ce - bf))}\right)}{\sqrt{2}\sqrt{e^2 - 4df}((cd - af)^2 - (bd - ae)(ce - bf))}$$

Rubi [A] time = 5.64, antiderivative size = 609, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, number of rules / integrand size = 0.143, Rules used = {1016, 1032, 724, 206}

$$\frac{2(a(-2acf + b^2f - bce + 2c^2d) + cx(abf - 2ace + bcd))}{(b^2 - 4ac)\sqrt{a + bx + cx^2}((cd - af)^2 - (bd - ae)(ce - bf))} + \frac{f(2d(ce - bf) - (e - \sqrt{e^2 - 4df})(cd - af)) \tanh^{-1}\left(\frac{2d(ce - bf) - (e - \sqrt{e^2 - 4df})(cd - af)}{\sqrt{2}\sqrt{e^2 - 4df}((cd - af)^2 - (bd - ae)(ce - bf))}\right)}{\sqrt{2}\sqrt{e^2 - 4df}((cd - af)^2 - (bd - ae)(ce - bf))}$$

Antiderivative was successfully verified.

[In] Int[x/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x]

[Out] (2*(a*(2*c^2*d - b*c*e + b^2*f - 2*a*c*f) + c*(b*c*d - 2*a*c*e + a*b*f)*x) / ((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[a + b*x + c*x^2]) + (f*(2*d*(c*e - b*f) - (c*d - a*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x] / (2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])*Sqrt[a + b*x + c*x^2])) / (Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - (f*(2*d*(c*e - b*f) - (c*d - a*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x] / (2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])*Sqrt[a + b*x + c*x^2])) / (Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1016

Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*c*d - 2*a*c*e + a*b*f))*x) / ((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e))) * (a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))]*(p + q + 2) - (2*f*((g*c)*(2

```
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a
(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x}{(a + bx + cx^2)^{3/2} (d + ex + fx^2)} dx = \frac{2(a(2c^2d - bce + b^2f - 2acf) + c(bcd - 2ace + abf)x)}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a + bx + cx^2}} - \frac{2 \int \frac{1}{2}(b^2 - 4ac)}{(b^2 - 4ac)}$$

$$= \frac{2(a(2c^2d - bce + b^2f - 2acf) + c(bcd - 2ace + abf)x)}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a + bx + cx^2}} - \frac{f(2d(ce - b^2) + 2af(bd - ae) - 2cf(bd - ae))}{(b^2 - 4ac)}$$

$$= \frac{2(a(2c^2d - bce + b^2f - 2acf) + c(bcd - 2ace + abf)x)}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a + bx + cx^2}} + \frac{(2f(2d(ce - b^2) + 2af(bd - ae) - 2cf(bd - ae)))}{(b^2 - 4ac)}$$

$$= \frac{2(a(2c^2d - bce + b^2f - 2acf) + c(bcd - 2ace + abf)x)}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a + bx + cx^2}} + \frac{f(2d(ce - b^2) + 2af(bd - ae) - 2cf(bd - ae))}{\sqrt{2}\sqrt{e^2 - 4df}}$$

Mathematica [A] time = 5.68, size = 770, normalized size = 1.26

```
2(1 - e/Sqrt[e^2 - 4*d*f])*(2*b^2*f + b*c*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x)
+ 2*c*(-2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x))/((b^2 - 4*a*c)*(4*a*f^2 +
c*(e - Sqrt[e^2 - 4*d*f])^2 + 2*b*f*(-e + Sqrt[e^2 - 4*d*f]))*Sqrt[a + x*(
b + c*x)]) + (2*(1 + e/Sqrt[e^2 - 4*d*f])*(2*b^2*f - b*c*(e + Sqrt[e^2 - 4*
d*f] - 2*f*x) - 2*c*(2*a*f + c*(e + Sqrt[e^2 - 4*d*f])*x))/((b^2 - 4*a*c)*
(4*a*f^2 - 2*b*f*(e + Sqrt[e^2 - 4*d*f]) + c*(e + Sqrt[e^2 - 4*d*f])^2)*Sqr
t[a + x*(b + c*x)]) - (Sqrt[2]*f^2*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f -
2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt
[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2
- 4*d*f]))]*Sqrt[a + x*(b + c*x)])]/(Sqrt[e^2 - 4*d*f]*(c*(e^2 - 2*d*f +
e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))^(3/2)) - (Sqr
t[2]*f^2*(-e + Sqrt[e^2 - 4*d*f])*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]
) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f + 2*c*(-e + Sqrt
```

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
[Out] (2*(1 - e/Sqrt[e^2 - 4*d*f])*(2*b^2*f + b*c*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x)
+ 2*c*(-2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x))/((b^2 - 4*a*c)*(4*a*f^2 +
c*(e - Sqrt[e^2 - 4*d*f])^2 + 2*b*f*(-e + Sqrt[e^2 - 4*d*f]))*Sqrt[a + x*(
b + c*x)]) + (2*(1 + e/Sqrt[e^2 - 4*d*f])*(2*b^2*f - b*c*(e + Sqrt[e^2 - 4*
d*f] - 2*f*x) - 2*c*(2*a*f + c*(e + Sqrt[e^2 - 4*d*f])*x))/((b^2 - 4*a*c)*
(4*a*f^2 - 2*b*f*(e + Sqrt[e^2 - 4*d*f]) + c*(e + Sqrt[e^2 - 4*d*f])^2)*Sqr
t[a + x*(b + c*x)]) - (Sqrt[2]*f^2*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f -
2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt
[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2
- 4*d*f]))]*Sqrt[a + x*(b + c*x)])]/(Sqrt[e^2 - 4*d*f]*(c*(e^2 - 2*d*f +
e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))^(3/2)) - (Sqr
t[2]*f^2*(-e + Sqrt[e^2 - 4*d*f])*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]
) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f + 2*c*(-e + Sqrt
```

```
[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e
^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]
*Sqrt[a + x*(b + c*x)])))/(Sqrt[e^2 - 4*d*f]*(c*(-e^2 + 2*d*f + e*Sqrt[e^2
- 4*d*f]) + f*(-2*a*f + b*(e - Sqrt[e^2 - 4*d*f])))^2)
```

IntegrateAlgebraic [C] time = 1.36, size = 608, normalized size = 1.00

RootSum[$\frac{e^2 f^2 - 2d^2 f^2 - 2d^2 f^2 + 4d^2 f^2 + 4d^2 f^2 + 2d^2 f^2 - 4d^2 f^2 + d^2 f^2 - d^2 f^2 + 4d^2 f^2}{-d^2 f^2 + 2ad^2 f^2 - ad^2 f^2 + b^2 d^2 f^2 - b^2 d^2 f^2}$], $\frac{2(2d^2 f^2 - ad^2 f^2 + abd^2 f^2 - 2ad^2 f^2 + 2d^2 f^2 - b^2 d^2 f^2)}{(4ac - f^2)\sqrt{a + bx + cx^2} (d^2 f^2 - 2ad^2 f^2 + ad^2 f^2 - b^2 d^2 f^2)}$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*(-2*a*c^2*d + a*b*c*e - a*b^2*f + 2*a^2*c*f - b*c^2*d*x + 2*a*c^2*e*x -
a*b*c*f*x))/((-b^2 + 4*a*c)*(c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*
d*f - a*b*e*f + a^2*f^2)*Sqrt[a + b*x + c*x^2]) + RootSum[b^2*d - a*b*e + a
^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*
#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (-b*c*d*e*Log[-(Sqrt[c]*x) + Sqrt[a +
b*x + c*x^2] - #1]) + b^2*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #
1] + a*c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^2*f^2*Log[-
(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*c^(3/2)*d*e*Log[-(Sqrt[c]*x)
+ Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*b*Sqrt[c]*d*f*Log[-(Sqrt[c]*x) + Sqrt[
a + b*x + c*x^2] - #1]*#1 - c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2]
- #1]*#1^2 + a*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(2*
b*Sqrt[c]*d - a*Sqrt[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2
- 2*f*#1^3) & ]/(-(c^2*d^2) + b*c*d*e - a*c*e^2 - b^2*d*f + 2*a*c*d*f + a
*b*e*f - a^2*f^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage2

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage2
```

maple [B] time = 0.02, size = 7163, normalized size = 11.76

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

```
mupad [F] time = 0.00, size = -1, normalized size = -0.00
```

$$\int \frac{x}{(cx^2 + bx + a)^{3/2} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(x/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

$$3.124 \quad \int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)} dx$$

Optimal. Leaf size=666

$$\frac{2(-cx(-2acf + b^2f - bce + 2c^2d) - bc(cd - 3af) - 2ac^2e + b^3(-f) + b^2ce)}{(b^2 - 4ac)\sqrt{a + bx + cx^2}((cd - af)^2 - (bd - ae)(ce - bf))} \frac{f(f(2af - b(\sqrt{e^2 - 4df} + e)))}{\sqrt{2}\sqrt{e^2 - 4df}((cd - af)^2 -$$

Rubi [A] time = 1.75, antiderivative size = 666, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, number of rules / integrand size = 0.148, Rules used = {974, 1032, 724, 206}

$$\frac{2(-cx(-2acf + b^2f - bce + 2c^2d) - bc(cd - 3af) - 2ac^2e + b^3(-f) + b^2ce)}{(b^2 - 4ac)\sqrt{a + bx + cx^2}((cd - af)^2 - (bd - ae)(ce - bf))} \frac{f(f(2af - b(\sqrt{e^2 - 4df} + e)))}{\sqrt{2}\sqrt{e^2 - 4df}((cd - af)^2 - (bd - ae)(ce - bf))} + \frac{f(f(2af - b(\sqrt{e^2 - 4df} + e)))}{\sqrt{2}\sqrt{e^2 - 4df}((cd - af)^2 - (bd - ae)(ce - bf))} \frac{f(f(2af - b(\sqrt{e^2 - 4df} + e)))}{\sqrt{2}\sqrt{e^2 - 4df}((cd - af)^2 - (bd - ae)(ce - bf))}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x]

[Out] (2*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f) - c*(2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[a + b*x + c*x^2]) - (f*(c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f])))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])]/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f])))] + (f*(c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f])))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])]/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f])))]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 974

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*

```
(p + 1) - c*e*(2*p + q + 4))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q,
0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(a + bx + cx^2)^{3/2} (d + ex + fx^2)} dx = \frac{2(b^2ce - 2ac^2e - b^3f - bc(cd - 3af) - c(2c^2d - bce + b^2f - 2acf)x)}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a + bx + cx^2}} + \dots$$

Mathematica [A] time = 5.04, size = 700, normalized size = 1.05

$$2f \left[\frac{-2(-2(2af+e)(\sqrt{c^2-4df}+e))+2b^2f-b(\sqrt{c^2-4df}+e-2fz)}{(f^2-4ae)\sqrt{c^2+4b^2+4c^2z}} + \frac{2(c(\sqrt{c^2-4df}-e)-2af+2b^2f+bc(\sqrt{c^2-4df}-e+2fz))}{(b^2-4ac)\sqrt{c^2+4b^2+4c^2z}} + \frac{\sqrt{2}f^2 \tanh^{-1}\left(\frac{af-\sqrt{c^2-4df}+e-2fz}{2\sqrt{c^2+4b^2+4c^2z}}\right)}{(f(2af-\sqrt{c^2-4df}+e)-(\sqrt{c^2-4df}-2af+e))^2} + \frac{\sqrt{2}f^2 \sqrt{f(2af+e)(\sqrt{c^2-4df}-e))+(-c\sqrt{c^2-4df}-2af+e)}{\tanh^{-1}\left(\frac{af-\sqrt{c^2-4df}+e-2fz}{2\sqrt{c^2+4b^2+4c^2z}}\right)} \right] \sqrt{c^2-4df}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
[Out] (2*f*((2*b^2*f + b*c*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x) + 2*c*(-2*a*f + c*(-e
+ Sqrt[e^2 - 4*d*f])*x))/((b^2 - 4*a*c)*(c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d
*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f])))*Sqrt[a + x*(b + c*x)]) - (2*
(2*b^2*f - b*c*(e + Sqrt[e^2 - 4*d*f] - 2*f*x) - 2*c*(2*a*f + c*(e + Sqrt[e
^2 - 4*d*f])*x))/((b^2 - 4*a*c)*(4*a*f^2 - 2*b*f*(e + Sqrt[e^2 - 4*d*f]) +
c*(e + Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + x*(b + c*x)]) + (Sqrt[2]*f^2*ArcTanh
[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x)
)/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e +
Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])))/(c*(e^2 - 2*d*f + e*Sqrt[e^2
- 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f])))^(3/2) - (Sqrt[2]*f^2*Sqr
t[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*
d*f]))]*ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2
- 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) +
```


$$\frac{f*(2*a*f + b*(-e + \sqrt{e^2 - 4*d*f}))]*\sqrt{a + x*(b + c*x)}}{(c*(-e^2 + 2*d*f + e*\sqrt{e^2 - 4*d*f}) + f*(-2*a*f + b*(e - \sqrt{e^2 - 4*d*f})))^2)}/\sqrt{e^2 - 4*d*f}$$

IntegrateAlgebraic [C] time = 0.00, size = 730, normalized size = 1.10

RootOf[...]

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

[Out]
$$\frac{(-2*(b*c^2*d - b^2*c*e + 2*a*c^2*e + b^3*f - 3*a*b*c*f + 2*c^3*d*x - b*c^2*e*x + b^2*c*f*x - 2*a*c^2*f*x))/((b^2 - 4*a*c)*(c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2)*\sqrt{a + b*x + c*x^2}) + \text{RootSum}[b^2*d - a*b*e + a^2*f - 4*b*\sqrt{c}*d*\#1 + 2*a*\sqrt{c}*e*\#1 + 4*c*d*\#1^2 + b*e*\#1^2 - 2*a*f*\#1^2 - 2*\sqrt{c}*e*\#1^3 + f*\#1^4 \& , (- (b*c*e^2*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1)] + b*c*d*f*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1] + b^2*e*f*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1] + a*c*e*f*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1] - 2*a*b*f^2*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1] + 2*c^(3/2)*e^2*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1]*\#1 - 2*c^(3/2)*d*f*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1]*\#1 - 2*b*\sqrt{c}*e*f*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1]*\#1 + 2*a*\sqrt{c}*f^2*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1]*\#1 - c*e*f*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1]*\#1^2 + b*f^2*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + b*x + c*x^2}] - \#1]*\#1^2)/(2*b*\sqrt{c}*d - a*\sqrt{c}*e - 4*c*d*\#1 - b*e*\#1 + 2*a*f*\#1 + 3*\sqrt{c}*e*\#1^2 - 2*f*\#1^3) \&]/(c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2)$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage2

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] sage2

maple [B] time = 0.02, size = 4099, normalized size = 6.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)

[Out]
$$\frac{-2/(-4*d*f+e^2)^{(1/2)}/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)*f^2/((x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)^2*c+(b*f-c*e-(-4*d*f+e^2)^{(1/2)}*c)*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/f^2)^{(1/2)}-4*f/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2-b^2)/((x+1/2*(e+(-4*d*f$$

$$\frac{1}{f+c^2e^2/f^2-(-4d*fe^2)*c^2/f^2-b^2} \left/ \left(\frac{x-1/2(-e+(-4d*fe^2)^{1/2})}{f} \right)^2 + \frac{(b*fe-c*fe+(-4d*fe^2)^{1/2})c}{(x-1/2(-e+(-4d*fe^2)^{1/2}))f} \right/ \frac{1}{f} + \frac{1}{2} \frac{(2a*f^2-b*ef-2c*df+c*e^2+(-4d*fe^2)^{1/2})b*fe-(-4d*fe^2)^{1/2}c*ef}{f^2} \right)^{1/2} \frac{b^2+2/(-4d*fe^2)^{1/2}f}{(2a*f^2-b*ef-2c*df+c*e^2+(-4d*fe^2)^{1/2})b*fe-(-4d*fe^2)^{1/2}c*ef} \left/ \left(\frac{x-1/2(-e+(-4d*fe^2)^{1/2})}{f} \right)^2 + \frac{(b*fe-c*fe+(-4d*fe^2)^{1/2})c}{(x-1/2(-e+(-4d*fe^2)^{1/2}))f} \right/ \frac{1}{f} + \frac{1}{2} \frac{(2a*f^2-b*ef-2c*df+c*e^2+(-4d*fe^2)^{1/2})b*fe-(-4d*fe^2)^{1/2}c*ef}{f^2} \right)^{1/2} \frac{b*c*ef-2/(-4d*fe^2)^{1/2}}{(2a*f^2-b*ef-2c*df+c*e^2+(-4d*fe^2)^{1/2})b*fe-(-4d*fe^2)^{1/2}c*ef} \right)^{1/2} \frac{f^2+2^{1/2}}{\left(\frac{x-1/2(-e+(-4d*fe^2)^{1/2})}{f} \right)^2 + \frac{(b*fe-c*fe+(-4d*fe^2)^{1/2})c}{(x-1/2(-e+(-4d*fe^2)^{1/2}))f} \right/ \frac{1}{f} + \frac{1}{2} \frac{(2a*f^2-b*ef-2c*df+c*e^2+(-4d*fe^2)^{1/2})b*fe-(-4d*fe^2)^{1/2}c*ef}{f^2} \right)^{1/2} \ln \left(\frac{(b*fe-c*fe+(-4d*fe^2)^{1/2})c}{(x-1/2(-e+(-4d*fe^2)^{1/2}))f} \right) \right/ \frac{1}{f} + \frac{1}{2} \frac{(2a*f^2-b*ef-2c*df+c*e^2+(-4d*fe^2)^{1/2})b*fe-(-4d*fe^2)^{1/2}c*ef}{f^2} + \frac{1}{2} \frac{(2a*f^2-b*ef-2c*df+c*e^2+(-4d*fe^2)^{1/2})b*fe-(-4d*fe^2)^{1/2}c*ef}{f^2} \right)^{1/2} \frac{4(x-1/2(-e+(-4d*fe^2)^{1/2}))}{f} \right)^2 + \frac{4(b*fe-c*fe+(-4d*fe^2)^{1/2})c}{(x-1/2(-e+(-4d*fe^2)^{1/2}))f} \right/ \frac{1}{f} + \frac{1}{2} \frac{(2a*f^2-b*ef-2c*df+c*e^2+(-4d*fe^2)^{1/2})b*fe-(-4d*fe^2)^{1/2}c*ef}{f^2} \right)^{1/2} \right) \right/ (x-1/2(-e+(-4d*fe^2)^{1/2})) \right/ \frac{1}{f}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d*f-e^2>0)', see 'assume?' for more details) Is 4*d*f-e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(cx^2 + bx + a)^{3/2} (fx^2 + ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)

[Out] int(1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)

[Out] Timed out

3.125 $\int \frac{1}{x(a+bx+cx^2)^{3/2}(d+ex+fx^2)} dx$

Optimal. Leaf size=816

$$\frac{2(b^2 + cxb - 2ac)}{a(b^2 - 4ac)d\sqrt{cx^2 + bx + a}} - \frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{cx^2+bx+a}}\right)}{a^{3/2}d} + \frac{f\left((e - \sqrt{e^2 - 4df})(f(be - af) - c(e^2 - df)) - 2(f(be^2 - aef) - c(e^2 - df))\right)}{\sqrt{2}d\sqrt{e^2 - 4df}\left((cd - af)^2 - (bd - af)^2\right)}$$

Rubi [A] time = 15.92, antiderivative size = 814, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 30, number of rules / integrand size = 0.233, Rules used = {6728, 740, 12, 724, 206, 1016, 1032}

$$\frac{2\sqrt{a}\sqrt{cx^2+bx+a}}{a^{3/2}d} - \frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{cx^2+bx+a}}\right)}{a^{3/2}d} + \frac{f\left((e - \sqrt{e^2 - 4df})(f(be - af) - c(e^2 - df)) - 2(f(be^2 - aef) - c(e^2 - df))\right)}{\sqrt{2}d\sqrt{e^2 - 4df}\left((cd - af)^2 - (bd - af)^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*d*Sqrt[a + b*x + c*x^2]) + (2*(c
*e*(2*a*c*e - b*(c*d + a*f)) + (b*e - a*f)*(2*c^2*d + b^2*f - c*(b*e + 2*a*
f)) + c*(2*c^2*d*e + b*f*(b*e - a*f) - b*c*(e^2 + d*f))*x)/((b^2 - 4*a*c)*
d*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[a + b*x + c*x^2]) - ArcTan
h[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/(a^(3/2)*d) - (f*(2*f*(b*e
^2 - b*d*f - a*e*f) - 2*c*(e^3 - 2*d*e*f) - (e - Sqrt[e^2 - 4*d*f])*(f*(b*e
- a*f) - c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b
*f - c*(e - Sqrt[e^2 - 4*d*f]))*x]/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f
+ 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]))/(Sqrt[2
]*d*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2
- 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) + (f*(2*f*(b*
e^2 - b*d*f - a*e*f) - 2*c*(e^3 - 2*d*e*f) - (e + Sqrt[e^2 - 4*d*f])*(f*(b*
e - a*f) - c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(
b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x]/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f
+ 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]))/(Sqrt[
2]*d*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2
- 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
```

```
) * x) * (a + b * x + c * x^2)^(p + 1)) / ((p + 1) * (b^2 - 4 * a * c) * (c * d^2 - b * d * e + a * e^2)), x] + Dist[1 / ((p + 1) * (b^2 - 4 * a * c) * (c * d^2 - b * d * e + a * e^2)), Int[(d + e * x)^m * Simp[b * c * d * e * (2 * p - m + 2) + b^2 * e^2 * (m + p + 2) - 2 * c^2 * d^2 * (2 * p + 3) - 2 * a * c * e^2 * (m + 2 * p + 3) - c * e * (2 * c * d - b * e) * (m + 2 * p + 4) * x, x] * (a + b * x + c * x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4 * a * c, 0] && NeQ[c * d^2 - b * d * e + a * e^2, 0] && NeQ[2 * c * d - b * e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1016

```
Int[((g_.) + (h_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b * x + c * x^2)^(p + 1) * (d + e * x + f * x^2)^(q + 1) * (g * c * (2 * a * c * e - b * (c * d + a * f)) + (g * b - a * h) * (2 * c^2 * d + b^2 * f - c * (b * e + 2 * a * f)) + c * (g * (2 * c^2 * d + b^2 * f - c * (b * e + 2 * a * f)) - h * (b * c * d - 2 * a * c * e + a * b * f)) * x)) / ((b^2 - 4 * a * c) * ((c * d - a * f)^2 - (b * d - a * e) * (c * e - b * f)) * (p + 1)), x] + Dist[1 / ((b^2 - 4 * a * c) * ((c * d - a * f)^2 - (b * d - a * e) * (c * e - b * f)) * (p + 1)), Int[(a + b * x + c * x^2)^(p + 1) * (d + e * x + f * x^2)^q * Simp[(b * h - 2 * g * c) * ((c * d - a * f)^2 - (b * d - a * e) * (c * e - b * f)) * (p + 1) + (b^2 * (g * f) - b * (h * c * d + g * c * e + a * h * f) + 2 * (g * c * (c * d - a * f) - a * (-h * c * e))) * (a * f * (p + 1) - c * d * (p + 2)) - e * ((g * c) * (2 * a * c * e - b * (c * d + a * f)) + (g * b - a * h) * (2 * c^2 * d + b^2 * f - c * (b * e + 2 * a * f))) * (p + q + 2) - (2 * f * ((g * c) * (2 * a * c * e - b * (c * d + a * f)) + (g * b - a * h) * (2 * c^2 * d + b^2 * f - c * (b * e + 2 * a * f)))) * (p + q + 2) - (b^2 * g * f - b * (h * c * d + g * c * e + a * h * f) + 2 * (g * c * (c * d - a * f) - a * (-h * c * e))) * (b * f * (p + 1) - c * e * (2 * p + q + 4))] * x - c * f * (b^2 * (g * f) - b * (h * c * d + g * c * e + a * h * f) + 2 * (g * c * (c * d - a * f) + a * h * c * e)) * (2 * p + 2 * q + 5) * x^2, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4 * a * c, 0] && NeQ[e^2 - 4 * d * f, 0] && LtQ[p, -1] && NeQ[(c * d - a * f)^2 - (b * d - a * e) * (c * e - b * f), 0] && !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_)) / (((a_.) + (b_.)*(x_) + (c_.)*(x_)^2) * Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4 * a * c, 2]}, Dist[(2 * c * g - h * (b - q)) / q, Int[1 / ((b - q + 2 * c * x) * Sqrt[d + e * x + f * x^2]), x], x] - Dist[(2 * c * g - h * (b + q)) / q, Int[1 / ((b + q + 2 * c * x) * Sqrt[d + e * x + f * x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4 * a * c, 0] && NeQ[e^2 - 4 * d * f, 0] && PosQ[b^2 - 4 * a * c]
```

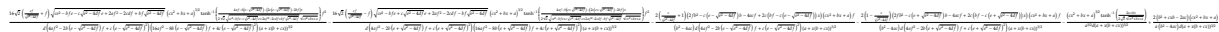
Rule 6728

```
Int[(u_) / ((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[{v = RationalFunctionExpand[u / (a + b * x^n + c * x^(2 * n)), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2 * n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(a+bx+cx^2)^{3/2}(d+ex+fx^2)} dx &= \int \left(\frac{1}{dx(a+bx+cx^2)^{3/2}} + \frac{-e-fx}{d(a+bx+cx^2)^{3/2}(d+ex+fx^2)} \right) dx \\
&= \frac{\int \frac{1}{x(a+bx+cx^2)^{3/2}} dx}{d} + \frac{\int \frac{-e-fx}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)} dx}{d} \\
&= \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)d\sqrt{a+bx+cx^2}} + \frac{2(ce(2ace-b(cd+af))+(be-af)(2d+ex+fx^2))}{(b^2-4ac)d\sqrt{a+bx+cx^2}} \\
&= \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)d\sqrt{a+bx+cx^2}} + \frac{2(ce(2ace-b(cd+af))+(be-af)(2d+ex+fx^2))}{(b^2-4ac)d\sqrt{a+bx+cx^2}} \\
&= \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)d\sqrt{a+bx+cx^2}} + \frac{2(ce(2ace-b(cd+af))+(be-af)(2d+ex+fx^2))}{(b^2-4ac)d\sqrt{a+bx+cx^2}} \\
&= \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)d\sqrt{a+bx+cx^2}} + \frac{2(ce(2ace-b(cd+af))+(be-af)(2d+ex+fx^2))}{(b^2-4ac)d\sqrt{a+bx+cx^2}}
\end{aligned}$$

Mathematica [A] time = 6.56, size = 1121, normalized size = 1.37



Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

[Out]
$$\begin{aligned}
& \frac{2(b^2-2ac+bcx)(a+bx+cx^2)}{a(b^2-4ac)d\sqrt{a+bx+cx^2}} - \frac{(2f(1+e/\sqrt{e^2-4df}))(2b^2f-4acf-bc(e-\sqrt{e^2-4df}))+2c(bf-c(e-\sqrt{e^2-4df}))x(a+bx+cx^2)}{(b^2-4ac)d(4af^2-2bf(e-\sqrt{e^2-4df}))+c(e-\sqrt{e^2-4df})^2(a+x(b+cx))^{3/2}} \\
& - \frac{(2f(1-e/\sqrt{e^2-4df}))(2b^2f-4acf-bc(e+\sqrt{e^2-4df}))+2c(bf-c(e+\sqrt{e^2-4df}))x(a+bx+cx^2)}{(b^2-4ac)d(4af^2-2bf(e+\sqrt{e^2-4df}))+c(e+\sqrt{e^2-4df})^2(a+x(b+cx))^{3/2}} \\
& - \frac{(a+bx+cx^2)^{3/2} \operatorname{ArcTanh}[(2a+bx)/(2\sqrt{a}\sqrt{a+bx+cx^2})]}{a^{3/2}d(a+x(b+cx))^{3/2}} + \frac{(16\sqrt{2}f^2(f+(ef)/\sqrt{e^2-4df})\sqrt{ce^2-2cdf-bef+2af^2-c\sqrt{e^2-4df}}+bf\sqrt{e^2-4df})(a+bx+cx^2)^{3/2} \operatorname{ArcTanh}[(4af-b(e-\sqrt{e^2-4df})-(-2bf+2c(e-\sqrt{e^2-4df})))x]/(2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-c\sqrt{e^2-4df}}+bf\sqrt{e^2-4df})\sqrt{a+bx+cx^2})]}{d(4af^2-2bf(e-\sqrt{e^2-4df}))+c(e-\sqrt{e^2-4df})^2(16af^2-8bf(e-\sqrt{e^2-4df}))+4c(e-\sqrt{e^2-4df})^2(a+x(b+cx))^{3/2}} \\
& - \frac{(16\sqrt{2}f^2(-f+(ef)/\sqrt{e^2-4df})\sqrt{ce^2-2cdf-bef+2af^2+c\sqrt{e^2-4df}}-bf\sqrt{e^2-4df})(a+bx+cx^2)^{3/2} \operatorname{ArcTanh}[(4af-b(e+\sqrt{e^2-4df})-(-2bf+2c(e+\sqrt{e^2-4df})))x]/(2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+c\sqrt{e^2-4df}}-bf\sqrt{e^2-4df})\sqrt{a+bx+cx^2})]}{d(4af^2-2bf(e+\sqrt{e^2-4df}))+c(e+\sqrt{e^2-4df})^2(16af^2-8bf(e+\sqrt{e^2-4df}))+4c(e+\sqrt{e^2-4df})^2(a+x(b+cx))^{3/2}}
\end{aligned}$$

IntegrateAlgebraic [C] time = 6.98, size = 1054, normalized size = 1.29

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
[Out] (2*(b^2*c^2*d - 2*a*c^3*d - b^3*c*e + 3*a*b*c^2*e + b^4*f - 4*a*b^2*c*f + 2
*a^2*c^2*f + b*c^3*d*x - b^2*c^2*e*x + 2*a*c^3*e*x + b^3*c*f*x - 3*a*b*c^2*
f*x))/(a*(b^2 - 4*a*c)*(c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f -
a*b*e*f + a^2*f^2)*Sqrt[a + b*x + c*x^2]) + (2*ArcTanh[(Sqrt[c]*x)/Sqrt[a]
- Sqrt[a + b*x + c*x^2]/Sqrt[a]])/(a^(3/2)*d) - RootSum[b^2*d - a*b*e + a^
2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#
1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (-b*c*e^3*Log[-(Sqrt[c]*x) + Sqrt[a +
b*x + c*x^2] - #1]) + 2*b*c*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2]
- #1] + b^2*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a*c*e^2*
f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - b^2*d*f^2*Log[-(Sqrt[c]*
x) + Sqrt[a + b*x + c*x^2] - #1] - a*c*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*
x + c*x^2] - #1] - 2*a*b*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #
1] + a^2*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*c^(3/2)*e^3
*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 4*c^(3/2)*d*e*f*Log[-(
Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*b*Sqrt[c]*e^2*f*Log[-(Sqrt[
c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*b*Sqrt[c]*d*f^2*Log[-(Sqrt[c]*x)
+ Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*a*Sqrt[c]*e*f^2*Log[-(Sqrt[c]*x) + Sq
rt[a + b*x + c*x^2] - #1]*#1 - c*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*
x^2] - #1]*#1^2 + c*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1
^2 + b*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - a*f^3*Lo
g[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(2*b*Sqrt[c]*d - a*Sqrt[
c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2*f*#1^3) & ]/(d*(
c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage0x

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
[Out] sage0*x
```

maple [B] time = 0.04, size = 4594, normalized size = 5.63

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)
[Out] 4*f^3/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^
2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/((x+1/2*(e+(-4*d*f+e^2)^(1
```


) * c) * (x - 1/2 * (-e + (-4*d*f + e^2)^(1/2)) / f) / f + 1/2 * (2*a*f^2 - b*e*f - 2*c*d*f + c*e^2 + (-4*d*f + e^2)^(1/2) * b*f - (-4*d*f + e^2)^(1/2) * c*e) / f^2)^(1/2) * x * c^2 * e - 4*f^2 / (-e + (-4*d*f + e^2)^(1/2)) / (2*a*f^2 - b*e*f - 2*c*d*f + c*e^2 + (-4*d*f + e^2)^(1/2) * b*f - (-4*d*f + e^2)^(1/2) * c*e) / (4*a*c - 4*c^2*d/f + c^2*e^2/f^2 - (-4*d*f + e^2) * c^2/f^2 - b^2) / ((x - 1/2 * (-e + (-4*d*f + e^2)^(1/2)) / f) / f + 1/2 * (2*a*f^2 - b*e*f - 2*c*d*f + c*e^2 + (-4*d*f + e^2)^(1/2) * b*f - (-4*d*f + e^2)^(1/2) * c*e) / f^2)^(1/2) * b * c - 4*f^3 / (-e + (-4*d*f + e^2)^(1/2)) / (-4*d*f + e^2)^(1/2) / (2*a*f^2 - b*e*f - 2*c*d*f + c*e^2 + (-4*d*f + e^2)^(1/2) * b*f - (-4*d*f + e^2)^(1/2) * c*e) / (4*a*c - 4*c^2*d/f + c^2*e^2/f^2 - (-4*d*f + e^2) * c^2/f^2 - b^2) / ((x - 1/2 * (-e + (-4*d*f + e^2)^(1/2)) / f) / f + 1/2 * (2*a*f^2 - b*e*f - 2*c*d*f + c*e^2 + (-4*d*f + e^2)^(1/2) * b*f - (-4*d*f + e^2)^(1/2) * c*e) / f^2)^(1/2) * b^2 + 4*f^2 / (-e + (-4*d*f + e^2)^(1/2)) / (-4*d*f + e^2)^(1/2) / (2*a*f^2 - b*e*f - 2*c*d*f + c*e^2 + (-4*d*f + e^2)^(1/2) * b*f - (-4*d*f + e^2)^(1/2) * c*e) / (4*a*c - 4*c^2*d/f + c^2*e^2/f^2 - (-4*d*f + e^2) * c^2/f^2 - b^2) / ((x - 1/2 * (-e + (-4*d*f + e^2)^(1/2)) / f) / f + 1/2 * (2*a*f^2 - b*e*f - 2*c*d*f + c*e^2 + (-4*d*f + e^2)^(1/2) * b*f - (-4*d*f + e^2)^(1/2) * c*e) / f^2)^(1/2) * b * c * e - 4*f^3 / (-e + (-4*d*f + e^2)^(1/2)) / (-4*d*f + e^2)^(1/2) / (2*a*f^2 - b*e*f - 2*c*d*f + c*e^2 + (-4*d*f + e^2)^(1/2) * b*f - (-4*d*f + e^2)^(1/2) * c*e) * 2^(1/2) / ((2*a*f^2 - b*e*f - 2*c*d*f + c*e^2 + (-4*d*f + e^2)^(1/2) * b*f - (-4*d*f + e^2)^(1/2) * c*e) / f^2)^(1/2) * ln(((b*f - c*e + (-4*d*f + e^2)^(1/2) * c) * (x - 1/2 * (-e + (-4*d*f + e^2)^(1/2)) / f) / f + (2*a*f^2 - b*e*f - 2*c*d*f + c*e^2 + (-4*d*f + e^2)^(1/2) * b*f - (-4*d*f + e^2)^(1/2) * c*e) / f^2 + 1/2 * 2^(1/2) * ((2*a*f^2 - b*e*f - 2*c*d*f + c*e^2 + (-4*d*f + e^2)^(1/2) * b*f - (-4*d*f + e^2)^(1/2) * c*e) / f^2)^(1/2) * (4 * (x - 1/2 * (-e + (-4*d*f + e^2)^(1/2)) / f) / f + 4 * (b*f - c*e + (-4*d*f + e^2)^(1/2) * c) * (x - 1/2 * (-e + (-4*d*f + e^2)^(1/2)) / f) / f + 2 * (2*a*f^2 - b*e*f - 2*c*d*f + c*e^2 + (-4*d*f + e^2)^(1/2) * b*f - (-4*d*f + e^2)^(1/2) * c*e) / f^2)^(1/2)) / (x - 1/2 * (-e + (-4*d*f + e^2)^(1/2)) / f) - 4*f / (-e + (-4*d*f + e^2)^(1/2)) / (e + (-4*d*f + e^2)^(1/2)) / a / (c*x^2 + b*x + a)^(1/2) + 8*f / (-e + (-4*d*f + e^2)^(1/2)) / (e + (-4*d*f + e^2)^(1/2)) * b/a / (4*a*c - b^2) / (c*x^2 + b*x + a)^(1/2) * c*x + 4*f / (-e + (-4*d*f + e^2)^(1/2)) / (e + (-4*d*f + e^2)^(1/2)) * b^2/a / (4*a*c - b^2) / (c*x^2 + b*x + a)^(1/2) + 4*f / (-e + (-4*d*f + e^2)^(1/2)) / (e + (-4*d*f + e^2)^(1/2)) / a^(3/2) * ln((b*x + 2*a + 2*(c*x^2 + b*x + a)^(1/2)) * a^(1/2)) / x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + bx + a)^{\frac{3}{2}}(fx^2 + ex + d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] integrate(1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 + e*x + d)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x(c x^2 + b x + a)^{\frac{3}{2}}(f x^2 + e x + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)

[Out] int(1/(x*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b x + c x^2)^{\frac{3}{2}}(d + e x + f x^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(1/(x*(a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

$$3.126 \quad \int \frac{x^4}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx$$

Optimal. Leaf size=140

$$-\frac{1}{4}\sqrt{-x^2-4x-3}x + \frac{5}{2}\sqrt{-x^2-4x-3} + \frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Rubi [A] time = 0.50, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 14, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {6728, 619, 216, 640, 742, 1028, 986, 12, 1026, 1161, 618, 204, 1027, 206}

$$-\frac{1}{4}\sqrt{-x^2-4x-3}x + \frac{5}{2}\sqrt{-x^2-4x-3} + \frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right) + \frac{11}{2}\sin^{-1}(x+2)$$

Antiderivative was successfully verified.

[In] Int[x^4/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]

[Out] (5*Sqrt[-3 - 4*x - x^2])/2 - (x*Sqrt[-3 - 4*x - x^2])/4 + (11*ArcSin[2 + x])/2 + ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/(2*Sqrt[2]) - ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/(2*Sqrt[2]) - (5*ArcTanh[x/Sqrt[-3 - 4*x - x^2]])/4

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b

+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 742

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]

Rule 986

Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e - b*f, 0] && NegQ[b^2 - 4*a*c]

Rule 1026

Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 - 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]

Rule 1027

Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[g, Subst[Int[1/(a + (c*d - a*f)*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] && EqQ[2*h*d - g*e, 0]

Rule 1028

Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := -Dist[(2*h*d - g*e)/e, Int[1/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] && NeQ[2*h*d - g*e, 0]

Rule 1161

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2

, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))

Rule 6728

Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^4}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx &= \int \left(\frac{5}{4\sqrt{-3-4x-x^2}} - \frac{x}{\sqrt{-3-4x-x^2}} + \frac{x^2}{2\sqrt{-3-4x-x^2}} - \frac{x^2}{4\sqrt{-3-4x-x^2}} \right) dx \\
 &= -\left(\frac{1}{4} \int \frac{15+8x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx \right) + \frac{1}{2} \int \frac{x^2}{\sqrt{-3-4x-x^2}} dx \\
 &= \sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} - \frac{1}{4} \int \frac{3+6x}{\sqrt{-3-4x-x^2}} dx + \frac{1}{2} \int \frac{x^2}{\sqrt{-3-4x-x^2}} dx \\
 &= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} + \frac{5}{4}\sin^{-1}(2+x) + \frac{1}{8} \int \frac{x^2}{\sqrt{-3-4x-x^2}} dx \\
 &= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} + \frac{13}{4}\sin^{-1}(2+x) - \tanh^{-1}\left(\frac{1-x}{\sqrt{-3-4x-x^2}}\right) \\
 &= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} + \frac{11}{2}\sin^{-1}(2+x) - \frac{5}{4}\tanh^{-1}\left(\frac{1-x}{\sqrt{-3-4x-x^2}}\right) \\
 &= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} + \frac{11}{2}\sin^{-1}(2+x) - \frac{5}{4}\tanh^{-1}\left(\frac{1-x}{\sqrt{-3-4x-x^2}}\right) \\
 &= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} + \frac{11}{2}\sin^{-1}(2+x) - \frac{5}{4}\tanh^{-1}\left(\frac{1-x}{\sqrt{-3-4x-x^2}}\right) \\
 &= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} + \frac{11}{2}\sin^{-1}(2+x) + \frac{\tan^{-1}\left(\frac{1-\sqrt{-3-4x-x^2}}{\sqrt{-3-4x-x^2}}\right)}{2\sqrt{2}}
 \end{aligned}$$

Mathematica [C] time = 0.53, size = 210, normalized size = 1.50

$$\frac{1}{24} \left(-6\sqrt{-x^2-4x-3} + 60\sqrt{-x^2-4x-3} - \sqrt{1-2i\sqrt{2}}(4\sqrt{2}+7i)\tanh^{-1}\left(\frac{-i\sqrt{2}x+2x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right) - \sqrt{1+2i\sqrt{2}}(4\sqrt{2}-7i)\tanh^{-1}\left(\frac{(2+i\sqrt{2})x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right) + 132\sin^{-1}(x+2) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]

[Out] (60*Sqrt[-3 - 4*x - x^2] - 6*x*Sqrt[-3 - 4*x - x^2] + 132*ArcSin[2 + x] - Sqrt[1 - (2*I)*Sqrt[2]]*(7*I + 4*Sqrt[2])*ArcTanh[(2 - (2*I)*Sqrt[2] + 2*x - I*Sqrt[2]*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]]) - Sqrt[1 + (2*I)*Sqrt[2]]*(-7*I + 4*Sqrt[2])*ArcTanh[(2 + (2*I)*Sqrt[2] + (2 + I*Sqrt[2])*x)/(Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])])/24

IntegrateAlgebraic [A] time = 0.42, size = 108, normalized size = 0.77

$$\frac{1}{4}\sqrt{-x^2-4x-3}(10-x) - \frac{\tan^{-1}\left(\frac{\sqrt{2}x+\frac{3}{\sqrt{2}}}{\sqrt{-x^2-4x-3}}\right)}{2\sqrt{2}} - 11 \tan^{-1}\left(\frac{\sqrt{-x^2-4x-3}}{x+3}\right) - \frac{5}{4} \tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^4/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]

[Out] ((10 - x)*Sqrt[-3 - 4*x - x^2])/4 - ArcTan[(3/Sqrt[2] + Sqrt[2]*x)/Sqrt[-3 - 4*x - x^2]]/(2*Sqrt[2]) - 11*ArcTan[Sqrt[-3 - 4*x - x^2]/(3 + x)] - (5*ArcTanh[x/Sqrt[-3 - 4*x - x^2]])/4

fricas [A] time = 1.28, size = 178, normalized size = 1.27

$$-\frac{1}{4}\sqrt{-x^2-4x-3}(x-10) + \frac{1}{8}\sqrt{2} \arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right) + \frac{1}{8}\sqrt{2} \arctan\left(\frac{-\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right) - \frac{11}{2} \arctan\left(\frac{\sqrt{-x^2-4x-3}(x+2)}{x^2+4x+3}\right) + \frac{5}{16} \log\left(\frac{-2\sqrt{-x^2-4x-3}x+4x+3}{x^2}\right) - \frac{5}{16} \log\left(\frac{2\sqrt{-x^2-4x-3}x-4x-3}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")

[Out] -1/4*sqrt(-x^2 - 4*x - 3)*(x - 10) + 1/8*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) + 1/8*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) - 11/2*arctan(sqrt(-x^2 - 4*x - 3)*(x + 2)/(x^2 + 4*x + 3)) + 5/16*log((-2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) - 5/16*log((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)

giac [A] time = 0.20, size = 188, normalized size = 1.34

$$-\frac{1}{4}\sqrt{-x^2-4x-3}(x-10) + \frac{1}{4}\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3(\sqrt{-x^2-4x-3}-1)}{x+2}+1\right)\right) + \frac{1}{4}\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right) + \frac{11}{2} \arcsin(x+2) - \frac{5}{8} \log\left(\frac{2(\sqrt{-x^2-4x-3}-1)}{x+2} + \frac{3(\sqrt{-x^2-4x-3}-1)^2}{(x+2)^2} + 1\right) + \frac{5}{8} \log\left(\frac{2(\sqrt{-x^2-4x-3}+1)}{x+2} + \frac{(\sqrt{-x^2-4x-3}-1)^2}{(x+2)^2} + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")

[Out] -1/4*sqrt(-x^2 - 4*x - 3)*(x - 10) + 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 11/2*arcsin(x + 2) - 5/8*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) + 5/8*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)

maple [A] time = 0.02, size = 159, normalized size = 1.14

$$-\frac{\sqrt{-x^2-4x-3}x}{4} + \frac{11 \arcsin(x+2)}{2} + \frac{5\sqrt{-x^2-4x-3}}{2} + \frac{\sqrt{3}\sqrt{4}\sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2}-12}}{24\sqrt{\frac{x^2}{(-x-\frac{3}{2})^2}-4}\sqrt{\frac{x}{(-x-\frac{3}{2})^2}+1}} \left(5 \operatorname{arctanh}\left(\frac{3x}{(-x-\frac{3}{2})\sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2}-12}}\right) + \sqrt{2} \arctan\left(\frac{\sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2}-12}\sqrt{2}}{6}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)

[Out] -1/4*x*(-x^2-4*x-3)^(1/2)+5/2*(-x^2-4*x-3)^(1/2)+11/2*arcsin(2+x)+1/24*3^(1/2)*4^(1/2)*(3/(-x-3/2)^2*x^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2))+5*arctanh(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(1/2)*x))/((1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^(1/2)/(1/(-x-3/2)*x+1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(2x^2 + 4x + 3)\sqrt{-x^2 - 4x - 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")

[Out] integrate(x^4/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{-x^2 - 4x - 3} (2x^2 + 4x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)

[Out] int(x^4/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{-(x+1)(x+3)} (2x^2 + 4x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)

[Out] Integral(x**4/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)

$$3.127 \quad \int \frac{x^3}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx$$

Optimal. Leaf size=115

$$-\frac{1}{2}\sqrt{-x^2-4x-3} + \frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{2\sqrt{2}} + \tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right) - 2\sin^{-1}(x+2)$$

Rubi [A] time = 0.42, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {6728, 619, 216, 640, 1028, 986, 12, 1026, 1161, 618, 204, 1027, 206}

$$-\frac{1}{2}\sqrt{-x^2-4x-3} + \frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{2\sqrt{2}} + \tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right) - 2\sin^{-1}(x+2)$$

Antiderivative was successfully verified.

[In] Int[x^3/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)), x]

[Out] -Sqrt[-3 - 4*x - x^2]/2 - 2*ArcSin[2 + x] + ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/(2*Sqrt[2]) - ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/(2*Sqrt[2]) + ArcTanh[x/Sqrt[-3 - 4*x - x^2]]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1026

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 -
4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1027

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
EqQ[2*h*d - g*e, 0]
```

Rule 1028

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(2*h*d - g*e)/e, Int[1/((a + b
*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g
, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
&& NeQ[2*h*d - g*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx &= \int \left(-\frac{1}{\sqrt{-3-4x-x^2}} + \frac{x}{2\sqrt{-3-4x-x^2}} + \frac{6+5x}{2\sqrt{-3-4x-x^2}(3+4x+2x^2)} \right) dx \\
&= \frac{1}{2} \int \frac{x}{\sqrt{-3-4x-x^2}} dx + \frac{1}{2} \int \frac{6+5x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx - \int \frac{1}{\sqrt{-3-4x-x^2}} dx \\
&= -\frac{1}{2} \sqrt{-3-4x-x^2} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}} dx, x, -4-2x \right) - \frac{5}{8} \int \frac{6+5x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx \\
&= -\frac{1}{2} \sqrt{-3-4x-x^2} - \sin^{-1}(2+x) + \frac{1}{8} \int \frac{-6-4x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx \\
&= -\frac{1}{2} \sqrt{-3-4x-x^2} - 2 \sin^{-1}(2+x) + \frac{5}{4} \tanh^{-1} \left(\frac{x}{\sqrt{-3-4x-x^2}} \right) + \frac{1}{2} \int \frac{6+5x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx \\
&= -\frac{1}{2} \sqrt{-3-4x-x^2} - 2 \sin^{-1}(2+x) + \tanh^{-1} \left(\frac{x}{\sqrt{-3-4x-x^2}} \right) + 4 \text{Subst} \left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}} dx, x, -4-2x \right) \\
&= -\frac{1}{2} \sqrt{-3-4x-x^2} - 2 \sin^{-1}(2+x) + \tanh^{-1} \left(\frac{x}{\sqrt{-3-4x-x^2}} \right) - \frac{1}{6} \text{Subst} \left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}} dx, x, -4-2x \right) \\
&= -\frac{1}{2} \sqrt{-3-4x-x^2} - 2 \sin^{-1}(2+x) + \tanh^{-1} \left(\frac{x}{\sqrt{-3-4x-x^2}} \right) + \frac{1}{3} \text{Subst} \left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}} dx, x, -4-2x \right) \\
&= -\frac{1}{2} \sqrt{-3-4x-x^2} - 2 \sin^{-1}(2+x) + \frac{\tan^{-1} \left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}} \right)}{2\sqrt{2}} - \frac{\tan^{-1} \left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}} \right)}{2\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 0.44, size = 192, normalized size = 1.67

$$\frac{1}{8} \left(-4 \left(\sqrt{-x^2-4x-3} + 4 \sin^{-1}(x+2) \right) + \frac{(5\sqrt{2}-2i) \tanh^{-1} \left(\frac{i\sqrt{2}x+2x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}} \sqrt{-x^2-4x-3}} \right)}{\sqrt{1-2i\sqrt{2}}} + \frac{(5\sqrt{2}+2i) \tanh^{-1} \left(\frac{(2-i\sqrt{2})x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}} \sqrt{-x^2-4x-3}} \right)}{\sqrt{1+2i\sqrt{2}}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(Sqrt[-3-4*x-x^2]*(3+4*x+2*x^2)),x]

[Out] (-4*(Sqrt[-3-4*x-x^2]+4*ArcSin[2+x]))+((-2*I+5*Sqrt[2])*ArcTanh[(2+(2*I)*Sqrt[2]+2*x+I*Sqrt[2]*x)/(Sqrt[2-(4*I)*Sqrt[2]]*Sqrt[-3-4*x-x^2])])/Sqrt[1-(2*I)*Sqrt[2]]+((2*I+5*Sqrt[2])*ArcTanh[(2-(2*I)*Sqrt[2]+(2-I*Sqrt[2])*x)/(Sqrt[2+(4*I)*Sqrt[2]]*Sqrt[-3-4*x-x^2])])/Sqrt[1+(2*I)*Sqrt[2]]/8

IntegrateAlgebraic [A] time = 0.37, size = 99, normalized size = 0.86

$$-\frac{1}{2} \sqrt{-x^2-4x-3} - \frac{\tan^{-1} \left(\frac{\sqrt{2}x+\frac{3}{\sqrt{2}}}{\sqrt{-x^2-4x-3}} \right)}{2\sqrt{2}} + 4 \tan^{-1} \left(\frac{\sqrt{-x^2-4x-3}}{x+3} \right) + \tanh^{-1} \left(\frac{x}{\sqrt{-x^2-4x-3}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3/(Sqrt[-3-4*x-x^2]*(3+4*x+2*x^2)),x]

[Out] $-1/2*\text{Sqrt}[-3 - 4*x - x^2] - \text{ArcTan}[(3/\text{Sqrt}[2] + \text{Sqrt}[2]*x)/\text{Sqrt}[-3 - 4*x - x^2]]/(2*\text{Sqrt}[2]) + 4*\text{ArcTan}[\text{Sqrt}[-3 - 4*x - x^2]/(3 + x)] + \text{ArcTanh}[x/\text{Sqrt}[-3 - 4*x - x^2]]$

fricas [A] time = 1.40, size = 175, normalized size = 1.52

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{\sqrt{2x+3}\sqrt{2x^2-4x-3}}{2(2x+3)}\right) + \frac{1}{8}\sqrt{2}\arctan\left(-\frac{\sqrt{2x-3}\sqrt{2x^2-4x-3}}{2(2x+3)}\right) - \frac{1}{2}\sqrt{-x^2-4x-3} + 2\arctan\left(\frac{\sqrt{-x^2-4x-3}(x+2)}{x^2+4x+3}\right) - \frac{1}{4}\log\left(\frac{2\sqrt{-x^2-4x-3}x+4x+3}{x^2}\right) + \frac{1}{4}\log\left(\frac{2\sqrt{-x^2-4x-3}x-4x-3}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")`

[Out] $1/8*\text{sqrt}(2)*\arctan(1/2*(\text{sqrt}(2)*x + 3*\text{sqrt}(2)*\text{sqrt}(-x^2 - 4*x - 3))/(2*x + 3)) + 1/8*\text{sqrt}(2)*\arctan(-1/2*(\text{sqrt}(2)*x - 3*\text{sqrt}(2)*\text{sqrt}(-x^2 - 4*x - 3))/(2*x + 3)) - 1/2*\text{sqrt}(-x^2 - 4*x - 3) + 2*\arctan(\text{sqrt}(-x^2 - 4*x - 3)*(x + 2)/(x^2 + 4*x + 3)) - 1/4*\log(-(2*\text{sqrt}(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) + 1/4*\log((2*\text{sqrt}(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)$

giac [A] time = 0.25, size = 185, normalized size = 1.61

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3(\sqrt{-x^2-4x-3}-1)}{x+2}\right)+1\right) + \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}\right)+1\right) - \frac{1}{2}\sqrt{-x^2-4x-3} - 2\arcsin(x+2) + \frac{1}{2}\log\left(\frac{2(\sqrt{-x^2-4x-3}-1)}{x+2} + \frac{3(\sqrt{-x^2-4x-3}-1)^2}{(x+2)^2} + 1\right) - \frac{1}{2}\log\left(\frac{2(\sqrt{-x^2-4x-3}+1)}{x+2} + \frac{3(\sqrt{-x^2-4x-3}+1)^2}{(x+2)^2} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")`

[Out] $1/4*\text{sqrt}(2)*\arctan(1/2*\text{sqrt}(2)*(3*(\text{sqrt}(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/4*\text{sqrt}(2)*\arctan(1/2*\text{sqrt}(2)*((\text{sqrt}(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) - 1/2*\text{sqrt}(-x^2 - 4*x - 3) - 2*\arcsin(x + 2) + 1/2*\log(2*(\text{sqrt}(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(\text{sqrt}(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) - 1/2*\log(2*(\text{sqrt}(-x^2 - 4*x - 3) + 1)/(x + 2) + 3*(\text{sqrt}(-x^2 - 4*x - 3) + 1)^2/(x + 2)^2 + 1)$

maple [A] time = 0.02, size = 144, normalized size = 1.25

$$-2\arcsin(x+2) - \frac{\sqrt{-x^2-4x-3}}{2} + \frac{\sqrt{3}\sqrt{4}\sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2}-12}\left(-4\operatorname{arctanh}\left(\frac{3x}{(-x-\frac{3}{2})\sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2}-12}}\right) + \sqrt{2}\arctan\left(\frac{\sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2}-12}\sqrt{2}}{6}\right)\right)}{24\sqrt{\frac{x^2}{(-x-\frac{3}{2})^2}-4}\left(\frac{x}{(-x-\frac{3}{2})}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)`

[Out] $-1/2*(-x^2-4*x-3)^(1/2) - 2*\arcsin(x+2) + 1/24*3^(1/2)*4^(1/2)*(3/(-x-3/2)^2*x^2-12)^(1/2)*(2^(1/2)*\arctan(1/6*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2)) - 4*\operatorname{arctanh}(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(1/2)*x))/((1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^(1/2)/(1/(-x-3/2)*x+1)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(2x^2 + 4x + 3)\sqrt{-x^2 - 4x - 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")`

[Out] `integrate(x^3/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{-x^2 - 4x - 3} (2x^2 + 4x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)

[Out] int(x^3/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{-(x+1)(x+3)} (2x^2 + 4x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2), x)

[Out] Integral(x**3/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)

$$3.128 \quad \int \frac{x^2}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx$$

Optimal. Leaf size=98

$$-\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{1}{2} \tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right) + \frac{1}{2} \sin^{-1}(x+2)$$

Rubi [A] time = 0.20, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1077, 619, 216, 1028, 986, 12, 1026, 1161, 618, 204, 1027, 206}

$$-\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{1}{2} \tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right) + \frac{1}{2} \sin^{-1}(x+2)$$

Antiderivative was successfully verified.

[In] Int[x^2/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]

[Out] ArcSin[2 + x]/2 - ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2] + ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2] - ArcTanh[x/Sqrt[-3 - 4*x - x^2]]/2

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b

+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 986

Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :=> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e - b*f, 0] && NegQ[b^2 - 4*a*c]

Rule 1026

Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :=> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 - 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]

Rule 1027

Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :=> Dist[g, Subst[Int[1/(a + (c*d - a*f)*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] && EqQ[2*h*d - g*e, 0]

Rule 1028

Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :=> -Dist[(2*h*d - g*e)/e, Int[1/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] && NeQ[2*h*d - g*e, 0]

Rule 1077

Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :=> Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C - b*C*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

Rule 1161

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :=> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx &= \frac{1}{2} \int \frac{1}{\sqrt{-3-4x-x^2}} dx + \frac{1}{2} \int \frac{-3-4x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx \\
&= -\left(\frac{1}{4} \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}} dx, x, -4-2x\right)\right) + \frac{1}{2} \int \frac{-6-4x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx \\
&= \frac{1}{2} \sin^{-1}(2+x) - \frac{1}{4} \int \frac{-6-4x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx + \frac{1}{4} \int \frac{-6-4x}{\sqrt{-3-4x-x^2}} dx \\
&= \frac{1}{2} \sin^{-1}(2+x) - \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{3}{2} \text{Subst}\left(\int \frac{1}{3-3x^2} dx, x, -4-2x\right) \\
&= \frac{1}{2} \sin^{-1}(2+x) - \frac{1}{2} \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - 8 \text{Subst}\left(\int \frac{1+3x}{-4-8x^2} dx, x, -4-2x\right) \\
&= \frac{1}{2} \sin^{-1}(2+x) - \frac{1}{2} \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{1}{3} \text{Subst}\left(\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2} dx, x, -4-2x\right) \\
&= \frac{1}{2} \sin^{-1}(2+x) - \frac{1}{2} \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - \frac{2}{3} \text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2} dx, x, -4-2x\right) \\
&= \frac{1}{2} \sin^{-1}(2+x) - \frac{\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{1}{2} \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)
\end{aligned}$$

Mathematica [C] time = 0.19, size = 159, normalized size = 1.62

$$\frac{1}{4} \left(-i\sqrt{1-2i\sqrt{2}} \tanh^{-1}\left(\frac{i\sqrt{2}x+2x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right) + i\sqrt{1+2i\sqrt{2}} \tanh^{-1}\left(\frac{(2-i\sqrt{2})x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right) + 2\sin^{-1}(x+2) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)), x]

[Out] (2*ArcSin[2 + x] - I*Sqrt[1 - (2*I)*Sqrt[2]]*ArcTanh[(2 + (2*I)*Sqrt[2] + 2*x + I*Sqrt[2]*x)/(Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]]) + I*Sqrt[1 + (2*I)*Sqrt[2]]*ArcTanh[(2 - (2*I)*Sqrt[2] + (2 - I*Sqrt[2])*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]])/4

IntegrateAlgebraic [A] time = 0.33, size = 82, normalized size = 0.84

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x+\frac{3}{\sqrt{2}}}{\sqrt{-x^2-4x-3}}\right)}{\sqrt{2}} - \tan^{-1}\left(\frac{\sqrt{-x^2-4x-3}}{x+3}\right) - \frac{1}{2} \tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)), x]

[Out] ArcTan[(3/Sqrt[2] + Sqrt[2]*x)/Sqrt[-3 - 4*x - x^2]]/Sqrt[2] - ArcTan[Sqrt[-3 - 4*x - x^2]/(3 + x)] - ArcTanh[x/Sqrt[-3 - 4*x - x^2]]/2

fricas [A] time = 1.04, size = 161, normalized size = 1.64

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right) - \frac{1}{4}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right) - \frac{1}{2}\arctan\left(\frac{\sqrt{-x^2-4x-3}(x+2)}{x^2+4x+3}\right) + \frac{1}{8}\log\left(-\frac{2\sqrt{-x^2-4x-3}x+4x+3}{x^2}\right) - \frac{1}{8}\log\left(\frac{2\sqrt{-x^2-4x-3}x-4x-3}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")

[Out] -1/4*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) - 1/4*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) - 1/2*arctan(sqrt(-x^2 - 4*x - 3)*(x + 2)/(x^2 + 4*x + 3)) + 1/8*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) - 1/8*log((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)

giac [B] time = 0.23, size = 171, normalized size = 1.74

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3(\sqrt{-x^2-4x-3}-1)}{x+2}+1\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)+\frac{1}{2}\arcsin(x+2)-\frac{1}{4}\log\left(\frac{2(\sqrt{-x^2-4x-3}-1)}{x+2}+\frac{3(\sqrt{-x^2-4x-3}-1)^2}{(x+2)^2}+1\right)+\frac{1}{4}\log\left(\frac{2(\sqrt{-x^2-4x-3}-1)}{x+2}+\frac{(\sqrt{-x^2-4x-3}-1)^2}{(x+2)^2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")

[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/2*arcsin(x + 2) - 1/4*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) + 1/4*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)

maple [A] time = 0.01, size = 130, normalized size = 1.33

$$\frac{\arcsin(x+2)}{2} - \frac{\sqrt{3}\sqrt{4}\sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2}-12}\left(-\operatorname{arctanh}\left(\frac{3x}{(-x-\frac{3}{2})\sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2}-12}}\right)+\sqrt{2}\arctan\left(\frac{\sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2}-12}\sqrt{2}}{6}\right)\right)}{12\sqrt{\frac{\frac{x^2}{(-x-\frac{3}{2})^2}-4}{\left(\frac{x}{-x-\frac{3}{2}}+1\right)^2}\left(\frac{x}{-x-\frac{3}{2}}+1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)

[Out] 1/2*arcsin(x+2)-1/12*3^(1/2)*4^(1/2)*(3/(-x-3/2)^2*x^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2))-arctanh(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(1/2)*x))/((1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^(1/2)/(1/(-x-3/2)*x+1)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(2x^2 + 4x + 3)\sqrt{-x^2 - 4x - 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")

[Out] integrate(x^2/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{-x^2 - 4x - 3} (2x^2 + 4x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)`

[Out] `int(x^2/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-(x+1)(x+3)}(2x^2+4x+3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)`

[Out] `Integral(x**2/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)`

$$3.129 \quad \int \frac{x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx$$

Optimal. Leaf size=68

$$\frac{\tan^{-1}\left(\frac{\frac{3\sqrt{-x-1}}{\sqrt{x+3}}+1}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1026, 1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]

[Out] ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2] - ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1026

Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4], x], x, (1 + ((e + Sqrt[e^2 - 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]

Rule 1161

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx &= 8 \operatorname{Subst} \left(\int \frac{1+3x^2}{-4-8x^2-36x^4} dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}} \right) \\
&= -\left(\frac{1}{3} \operatorname{Subst} \left(\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2} dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}} \right) \right) - \frac{1}{3} \operatorname{Subst} \left(\int \frac{1}{\frac{1}{3}+x^2} dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}} \right) \\
&= \frac{2}{3} \operatorname{Subst} \left(\int \frac{1}{-\frac{8}{9}-x^2} dx, x, \frac{2}{3} \left(-1 + \frac{3+x}{\sqrt{-3-4x-x^2}} \right) \right) + \frac{2}{3} \operatorname{Subst} \left(\int \frac{1}{x^2} dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}} \right) \\
&= \frac{\tan^{-1} \left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}} \right)}{\sqrt{2}} - \frac{\tan^{-1} \left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}} \right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 0.17, size = 174, normalized size = 2.56

$$\frac{(1-i\sqrt{2})\sqrt{1-2i\sqrt{2}} \tanh^{-1} \left(\frac{(2-i\sqrt{2})x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}} \sqrt{-x^2-4x-3}} \right) + (1+i\sqrt{2})\sqrt{1+2i\sqrt{2}} \tanh^{-1} \left(\frac{(2+i\sqrt{2})x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}} \sqrt{-x^2-4x-3}} \right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]

[Out] ((1 - I*Sqrt[2])*Sqrt[1 - (2*I)*Sqrt[2]]*ArcTanh[(2 - (2*I)*Sqrt[2] + (2 - I*Sqrt[2])*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]]) + (1 + I*Sqrt[2])*Sqrt[1 + (2*I)*Sqrt[2]]*ArcTanh[(2 + (2*I)*Sqrt[2] + (2 + I*Sqrt[2])*x)/(Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])])/(6*Sqrt[2])

IntegrateAlgebraic [A] time = 0.25, size = 38, normalized size = 0.56

$$\frac{\tan^{-1} \left(\frac{\sqrt{2}x + \frac{3}{\sqrt{2}}}{\sqrt{-x^2-4x-3}} \right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]

[Out] -(ArcTan[(3/Sqrt[2] + Sqrt[2]*x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2])

fricas [A] time = 0.87, size = 50, normalized size = 0.74

$$\frac{1}{4} \sqrt{2} \arctan \left(\frac{\sqrt{2} (6x^2 + 20x + 15) \sqrt{-x^2 - 4x - 3}}{4(2x^3 + 11x^2 + 18x + 9)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")

[Out] 1/4*sqrt(2)*arctan(1/4*sqrt(2)*(6*x^2 + 20*x + 15)*sqrt(-x^2 - 4*x - 3)/(2*x^3 + 11*x^2 + 18*x + 9))

giac [A] time = 0.18, size = 68, normalized size = 1.00

$$\frac{1}{2} \sqrt{2} \arctan \left(\frac{1}{2} \sqrt{2} \left(\frac{3(\sqrt{-x^2-4x-3}-1)}{x+2} + 1 \right) \right) + \frac{1}{2} \sqrt{2} \arctan \left(\frac{1}{2} \sqrt{2} \left(\frac{\sqrt{-x^2-4x-3}-1}{x+2} + 1 \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")

[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))

maple [A] time = 0.01, size = 92, normalized size = 1.35

$$\frac{\sqrt{3} \sqrt{4} \sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2} - 12} \sqrt{2} \arctan\left(\frac{\sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2} - 12} \sqrt{2}}{6}\right)}{12 \sqrt{\frac{\frac{x^2}{(-x-\frac{3}{2})^2} - 4}{\left(\frac{x}{-x-\frac{3}{2}} + 1\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)

[Out] 1/12*3^(1/2)*4^(1/2)/((1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^(1/2)/(1/(-x-3/2)*x+1)*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2)*arctan(1/6*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(2x^2 + 4x + 3)\sqrt{-x^2 - 4x - 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")

[Out] integrate(x/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{-x^2 - 4x - 3} (2x^2 + 4x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((-4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)

[Out] int(x/((-4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-(x+1)(x+3)} (2x^2 + 4x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)

[Out] Integral(x/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)

$$3.130 \quad \int \frac{1}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx$$

Optimal. Leaf size=95

$$-\frac{1}{3}\sqrt{2} \tan^{-1}\left(\frac{1 - \frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right) + \frac{1}{3}\sqrt{2} \tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}} + 1}{\sqrt{2}}\right) + \frac{1}{3} \tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Rubi [A] time = 0.11, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {986, 12, 1026, 1161, 618, 204, 1027, 206}

$$-\frac{1}{3}\sqrt{2} \tan^{-1}\left(\frac{1 - \frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right) + \frac{1}{3}\sqrt{2} \tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}} + 1}{\sqrt{2}}\right) + \frac{1}{3} \tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]

[Out] -(Sqrt[2]*ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/3 + (Sqrt[2]*ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/3 + ArcTanh[x/Sqrt[-3 - 4*x - x^2]]/3

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 986

Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e - b*f, 0] && NegQ[b^2 - 4*a*c]

Rule 1026

```
Int[(x_)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*
(x_)^2]), x_Symbol] := Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 -
4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1027

```
Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*
(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
EqQ[2*h*d - g*e, 0]
```

Rule 1161

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx &= -\left(\frac{1}{6} \int \frac{-6-4x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx\right) + \frac{1}{6} \int -\frac{4x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx \\
&= -\left(\frac{2}{3} \int \frac{x}{\sqrt{-3-4x-x^2} (3+4x+2x^2)} dx\right) + \text{Subst}\left(\int \frac{1}{3-3x^2} dx, x, \frac{x}{\sqrt{-3-4x-x^2}}\right) \\
&= \frac{1}{3} \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - \frac{16}{3} \text{Subst}\left(\int \frac{1+3x^2}{-4-8x^2-36x^4} dx, x, \frac{1}{\sqrt{-3-4x-x^2}}\right) \\
&= \frac{1}{3} \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{2}{9} \text{Subst}\left(\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2} dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right) \\
&= \frac{1}{3} \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - \frac{4}{9} \text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2} dx, x, \frac{2}{3}\left(-1+\frac{3}{\sqrt{-3-4x-x^2}}\right)\right) \\
&= -\frac{1}{3}\sqrt{2} \tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) + \frac{1}{3}\sqrt{2} \tan^{-1}\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) + \frac{1}{3} \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)
\end{aligned}$$

Mathematica [C] time = 0.10, size = 150, normalized size = 1.58

$$\frac{1}{6}i \left(\sqrt{1-2i\sqrt{2}} \tanh^{-1}\left(\frac{(2-i\sqrt{2})x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right) - \sqrt{1+2i\sqrt{2}} \tanh^{-1}\left(\frac{(2+i\sqrt{2})x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)), x]
```

```
[Out] (I/6)*(Sqrt[1 - (2*I)*Sqrt[2]]*ArcTanh[(2 - (2*I)*Sqrt[2] + (2 - I*Sqrt[2])
*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])] - Sqrt[1 + (2*I)*Sqrt[2]
```

]])*ArcTanh[(2 + (2*I)*Sqrt[2] + (2 + I*Sqrt[2])*x)/(Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]])

IntegrateAlgebraic [A] time = 0.31, size = 62, normalized size = 0.65

$$\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}x + \frac{3}{\sqrt{2}}}{\sqrt{-x^2 - 4x - 3}}\right) + \frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2 - 4x - 3}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]

[Out] (Sqrt[2]*ArcTan[(3/Sqrt[2] + Sqrt[2]*x)/Sqrt[-3 - 4*x - x^2]])/3 + ArcTanh[x/Sqrt[-3 - 4*x - x^2]]/3

fricas [A] time = 0.78, size = 132, normalized size = 1.39

$$-\frac{1}{6}\sqrt{2}\arctan\left(\frac{\sqrt{2}x + 3\sqrt{-x^2 - 4x - 3}}{2(2x + 3)}\right) - \frac{1}{6}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x - 3\sqrt{-x^2 - 4x - 3}}{2(2x + 3)}\right) - \frac{1}{12}\log\left(-\frac{2\sqrt{-x^2 - 4x - 3}x + 4x + 3}{x^2}\right) + \frac{1}{12}\log\left(\frac{2\sqrt{-x^2 - 4x - 3}x - 4x - 3}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")

[Out] -1/6*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) - 1/6*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) - 1/12*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) + 1/12*log((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)

giac [B] time = 0.19, size = 165, normalized size = 1.74

$$-\frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3(\sqrt{-x^2 - 4x - 3} - 1)}{x + 2} + 1\right)\right) - \frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2 - 4x - 3} - 1}{x + 2} + 1\right)\right) + \frac{1}{6}\log\left(\frac{2(\sqrt{-x^2 - 4x - 3} - 1)}{x + 2} + \frac{3(\sqrt{-x^2 - 4x - 3} - 1)^2}{(x + 2)^2} + 1\right) - \frac{1}{6}\log\left(\frac{2(\sqrt{-x^2 - 4x - 3} + 1)}{x + 2} + \frac{(\sqrt{-x^2 - 4x - 3} + 1)^2}{(x + 2)^2} + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")

[Out] -1/3*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) - 1/3*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/6*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) - 1/6*log(2*(sqrt(-x^2 - 4*x - 3) + 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) + 1)^2/(x + 2)^2 + 3)

maple [A] time = 0.01, size = 121, normalized size = 1.27

$$\frac{\sqrt{3}\sqrt{4}\sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2}-12}\left(\operatorname{arctanh}\left(\frac{3x}{(-x-\frac{3}{2})\sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2}-12}}\right)+\sqrt{2}\arctan\left(\frac{\sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2}-12}\sqrt{2}}{6}\right)\right)}{18\sqrt{\frac{\frac{x^2}{(-x-\frac{3}{2})^2}-4}{\left(\frac{x}{-x-\frac{3}{2}}+1\right)^2}}\left(\frac{x}{-x-\frac{3}{2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)

[Out] -1/18*3^(1/2)*4^(1/2)*(3/(-x-3/2)^2*x^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2))+arctanh(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(1/2)*x))/((1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^(1/2)/(1/(-x-3/2)*x+1)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2x^2 + 4x + 3)\sqrt{-x^2 - 4x - 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-x^2 - 4x - 3} (2x^2 + 4x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)

[Out] int(1/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(x+1)(x+3)} (2x^2 + 4x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)

[Out] Integral(1/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)

$$3.131 \quad \int \frac{1}{x\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx$$

Optimal. Leaf size=130

$$-\frac{\tan^{-1}\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right)}{3\sqrt{3}} + \frac{1}{9}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right) - \frac{1}{9}\sqrt{2}\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right) - \frac{4}{9}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Rubi [A] time = 0.42, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {6728, 724, 204, 1028, 986, 12, 1026, 1161, 618, 1027, 206}

$$-\frac{\tan^{-1}\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right)}{3\sqrt{3}} + \frac{1}{9}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right) - \frac{1}{9}\sqrt{2}\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right) - \frac{4}{9}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]

[Out] -ArcTan[(3 + 2*x)/(Sqrt[3]*Sqrt[-3 - 4*x - x^2])]/(3*Sqrt[3]) + (Sqrt[2]*ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/9 - (Sqrt[2]*ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/9 - (4*ArcTanh[x/Sqrt[-3 - 4*x - x^2]])/9

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 986

Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)]

, 2]], Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e - b*f, 0] && NegQ[b^2 - 4*a*c]

Rule 1026

Int[(x_)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 - 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]

Rule 1027

Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] && EqQ[2*h*d - g*e, 0]

Rule 1028

Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] :> -Dist[(2*h*d - g*e)/e, Int[1/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] && NeQ[2*h*d - g*e, 0]

Rule 1161

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))

Rule 6728

Int[(u_)/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x_Symbol] :> With[{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx &= \int \left(\frac{1}{3x\sqrt{-3-4x-x^2}} - \frac{2(2+x)}{3\sqrt{-3-4x-x^2}(3+4x+2x^2)} \right) dx \\
&= \frac{1}{3} \int \frac{1}{x\sqrt{-3-4x-x^2}} dx - \frac{2}{3} \int \frac{2+x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx \\
&= \frac{1}{6} \int \frac{-6-4x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx - \frac{1}{3} \int \frac{1}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx \\
&= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{1}{18} \int \frac{-6-4x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx - \frac{1}{3} \int \frac{1}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx \\
&= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} - \frac{1}{3} \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{2}{9} \int \frac{1}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx \\
&= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} - \frac{4}{9} \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{16}{9} \text{Subst}\left(\int \frac{1}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx, x, \frac{x}{\sqrt{-3-4x-x^2}}\right) \\
&= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} - \frac{4}{9} \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - \frac{2}{27} \text{Subst}\left(\int \frac{1}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx, x, \frac{x}{\sqrt{-3-4x-x^2}}\right) \\
&= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} - \frac{4}{9} \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{4}{27} \text{Subst}\left(\int \frac{1}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx, x, \frac{x}{\sqrt{-3-4x-x^2}}\right) \\
&= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{1}{9}\sqrt{2} \tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) - \frac{1}{9}\sqrt{2} \tan^{-1}\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [C] time = 0.44, size = 200, normalized size = 1.54

$$\frac{1}{54} \left(-6\sqrt{3} \tan^{-1}\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right) - 3\sqrt{1-2i\sqrt{2}}(\sqrt{2}+2i) \tanh^{-1}\left(\frac{(2-i\sqrt{2})x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right) - 3\sqrt{1+2i\sqrt{2}}(\sqrt{2}-2i) \tanh^{-1}\left(\frac{(2+i\sqrt{2})x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)), x]

[Out] (-6*Sqrt[3]*ArcTan[(3 + 2*x)/(Sqrt[3]*Sqrt[-3 - 4*x - x^2])] - 3*Sqrt[1 - (2*I)*Sqrt[2]]*(2*I + Sqrt[2])*ArcTanh[(2 - (2*I)*Sqrt[2] + (2 - I*Sqrt[2])*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])] - 3*Sqrt[1 + (2*I)*Sqrt[2]]*(-2*I + Sqrt[2])*ArcTanh[(2 + (2*I)*Sqrt[2] + (2 + I*Sqrt[2])*x)/(Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])])/54

IntegrateAlgebraic [A] time = 0.33, size = 97, normalized size = 0.75

$$-\frac{1}{9}\sqrt{2} \tan^{-1}\left(\frac{\sqrt{2}x + \frac{3}{\sqrt{2}}}{\sqrt{-x^2-4x-3}}\right) + \frac{2 \tan^{-1}\left(\frac{\sqrt{3}\sqrt{-x^2-4x-3}}{x+3}\right)}{3\sqrt{3}} - \frac{4}{9} \tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)), x]

[Out] -1/9*(Sqrt[2]*ArcTan[(3/Sqrt[2] + Sqrt[2]*x)/Sqrt[-3 - 4*x - x^2]]) + (2*ArcTan[(Sqrt[3]*Sqrt[-3 - 4*x - x^2])/(3 + x)]/(3*Sqrt[3]) - (4*ArcTanh[x/Sqrt[-3 - 4*x - x^2]]))/9

fricas [A] time = 1.05, size = 170, normalized size = 1.31

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-x^2-4x-3}(2x+3)}{3(x^2+4x+3)}\right)+\frac{1}{18}\sqrt{2}\arctan\left(\frac{\sqrt{2x+3}\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)+\frac{1}{18}\sqrt{2}\arctan\left(-\frac{\sqrt{2x-3}\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)+\frac{1}{9}\log\left(-\frac{2\sqrt{-x^2-4x-3}x+4x+3}{x^2}\right)-\frac{1}{9}\log\left(\frac{2\sqrt{-x^2-4x-3}x-4x-3}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")

[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(-x^2 - 4*x - 3)*(2*x + 3)/(x^2 + 4*x + 3)) + 1/18*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) + 1/18*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) + 1/9*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) - 1/9*log((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)

giac [A] time = 0.20, size = 199, normalized size = 1.53

$$\frac{1}{9}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3(\sqrt{-x^2-4x-3}-1)}{x+2}+1\right)\right)+\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2(\sqrt{-x^2-4x-3}-1)}{x+2}+1\right)\right)+\frac{1}{9}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)-\frac{2}{9}\log\left(\frac{2(\sqrt{-x^2-4x-3}-1)}{x+2}+\frac{3(\sqrt{-x^2-4x-3}-1)^2}{(x+2)^2}+1\right)+\frac{2}{9}\log\left(\frac{2(\sqrt{-x^2-4x-3}-1)}{x+2}+\frac{(\sqrt{-x^2-4x-3}-1)^2}{(x+2)^2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")

[Out] 1/9*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/9*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) - 2/9*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) + 2/9*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3))

maple [A] time = 0.02, size = 152, normalized size = 1.17

$$\frac{\sqrt{3}\arctan\left(\frac{(-4x-6)\sqrt{3}}{6\sqrt{-x^2-4x-3}}\right)}{9} + \frac{\sqrt{3}\sqrt{4}\sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2}-12}\left(4\operatorname{arctanh}\left(\frac{3x}{(-x-\frac{3}{2})\sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2}-12}}\right)+\sqrt{2}\arctan\left(\frac{\sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2}-12}\sqrt{2}}{6}\right)\right)}{54\sqrt{\frac{\frac{x^2}{(-x-\frac{3}{2})^2}-4}{\left(\frac{x}{-x-\frac{3}{2}}+1\right)^2}}\left(\frac{x}{-x-\frac{3}{2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)

[Out] 1/54*3^(1/2)*4^(1/2)*(3/(-x-3/2)^2*x^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2))+4*arctanh(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(1/2)*x))/((1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^(1/2)/(1/(-x-3/2)*x+1)+1/9*3^(1/2)*arctan(1/6*(-6-4*x)*3^(1/2)/(-x^2-4*x-3)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2x^2 + 4x + 3)\sqrt{-x^2 - 4x - 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\sqrt{-x^2 - 4x - 3}(2x^2 + 4x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)`

[Out] `int(1/(x*(- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{-(x+1)(x+3)}(2x^2+4x+3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)`

[Out] `Integral(1/(x*sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)`

$$3.132 \quad \int \frac{1}{x^2 \sqrt{-3-4x-x^2} (3+4x+2x^2)} dx$$

Optimal. Leaf size=151

$$\frac{\sqrt{-x^2-4x-3}}{9x} + \frac{2 \tan^{-1}\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right)}{3\sqrt{3}} + \frac{2}{27}\sqrt{2} \tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right) - \frac{2}{27}\sqrt{2} \tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right) + \frac{10}{27} \tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Rubi [A] time = 0.45, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6728, 730, 724, 204, 1028, 986, 12, 1026, 1161, 618, 1027, 206}

$$\frac{\sqrt{-x^2-4x-3}}{9x} + \frac{2 \tan^{-1}\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right)}{3\sqrt{3}} + \frac{2}{27}\sqrt{2} \tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right) - \frac{2}{27}\sqrt{2} \tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right) + \frac{10}{27} \tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)), x]
```

```
[Out] Sqrt[-3 - 4*x - x^2]/(9*x) + (2*ArcTan[(3 + 2*x)/(Sqrt[3]*Sqrt[-3 - 4*x - x^2])]/(3*Sqrt[3]) + (2*Sqrt[2]*ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/27 - (2*Sqrt[2]*ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/27 + (10*ArcTanh[x/Sqrt[-3 - 4*x - x^2]])/27
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c
```

```
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
  Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
  m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 986

```
Int[1/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*
(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1026

```
Int[(x_)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)
*(x_)^2]), x_Symbol] := Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 -
4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1027

```
Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_
)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
EqQ[2*h*d - g*e, 0]
```

Rule 1028

```
Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_
)*(x_) + (f_)*(x_)^2]), x_Symbol] := -Dist[(2*h*d - g*e)/e, Int[1/((a + b
*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g
, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
&& NeQ[2*h*d - g*e, 0]
```

Rule 1161

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 6728

```
Int[(u_)/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x_Symbol] := With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx &= \int \left(\frac{1}{3x^2\sqrt{-3-4x-x^2}} - \frac{4}{9x\sqrt{-3-4x-x^2}} + \frac{2(5+4x)}{9\sqrt{-3-4x-x^2}(3+4x+2x^2)} \right) dx \\
&= \frac{2}{9} \int \frac{5+4x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx + \frac{1}{3} \int \frac{1}{x^2\sqrt{-3-4x-x^2}} dx - \frac{4}{9} \int \frac{1}{x\sqrt{-3-4x-x^2}} dx \\
&= \frac{\sqrt{-3-4x-x^2}}{9x} - \frac{2}{9} \int \frac{1}{x\sqrt{-3-4x-x^2}} dx - \frac{2}{9} \int \frac{1}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx \\
&= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{4 \tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{9\sqrt{3}} + \frac{1}{27} \int \frac{-6-4x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx \\
&= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{2 \tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{4}{9} \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) \\
&= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{2 \tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{10}{27} \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) \\
&= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{2 \tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{10}{27} \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) \\
&= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{2 \tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{10}{27} \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) \\
&= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{2 \tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{10}{27} \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) \\
&= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{2 \tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{2}{27} \sqrt{2} \tan^{-1}\left(\frac{1 - \frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [C] time = 0.41, size = 225, normalized size = 1.49

$$\frac{3\left(\sqrt{-x^2-4x-3} + 2\sqrt{3}x \tan^{-1}\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right)\right) + \sqrt{1-2i\sqrt{2}}(2\sqrt{2}+i)x \tanh^{-1}\left(\frac{-i\sqrt{2}x+2x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right) + \sqrt{1+2i\sqrt{2}}(2\sqrt{2}-i)x \tanh^{-1}\left(\frac{(2+i\sqrt{2})x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right)}{27x}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)), x]

[Out] (3*(Sqrt[-3 - 4*x - x^2] + 2*Sqrt[3]*x*ArcTan[(3 + 2*x)/(Sqrt[3]*Sqrt[-3 - 4*x - x^2])]) + Sqrt[1 - (2*I)*Sqrt[2]]*(I + 2*Sqrt[2])*x*ArcTanh[(2 - (2*I)*Sqrt[2] + 2*x - I*Sqrt[2]*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]]) + Sqrt[1 + (2*I)*Sqrt[2]]*(-I + 2*Sqrt[2])*x*ArcTanh[(2 + (2*I)*Sqrt[2] + (2 + I*Sqrt[2])*x)/(Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])])/(27*x)

IntegrateAlgebraic [A] time = 0.43, size = 118, normalized size = 0.78

$$\frac{\sqrt{-x^2-4x-3}}{9x} - \frac{2}{27} \sqrt{2} \tan^{-1}\left(\frac{\sqrt{2}x + \frac{3}{\sqrt{2}}}{\sqrt{-x^2-4x-3}}\right) - \frac{4 \tan^{-1}\left(\frac{\sqrt{3}\sqrt{-x^2-4x-3}}{x+3}\right)}{3\sqrt{3}} + \frac{10}{27} \tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^2*Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)), x]

[Out] $\text{Sqrt}[-3 - 4*x - x^2]/(9*x) - (2*\text{Sqrt}[2]*\text{ArcTan}[(3/\text{Sqrt}[2] + \text{Sqrt}[2]*x)/\text{Sqrt}[-3 - 4*x - x^2]])/27 - (4*\text{ArcTan}[(\text{Sqrt}[3]*\text{Sqrt}[-3 - 4*x - x^2])/(3 + x)])/(3*\text{Sqrt}[3]) + (10*\text{ArcTanh}[x/\text{Sqrt}[-3 - 4*x - x^2]])/27$

fricas [A] time = 1.09, size = 194, normalized size = 1.28

$$\frac{12\sqrt{3}x \arctan\left(\frac{\sqrt{3}\sqrt{-x^2-4x-3}(2x+3)}{3(x^2+4x+3)}\right) - 2\sqrt{2}x \arctan\left(\frac{\sqrt{2}x+3\sqrt{-x^2-4x-3}}{2(2x+3)}\right) - 2\sqrt{2}x \arctan\left(\frac{-\sqrt{2}x-3\sqrt{-x^2-4x-3}}{2(2x+3)}\right) + 5x \log\left(\frac{2\sqrt{-x^2-4x-3}x+4x+3}{x^2}\right) - 5x \log\left(\frac{2\sqrt{-x^2-4x-3}x-4x-3}{x^2}\right) - 6\sqrt{-x^2-4x-3}}{54x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2), x, algorithm="fricas")

[Out] $-1/54*(12*\text{sqrt}(3)*x*\arctan(1/3*\text{sqrt}(3)*\text{sqrt}(-x^2 - 4*x - 3)*(2*x + 3)/(x^2 + 4*x + 3)) - 2*\text{sqrt}(2)*x*\arctan(1/2*(\text{sqrt}(2)*x + 3*\text{sqrt}(2)*\text{sqrt}(-x^2 - 4*x - 3))/(2*x + 3)) - 2*\text{sqrt}(2)*x*\arctan(-1/2*(\text{sqrt}(2)*x - 3*\text{sqrt}(2)*\text{sqrt}(-x^2 - 4*x - 3))/(2*x + 3)) + 5*x*\log(-(2*\text{sqrt}(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) - 5*x*\log((2*\text{sqrt}(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2) - 6*\text{sqrt}(-x^2 - 4*x - 3))/x$

giac [B] time = 0.48, size = 269, normalized size = 1.78

$$\frac{2}{27}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\frac{3(\sqrt{-x^2-4x-3}-1)}{x+2}\right) - \frac{4}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\frac{2(\sqrt{-x^2-4x-3}-1)}{x+2}\right) + \frac{2}{27}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\frac{(\sqrt{-x^2-4x-3}-1)}{x+2}\right) - \frac{\sqrt{2}\sqrt{4x+3}}{18\left(\frac{x^2-4x-3}{x+2} + \frac{(\sqrt{-x^2-4x-3})^2}{x+2} + 1\right)} + \frac{5}{27}\log\left(\frac{2(\sqrt{-x^2-4x-3}-1)}{x+2} + \frac{3(\sqrt{-x^2-4x-3}-1)^2}{(x+2)^2}\right) + \frac{5}{27}\log\left(\frac{2(\sqrt{-x^2-4x-3}+1)}{x+2} + \frac{(\sqrt{-x^2-4x-3}+1)^2}{(x+2)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2), x, algorithm="giac")

[Out] $2/27*\text{sqrt}(2)*\arctan(1/2*\text{sqrt}(2)*(3*(\text{sqrt}(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) - 4/9*\text{sqrt}(3)*\arctan(1/3*\text{sqrt}(3)*(2*(\text{sqrt}(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 2/27*\text{sqrt}(2)*\arctan(1/2*\text{sqrt}(2)*((\text{sqrt}(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) - 1/18*((\text{sqrt}(-x^2 - 4*x - 3) - 1)/(x + 2) + 2)/((\text{sqrt}(-x^2 - 4*x - 3) - 1)/(x + 2) + (\text{sqrt}(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) + 5/27*\log(2*(\text{sqrt}(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(\text{sqrt}(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) - 5/27*\log(2*(\text{sqrt}(-x^2 - 4*x - 3) + 1)/(x + 2) + (\text{sqrt}(-x^2 - 4*x - 3) + 1)^2/(x + 2)^2 + 3)$

maple [A] time = 0.02, size = 169, normalized size = 1.12

$$\frac{2\sqrt{3}\arctan\left(\frac{(-4x-6)\sqrt{3}}{6\sqrt{-x^2-4x-3}}\right)}{9} + \frac{\sqrt{-x^2-4x-3}}{9x} + \frac{\sqrt{3}\sqrt{4}\sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2}-12}\left(-5\operatorname{arctanh}\left(\frac{3x}{(-x-\frac{3}{2})\sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2}-12}}\right) + \sqrt{2}\arctan\left(\frac{\sqrt{\frac{3x^2}{(-x-\frac{3}{2})^2}-12}\sqrt{2}}{6}\right)\right)}{81\sqrt{\frac{x^2}{(-x-\frac{3}{2})^2}-4}\left(\frac{x}{(-x-\frac{3}{2})}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2), x)

[Out] $1/81*3^(1/2)*4^(1/2)*(3/(-x-3/2)^2*x^2-12)^(1/2)*(2^(1/2)*\arctan(1/6*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2))-5*\operatorname{arctanh}(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(1/2)*x))/((1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^(1/2)/(1/(-x-3/2)*x+1)+1/9*(-x^2-4*x-3)^(1/2)/x-2/9*3^(1/2)*\arctan(1/6*(-4*x-6)*3^(1/2)/(-x^2-4*x-3)^(1/2))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2x^2 + 4x + 3)\sqrt{-x^2 - 4x - 3}x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2 \sqrt{-x^2 - 4x - 3} (2x^2 + 4x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)

[Out] int(1/(x^2*(- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{-(x+1)(x+3)} (2x^2 + 4x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)

[Out] Integral(1/(x**2*sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)

$$3.133 \quad \int (2 + 3x)^2 (30 + 31x - 12x^2)^2 \sqrt{6 + 17x + 12x^2} dx$$

Optimal. Leaf size=149

$$-\frac{1}{32}(10-3x)(12x^2+17x+6)^{7/2} - \frac{873(12x^2+17x+6)^{7/2}}{1792} + \frac{25091(24x+17)(12x^2+17x+6)^{5/2}}{24576} - \frac{125455(24x+17)\sqrt{12x^2+17x+6}}{4718592} - \frac{125455 \tanh^{-1}\left(\frac{24x+17}{4\sqrt{3}\sqrt{12x^2+17x+6}}\right)}{603979776\sqrt{3}}$$

Rubi [A] time = 0.09, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1002, 742, 640, 612, 621, 206}

$$-\frac{1}{32}(10-3x)(12x^2+17x+6)^{7/2} - \frac{873(12x^2+17x+6)^{7/2}}{1792} + \frac{25091(24x+17)(12x^2+17x+6)^{5/2}}{24576} - \frac{125455(24x+17)(12x^2+17x+6)^{3/2}}{4718592} + \frac{125455(24x+17)\sqrt{12x^2+17x+6}}{150994944} - \frac{125455 \tanh^{-1}\left(\frac{24x+17}{4\sqrt{3}\sqrt{12x^2+17x+6}}\right)}{603979776\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x)^2*(30 + 31*x - 12*x^2)^2*Sqrt[6 + 17*x + 12*x^2], x]

[Out] (125455*(17 + 24*x)*Sqrt[6 + 17*x + 12*x^2])/150994944 - (125455*(17 + 24*x)*(6 + 17*x + 12*x^2)^(3/2))/4718592 + (25091*(17 + 24*x)*(6 + 17*x + 12*x^2)^(5/2))/24576 - (873*(6 + 17*x + 12*x^2)^(7/2))/1792 - ((10 - 3*x)*(6 + 17*x + 12*x^2)^(7/2))/32 - (125455*ArcTanh[(17 + 24*x)/(4*Sqrt[3]*Sqrt[6 + 17*x + 12*x^2])])/(603979776*Sqrt[3])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 742

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad

ratricQ[a, b, c, d, e, m, p, x]

Rule 1002

Int[((g_) + (h_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_ + (e_)*(x_) + (f_)*(x_)^2)^(m_)), x_Symbol] := Int[((d*g)/a + (f*h*x)/c)^(m*(a + b*x + c*x^2)^(m + p), x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x] && EqQ[c*g^2 - b*g*h + a*h^2, 0] && EqQ[c^2*d*g^2 - a*c*e*g*h + a^2*f*h^2, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int (2 + 3x)^2 (30 + 31x - 12x^2)^2 \sqrt{6 + 17x + 12x^2} dx &= \int (10 - 3x)^2 (6 + 17x + 12x^2)^{5/2} dx \\ &= -\frac{1}{32}(10 - 3x)(6 + 17x + 12x^2)^{7/2} + \frac{1}{96} \int \left(11331 - \frac{7857x}{2}\right) \sqrt{6 + 17x + 12x^2} dx \\ &= -\frac{873(6 + 17x + 12x^2)^{7/2}}{1792} - \frac{1}{32}(10 - 3x)(6 + 17x + 12x^2)^{5/2} \\ &= \frac{25091(17 + 24x)(6 + 17x + 12x^2)^{5/2}}{24576} - \frac{873(6 + 17x + 12x^2)^{3/2}}{1792} \\ &= -\frac{125455(17 + 24x)(6 + 17x + 12x^2)^{3/2}}{4718592} + \frac{25091(17 + 24x)(6 + 17x + 12x^2)^{5/2}}{24576} \\ &= \frac{125455(17 + 24x)\sqrt{6 + 17x + 12x^2}}{150994944} - \frac{125455(17 + 24x)(6 + 17x + 12x^2)^{3/2}}{4718592} \\ &= \frac{125455(17 + 24x)\sqrt{6 + 17x + 12x^2}}{150994944} - \frac{125455(17 + 24x)(6 + 17x + 12x^2)^{3/2}}{4718592} \\ &= \frac{125455(17 + 24x)\sqrt{6 + 17x + 12x^2}}{150994944} - \frac{125455(17 + 24x)(6 + 17x + 12x^2)^{3/2}}{4718592} \end{aligned}$$

Mathematica [A] time = 0.19, size = 87, normalized size = 0.58

$$\frac{12\sqrt{12x^2 + 17x + 6} (171228266496x^7 - 732816211968x^6 - 1190083166208x^5 + 3438453030912x^4 + 8974844476416x^3 + 7899203409792x^2 + 3132157281976x + 474999091769) - 878185\sqrt{3} \operatorname{tanh}^{-1}\left(\frac{24x+17}{4\sqrt{36x^2+51x+18}}\right)}{12683575296}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x)^2*(30 + 31*x - 12*x^2)^2*Sqrt[6 + 17*x + 12*x^2], x]

[Out] (12*Sqrt[6 + 17*x + 12*x^2]*(474999091769 + 3132157281976*x + 7899203409792*x^2 + 8974844476416*x^3 + 3438453030912*x^4 - 1190083166208*x^5 - 732816211968*x^6 + 171228266496*x^7) - 878185*Sqrt[3]*ArcTanh[(17 + 24*x)/(4*Sqrt[18 + 51*x + 36*x^2]]))/12683575296

IntegrateAlgebraic [A] time = 0.73, size = 92, normalized size = 0.62

$$\frac{\sqrt{12x^2 + 17x + 6} (171228266496x^7 - 732816211968x^6 - 1190083166208x^5 + 3438453030912x^4 + 8974844476416x^3 + 7899203409792x^2 + 3132157281976x + 474999091769) - 125455 \operatorname{tanh}^{-1}\left(\frac{2\sqrt{12x^2+17x+6}}{\sqrt{3}(4x+3)}\right)}{1056964608 - 301989888\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + 3*x)^2*(30 + 31*x - 12*x^2)^2*Sqrt[6 + 17*x + 12*x^2], x]

[Out] (Sqrt[6 + 17*x + 12*x^2]*(474999091769 + 3132157281976*x + 7899203409792*x^2 + 8974844476416*x^3 + 3438453030912*x^4 - 1190083166208*x^5 - 732816211968*x^6 + 171228266496*x^7) - 125455*Sqrt[3]*ArcTanh[(17 + 24*x)/(4*Sqrt[18 + 51*x + 36*x^2]]))/12683575296

$(8*x^6 + 171228266496*x^7)/1056964608 - (125455*ArcTanh[(2*sqrt(6 + 17*x + 12*x^2))/(sqrt(3)*(3 + 4*x))])/(301989888*sqrt(3))$

fricas [A] time = 0.95, size = 88, normalized size = 0.59

$\frac{1}{1056964608} (171228266496x^7 - 732816211968x^6 - 1190083166208x^5 + 3438453030912x^4 + 8974844476416x^3 + 7899203409792x^2 + 3132157281976x + 474999091769) \sqrt{12x^2 + 17x + 6} + \frac{125455}{3623878656} \sqrt{3} \log(-8\sqrt{3}\sqrt{12x^2 + 17x + 6}(24x + 17) + 1152x^2 + 1632x + 577)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*x)^2*(-12*x^2+31*x+30)^2*(12*x^2+17*x+6)^(1/2),x, algorithm="fricas")

[Out] 1/1056964608*(171228266496*x^7 - 732816211968*x^6 - 1190083166208*x^5 + 3438453030912*x^4 + 8974844476416*x^3 + 7899203409792*x^2 + 3132157281976*x + 474999091769)*sqrt(12*x^2 + 17*x + 6) + 125455/3623878656*sqrt(3)*log(-8*sqrt(3)*sqrt(12*x^2 + 17*x + 6)*(24*x + 17) + 1152*x^2 + 1632*x + 577)

giac [A] time = 0.30, size = 85, normalized size = 0.57

$\frac{1}{1056964608} (8(48(24(96(24(168x - 719)x - 56047)x + 3886417)x + 973832951)x + 20570842213)x + 391519660247)x + 474999091769) \sqrt{12x^2 + 17x + 6} + \frac{125455}{1811939328} \sqrt{3} \log(-4\sqrt{3}(2\sqrt{3}x - \sqrt{12x^2 + 17x + 6}) - 17)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*x)^2*(-12*x^2+31*x+30)^2*(12*x^2+17*x+6)^(1/2),x, algorithm="giac")

[Out] 1/1056964608*(8*(48*(24*(96*(24*(48*(168*x - 719)*x - 56047)*x + 3886417)*x + 973832951)*x + 20570842213)*x + 391519660247)*x + 474999091769)*sqrt(12*x^2 + 17*x + 6) + 125455/1811939328*sqrt(3)*log(abs(-4*sqrt(3)*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x + 6)) - 17))

maple [A] time = 0.02, size = 147, normalized size = 0.99

$\frac{27}{2} (12x^2 + 17x + 6)^{\frac{3}{2}} x^5 - \frac{8613}{112} (12x^2 + 17x + 6)^{\frac{3}{2}} x^4 + \frac{14991}{1792} (12x^2 + 17x + 6)^{\frac{3}{2}} x^3 + \frac{4267751}{14336} (12x^2 + 17x + 6)^{\frac{3}{2}} x^2 + \frac{129220757}{458752} (12x^2 + 17x + 6)^{\frac{3}{2}} x + \frac{125455\sqrt{12} \ln\left(\frac{(12x^2 + 17x + 6)\sqrt{12x^2 + 17x + 6} + \sqrt{12x^2 + 17x + 6}}{12}\right)}{3623878656} + \frac{2473875847}{33030144} (12x^2 + 17x + 6)^{\frac{3}{2}} + \frac{125455(24x + 17)\sqrt{12x^2 + 17x + 6}}{150994944}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2+3*x)^2*(-12*x^2+31*x+30)^2*(12*x^2+17*x+6)^(1/2),x)

[Out] 27/2*x^5*(12*x^2+17*x+6)^(3/2)-8613/112*x^4*(12*x^2+17*x+6)^(3/2)+14991/1792*x^3*(12*x^2+17*x+6)^(3/2)+4267751/14336*x^2*(12*x^2+17*x+6)^(3/2)+129220757/458752*x*(12*x^2+17*x+6)^(3/2)+2473875847/33030144*(12*x^2+17*x+6)^(3/2)-125455/3623878656*ln(1/12*(17/2+12*x)*12^(1/2)+(12*x^2+17*x+6)^(1/2))*12^(1/2)+125455/150994944*(17+24*x)*(12*x^2+17*x+6)^(1/2)

maxima [A] time = 1.00, size = 155, normalized size = 1.04

$\frac{27}{2} (12x^2 + 17x + 6)^{\frac{3}{2}} x^5 - \frac{8613}{112} (12x^2 + 17x + 6)^{\frac{3}{2}} x^4 + \frac{14991}{1792} (12x^2 + 17x + 6)^{\frac{3}{2}} x^3 + \frac{4267751}{14336} (12x^2 + 17x + 6)^{\frac{3}{2}} x^2 + \frac{129220757}{458752} (12x^2 + 17x + 6)^{\frac{3}{2}} x + \frac{125455\sqrt{12} \ln\left(\frac{(12x^2 + 17x + 6)\sqrt{12x^2 + 17x + 6} + \sqrt{12x^2 + 17x + 6}}{12}\right)}{3623878656} + \frac{2473875847}{33030144} (12x^2 + 17x + 6)^{\frac{3}{2}} + \frac{125455(24x + 17)\sqrt{12x^2 + 17x + 6}}{150994944}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*x)^2*(-12*x^2+31*x+30)^2*(12*x^2+17*x+6)^(1/2),x, algorithm="maxima")

[Out] 27/2*(12*x^2 + 17*x + 6)^(3/2)*x^5 - 8613/112*(12*x^2 + 17*x + 6)^(3/2)*x^4 + 14991/1792*(12*x^2 + 17*x + 6)^(3/2)*x^3 + 4267751/14336*(12*x^2 + 17*x + 6)^(3/2)*x^2 + 129220757/458752*(12*x^2 + 17*x + 6)^(3/2)*x + 2473875847/33030144*(12*x^2 + 17*x + 6)^(3/2) + 125455/6291456*sqrt(12*x^2 + 17*x + 6)*x - 125455/1811939328*sqrt(3)*log(4*sqrt(3)*sqrt(12*x^2 + 17*x + 6) + 24*x + 17) + 2132735/150994944*sqrt(12*x^2 + 17*x + 6)

mupad [B] time = 5.08, size = 187, normalized size = 1.26

$\frac{4267751x^2(12x^2 + 17x + 6)^{\frac{3}{2}}}{14336} + \frac{14991x^3(12x^2 + 17x + 6)^{\frac{3}{2}}}{1792} + \frac{8613x^4(12x^2 + 17x + 6)^{\frac{3}{2}}}{112} + \frac{27x^5(12x^2 + 17x + 6)^{\frac{3}{2}}}{2} - \frac{146030443\sqrt{12} \ln\left(\frac{\sqrt{12x^2 + 17x + 6} + \sqrt{12x^2 + 17x + 6}}{12}\right)}{8880384} + \frac{438091328\sqrt{3} \ln\left(\frac{2}{3} + \frac{17}{12}\sqrt{12x^2 + 17x + 6}\right)}{22576} + \frac{2473875847\sqrt{12x^2 + 17x + 6}(1152x^2 + 408x - 291)}{3178893824} + \frac{129220757x(12x^2 + 17x + 6)^{\frac{3}{2}}}{458752} + \frac{42058889399\sqrt{12} \ln\left(2\sqrt{12x^2 + 17x + 6} + \frac{\sqrt{12x^2 + 17x + 6}}{12}\right)}{2367036862}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x + 2)^2*(17*x + 12*x^2 + 6)^(1/2)*(31*x - 12*x^2 + 30)^2,x)`

[Out] $(4267751*x^2*(17*x + 12*x^2 + 6)^{(3/2)})/14336 + (14991*x^3*(17*x + 12*x^2 + 6)^{(3/2)})/1792 - (8613*x^4*(17*x + 12*x^2 + 6)^{(3/2)})/112 + (27*x^5*(17*x + 12*x^2 + 6)^{(3/2)})/2 - (146030443*12^{(1/2)}*\log((17*x + 12*x^2 + 6)^{(1/2)} + (12^{(1/2)}*(12*x + 17/2))/12))/88080384 + (438091329*(x/2 + 17/48)*(17*x + 12*x^2 + 6)^{(1/2)})/229376 + (2473875847*(17*x + 12*x^2 + 6)^{(1/2)}*(408*x + 1152*x^2 - 291))/3170893824 + (129220757*x*(17*x + 12*x^2 + 6)^{(3/2)})/458752 + (42055889399*12^{(1/2)}*\log(2*(17*x + 12*x^2 + 6)^{(1/2)} + (12^{(1/2)}*(24*x + 17))/12))/25367150592$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{(3x + 2)(4x + 3)} (3x - 10)^2 (3x + 2)^2 (4x + 3)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2+3*x)**2*(-12*x**2+31*x+30)**2*(12*x**2+17*x+6)**(1/2),x)`

[Out] `Integral(sqrt((3*x + 2)*(4*x + 3))*(3*x - 10)**2*(3*x + 2)**2*(4*x + 3)**2, x)`

$$3.134 \quad \int (2 + 3x) (30 + 31x - 12x^2) \sqrt{6 + 17x + 12x^2} dx$$

Optimal. Leaf size=103

$$-\frac{1}{20} (12x^2 + 17x + 6)^{5/2} + \frac{97}{768} (24x+17) (12x^2 + 17x + 6)^{3/2} - \frac{97(24x+17)\sqrt{12x^2+17x+6}}{24576} + \frac{97 \tanh^{-1}\left(\frac{24x+17}{4\sqrt{3}\sqrt{12x^2+17x+6}}\right)}{98304\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1002, 640, 612, 621, 206}

$$-\frac{1}{20} (12x^2 + 17x + 6)^{5/2} + \frac{97}{768} (24x+17) (12x^2 + 17x + 6)^{3/2} - \frac{97(24x+17)\sqrt{12x^2+17x+6}}{24576} + \frac{97 \tanh^{-1}\left(\frac{24x+17}{4\sqrt{3}\sqrt{12x^2+17x+6}}\right)}{98304\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x)*(30 + 31*x - 12*x^2)*Sqrt[6 + 17*x + 12*x^2], x]

[Out] (-97*(17 + 24*x)*Sqrt[6 + 17*x + 12*x^2])/24576 + (97*(17 + 24*x)*(6 + 17*x + 12*x^2)^(3/2))/768 - (6 + 17*x + 12*x^2)^(5/2)/20 + (97*ArcTanh[(17 + 24*x)/(4*Sqrt[3]*Sqrt[6 + 17*x + 12*x^2])])/(98304*Sqrt[3])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1002

Int[((g_) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(m_.), x_Symbol] := Int[((d*g)/a + (f*h*x)/c)^(m*(a + b*x + c*x^2)^(m + p)), x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x] && EqQ[c*g^2 - b*g*h + a*h^2, 0] && EqQ[c^2*d*g^2 - a*c*e*g*h + a^2*f*h^2, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
\int (2+3x)(30+31x-12x^2)\sqrt{6+17x+12x^2} dx &= \int (10-3x)(6+17x+12x^2)^{3/2} dx \\
&= -\frac{1}{20}(6+17x+12x^2)^{5/2} + \frac{97}{8} \int (6+17x+12x^2)^{3/2} dx \\
&= \frac{97}{768}(17+24x)(6+17x+12x^2)^{3/2} - \frac{1}{20}(6+17x+12x^2)^{5/2} \\
&= -\frac{97(17+24x)\sqrt{6+17x+12x^2}}{24576} + \frac{97}{768}(17+24x)(6+17x \\
&= -\frac{97(17+24x)\sqrt{6+17x+12x^2}}{24576} + \frac{97}{768}(17+24x)(6+17x \\
&= -\frac{97(17+24x)\sqrt{6+17x+12x^2}}{24576} + \frac{97}{768}(17+24x)(6+17x
\end{aligned}$$

Mathematica [A] time = 0.03, size = 72, normalized size = 0.70

$$\frac{485\sqrt{3} \tanh^{-1}\left(\frac{24x+17}{4\sqrt{36x^2+51x+18}}\right) + 12\sqrt{12x^2+17x+6}(-884736x^4 + 1963008x^3 + 6837888x^2 + 5455144x + 1353611)}{1474560}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x)*(30 + 31*x - 12*x^2)*Sqrt[6 + 17*x + 12*x^2], x]

[Out] (12*Sqrt[6 + 17*x + 12*x^2]*(1353611 + 5455144*x + 6837888*x^2 + 1963008*x^3 - 884736*x^4) + 485*Sqrt[3]*ArcTanh[(17 + 24*x)/(4*Sqrt[18 + 51*x + 36*x^2])])/1474560

IntegrateAlgebraic [A] time = 0.44, size = 77, normalized size = 0.75

$$\frac{97 \tanh^{-1}\left(\frac{2\sqrt{12x^2+17x+6}}{\sqrt{3}(4x+3)}\right)}{49152\sqrt{3}} + \frac{\sqrt{12x^2+17x+6}(-884736x^4 + 1963008x^3 + 6837888x^2 + 5455144x + 1353611)}{122880}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + 3*x)*(30 + 31*x - 12*x^2)*Sqrt[6 + 17*x + 12*x^2], x]

[Out] (Sqrt[6 + 17*x + 12*x^2]*(1353611 + 5455144*x + 6837888*x^2 + 1963008*x^3 - 884736*x^4))/122880 + (97*ArcTanh[(2*Sqrt[6 + 17*x + 12*x^2])/(Sqrt[3]*(3 + 4*x))])/ (49152*Sqrt[3])

fricas [A] time = 0.87, size = 73, normalized size = 0.71

$$-\frac{1}{122880}(884736x^4 - 1963008x^3 - 6837888x^2 - 5455144x - 1353611)\sqrt{12x^2+17x+6} + \frac{97}{589824}\sqrt{3} \log\left(8\sqrt{3}\sqrt{12x^2+17x+6}(24x+17) + 1152x^2 + 1632x + 577\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*x)*(-12*x^2+31*x+30)*(12*x^2+17*x+6)^(1/2), x, algorithm="fricas")

[Out] -1/122880*(884736*x^4 - 1963008*x^3 - 6837888*x^2 - 5455144*x - 1353611)*sqrt(12*x^2 + 17*x + 6) + 97/589824*sqrt(3)*log(8*sqrt(3)*sqrt(12*x^2 + 17*x + 6)*(24*x + 17) + 1152*x^2 + 1632*x + 577)

giac [A] time = 0.20, size = 70, normalized size = 0.68

$$-\frac{1}{122880}(8(48(72(32x-71)x-17807)x-681893)x-1353611)\sqrt{12x^2+17x+6} - \frac{97}{294912}\sqrt{3} \log\left(\left|-4\sqrt{3}\left(2\sqrt{3}x - \sqrt{12x^2+17x+6}\right) - 17\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*x)*(-12*x^2+31*x+30)*(12*x^2+17*x+6)^(1/2),x, algorithm="giac")

[Out] -1/122880*(8*(48*(72*(32*x - 71)*x - 17807)*x - 681893)*x - 1353611)*sqrt(12*x^2 + 17*x + 6) - 97/294912*sqrt(3)*log(abs(-4*sqrt(3)*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x + 6)) - 17))

maple [A] time = 0.01, size = 96, normalized size = 0.93

$$-\frac{3(12x^2+17x+6)^{\frac{3}{2}}x^2}{5} + \frac{349(12x^2+17x+6)^{\frac{3}{2}}x}{160} + \frac{97\sqrt{12}\ln\left(\frac{(12x+\frac{17}{2})\sqrt{12}}{12} + \sqrt{12x^2+17x+6}\right)}{589824} + \frac{7093(12x^2+17x+6)^{\frac{3}{2}}}{3840} - \frac{97(24x+17)\sqrt{12x^2+17x+6}}{24576}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2+3*x)*(-12*x^2+31*x+30)*(12*x^2+17*x+6)^(1/2),x)

[Out] -3/5*(12*x^2+17*x+6)^(3/2)*x^2+349/160*(12*x^2+17*x+6)^(3/2)*x+7093/3840*(12*x^2+17*x+6)^(3/2)-97/24576*(24*x+17)*(12*x^2+17*x+6)^(1/2)+97/589824*12^(1/2)*ln(1/12*(12*x+17/2)*12^(1/2)+(12*x^2+17*x+6)^(1/2))

maxima [A] time = 0.98, size = 104, normalized size = 1.01

$$-\frac{3}{5}(12x^2+17x+6)^{\frac{3}{2}}x^2 + \frac{349}{160}(12x^2+17x+6)^{\frac{3}{2}}x + \frac{7093}{3840}(12x^2+17x+6)^{\frac{3}{2}} - \frac{97}{1024}\sqrt{12x^2+17x+6}x + \frac{97}{294912}\sqrt{3}\log\left(4\sqrt{3}\sqrt{12x^2+17x+6}+24x+17\right) - \frac{1649}{24576}\sqrt{12x^2+17x+6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*x)*(-12*x^2+31*x+30)*(12*x^2+17*x+6)^(1/2),x, algorithm="maxima")

[Out] -3/5*(12*x^2 + 17*x + 6)^(3/2)*x^2 + 349/160*(12*x^2 + 17*x + 6)^(3/2)*x + 7093/3840*(12*x^2 + 17*x + 6)^(3/2) - 97/1024*sqrt(12*x^2 + 17*x + 6)*x + 97/294912*sqrt(3)*log(4*sqrt(3)*sqrt(12*x^2 + 17*x + 6) + 24*x + 17) - 1649/24576*sqrt(12*x^2 + 17*x + 6)

mupad [B] time = 4.69, size = 136, normalized size = 1.32

$$\frac{3753\left(\frac{x}{2} + \frac{17}{48}\right)\sqrt{12x^2+17x+6}}{80} - \frac{417\sqrt{12}\ln\left(\sqrt{12x^2+17x+6} + \frac{\sqrt{12}\left(\frac{x}{2} + \frac{17}{48}\right)}{12}\right)}{10240} - \frac{3x^2(12x^2+17x+6)^{\frac{3}{2}}}{5} + \frac{7093\sqrt{12x^2+17x+6}(1152x^2+408x-291)}{368640} + \frac{349x(12x^2+17x+6)^{\frac{3}{2}}}{160} + \frac{120581\sqrt{12}\ln\left(2\sqrt{12x^2+17x+6} + \frac{\sqrt{12}\left(\frac{x}{2} + \frac{17}{48}\right)}{12}\right)}{2949120}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x + 2)*(17*x + 12*x^2 + 6)^(1/2)*(31*x - 12*x^2 + 30),x)

[Out] (3753*(x/2 + 17/48)*(17*x + 12*x^2 + 6)^(1/2))/80 - (417*12^(1/2)*log((17*x + 12*x^2 + 6)^(1/2) + (12^(1/2)*(12*x + 17/2))/12))/10240 - (3*x^2*(17*x + 12*x^2 + 6)^(3/2))/5 + (7093*(17*x + 12*x^2 + 6)^(1/2)*(408*x + 1152*x^2 - 291))/368640 + (349*x*(17*x + 12*x^2 + 6)^(3/2))/160 + (120581*12^(1/2)*log(2*(17*x + 12*x^2 + 6)^(1/2) + (12^(1/2)*(24*x + 17))/12))/2949120

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int(-152x\sqrt{12x^2+17x+6})dx - \int(-69x^2\sqrt{12x^2+17x+6})dx - \int 36x^3\sqrt{12x^2+17x+6}dx - \int(-60\sqrt{12x^2+17x+6})dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*x)*(-12*x**2+31*x+30)*(12*x**2+17*x+6)**(1/2),x)

[Out] -Integral(-152*x*sqrt(12*x**2 + 17*x + 6), x) - Integral(-69*x**2*sqrt(12*x**2 + 17*x + 6), x) - Integral(36*x**3*sqrt(12*x**2 + 17*x + 6), x) - Integral(-60*sqrt(12*x**2 + 17*x + 6), x)

$$3.135 \quad \int \frac{\sqrt{6+17x+12x^2}}{(2+3x)(30+31x-12x^2)} dx$$

Optimal. Leaf size=28

$$\frac{1}{42} \tanh^{-1} \left(\frac{291x + 206}{84\sqrt{12x^2 + 17x + 6}} \right)$$

Rubi [A] time = 0.05, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {1002, 724, 206}

$$\frac{1}{42} \tanh^{-1} \left(\frac{291x + 206}{84\sqrt{12x^2 + 17x + 6}} \right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)*(30 + 31*x - 12*x^2)),x]

[Out] ArcTanh[(206 + 291*x)/(84*Sqrt[6 + 17*x + 12*x^2])]/42

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1002

Int[((g_) + (h_.)*(x_)^(m_.))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(m_.), x_Symbol] := Int[((d*g)/a + (f*h*x)/c)^(m*(a + b*x + c*x^2)^(m + p)), x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x] && EqQ[c*g^2 - b*g*h + a*h^2, 0] && EqQ[c^2*d*g^2 - a*c*e*g*h + a^2*f*h^2, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{6+17x+12x^2}}{(2+3x)(30+31x-12x^2)} dx &= \int \frac{1}{(10-3x)\sqrt{6+17x+12x^2}} dx \\ &= - \left(2 \text{Subst} \left(\int \frac{1}{7056-x^2} dx, x, \frac{-206-291x}{\sqrt{6+17x+12x^2}} \right) \right) \\ &= \frac{1}{42} \tanh^{-1} \left(\frac{206+291x}{84\sqrt{6+17x+12x^2}} \right) \end{aligned}$$

Mathematica [A] time = 0.10, size = 37, normalized size = 1.32

$$\frac{1}{42} \log \left(84\sqrt{12x^2 + 17x + 6} + 291x + 206 \right) - \frac{1}{42} \log(10 - 3x)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)*(30 + 31*x - 12*x^2)),x]

[Out] -1/42*Log[10 - 3*x] + Log[206 + 291*x + 84*Sqrt[6 + 17*x + 12*x^2]]/42

IntegrateAlgebraic [A] time = 0.30, size = 30, normalized size = 1.07

$$\frac{1}{21} \tanh^{-1} \left(\frac{6\sqrt{12x^2 + 17x + 6}}{7(3x + 2)} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)*(30 + 31*x - 12*x^2)),x]

[Out] ArcTanh[(6*Sqrt[6 + 17*x + 12*x^2])/(7*(2 + 3*x))]/21

fricas [B] time = 0.63, size = 53, normalized size = 1.89

$$\frac{1}{84} \log \left(\frac{291x + 84\sqrt{12x^2 + 17x + 6} + 206}{x} \right) - \frac{1}{84} \log \left(\frac{291x - 84\sqrt{12x^2 + 17x + 6} + 206}{x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)/(-12*x^2+31*x+30),x, algorithm="fricas")

[Out] 1/84*log((291*x + 84*sqrt(12*x^2 + 17*x + 6) + 206)/x) - 1/84*log((291*x - 84*sqrt(12*x^2 + 17*x + 6) + 206)/x)

giac [B] time = 0.27, size = 63, normalized size = 2.25

$$\frac{1}{42} \log \left(\left| -6\sqrt{3}x + 20\sqrt{3} + 3\sqrt{12x^2 + 17x + 6} + 42 \right| \right) - \frac{1}{42} \log \left(\left| -6\sqrt{3}x + 20\sqrt{3} + 3\sqrt{12x^2 + 17x + 6} - 42 \right| \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)/(-12*x^2+31*x+30),x, algorithm="giac")

[Out] 1/42*log(abs(-6*sqrt(3)*x + 20*sqrt(3) + 3*sqrt(12*x^2 + 17*x + 6) + 42)) - 1/42*log(abs(-6*sqrt(3)*x + 20*sqrt(3) + 3*sqrt(12*x^2 + 17*x + 6) - 42))

maple [B] time = 0.02, size = 163, normalized size = 5.82

$$\frac{\operatorname{arctanh} \left(\frac{97x+206}{28\sqrt{97x+12}\left(x-\frac{10}{3}\right)-\frac{382}{3}} \right)}{42} - \frac{97\sqrt{12} \ln \left(\frac{(12x+\frac{17}{2})\sqrt{12} + \sqrt{97x+12}\left(x-\frac{10}{3}\right)-\frac{382}{3}}{14112} \right)}{14112} + \frac{\sqrt{12} \ln \left(\frac{(12x+\frac{17}{2})\sqrt{12} + \sqrt{x+12}\left(x+\frac{2}{3}\right)+\frac{2}{3}}{288} \right)}{288} + \frac{\sqrt{12} \ln \left(\frac{(12x+\frac{17}{2})\sqrt{12} + \sqrt{-x+12}\left(x+\frac{3}{4}\right)-\frac{3}{4}}{294} \right)}{294} - \frac{4\sqrt{-x+12}\left(x+\frac{3}{4}\right)-\frac{3}{4}}{49} + \frac{\sqrt{x+12}\left(x+\frac{2}{3}\right)+\frac{2}{3}}{12} - \frac{\sqrt{97x+12}\left(x-\frac{10}{3}\right)-\frac{382}{3}}{588}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((12*x^2+17*x+6)^(1/2)/(2+3*x)/(-12*x^2+31*x+30),x)

[Out] -4/49*(12*(x+3/4)^2-x-3/4)^(1/2)+1/294*ln(1/12*(12*x+17/2)*12^(1/2)+(12*(x+3/4)^2-x-3/4)^(1/2))*12^(1/2)+1/12*(12*(x+2/3)^2+x+2/3)^(1/2)+1/288*ln(1/12*(12*x+17/2)*12^(1/2)+(12*(x+2/3)^2+x+2/3)^(1/2))*12^(1/2)-1/588*(12*(x-10/3)^2+97*x-382/3)^(1/2)-97/14112*ln(1/12*(12*x+17/2)*12^(1/2)+(12*(x-10/3)^2+97*x-382/3)^(1/2))*12^(1/2)+1/42*arctanh(1/28*(206/3+97*x)/(12*(x-10/3)^2+97*x-382/3)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{\sqrt{12x^2 + 17x + 6}}{(12x^2 - 31x - 30)(3x + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)/(-12*x^2+31*x+30),x, algorithm="maxima")

[Out] -integrate(sqrt(12*x^2 + 17*x + 6)/((12*x^2 - 31*x - 30)*(3*x + 2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{12x^2 + 17x + 6}}{(3x + 2)(-12x^2 + 31x + 30)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)*(31*x - 12*x^2 + 30)),x)

[Out] int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)*(31*x - 12*x^2 + 30)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{\sqrt{12x^2 + 17x + 6}}{36x^3 - 69x^2 - 152x - 60} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((12*x**2+17*x+6)**(1/2)/(2+3*x)/(-12*x**2+31*x+30),x)

[Out] -Integral(sqrt(12*x**2 + 17*x + 6)/(36*x**3 - 69*x**2 - 152*x - 60), x)

$$3.136 \quad \int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^2(30+31x-12x^2)^2} dx$$

Optimal. Leaf size=84

$$-\frac{388x+275}{98(10-3x)\sqrt{12x^2+17x+6}} + \frac{3137\sqrt{12x^2+17x+6}}{38416(10-3x)} + \frac{97 \tanh^{-1}\left(\frac{291x+206}{84\sqrt{12x^2+17x+6}}\right)}{3226944}$$

Rubi [A] time = 0.08, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {1002, 740, 806, 724, 206}

$$-\frac{388x+275}{98(10-3x)\sqrt{12x^2+17x+6}} + \frac{3137\sqrt{12x^2+17x+6}}{38416(10-3x)} + \frac{97 \tanh^{-1}\left(\frac{291x+206}{84\sqrt{12x^2+17x+6}}\right)}{3226944}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^2*(30 + 31*x - 12*x^2)^2), x]

[Out] -(275 + 388*x)/(98*(10 - 3*x)*Sqrt[6 + 17*x + 12*x^2]) + (3137*Sqrt[6 + 17*x + 12*x^2])/(38416*(10 - 3*x)) + (97*ArcTanh[(206 + 291*x)/(84*Sqrt[6 + 17*x + 12*x^2])])/3226944

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 740

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]

Rule 806

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]

Rule 1002

```
Int[((g_) + (h_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_
) + (e_)*(x_) + (f_)*(x_)^2)^(m_), x_Symbol] := Int[((d*g)/a + (f*h*x)/c
)^(m*(a + b*x + c*x^2)^(m + p), x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x]
&& EqQ[c*g^2 - b*g*h + a*h^2, 0] && EqQ[c^2*d*g^2 - a*c*e*g*h + a^2*f*h^2,
0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^2(30+31x-12x^2)^2} dx &= \int \frac{1}{(10-3x)^2(6+17x+12x^2)^{3/2}} dx \\ &= -\frac{275+388x}{98(10-3x)\sqrt{6+17x+12x^2}} - \frac{1}{882} \int \frac{-\frac{14859}{2} - 10476x}{(10-3x)^2\sqrt{6+17x+12x^2}} dx \\ &= -\frac{275+388x}{98(10-3x)\sqrt{6+17x+12x^2}} + \frac{3137\sqrt{6+17x+12x^2}}{38416(10-3x)} + \frac{97 \int \frac{1}{(10-3x)\sqrt{6+17x+12x^2}} dx}{76832} \\ &= -\frac{275+388x}{98(10-3x)\sqrt{6+17x+12x^2}} + \frac{3137\sqrt{6+17x+12x^2}}{38416(10-3x)} - \frac{97 \operatorname{Subst}\left(\int \frac{1}{705}\right)}{76832} \\ &= -\frac{275+388x}{98(10-3x)\sqrt{6+17x+12x^2}} + \frac{3137\sqrt{6+17x+12x^2}}{38416(10-3x)} + \frac{97 \tanh^{-1}\left(\frac{2}{84\sqrt{6+17x+12x^2}}\right)}{32269} \end{aligned}$$

Mathematica [A] time = 0.24, size = 114, normalized size = 1.36

$$\frac{\sqrt{12x^2+17x+6} \left(97(36x^3-69x^2-152x-60) \tanh^{-1}\left(\frac{7\sqrt{3x+2}}{6\sqrt{4x+3}}\right) - 42\sqrt{3x+2}\sqrt{4x+3} (37644x^2-98767x-88978) \right)}{1613472(3x-10)(3x+2)^{3/2}(4x+3)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^2*(30 + 31*x - 12*x^2)^2), x]

[Out] (Sqrt[6 + 17*x + 12*x^2]*(-42*Sqrt[2 + 3*x]*Sqrt[3 + 4*x]*(-88978 - 98767*x + 37644*x^2) + 97*(-60 - 152*x - 69*x^2 + 36*x^3)*ArcTanh[(7*Sqrt[2 + 3*x])/(6*Sqrt[3 + 4*x])]))/(1613472*(-10 + 3*x)*(2 + 3*x)^(3/2)*(3 + 4*x)^(3/2))

IntegrateAlgebraic [A] time = 0.37, size = 80, normalized size = 0.95

$$\frac{\sqrt{12x^2+17x+6} (-37644x^2+98767x+88978)}{38416(3x-10)(3x+2)(4x+3)} + \frac{97 \tanh^{-1}\left(\frac{6\sqrt{12x^2+17x+6}}{7(3x+2)}\right)}{1613472}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^2*(30 + 31*x - 12*x^2)^2), x]

[Out] ((88978 + 98767*x - 37644*x^2)*Sqrt[6 + 17*x + 12*x^2])/(38416*(-10 + 3*x)*(2 + 3*x)*(3 + 4*x)) + (97*ArcTanh[(6*Sqrt[6 + 17*x + 12*x^2])/(7*(2 + 3*x))])/1613472

fricas [A] time = 0.98, size = 126, normalized size = 1.50

$$\frac{97(36x^3-69x^2-152x-60) \log\left(\frac{291x+84\sqrt{12x^2+17x+6}+206}{x}\right) - 97(36x^3-69x^2-152x-60) \log\left(\frac{291x-84\sqrt{12x^2+17x+6}+206}{x}\right) - 168(37644x^2-98767x-88978)\sqrt{12x^2+17x+6}}{6453888(36x^3-69x^2-152x-60)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^2/(-12*x^2+31*x+30)^2,x, algorithm="fricas")

[Out] 1/6453888*(97*(36*x^3 - 69*x^2 - 152*x - 60)*log((291*x + 84*sqrt(12*x^2 + 17*x + 6) + 206)/x) - 97*(36*x^3 - 69*x^2 - 152*x - 60)*log((291*x - 84*sqrt(12*x^2 + 17*x + 6) + 206)/x) - 168*(37644*x^2 - 98767*x - 88978)*sqrt(12*x^2 + 17*x + 6))/(36*x^3 - 69*x^2 - 152*x - 60)

giac [B] time = 0.27, size = 159, normalized size = 1.89

$$\frac{1}{9680832} \sqrt{3} \left(\sqrt{3} \left(175672 \sqrt{3} + 97 \log \left(\frac{7\sqrt{3} - 12}{7\sqrt{3} + 12} \right) \operatorname{sgn} \left(\frac{1}{3x+2} \right) - \left(97 \sqrt{3} \log \left(\frac{-28\sqrt{3} + 24\sqrt{\frac{1}{3x+2} + 4}}{4(7\sqrt{3} + 6\sqrt{\frac{1}{3x+2} + 4})} \right) + 134456 \sqrt{\frac{1}{3x+2} + 4} + \frac{28 \left(\frac{221183}{3x+2} - 18436 \right)}{12 \left(\frac{1}{3x+2} + 4 \right)^{\frac{3}{2}} - 49 \sqrt{\frac{1}{3x+2} + 4}} \right) \operatorname{sgn} \left(\frac{1}{3x+2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^2/(-12*x^2+31*x+30)^2,x, algorithm="giac")

[Out] 1/9680832*sqrt(3)*(sqrt(3)*(175672*sqrt(3) + 97*log((7*sqrt(3) - 12)/(7*sqrt(3) + 12)))*sgn(1/(3*x + 2)) - (97*sqrt(3)*log(1/4*abs(-28*sqrt(3) + 24*sqrt(1/(3*x + 2) + 4)))/(7*sqrt(3) + 6*sqrt(1/(3*x + 2) + 4))) + 134456*sqrt(1/(3*x + 2) + 4) + 28*(221183/(3*x + 2) - 18436)/(12*(1/(3*x + 2) + 4)^(3/2) - 49*sqrt(1/(3*x + 2) + 4)))*sgn(1/(3*x + 2)))

maple [B] time = 0.02, size = 245, normalized size = 2.92

$$\frac{97 \operatorname{arctanh} \left(\frac{7\sqrt{3}-12}{7\sqrt{3}+12} \right)}{322644} - \frac{21057\sqrt{3} \ln \left(\frac{12+\sqrt{3}}{12-\sqrt{3}} \sqrt{\frac{12+\sqrt{3}}{12-\sqrt{3}}} + \sqrt{\frac{12+\sqrt{3}}{12-\sqrt{3}}} \sqrt{\frac{12+\sqrt{3}}{12-\sqrt{3}}} \right)}{813189888} + \frac{\sqrt{3} \ln \left(\frac{12+\sqrt{3}}{12-\sqrt{3}} \sqrt{\frac{12+\sqrt{3}}{12-\sqrt{3}}} + \sqrt{\frac{12+\sqrt{3}}{12-\sqrt{3}}} \sqrt{\frac{12+\sqrt{3}}{12-\sqrt{3}}} \right)}{4912} + \frac{16\sqrt{3} \ln \left(\frac{12+\sqrt{3}}{12-\sqrt{3}} \sqrt{\frac{12+\sqrt{3}}{12-\sqrt{3}}} + \sqrt{\frac{12+\sqrt{3}}{12-\sqrt{3}}} \sqrt{\frac{12+\sqrt{3}}{12-\sqrt{3}}} \right)}{117649} + \frac{288\sqrt{12\sqrt{\frac{1}{3x+2}+4} + \frac{1}{3x+2}}}{117649} + \frac{\sqrt{12\sqrt{\frac{1}{3x+2}+4} + \frac{1}{3x+2}}}{288} + \frac{97\sqrt{97x+12}\sqrt{\frac{12+\sqrt{3}}{12-\sqrt{3}}}}{4517216} + \frac{(x+12)\sqrt{\frac{12+\sqrt{3}}{12-\sqrt{3}}}}{72\sqrt{\frac{1}{3x+2}+4}} + \frac{32\sqrt{12\sqrt{\frac{1}{3x+2}+4} + \frac{1}{3x+2}}}{2401\sqrt{\frac{1}{3x+2}+4}} + \frac{(24x+17)\sqrt{97x+12}\sqrt{\frac{12+\sqrt{3}}{12-\sqrt{3}}}}{135531648}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((12*x^2+17*x+6)^(1/2)/(2+3*x)^2/(-12*x^2+31*x+30)^2,x)

[Out] 384/117649*(-x+12*(x+3/4)^2-3/4)^(1/2)-16/117649*12^(1/2)*ln(1/12*(12*x+17/2)*12^(1/2)+(-x+12*(x+3/4)^2-3/4)^(1/2))+1/288*(x+12*(x+2/3)^2+2/3)^(1/2)+1/6912*12^(1/2)*ln(1/12*(12*x+17/2)*12^(1/2)+(x+12*(x+2/3)^2+2/3)^(1/2))-97/45177216*(97*x+12*(x-10/3)^2-382/3)^(1/2)-7057/813189888*12^(1/2)*ln(1/12*(12*x+17/2)*12^(1/2)+(97*x+12*(x-10/3)^2-382/3)^(1/2))+97/3226944*arctanh(1/28*(97*x+206/3)/(97*x+12*(x-10/3)^2-382/3)^(1/2))-1/72/(x+2/3)^2*(x+12*(x+2/3)^2+2/3)^(3/2)+32/2401/(x+3/4)^2*(-x+12*(x+3/4)^2-3/4)^(3/2)-1/67765824/(x-10/3)*(97*x+12*(x-10/3)^2-382/3)^(3/2)+1/135531648*(24*x+17)*(97*x+12*(x-10/3)^2-382/3)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{12x^2 + 17x + 6}}{(12x^2 - 31x - 30)^2(3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^2/(-12*x^2+31*x+30)^2,x, algorithm="maxima")

[Out] integrate(sqrt(12*x^2 + 17*x + 6)/((12*x^2 - 31*x - 30)^2*(3*x + 2)^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{12x^2 + 17x + 6}}{(3x + 2)^2(-12x^2 + 31x + 30)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)^2*(31*x - 12*x^2 + 30)^2), x)
```

```
[Out] int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)^2*(31*x - 12*x^2 + 30)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(3x+2)(4x+3)}}{(3x-10)^2(3x+2)^2(4x+3)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x**2+17*x+6)**(1/2)/(2+3*x)**2/(-12*x**2+31*x+30)**2, x)
```

```
[Out] Integral(sqrt((3*x + 2)*(4*x + 3))/((3*x - 10)**2*(3*x + 2)**2*(4*x + 3)**2), x)
```


$$3.137 \quad \int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^3(30+31x-12x^2)^3} dx$$

Optimal. Leaf size=139

$$\frac{388x + 275}{294(10 - 3x)^2(12x^2 + 17x + 6)^{3/2}} - \frac{1634466587\sqrt{12x^2 + 17x + 6}}{7589772288(10 - 3x)} - \frac{50555899\sqrt{12x^2 + 17x + 6}}{19361664(10 - 3x)^2} + \frac{1042556x + 738029}{8232(10 - 3x)^2\sqrt{12x^2 + 17x + 6}} + \frac{40325 \tanh^{-1}\left(\frac{291x+206}{84\sqrt{12x^2+17x+6}}\right)}{637540872192}$$

Rubi [A] time = 0.12, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {1002, 740, 822, 834, 806, 724, 206}

$$\frac{388x + 275}{294(10 - 3x)^2(12x^2 + 17x + 6)^{3/2}} - \frac{1634466587\sqrt{12x^2 + 17x + 6}}{7589772288(10 - 3x)} - \frac{50555899\sqrt{12x^2 + 17x + 6}}{19361664(10 - 3x)^2} + \frac{1042556x + 738029}{8232(10 - 3x)^2\sqrt{12x^2 + 17x + 6}} + \frac{40325 \tanh^{-1}\left(\frac{291x+206}{84\sqrt{12x^2+17x+6}}\right)}{637540872192}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^3*(30 + 31*x - 12*x^2)^3), x]

[Out] -(275 + 388*x)/(294*(10 - 3*x)^2*(6 + 17*x + 12*x^2)^(3/2)) + (738029 + 1042556*x)/(8232*(10 - 3*x)^2*Sqrt[6 + 17*x + 12*x^2]) - (50555899*Sqrt[6 + 17*x + 12*x^2])/(19361664*(10 - 3*x)^2) - (1634466587*Sqrt[6 + 17*x + 12*x^2])/(7589772288*(10 - 3*x)) + (40325*ArcTanh[(206 + 291*x)/(84*Sqrt[6 + 17*x + 12*x^2])])/637540872192

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 740

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]

Rule 806

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +

$2*p + 3], 0]$

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1002

```
Int[((g_) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(m_.), x_Symbol] := Int[((d*g)/a + (f*h*x)/c)^(m*(a + b*x + c*x^2)^(m + p)), x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x] && EqQ[c*g^2 - b*g*h + a*h^2, 0] && EqQ[c^2*d*g^2 - a*c*e*g*h + a^2*f*h^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^3(30+31x-12x^2)^3} dx &= \int \frac{1}{(10-3x)^3(6+17x+12x^2)^{5/2}} dx \\
&= -\frac{275+388x}{294(10-3x)^2(6+17x+12x^2)^{3/2}} - \frac{\int \frac{\frac{109953}{2}-41904x}{(10-3x)^3(6+17x+12x^2)^{3/2}} dx}{2646} \\
&= -\frac{275+388x}{294(10-3x)^2(6+17x+12x^2)^{3/2}} + \frac{738029+1042556x}{8232(10-3x)^2\sqrt{6+17x+12x^2}} \\
&= -\frac{275+388x}{294(10-3x)^2(6+17x+12x^2)^{3/2}} + \frac{738029+1042556x}{8232(10-3x)^2\sqrt{6+17x+12x^2}} \\
&= -\frac{275+388x}{294(10-3x)^2(6+17x+12x^2)^{3/2}} + \frac{738029+1042556x}{8232(10-3x)^2\sqrt{6+17x+12x^2}} \\
&= -\frac{275+388x}{294(10-3x)^2(6+17x+12x^2)^{3/2}} + \frac{738029+1042556x}{8232(10-3x)^2\sqrt{6+17x+12x^2}} \\
&= -\frac{275+388x}{294(10-3x)^2(6+17x+12x^2)^{3/2}} + \frac{738029+1042556x}{8232(10-3x)^2\sqrt{6+17x+12x^2}}
\end{aligned}$$

Mathematica [A] time = 0.37, size = 131, normalized size = 0.94

$$\frac{\sqrt{12x^2+17x+6} \left(40325(-36x^3+69x^2+152x+60)^2 \tanh^{-1}\left(\frac{\sqrt{3x+2}}{6\sqrt{4x+3}}\right) + 42\sqrt{3x+2}\sqrt{4x+3} (706089565584x^5 - 3206824169544x^4 - 1096520427663x^3 + 9848047480070x^2 + 10124325497244x + 2773753482408) \right)}{318770436096(10-3x)^2(3x+2)^{5/2}(4x+3)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^3*(30 + 31*x - 12*x^2)^3), x]

[Out] (Sqrt[6 + 17*x + 12*x^2]*(42*Sqrt[2 + 3*x]*Sqrt[3 + 4*x]*(2773753482408 + 10124325497244*x + 9848047480070*x^2 - 1096520427663*x^3 - 3206824169544*x^4 + 706089565584*x^5) + 40325*(60 + 152*x + 69*x^2 - 36*x^3)^2*ArcTanh[(7*Sqrt[2 + 3*x])/(6*Sqrt[3 + 4*x])]))/(318770436096*(10 - 3*x)^2*(2 + 3*x)^(5/2)*(3 + 4*x)^(5/2))

IntegrateAlgebraic [A] time = 0.39, size = 95, normalized size = 0.68

$$\frac{40325 \tanh^{-1}\left(\frac{6\sqrt{12x^2+17x+6}}{7(3x+2)}\right)}{318770436096} + \frac{\sqrt{12x^2+17x+6} (706089565584x^5 - 3206824169544x^4 - 1096520427663x^3 + 9848047480070x^2 + 10124325497244x + 2773753482408)}{7589772288(3x-10)^2(3x+2)^2(4x+3)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^3*(30 + 31*x - 12*x^2)^3), x]

[Out] (Sqrt[6 + 17*x + 12*x^2]*(2773753482408 + 10124325497244*x + 9848047480070*x^2 - 1096520427663*x^3 - 3206824169544*x^4 + 706089565584*x^5))/(7589772288*(-10 + 3*x)^2*(2 + 3*x)^2*(3 + 4*x)^2) + (40325*ArcTanh[(6*Sqrt[6 + 17*x + 12*x^2])/(7*(2 + 3*x))])/318770436096

fricas [A] time = 1.74, size = 186, normalized size = 1.34

$$\frac{40325(1296x^5 - 4968x^4 - 6183x^3 + 16656x^2 + 31384x + 18240) \log\left(\frac{2x+10\sqrt{12x^2+17x+6}+20}{7}\right) - 40325(1296x^5 - 4968x^4 - 6183x^3 + 16656x^2 + 31384x + 18240) \log\left(\frac{2x-10\sqrt{12x^2+17x+6}+20}{7}\right) + 168(706089565584x^5 - 3206824169544x^4 - 1096520427663x^3 + 9848047480070x^2 + 10124325497244x + 2773753482408)\sqrt{12x^2+17x+6}}{1275081744384(1296x^5 - 4968x^4 - 6183x^3 + 16656x^2 + 31384x + 18240)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^3/(-12*x^2+31*x+30)^3,x, algorithm="fricas")

[Out] 1/1275081744384*(40325*(1296*x^6 - 4968*x^5 - 6183*x^4 + 16656*x^3 + 31384*x^2 + 18240*x + 3600)*log((291*x + 84*sqrt(12*x^2 + 17*x + 6) + 206)/x) - 40325*(1296*x^6 - 4968*x^5 - 6183*x^4 + 16656*x^3 + 31384*x^2 + 18240*x + 3600)*log((291*x - 84*sqrt(12*x^2 + 17*x + 6) + 206)/x) + 168*(706089565584*x^5 - 3206824169544*x^4 - 1096520427663*x^3 + 9848047480070*x^2 + 10124325497244*x + 2773753482408)*sqrt(12*x^2 + 17*x + 6))/(1296*x^6 - 4968*x^5 - 6183*x^4 + 16656*x^3 + 31384*x^2 + 18240*x + 3600)

giac [A] time = 0.26, size = 232, normalized size = 1.67

$$\frac{\sqrt{5} (282273 \sqrt{3} (2\sqrt{5}x - \sqrt{12x^2 + 17x + 6})^3 - 11460924 (2\sqrt{5}x - \sqrt{12x^2 + 17x + 6})^2 - 37551180 \sqrt{3} (2\sqrt{5}x - \sqrt{12x^2 + 17x + 6}) - 83365264)}{159385218048 (3(2\sqrt{5}x - \sqrt{12x^2 + 17x + 6})^2 - 40\sqrt{3} (2\sqrt{5}x - \sqrt{12x^2 + 17x + 6}) - 188)} + \frac{8(2860316794x + 6078171227)x + 34383350229x + 8090114146}{2213683584 (12x^2 + 17x + 6)^2} + \frac{40325}{637540872192} \log\left(\frac{-6\sqrt{3}x + 20\sqrt{3} + 3\sqrt{12x^2 + 17x + 6} + 42}{-6\sqrt{3}x + 20\sqrt{3} + 3\sqrt{12x^2 + 17x + 6} - 42}\right) + \frac{40325}{637540872192} \log\left(\frac{-6\sqrt{3}x + 20\sqrt{3} + 3\sqrt{12x^2 + 17x + 6} - 42}{-6\sqrt{3}x + 20\sqrt{3} + 3\sqrt{12x^2 + 17x + 6} + 42}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^3/(-12*x^2+31*x+30)^3,x, algorithm="giac")

[Out] 1/159385218048*sqrt(3)*(282273*sqrt(3)*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x + 6))^3 - 11460924*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x + 6))^2 - 37551180*sqrt(3)*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x + 6)) - 83365264)/(3*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x + 6))^2 - 40*sqrt(3)*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x + 6)) - 188)^2 + 1/2213683584*((8*(2860316794*x + 6078171227)*x + 34383350229)*x + 8090114146)/(12*x^2 + 17*x + 6)^(3/2) + 40325/637540872192*log(abs(-6*sqrt(3)*x + 20*sqrt(3) + 3*sqrt(12*x^2 + 17*x + 6) + 42)) - 40325/637540872192*log(abs(-6*sqrt(3)*x + 20*sqrt(3) + 3*sqrt(12*x^2 + 17*x + 6) - 42))

maple [B] time = 0.02, size = 306, normalized size = 2.20

$$\frac{40325 \arctan\left(\frac{-6\sqrt{3}x + 20\sqrt{3} + 3\sqrt{12x^2 + 17x + 6} + 42}{-6\sqrt{3}x + 20\sqrt{3} + 3\sqrt{12x^2 + 17x + 6} - 42}\right) + 40325 \arctan\left(\frac{-6\sqrt{3}x + 20\sqrt{3} + 3\sqrt{12x^2 + 17x + 6} - 42}{-6\sqrt{3}x + 20\sqrt{3} + 3\sqrt{12x^2 + 17x + 6} + 42}\right) + \frac{1}{2213683584} \left((8(2860316794x + 6078171227)x + 34383350229)x + 8090114146 \right) (12x^2 + 17x + 6)^{-3/2} + \frac{1}{159385218048} \sqrt{3} \left((282273 \sqrt{3} (2\sqrt{3}x - \sqrt{12x^2 + 17x + 6})^3 - 11460924 (2\sqrt{3}x - \sqrt{12x^2 + 17x + 6})^2 - 37551180 \sqrt{3} (2\sqrt{3}x - \sqrt{12x^2 + 17x + 6}) - 83365264) \right) (3(2\sqrt{3}x - \sqrt{12x^2 + 17x + 6})^2 - 40\sqrt{3} (2\sqrt{3}x - \sqrt{12x^2 + 17x + 6}) - 188)^{-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((12*x^2+17*x+6)^(1/2)/(2+3*x)^3/(-12*x^2+31*x+30)^3,x)

[Out] -1/2592/(x+2/3)^3*(x+12*(x+2/3)^2+2/3)^(3/2)-1261/31239502737408/(x-10/3)*(97*x+12*(x-10/3)^2-382/3)^(3/2)+1261/62479005474816*(24*x+17)*(97*x+12*(x-10/3)^2-382/3)^(1/2)+1/79692609024/(x-10/3)^2*(97*x+12*(x-10/3)^2-382/3)^(3/2)-230400/5764801/(x+3/4)^2*(-x+12*(x+3/4)^2-3/4)^(3/2)-128/352947/(x+3/4)^3*(-x+12*(x+3/4)^2-3/4)^(3/2)-570457/31239502737408*12^(1/2)*ln(1/12*(12*x+17/2)*12^(1/2)+(97*x+12*(x-10/3)^2-382/3)^(1/2))+47/1152/(x+2/3)^2*(x+12*(x+2/3)^2+2/3)^(3/2)-23/110592*12^(1/2)*ln(1/12*(12*x+17/2)*12^(1/2)+(x+12*(x+2/3)^2+2/3)^(1/2))+58752/282475249*12^(1/2)*ln(1/12*(12*x+17/2)*12^(1/2)+(-x+12*(x+3/4)^2-3/4)^(1/2))-40325/8925572210688*(97*x+12*(x-10/3)^2-382/3)^(1/2)+40325/637540872192*arctanh(1/28*(97*x+206/3)/(97*x+12*(x-10/3)^2-382/3)^(1/2))-23/4608*(x+12*(x+2/3)^2+2/3)^(1/2)-1410048/282475249*(-x+12*(x+3/4)^2-3/4)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{12x^2 + 17x + 6}}{(12x^2 - 31x - 30)^3 (3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^3/(-12*x^2+31*x+30)^3,x, algorithm="maxima")

[Out] -integrate(sqrt(12*x^2 + 17*x + 6)/((12*x^2 - 31*x - 30)^3*(3*x + 2)^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{12x^2 + 17x + 6}}{(3x + 2)^3 (-12x^2 + 31x + 30)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)^3*(31*x - 12*x^2 + 30)^3), x)

[Out] int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)^3*(31*x - 12*x^2 + 30)^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{12x^2 + 17x + 6}}{46656x^9 - 268272x^8 - 76788x^7 + 1703619x^6 + 1218456x^5 - 3669588x^4 - 6898688x^3 - 4903920x^2 - 1641600x - 216000} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((12*x**2+17*x+6)**(1/2)/(2+3*x)**3/(-12*x**2+31*x+30)**3, x)

[Out] -Integral(sqrt(12*x**2 + 17*x + 6)/(46656*x**9 - 268272*x**8 - 76788*x**7 + 1703619*x**6 + 1218456*x**5 - 3669588*x**4 - 6898688*x**3 - 4903920*x**2 - 1641600*x - 216000), x)

$$3.138 \quad \int (-3 + 2x) (-3x + x^2)^{2/3} dx$$

Optimal. Leaf size=15

$$\frac{3}{5} (x^2 - 3x)^{5/3}$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {629}

$$\frac{3}{5} (x^2 - 3x)^{5/3}$$

Antiderivative was successfully verified.

[In] Int[(-3 + 2*x)*(-3*x + x^2)^(2/3), x]

[Out] (3*(-3*x + x^2)^(5/3))/5

Rule 629

Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rubi steps

$$\int (-3 + 2x) (-3x + x^2)^{2/3} dx = \frac{3}{5} (-3x + x^2)^{5/3}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 0.87

$$\frac{3}{5} ((x - 3)x)^{5/3}$$

Antiderivative was successfully verified.

[In] Integrate[(-3 + 2*x)*(-3*x + x^2)^(2/3), x]

[Out] (3*((-3 + x)*x)^(5/3))/5

IntegrateAlgebraic [A] time = 0.02, size = 13, normalized size = 0.87

$$\frac{3}{5} ((x - 3)x)^{5/3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(-3 + 2*x)*(-3*x + x^2)^(2/3), x]

[Out] (3*((-3 + x)*x)^(5/3))/5

fricas [A] time = 0.60, size = 11, normalized size = 0.73

$$\frac{3}{5} (x^2 - 3x)^{5/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3+2*x)*(x^2-3*x)^(2/3), x, algorithm="fricas")

[Out] $3/5*(x^2 - 3*x)^{(5/3)}$

giac [A] time = 0.15, size = 11, normalized size = 0.73

$$\frac{3}{5}(x^2 - 3x)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3+2*x)*(x^2-3*x)^(2/3),x, algorithm="giac")

[Out] $3/5*(x^2 - 3*x)^{(5/3)}$

maple [A] time = 0.00, size = 16, normalized size = 1.07

$$\frac{3(x-3)(x^2-3x)^{\frac{2}{3}}x}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-3+2*x)*(x^2-3*x)^(2/3),x)

[Out] $3/5*(x-3)*x*(x^2-3*x)^{(2/3)}$

maxima [A] time = 0.43, size = 11, normalized size = 0.73

$$\frac{3}{5}(x^2 - 3x)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3+2*x)*(x^2-3*x)^(2/3),x, algorithm="maxima")

[Out] $3/5*(x^2 - 3*x)^{(5/3)}$

mupad [B] time = 3.72, size = 15, normalized size = 1.00

$$\frac{3x(x^2-3x)^{\frac{2}{3}}(x-3)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x - 3)*(x^2 - 3*x)^(2/3),x)

[Out] $(3*x*(x^2 - 3*x)^{(2/3)}*(x - 3))/5$

sympy [B] time = 0.31, size = 31, normalized size = 2.07

$$\frac{3x^2(x^2-3x)^{\frac{2}{3}}}{5} - \frac{9x(x^2-3x)^{\frac{2}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3+2*x)*(x**2-3*x)**(2/3),x)

[Out] $3*x**2*(x**2 - 3*x)**(2/3)/5 - 9*x*(x**2 - 3*x)**(2/3)/5$

$$3.139 \quad \int((-3+x)x)^{2/3}(-3+2x) dx$$

Optimal. Leaf size=16

$$\frac{3}{5}(-((3-x)x))^{5/3}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1588}

$$\frac{3}{5}(-3-x)x^{5/3}$$

Antiderivative was successfully verified.

[In] Int[((-3 + x)*x)^(2/3)*(-3 + 2*x), x]

[Out] (3*(-((3 - x)*x))^(5/3))/5

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int((-3+x)x)^{2/3}(-3+2x) dx = \frac{3}{5}(-((3-x)x))^{5/3}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{3}{5}((x-3)x)^{5/3}$$

Antiderivative was successfully verified.

[In] Integrate[((-3 + x)*x)^(2/3)*(-3 + 2*x), x]

[Out] (3*(-(-3 + x)*x)^(5/3))/5

IntegrateAlgebraic [A] time = 0.01, size = 13, normalized size = 0.81

$$\frac{3}{5}((x-3)x)^{5/3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((-3 + x)*x)^(2/3)*(-3 + 2*x), x]

[Out] (3*(-(-3 + x)*x)^(5/3))/5

fricas [A] time = 0.67, size = 11, normalized size = 0.69

$$\frac{3}{5}(x^2 - 3x)^{5/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((−3+x)*x)^(2/3)*(−3+2*x),x, algorithm="fricas")

[Out] 3/5*(x^2 - 3*x)^(5/3)

giac [A] time = 0.15, size = 11, normalized size = 0.69

$$\frac{3}{5} (x^2 - 3x)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((−3+x)*x)^(2/3)*(−3+2*x),x, algorithm="giac")

[Out] 3/5*(x^2 - 3*x)^(5/3)

maple [A] time = 0.00, size = 14, normalized size = 0.88

$$\frac{3(x-3)((x-3)x)^{\frac{2}{3}}x}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x-3)*x)^(2/3)*(−3+2*x),x)

[Out] 3/5*(x-3)*x*((x-3)*x)^(2/3)

maxima [A] time = 0.43, size = 9, normalized size = 0.56

$$\frac{3}{5} ((x-3)x)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((−3+x)*x)^(2/3)*(−3+2*x),x, algorithm="maxima")

[Out] 3/5*((x - 3)*x)^(5/3)

mupad [B] time = 3.66, size = 13, normalized size = 0.81

$$\frac{3x(x(x-3))^{2/3}(x-3)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x - 3)*(x*(x - 3))^(2/3),x)

[Out] (3*x*(x*(x - 3))^(2/3)*(x - 3))/5

sympy [A] time = 4.25, size = 10, normalized size = 0.62

$$\frac{3(x(x-3))^{\frac{5}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((−3+x)*x)**(2/3)*(−3+2*x),x)

[Out] 3*(x*(x - 3))**(5/3)/5

$$3.140 \quad \int \frac{x(9-9x+2x^2)}{\sqrt[3]{-3x+x^2}} dx$$

Optimal. Leaf size=15

$$\frac{3}{5}(x^2 - 3x)^{5/3}$$

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1631, 629}

$$\frac{3}{5}(x^2 - 3x)^{5/3}$$

Antiderivative was successfully verified.

[In] Int[(x*(9 - 9*x + 2*x^2))/(-3*x + x^2)^(1/3), x]

[Out] (3*(-3*x + x^2)^(5/3))/5

Rule 629

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1631

Int[(Pq_)*((e_)*(x_)^(m_))*((b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Dist[e, Int[(e*x)^(m - 1)*PolynomialQuotient[Pq, b + c*x, x]*(b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{b, c, e, m, p}, x] && PolyQ[Pq, x] && EqQ[PolynomialRemainder[Pq, b + c*x, x], 0]

Rubi steps

$$\begin{aligned} \int \frac{x(9-9x+2x^2)}{\sqrt[3]{-3x+x^2}} dx &= \int (-3+2x)(-3x+x^2)^{2/3} dx \\ &= \frac{3}{5}(-3x+x^2)^{5/3} \end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 0.87

$$\frac{3}{5}((x - 3)x)^{5/3}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(9 - 9*x + 2*x^2))/(-3*x + x^2)^(1/3), x]

[Out] (3*((-3 + x)*x)^(5/3))/5

IntegrateAlgebraic [A] time = 0.02, size = 13, normalized size = 0.87

$$\frac{3}{5}((x - 3)x)^{5/3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x*(9 - 9*x + 2*x^2))/(-3*x + x^2)^(1/3), x]

[Out] $(3*((-3 + x)*x)^{(5/3)})/5$

fricas [A] time = 0.99, size = 11, normalized size = 0.73

$$\frac{3}{5} (x^2 - 3x)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(2*x^2-9*x+9)/(x^2-3*x)^(1/3),x, algorithm="fricas")`

[Out] $3/5*(x^2 - 3*x)^{(5/3)}$

giac [A] time = 0.16, size = 11, normalized size = 0.73

$$\frac{3}{5} (x^2 - 3x)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(2*x^2-9*x+9)/(x^2-3*x)^(1/3),x, algorithm="giac")`

[Out] $3/5*(x^2 - 3*x)^{(5/3)}$

maple [A] time = 0.00, size = 20, normalized size = 1.33

$$\frac{3(x-3)^2 x^2}{5(x^2-3x)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(2*x^2-9*x+9)/(x^2-3*x)^(1/3),x)`

[Out] $3/5*(x-3)^2*x^2/(x^2-3*x)^{(1/3)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - 9x + 9)x}{(x^2 - 3x)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(2*x^2-9*x+9)/(x^2-3*x)^(1/3),x, algorithm="maxima")`

[Out] `integrate((2*x^2 - 9*x + 9)*x/(x^2 - 3*x)^(1/3), x)`

mupad [B] time = 3.68, size = 15, normalized size = 1.00

$$\frac{3x(x^2 - 3x)^{\frac{2}{3}}(x - 3)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*(2*x^2 - 9*x + 9))/(x^2 - 3*x)^(1/3),x)`

[Out] $(3*x*(x^2 - 3*x)^{(2/3)}*(x - 3))/5$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(x-3)(2x-3)}{\sqrt[3]{x(x-3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(2*x**2-9*x+9)/(x**2-3*x)**(1/3),x)`

[Out] `Integral(x*(x - 3)*(2*x - 3)/(x*(x - 3))**(1/3), x)`

$$3.141 \quad \int \frac{x(9-9x+2x^2)}{\sqrt[3]{(-3+x)x}} dx$$

Optimal. Leaf size=15

$$\frac{3}{5}(x^2 - 3x)^{5/3}$$

Rubi [A] time = 0.06, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1985, 1631, 629}

$$\frac{3}{5}(x^2 - 3x)^{5/3}$$

Antiderivative was successfully verified.

[In] Int[(x*(9 - 9*x + 2*x^2))/((-3 + x)*x)^(1/3),x]

[Out] (3*(-3*x + x^2)^(5/3))/5

Rule 629

Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1631

Int[(Pq_)*((e_.)*(x_)^(m_.))*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[e, Int[(e*x)^(m - 1)*PolynomialQuotient[Pq, b + c*x, x]*(b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{b, c, e, m, p}, x] && PolyQ[Pq, x] && EqQ[PolynomialRemainder[Pq, b + c*x, x], 0]

Rule 1985

Int[(u_)^(p_.)*(v_)^(q_.)*(z_)^(m_.), x_Symbol] :> Int[ExpandToSum[z, x]^m*ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{m, p, q}, x] && LinearQ[z, x] && QuadraticQ[{u, v}, x] && !(LinearMatchQ[z, x] && QuadraticMatchQ[{u, v}, x])

Rubi steps

$$\begin{aligned} \int \frac{x(9-9x+2x^2)}{\sqrt[3]{(-3+x)x}} dx &= \int \frac{x(9-9x+2x^2)}{\sqrt[3]{-3x+x^2}} dx \\ &= \int (-3+2x)(-3x+x^2)^{2/3} dx \\ &= \frac{3}{5}(-3x+x^2)^{5/3} \end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 0.87

$$\frac{3}{5}((x-3)x)^{5/3}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(9 - 9*x + 2*x^2))/((-3 + x)*x)^(1/3),x]

[Out] $(3*((-3 + x)*x)^{(5/3)})/5$

IntegrateAlgebraic [A] time = 0.01, size = 13, normalized size = 0.87

$$\frac{3}{5}((x-3)x)^{5/3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x*(9 - 9*x + 2*x^2))/((-3 + x)*x)^(1/3),x]

[Out] $(3*((-3 + x)*x)^{(5/3)})/5$

fricas [A] time = 1.02, size = 11, normalized size = 0.73

$$\frac{3}{5}(x^2 - 3x)^{5/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*x^2-9*x+9)/((-3+x)*x)^(1/3),x, algorithm="fricas")

[Out] $3/5*(x^2 - 3*x)^{(5/3)}$

giac [A] time = 0.20, size = 11, normalized size = 0.73

$$\frac{3}{5}(x^2 - 3x)^{5/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*x^2-9*x+9)/((-3+x)*x)^(1/3),x, algorithm="giac")

[Out] $3/5*(x^2 - 3*x)^{(5/3)}$

maple [A] time = 0.00, size = 18, normalized size = 1.20

$$\frac{3(x-3)^2 x^2}{5((x-3)x)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(2*x^2-9*x+9)/((x-3)*x)^(1/3),x)

[Out] $3/5*(x-3)^2*x^2/((x-3)*x)^{(1/3)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - 9x + 9)x}{((x-3)x)^{1/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*x^2-9*x+9)/((-3+x)*x)^(1/3),x, algorithm="maxima")

[Out] integrate((2*x^2 - 9*x + 9)*x/((x - 3)*x)^(1/3), x)

mupad [B] time = 3.61, size = 13, normalized size = 0.87

$$\frac{3x(x(x-3))^{2/3}(x-3)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(2*x^2 - 9*x + 9))/(x*(x - 3))^(1/3), x)
```

```
[Out] (3*x*(x*(x - 3))^(2/3)*(x - 3))/5
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(x-3)(2x-3)}{\sqrt[3]{x(x-3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x**2-9*x+9)/((-3+x)*x)**(1/3), x)
```

```
[Out] Integral(x*(x - 3)*(2*x - 3)/(x*(x - 3))**(1/3), x)
```

$$3.142 \quad \int \frac{g+hx}{\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2} (g^2+3h^2x^2)} dx$$

Optimal. Leaf size=242

$$\frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}} \log(g^2+3h^2x^2)}{6 \cdot 2^{2/3} h \sqrt[3]{9cx^2-\frac{cg^2}{h^2}}} - \frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}} \log\left(\left(1-\frac{3hx}{g}\right)^{2/3} + \sqrt[3]{2} \sqrt[3]{\frac{3hx}{g}+1}\right)}{2 \cdot 2^{2/3} h \sqrt[3]{9cx^2-\frac{cg^2}{h^2}}} + \frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}} \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2^{2/3}\left(1-\frac{3hx}{g}\right)^{2/3}}{\sqrt{3} \sqrt[3]{\frac{3hx}{g}+1}}\right)}{2^{2/3} \sqrt{3} h \sqrt[3]{9cx^2-\frac{cg^2}{h^2}}}$$

Rubi [A] time = 0.09, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1009, 1008}

$$\frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}} \log(g^2+3h^2x^2)}{6 \cdot 2^{2/3} h \sqrt[3]{9cx^2-\frac{cg^2}{h^2}}} - \frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}} \log\left(\left(1-\frac{3hx}{g}\right)^{2/3} + \sqrt[3]{2} \sqrt[3]{\frac{3hx}{g}+1}\right)}{2 \cdot 2^{2/3} h \sqrt[3]{9cx^2-\frac{cg^2}{h^2}}} + \frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}} \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2^{2/3}\left(1-\frac{3hx}{g}\right)^{2/3}}{\sqrt{3} \sqrt[3]{\frac{3hx}{g}+1}}\right)}{2^{2/3} \sqrt{3} h \sqrt[3]{9cx^2-\frac{cg^2}{h^2}}}$$

Antiderivative was successfully verified.

[In] Int[(g + h*x)/((-((c*g^2)/h^2) + 9*c*x^2)^(1/3)*(g^2 + 3*h^2*x^2)), x]

[Out] ((1 - (9*h^2*x^2)/g^2)^(1/3)*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - (3*h*x)/g)^(2/3))/(Sqrt[3]*(1 + (3*h*x)/g)^(1/3))]/(2^(2/3)*Sqrt[3]*h*(-((c*g^2)/h^2) + 9*c*x^2)^(1/3)) + ((1 - (9*h^2*x^2)/g^2)^(1/3)*Log[g^2 + 3*h^2*x^2])/(6*2^(2/3)*h*(-((c*g^2)/h^2) + 9*c*x^2)^(1/3)) - ((1 - (9*h^2*x^2)/g^2)^(1/3)*Log[(1 - (3*h*x)/g)^(2/3) + 2^(1/3)*(1 + (3*h*x)/g)^(1/3)])/(2*2^(2/3)*h*(-((c*g^2)/h^2) + 9*c*x^2)^(1/3))

Rule 1008

Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)^(1/3)*((d_) + (f_.)*(x_)^2)), x_Symbol] :> Simp[(Sqrt[3]*h*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - (3*h*x)/g)^(2/3))/(Sqrt[3]*(1 + (3*h*x)/g)^(1/3))]/(2^(2/3)*a^(1/3)*f), x] + (-Simp[(3*h*Log[(1 - (3*h*x)/g)^(2/3) + 2^(1/3)*(1 + (3*h*x)/g)^(1/3)])/(2^(5/3)*a^(1/3)*f), x] + Simp[(h*Log[d + f*x^2])/(2^(5/3)*a^(1/3)*f), x]) /; FreeQ[{a, c, d, f, g, h}, x] && EqQ[c*d + 3*a*f, 0] && EqQ[c*g^2 + 9*a*h^2, 0] && GtQ[a, 0]

Rule 1009

Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)^(1/3)*((d_) + (f_.)*(x_)^2)), x_Symbol] :> Dist[(1 + (c*x^2)/a)^(1/3)/(a + c*x^2)^(1/3), Int[(g + h*x)/((1 + (c*x^2)/a)^(1/3)*(d + f*x^2)), x], x] /; FreeQ[{a, c, d, f, g, h}, x] && EqQ[c*d + 3*a*f, 0] && EqQ[c*g^2 + 9*a*h^2, 0] && !GtQ[a, 0]

Rubi steps

$$\int \frac{g + hx}{\sqrt[3]{-\frac{cg^2}{h^2} + 9cx^2} (g^2 + 3h^2x^2)} dx = \frac{\sqrt[3]{1 - \frac{9h^2x^2}{g^2}} \int \frac{g+hx}{(g^2+3h^2x^2)\sqrt[3]{1-\frac{9h^2x^2}{g^2}}} dx}{\sqrt[3]{-\frac{cg^2}{h^2} + 9cx^2}}$$

$$= \frac{\sqrt[3]{1 - \frac{9h^2x^2}{g^2}} \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2^{2/3}\left(1 - \frac{3hx}{g}\right)^{2/3}}{\sqrt{3}\sqrt[3]{1 + \frac{3hx}{g}}}\right)}{2^{2/3}\sqrt{3}h\sqrt[3]{-\frac{cg^2}{h^2} + 9cx^2}} + \frac{\sqrt[3]{1 - \frac{9h^2x^2}{g^2}} \log(g^2 + 3h^2x^2)}{6 \cdot 2^{2/3}h\sqrt[3]{-\frac{cg^2}{h^2} + 9cx^2}} - \frac{\sqrt[3]{-\frac{cg^2}{h^2} + 9cx^2}}{6 \cdot 2^{2/3}h\sqrt[3]{-\frac{cg^2}{h^2} + 9cx^2}}$$

Mathematica [C] time = 0.56, size = 268, normalized size = 1.11

$$\frac{h^2x \left(-hx \sqrt[3]{1 - \frac{9h^2x^2}{g^2}} F_1\left(1; \frac{1}{3}, 1; 2; \frac{9h^2x^2}{g^2}, -\frac{3h^2x^2}{g^2}\right) - \frac{2g^5 F_1\left(\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, \frac{9h^2x^2}{g^2}, -\frac{3h^2x^2}{g^2}\right)}{(g^2+3h^2x^2) \left(g^2 F_1\left(\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, \frac{9h^2x^2}{g^2}, -\frac{3h^2x^2}{g^2}\right) + 2h^2x^2 \left(F_1\left(\frac{3}{2}, \frac{4}{3}, 1, \frac{5}{2}, \frac{9h^2x^2}{g^2}, -\frac{3h^2x^2}{g^2}\right) - F_1\left(\frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, \frac{9h^2x^2}{g^2}, -\frac{3h^2x^2}{g^2}\right) \right) \right)}{2cg^2(g^2 - 9h^2x^2)} \right) \left(c \left(9x^2 - \frac{g^2}{h^2} \right) \right)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(g + h*x)/((-((c*g^2)/h^2) + 9*c*x^2)^(1/3)*(g^2 + 3*h^2*x^2)), x]

[Out] (h^2*x*(c*(-(g^2/h^2) + 9*x^2))^(2/3)*(-(h*x*(1 - (9*h^2*x^2)/g^2)^(1/3)*AppellF1[1, 1/3, 1, 2, (9*h^2*x^2)/g^2, (-3*h^2*x^2)/g^2]) - (2*g^5*AppellF1[1/2, 1/3, 1, 3/2, (9*h^2*x^2)/g^2, (-3*h^2*x^2)/g^2]))/((g^2 + 3*h^2*x^2)*(g^2*AppellF1[1/2, 1/3, 1, 3/2, (9*h^2*x^2)/g^2, (-3*h^2*x^2)/g^2] + 2*h^2*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (9*h^2*x^2)/g^2, (-3*h^2*x^2)/g^2] + AppellF1[3/2, 4/3, 1, 5/2, (9*h^2*x^2)/g^2, (-3*h^2*x^2)/g^2]))))/(2*c*g^2*(g^2 - 9*h^2*x^2))

IntegrateAlgebraic [B] time = 1.08, size = 499, normalized size = 2.06

$$\frac{\log\left(\sqrt[3]{2c^2g^2 - 6\sqrt[3]{2c^2g}hx + 9\sqrt[3]{2c^2g^2}h^2x^2 - 3 \cdot 2^{2/3}\sqrt[3]{c}\sqrt[3]{h^3x^3}\sqrt[3]{9cx^2 - \frac{cg^2}{h^2}} + 2^{2/3}\sqrt[3]{c}g^{4/3}h^{2/3}\sqrt[3]{9cx^2 - \frac{cg^2}{h^2}} + 2c^{2/3}h^{4/3}\left(9cx^2 - \frac{cg^2}{h^2}\right)^{2/3}\right)}{6 \cdot 2^{2/3}\sqrt[3]{c}g^{2/3}\sqrt[3]{h}} - \frac{\log\left(\sqrt[3]{9cx^2 - \frac{cg^2}{h^2}}\right)}{3 \cdot 2^{2/3}\sqrt[3]{c}g^{2/3}\sqrt[3]{h}} + \frac{\log\left(g^{2/3}\left(9cx^2 - \frac{cg^2}{h^2}\right)^{2/3}\right)}{6 \cdot 2^{2/3}\sqrt[3]{c}g^{2/3}\sqrt[3]{h}} + \frac{\log\left(-2\sqrt[3]{g}h^{2/3}\sqrt[3]{9cx^2 - \frac{cg^2}{h^2}} + 2^{2/3}\sqrt[3]{c}g - 3 \cdot 2^{2/3}\sqrt[3]{c}hx\right)}{3 \cdot 2^{2/3}\sqrt[3]{c}g^{2/3}\sqrt[3]{h}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}\sqrt[3]{g^{2/3}}\sqrt[3]{9cx^2 - \frac{cg^2}{h^2}}}{\sqrt[3]{9cx^2 - \frac{cg^2}{h^2}} - \frac{2^{2/3}\sqrt[3]{c}g - 3 \cdot 2^{2/3}\sqrt[3]{c}hx}{2^{2/3}\sqrt[3]{c}g^{2/3}\sqrt[3]{h}}}\right)}{2^{2/3}\sqrt[3]{c}g^{2/3}\sqrt[3]{h}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(g + h*x)/((-((c*g^2)/h^2) + 9*c*x^2)^(1/3)*(g^2 + 3*h^2*x^2)), x]

[Out] ArcTan[(Sqrt[3]*g^(1/3)*h^(2/3)*(-(c*g^2)/h^2) + 9*c*x^2)^(1/3)]/(2^(2/3)*c^(1/3)*g - 3*2^(2/3)*c^(1/3)*h*x + g^(1/3)*h^(2/3)*(-(c*g^2)/h^2) + 9*c*x^2)^(1/3)]/(2^(2/3)*Sqrt[3]*c^(1/3)*g^(2/3)*h^(1/3)) - Log[g^(1/3)*(-(c*g^2)/h^2) + 9*c*x^2)^(1/3)]/(3*2^(2/3)*c^(1/3)*g^(2/3)*h^(1/3)) + Log[g^(2/3)*(-(c*g^2)/h^2) + 9*c*x^2)^(2/3)]/(6*2^(2/3)*c^(1/3)*g^(2/3)*h^(1/3)) + Log[2^(2/3)*c^(1/3)*g - 3*2^(2/3)*c^(1/3)*h*x - 2*g^(1/3)*h^(2/3)*(-(c*g^2)/h^2) + 9*c*x^2)^(1/3)]/(3*2^(2/3)*c^(1/3)*g^(2/3)*h^(1/3)) - Log[2^(1/3)*c^(2/3)*g^2 - 6*2^(1/3)*c^(2/3)*g*h*x + 9*2^(1/3)*c^(2/3)*h^2*x^2 + 2^(2/3)*c^(1/3)*g^(4/3)*h^(2/3)*(-(c*g^2)/h^2) + 9*c*x^2)^(1/3) - 3*2^(2/3)*c^(1/3)*g^(1/3)*h^(5/3)*x*(-(c*g^2)/h^2) + 9*c*x^2)^(1/3) + 2*g^(2/3)*h^(4/3)*(-(c*g^2)/h^2) + 9*c*x^2)^(2/3)]/(6*2^(2/3)*c^(1/3)*g^(2/3)*h^(1/3))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2), x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{hx + g}{(3h^2x^2 + g^2)\left(9cx^2 - \frac{cg^2}{h^2}\right)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x, algorithm="giac")

[Out] integrate((h*x + g)/((3*h^2*x^2 + g^2)*(9*c*x^2 - c*g^2/h^2)^(1/3)), x)

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{hx + g}{\left(9cx^2 - \frac{cg^2}{h^2}\right)^{\frac{1}{3}} (3h^2x^2 + g^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x)

[Out] int((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{hx + g}{(3h^2x^2 + g^2)\left(9cx^2 - \frac{cg^2}{h^2}\right)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x, algorithm="maxima")

[Out] integrate((h*x + g)/((3*h^2*x^2 + g^2)*(9*c*x^2 - c*g^2/h^2)^(1/3)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{g + hx}{(g^2 + 3h^2x^2)\left(9cx^2 - \frac{cg^2}{h^2}\right)^{1/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g + h*x)/((g^2 + 3*h^2*x^2)*(9*c*x^2 - (c*g^2)/h^2)^(1/3)),x)

[Out] int((g + h*x)/((g^2 + 3*h^2*x^2)*(9*c*x^2 - (c*g^2)/h^2)^(1/3)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{g + hx}{\sqrt[3]{c\left(-\frac{g}{h} + 3x\right)\left(\frac{g}{h} + 3x\right)} (g^2 + 3h^2x^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)/(-c*g**2/h**2+9*c*x**2)**(1/3)/(3*h**2*x**2+g**2),x)

[Out] Integral((g + h*x)/((c*(-g/h + 3*x)*(g/h + 3*x))**(1/3)*(g**2 + 3*h**2*x**2)), x)

$$3.143 \quad \int \frac{g+hx}{\sqrt[3]{\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2} \left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2} + \frac{bfx}{c} + fx^2 \right)} dx$$

Optimal. Leaf size=488

$$\frac{3^{2/3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}} \log\left(\frac{f\left(b^2h^2-bcgh+c^2g^2\right)}{3c^2h^2} + \frac{bfx}{c} + fx^2\right) - 3^{2/3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}} \log\left(\left(1 - \frac{3f}{c}\right)\right)}{2f\sqrt[3]{-\frac{(cg-2bh)(bh+cg)}{ch^2} + 9bx + 9cx^2}} - \frac{3^{2/3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}} \log\left(\left(1 - \frac{3f}{c}\right)\right)}{2f\sqrt[3]{-\frac{(cg-2bh)(bh+cg)}{ch^2} + 9bx + 9cx^2}}$$

Rubi [A] time = 0.36, antiderivative size = 488, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 104, number of rules / integrand size = 0.019, Rules used = {1041, 1040}

$$\frac{3^{2/3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}} \log\left(\frac{f\left(b^2h^2-bcgh+c^2g^2\right)}{3c^2h^2} + \frac{bfx}{c} + fx^2\right) - 3^{2/3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}} \log\left(\left(1 - \frac{3f}{c}\right)\right)}{2f\sqrt[3]{-\frac{(cg-2bh)(bh+cg)}{ch^2} + 9bx + 9cx^2}} + \frac{3^{2/3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}} \log\left(\left(1 - \frac{3f}{c}\right)\right)}{2f\sqrt[3]{-\frac{(cg-2bh)(bh+cg)}{ch^2} + 9bx + 9cx^2}} + \frac{3\sqrt[3]{h}\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}} \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2^{2/3}\left(1 - \frac{3f}{c}\right)^{2/3}}{\sqrt{3}\sqrt[3]{\frac{3h(b+2cx)}{2cg-bh} + 1}}\right)}{f\sqrt[3]{-\frac{(cg-2bh)(bh+cg)}{ch^2} + 9bx + 9cx^2}}$$

Antiderivative was successfully verified.

[In] Int[(g + h*x)/(((c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(9*c*h^2) + b*x + c*x^2)^(1/3)*((f*(b^2 - (c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(3*h^2)))/c^2 + (b*f*x)/c + f*x^2), x]

[Out] (3*3^(1/6)*h*((c*h^2*((c*g - 2*b*h)*(c*g + b*h))/(c*h^2) - 9*b*x - 9*c*x^2))/(2*c*g - b*h)^2)^(1/3)*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - (3*h*(b + 2*c*x)))/(2*c*g - b*h))^(2/3)]/(Sqrt[3]*(1 + (3*h*(b + 2*c*x))/(2*c*g - b*h))^(1/3))]/(f*(-((c*g - 2*b*h)*(c*g + b*h))/(c*h^2)) + 9*b*x + 9*c*x^2)^(1/3) + (3^(2/3)*h*((c*h^2*((c*g - 2*b*h)*(c*g + b*h))/(c*h^2) - 9*b*x - 9*c*x^2))/(2*c*g - b*h)^2)^(1/3)*Log[(f*(c^2*g^2 - b*c*g*h + b^2*h^2))/(3*c^2*h^2) + (b*f*x)/c + f*x^2]/(2*f*(-((c*g - 2*b*h)*(c*g + b*h))/(c*h^2)) + 9*b*x + 9*c*x^2)^(1/3) - (3*3^(2/3)*h*((c*h^2*((c*g - 2*b*h)*(c*g + b*h))/(c*h^2) - 9*b*x - 9*c*x^2))/(2*c*g - b*h)^2)^(1/3)*Log[(1 - (3*h*(b + 2*c*x))/(2*c*g - b*h))^(2/3) + 2^(1/3)*(1 + (3*h*(b + 2*c*x))/(2*c*g - b*h))^(1/3)]/(2*f*(-((c*g - 2*b*h)*(c*g + b*h))/(c*h^2)) + 9*b*x + 9*c*x^2)^(1/3)

Rule 1040

Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] := With[{q = ((-9*c*h^2)/(2*c*g - b*h)^2)^(1/3)}, Simp[(Sqrt[3]*h*q*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - (3*h*(b + 2*c*x))/(2*c*g - b*h))^(2/3)]/(Sqrt[3]*(1 + (3*h*(b + 2*c*x))/(2*c*g - b*h))^(1/3)))]/f, x] + (-Simp[(3*h*q*Log[(1 - (3*h*(b + 2*c*x))/(2*c*g - b*h))^(2/3) + 2^(1/3)*(1 + (3*h*(b + 2*c*x))/(2*c*g - b*h))^(1/3)]/(2*f), x] + Simp[(h*q*Log[d + e*x + f*x^2])/(2*f), x])]/; FreeQ[{a, b, c, d, e, f, g, h}, x] && EqQ[c*e - b*f, 0] && EqQ[c^2*d - f*(b^2 - 3*a*c), 0] && EqQ[c^2*g^2 - b*c*g*h - 2*b^2*h^2 + 9*a*c*h^2, 0] && GtQ[(-9*c*h^2)/(2*c*g - b*h)^2, 0]

Rule 1041

Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] := With[{q = -(c/(b^2 - 4*a*c))}, Dist[(q*(a + b*x + c*x^2)^(1/3)/(a + b*x + c*x^2)^(1/3), Int[(g + h*x)/((q*a + b*q*x + c*q*x^2)^(1/3)*(d + e*x + f*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && EqQ[c*e - b*f, 0] && EqQ[c^2*d - f*(b^2 - 3*a*c), 0] && EqQ[c^2*g^2 - b*c*g*h - 2*b^2*h^2 + 9*a*c*h^2, 0] && !GtQ[4*a - b^2/c,

0]

Rubi steps

$$\int \frac{g + hx}{\sqrt[3]{\frac{-c^2g^2 + bcgh + 2b^2h^2}{9ch^2} + bx + cx^2} \left(\frac{f\left(b^2 - \frac{-c^2g^2 + bcgh + 2b^2h^2}{3h^2}\right)}{c^2} + \frac{bfx}{c} + fx^2 \right)} dx = \frac{\sqrt[3]{\frac{c\left(\frac{-c^2g^2 + bcgh + 2b^2h^2}{9ch^2} + bx + cx^2\right)}{b^2 - \frac{4(-c^2g^2 + bcgh + 2b^2h^2)}{9h^2}}} \int \frac{f\left(b^2 - \frac{-c^2g^2 + bcgh + 2b^2h^2}{3h^2}\right)}{\left(\frac{f\left(b^2 - \frac{-c^2g^2 + bcgh + 2b^2h^2}{3h^2}\right)}{c^2} + \frac{bfx}{c} + fx^2\right)} dx}{3\sqrt[6]{3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(cg+bh)}{ch^2} - 9bx - 9cx^2\right)}{(2cg-bh)^2}} \tan^{-1}\left(\frac{f\sqrt[3]{-\frac{(cg-2bh)(cg+bh)}{ch^2}} + 9bx + 9cx^2}}{f\sqrt[3]{-\frac{(cg-2bh)(cg+bh)}{ch^2}} + 9bx + 9cx^2}\right)}$$

Mathematica [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{g + hx}{\sqrt[3]{\frac{-c^2g^2 + bcgh + 2b^2h^2}{9ch^2} + bx + cx^2} \left(\frac{f\left(b^2 - \frac{-c^2g^2 + bcgh + 2b^2h^2}{3h^2}\right)}{c^2} + \frac{bfx}{c} + fx^2 \right)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(g + h*x)/(((-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(9*c*h^2) + b*x + c*x^2)^(1/3)*((f*(b^2 - (-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(3*h^2)))/c^2 + (b*f*x)/c + f*x^2)),x]
```

```
[Out] Integrate[(g + h*x)/(((-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(9*c*h^2) + b*x + c*x^2)^(1/3)*((f*(b^2 - (-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(3*h^2)))/c^2 + (b*f*x)/c + f*x^2)), x]
```

IntegrateAlgebraic [C] time = 6.23, size = 1038, normalized size = 2.13

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(g + h*x)/(((-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(9*c*h^2) + b*x + c*x^2)^(1/3)*((f*(b^2 - (-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(3*h^2)))/c^2 + (b*f*x)/c + f*x^2)),x]
```

```
[Out] (-3*(-3)^(1/6)*c^(1/3)*h^(5/3)*ArcTanh[(((I)*c^(2/3)*g)/(Sqrt[3]*h^(2/3)*(2*c*g - b*h)^(1/3)) + ((2*I)*b*h^(1/3))/(Sqrt[3]*c^(1/3)*(2*c*g - b*h)^(1/3)) + (c*g - 2*b*h)/(c^(1/3)*h^(2/3)*(2*c*g - b*h)^(1/3)) - (3*c^(2/3)*h^(1/3)*x)/(2*c*g - b*h)^(1/3) + (I*Sqrt[3]*c^(2/3)*h^(1/3)*x)/(2*c*g - b*h)^(1/3) + (I*((2*b^2)/c - (c*g^2)/h^2 + (b*g)/h + 9*b*x + 9*c*x^2)^(1/3))/Sqrt[3]]/(((2*b^2)/c - (c*g^2)/h^2 + (b*g)/h + 9*b*x + 9*c*x^2)^(1/3))]/(f*(2*c*g - b*h)^(2/3)) + ((-3)^(2/3)*c^(1/3)*h^(5/3)*Log[3*c*g - 6*b*h - (6*I)*Sqrt[3]*b*h - 9*c*h*x + Sqrt[3]*c*((3*I)*g - (9*I)*h*x) + 6*c^(1/3)*h^(2/3)*(2*c*g - b*h)^(1/3)*((2*b^2)/c - (c*g^2)/h^2 + (b*g)/h + 9*b*x + 9*c*x^2)^(1/3)])/((f*(2*c*g - b*h)^(2/3)) - ((-3)^(2/3)*c^(1/3)*h^(5/3)*Log[-9*c^2*g^2 + 36*b*c*g*h - 36*b^2*h^2 + (36*I)*Sqrt[3]*b^2*h^2 + 54*c^2*g*h*x - 108*b*c*h^2])
```

$$2*x - 81*c^2*h^2*x^2 + \text{Sqrt}[3]*c*((-36*I)*b*g*h + (108*I)*b*h^2*x) - 9*c^{(4/3)}*g*h^{(2/3)}*(2*c*g - b*h)^{(1/3)}*((2*b^2)/c - (c*g^2)/h^2 + (b*g)/h + 9*b*x + 9*c*x^2)^{(1/3)} + 18*b*c^{(1/3)}*h^{(5/3)}*(2*c*g - b*h)^{(1/3)}*((2*b^2)/c - (c*g^2)/h^2 + (b*g)/h + 9*b*x + 9*c*x^2)^{(1/3)} + (18*I)*\text{Sqrt}[3]*b*c^{(1/3)}*h^{(5/3)}*(2*c*g - b*h)^{(1/3)}*((2*b^2)/c - (c*g^2)/h^2 + (b*g)/h + 9*b*x + 9*c*x^2)^{(1/3)} + 27*c^{(4/3)}*h^{(5/3)}*(2*c*g - b*h)^{(1/3)}*x*((2*b^2)/c - (c*g^2)/h^2 + (b*g)/h + 9*b*x + 9*c*x^2)^{(1/3)} + 18*c^{(2/3)}*h^{(4/3)}*(2*c*g - b*h)^{(2/3)}*((2*b^2)/c - (c*g^2)/h^2 + (b*g)/h + 9*b*x + 9*c*x^2)^{(2/3)} + \text{Sqrt}[3]*c^2*((9*I)*g^2 - (54*I)*g*h*x + (81*I)*h^2*x^2) + \text{Sqrt}[3]*c^{(4/3)}*((-9*I)*g*h^{(2/3)}*(2*c*g - b*h)^{(1/3)}*((2*b^2)/c - (c*g^2)/h^2 + (b*g)/h + 9*b*x + 9*c*x^2)^{(1/3)} + (27*I)*h^{(5/3)}*(2*c*g - b*h)^{(1/3)}*x*((2*b^2)/c - (c*g^2)/h^2 + (b*g)/h + 9*b*x + 9*c*x^2)^{(1/3))]/(2*f*(2*c*g - b*h)^{(2/3)})$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/(f*(b^2+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x, algorithm m="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3(hx + g)}{\left(cx^2 + bx - \frac{c^2g^2 - bcgh - 2b^2h^2}{9ch^2}\right)^{\frac{1}{3}} \left(3fx^2 + \frac{3bfx}{c} + \frac{\left(3b^2 + \frac{c^2g^2 - bcgh - 2b^2h^2}{h^2}\right)f}{c^2}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/(f*(b^2+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x, algorithm m="giac")

[Out] integrate(3*(h*x + g)/((c*x^2 + b*x - 1/9*(c^2*g^2 - b*c*g*h - 2*b^2*h^2)/c*h^2)^(1/3)*(3*f*x^2 + 3*b*f*x/c + (3*b^2 + (c^2*g^2 - b*c*g*h - 2*b^2*h^2)/h^2)*f/c^2)), x)

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{hx + g}{\left(cx^2 + bx + \frac{2b^2h^2 + bcgh - c^2g^2}{9ch^2}\right)^{\frac{1}{3}} \left(fx^2 + \frac{bfx}{c} + \frac{\left(b^2 + \frac{-2b^2h^2 - bcgh + c^2g^2}{3h^2}\right)f}{c^2}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/(f*(b^2+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x)

[Out] int((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/(f*(b^2+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$3 \int \frac{hx + g}{\left(cx^2 + bx - \frac{c^2g^2 - bcgh - 2b^2h^2}{9ch^2}\right)^{\frac{1}{3}} \left(3fx^2 + \frac{3bfx}{c} + \frac{\left(3b^2 + \frac{c^2g^2 - bcgh - 2b^2h^2}{h^2}\right)f}{c^2}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/(
f*(b^2+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x, algorith
m="maxima")
```

```
[Out] 3*integrate((h*x + g)/((c*x^2 + b*x - 1/9*(c^2*g^2 - b*c*g*h - 2*b^2*h^2)/(
c*h^2))^(1/3)*(3*f*x^2 + 3*b*f*x/c + (3*b^2 + (c^2*g^2 - b*c*g*h - 2*b^2*h^
2)/h^2)*f/c^2)), x)
```

```
mupad [F] time = 0.00, size = -1, normalized size = -0.00
```

$$\int \frac{g + h x}{\left(b x + c x^2 + \frac{2 b^2 h^2 + b c g h - c^2 g^2}{c h^2} \right)^{1/3} \left(f x^2 - \frac{f \left(\frac{2 b^2 h^2 + b c g h - c^2 g^2}{3 h^2} - b^2 \right)}{c^2} + \frac{b f x}{c} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)/((b*x + c*x^2 + ((2*b^2*h^2)/9 - (c^2*g^2)/9 + (b*c*g*h)/9)/(
c*h^2))^(1/3)*(f*x^2 - (f*((2*b^2*h^2)/3 - (c^2*g^2)/3 + (b*c*g*h)/3)/h^2
- b^2))/c^2 + (b*f*x)/c),x)
```

```
[Out] int((g + h*x)/((b*x + c*x^2 + ((2*b^2*h^2)/9 - (c^2*g^2)/9 + (b*c*g*h)/9)/(
c*h^2))^(1/3)*(f*x^2 - (f*((2*b^2*h^2)/3 - (c^2*g^2)/3 + (b*c*g*h)/3)/h^2
- b^2))/c^2 + (b*f*x)/c), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$3 \cdot 3^{1/3} \cdot h^2 \int \frac{g}{\left(b x + c x^2 + \frac{2 b^2 h^2 + b c g h - c^2 g^2}{c h^2} \right)^{1/3} \left(f x^2 - \frac{f \left(\frac{2 b^2 h^2 + b c g h - c^2 g^2}{3 h^2} - b^2 \right)}{c^2} + \frac{b f x}{c} \right)} dx + \int \frac{h x}{\left(b x + c x^2 + \frac{2 b^2 h^2 + b c g h - c^2 g^2}{c h^2} \right)^{1/3} \left(f x^2 - \frac{f \left(\frac{2 b^2 h^2 + b c g h - c^2 g^2}{3 h^2} - b^2 \right)}{c^2} + \frac{b f x}{c} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(1/9*(2*b**2*h**2+b*c*g*h-c**2*g**2)/c/h**2+b*x+c*x**2)**
(1/3)/(f*(b**2+1/3*(-2*b**2*h**2-b*c*g*h+c**2*g**2)/h**2)/c**2+b*f*x/c+f*x*
**2),x)
```

```
[Out] 3*3**(2/3)*c**2*h**2*(Integral(g/(b**2*h**2*(2*b**2/c + b*g/h + 9*b*x - c*g
**2/h**2 + 9*c*x**2)**(1/3) - b*c*g*h*(2*b**2/c + b*g/h + 9*b*x - c*g**2/h**
*2 + 9*c*x**2)**(1/3) + 3*b*c*h**2*x*(2*b**2/c + b*g/h + 9*b*x - c*g**2/h**2
+ 9*c*x**2)**(1/3) + c**2*g**2*(2*b**2/c + b*g/h + 9*b*x - c*g**2/h**2 +
9*c*x**2)**(1/3) + 3*c**2*h**2*x**2*(2*b**2/c + b*g/h + 9*b*x - c*g**2/h**2
+ 9*c*x**2)**(1/3)), x) + Integral(h*x/(b**2*h**2*(2*b**2/c + b*g/h + 9*b*
x - c*g**2/h**2 + 9*c*x**2)**(1/3) - b*c*g*h*(2*b**2/c + b*g/h + 9*b*x - c*
g**2/h**2 + 9*c*x**2)**(1/3) + 3*b*c*h**2*x*(2*b**2/c + b*g/h + 9*b*x - c*g
**2/h**2 + 9*c*x**2)**(1/3) + c**2*g**2*(2*b**2/c + b*g/h + 9*b*x - c*g**2/
h**2 + 9*c*x**2)**(1/3) + 3*c**2*h**2*x**2*(2*b**2/c + b*g/h + 9*b*x - c*g*
**2/h**2 + 9*c*x**2)**(1/3)), x))/f
```


Chapter 4

Appendix

Local contents

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4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

Mathematica format Mathematica_syntax_CAS_integration_elementary_version.zip

Maple and Mupad format Maple_syntax_CAS_integration_elementary_version.zip

Sympy format SYMPY_syntax_CAS_integration_elementary_version.zip

Sage math format SAGE_syntax_CAS_integration_elementary_version.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
        "B"],
      "C"],
    If[FreeQ[result,Integrate] && FreeQ[result,Int],
      "C",
      "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
```



```
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
    If[ListQ[expn],
      Max[Map[ExpnType, expn]],
      If[Head[expn]===Power,
        If[IntegerQ[expn[[2]]],
          ExpnType[expn[[1]]],
          If[Head[expn[[2]]]===Rational,
            If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
              1,
              Max[ExpnType[expn[[1]], 2]],
            Max[ExpnType[expn[[1]], ExpnType[expn[[2]], 3]]],
          If[Head[expn]===Plus || Head[expn]===Times,
            Max[ExpnType[First[expn]], ExpnType[Rest[expn]]],
            If[ElementaryFunctionQ[Head[expn]],
              Max[3, ExpnType[expn[[1]]]],
              If[SpecialFunctionQ[Head[expn]],
                Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 4]],
                If[HypergeometricFunctionQ[Head[expn]],
                  Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 5]],
                  If[AppellFunctionQ[Head[expn]],
                    Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 6]],
                    If[Head[expn]===RootSum,
                      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
                      If[Head[expn]===Integrate || Head[expn]===Int,
                        Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
                        9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
    Exp, Log,
    Sin, Cos, Tan, Cot, Sec, Csc,
    ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
    Sinh, Cosh, Tanh, Coth, Sech, Csch,
    ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
  }, func]
```

```
SpecialFunctionQ[func_] :=
  MemberQ[{
    Erf, Erfc, Erfi,
    FresnelS, FresnelC,
    ExpIntegralE, ExpIntegralEi, LogIntegral,
    SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
    Gamma, LogGamma, PolyGamma,
    Zeta, PolyLog, ProductLog,
    EllipticF, EllipticE, EllipticPi
  }, func]
```

```
HypergeometricFunctionQ[func_] :=
```

```
MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#                    if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
#                    see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

    leaf_count_result:=leafcount(result);
    #do NOT call ExpnType() if leaf size is too large. Recursion problem
    if leaf_count_result > 500000 then
        return "B";
    fi;

    leaf_count_optimal:=leafcount(optimal);

    ExpnType_result:=ExpnType(result);
    ExpnType_optimal:=ExpnType(optimal);

    if debug then
        print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
ExpnType_optimal);
    fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;

```

```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do
not as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function

```

```

# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
    9
  end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

```

```

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:

```

4.2.3 Sympy grading function

```

#Dec 24, 2019. Nasser M. Abbasi:
#      Port of original Maple grading function by
#      Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#      added 'exp_polar'
from sympy import *

def leaf_count(expr):
  #sympy do not have leaf count function. This is approximation
  return round(1.7*count_ops(expr))

def is_sqrt(expr):
  if isinstance(expr,Pow):
    if expr.args[1] == Rational(1,2):
      return True
    else:
      return False
  else:
    return False

def is_elementary_function(func):
  return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
                 asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
                 asinh,acosh,atanh,acoth,asech,acsch
                 ]

def is_special_function(func):
  return func in [ erf,erfc,erfi,
                 fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
                 gamma,loggamma,digamma,zeta,polylog,LambertW,
                 elliptic_f,elliptic_e,elliptic_pi,exp_polar
                 ]

def is_hypergeometric_function(func):
  return func in [hyper]

def is_appell_function(func):
  return func in [appellf1]

def is_atom(expn):

```

```

try:
    if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
    else:
        return False

except AttributeError as error:
    return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'^^')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+`') or
    type(expn,'*`)
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
    ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function

```

```

def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```

4.2.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fracas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: # isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False

```

```

else:
    return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M',
        hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):

```



```

        return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
:
            return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
        return False

except AttributeError as error:
    return False

def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer
)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(
expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn)))
    elif is_elementary_function(expn.operator()): #is_elementary_function(expn
.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(4,m1) #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(5,m1) #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))

```

```

    return max(6,m1)    #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
    #is checked before calling the grading function that is passed.
    #but kept it here.
    m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
    return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```